Signal Analysis Using The Solitary Chirplet

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Abstract:
In the present work, the solitary chirplet, formed by modulating a frequency modulated signal with a hyperbolic secant envelope is formulated. It is seen that this solitary chirplet possesses a high number of vanishing moments. Sample analysis of sinusoidal and FM signals using the solitart chirplet confirms its efficacy in detecting frequency changes and discontinuities, following which the analysis of an earthquake signal is presented. It is seen that the proposed solitary chirplets come in handy while analyzing frequency variations and breaks in signals.

Keywords: Chirplet, Chirping, Solitons, Signal Analysis

1. Introduction
The phenomenon of chirping, characterized by a time varying frequency, essentially is composed of segregation of frequencies in a signal, owing to the differences in the propagating velocities of the various frequency components. While this phenomenon has been put to good use in optical fibers to generate distortion-less long-range waves called solitons, 'chirplets' have also been used as a class of wavelets to analyze images that exhibit progressive variations of distance due to perspective [1-2].

In the present work, a solitary chirplet, formed by modulating a frequency modulated (FM) signal with a hyperbolic secant (sech) pulse, is proposed. It is seen that, owing to the compactness and smoothness of the sech pulse, the solitary chirplet has a large number of vanishing moments, making them ideal choices for detection of bursts and discontinuities [2-5]. Following this, the analysis of various signals such as sine and FM are explored. Finally, an earthquake signal is analyzed using the chirplet. It is seen that the proposed solitary chirplets come in handy while analyzing frequency variations and breaks in signals.

2. Formulation of the Solitary Chirplet
Most optical soliton solutions derived from the Nonlinear Schrodinger Equation consist of a temporal hyperbolic secant function based profile, defined as follows [1]:

$$A(t) = A_0 sech\left(\frac{t-S}{W}\right) \tag{1}$$

where A_0 denotes the peak amplitude and S and W denote the pulse shift (time offset) and width (measured at half-peak value) respectively. This signal represents a bell-shaped curve and is plotted in Fig. 1.

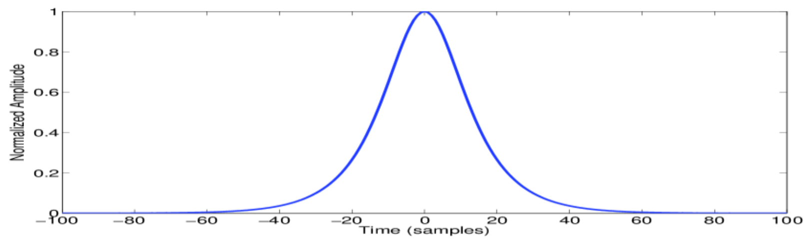

Figure 1 The Hyperbolic Secant Signal

This sech signal is used to modulate a Frequency Modulated signal, with the resultant signal ϕ(t) defined as the father wavelet, or the scaling function. The Father Wavelet ϕ thus defined is used as the basis to form the 'Mother Wavelet' ψ, such that the following criteria are satisfied [6-9]:

1. ψ(t) belongs to a subspace of the space $L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, the space of absolutely and square integrable measurable functions.
2. ϕ(t) and ψ(t) are orthogonal to each other.
3. ψ(t) has zero mean, i.e. the following holds: $\int_{-\infty}^{\infty} \psi(t)dt = 0$.
4. ψ(t) has unity square norm, as per the following equation: $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1$.
5. It is preferable, but not a mandatory criterion to ensure that ψ(t) possesses a higher number M vanishing moments. In other words, for all m<M, $\int_{-\infty}^{\infty} t^m \psi(t)dt = 0$.

The Mother Wavelet ψ is used to define the daughter wavelets $\psi_{(a,b)}(t)$ in the following fashion with a > 0 denoting the 'scale' and b ϵ ℝ denoting the 'shift': $\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$.

Based on the above procedure, the father and mother 'solitary chirplets' have been formed using the MATLAB Wavelet Toolbox [9]. The Father and Mother Wavelet Signals are plotted in Fig. 2, along with the decomposition and reconstruction low/high pass filter coefficients.

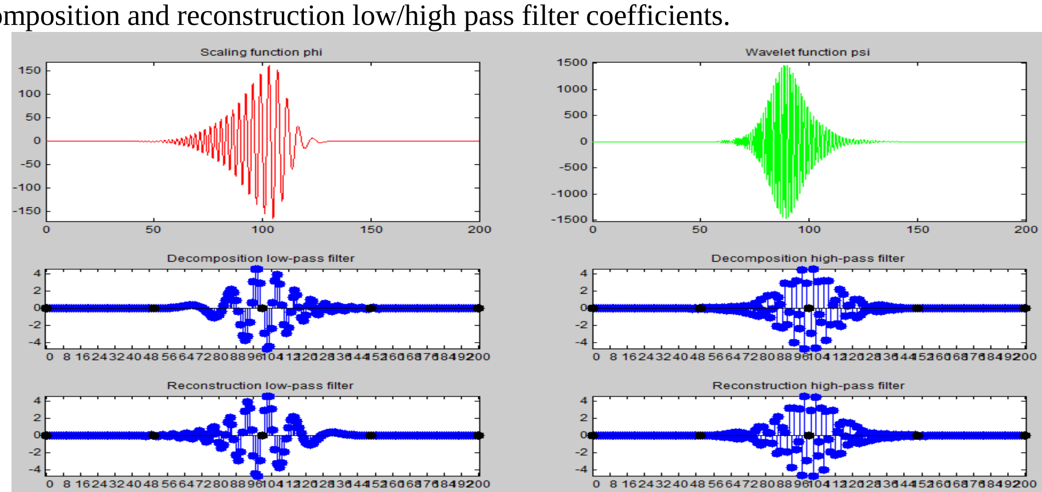

Figure 2 Father and Mother Solitary Chirplets

One of the preferable but not mandatory criteria mentioned above in the mother wavelet formulation is the presence of vanishing higher moments, where the 'm'th moment of the mother wavelet ψ is given by Eq. 4. Physically, the existence of vanishing higher moments signifies that the wavelet has a compact, continuous, smooth structure, and that the analysis of bursts in signals with such wavelets can be carried out with minimal filtering [6-9].

In order to investigate and characterize the performance of the solitary chirplet, the moments upto the tenth order of the mother solitary chirplet (SCL) are computed and compared with the corresponding moments of established wavelets, namely Daubechies 4 (DB4), Biorthogonal 4.4 (BIOR4.4), Coiflet 4 (COIF4) and the Discrete Meyer Wavelet (DMEY) [10]. The moments are tabulated in Table 1.

Table 1 Moments of Various Wavelets upto the Tenth Order

Moments	DB4	BIOR4.4	COIF4	DMEY	SCL
First	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Second	1.33E-01	1.09E-01	4.26E-02	9.90E-03	1.20E-02
Third	2.05E-02	5.41E-02	1.96E-02	3.30E-03	1.70E-03
Fourth	1.13E-01	9.95E-02	3.74E-02	7.60E-03	2.90E-03
Fifth	3.25E-02	8.78E-02	3.19E-02	5.40E-03	9.70E-04
Sixth	1.05E-01	1.12E-01	4.10E-02	7.30E-03	9.88E-04
Seventh	4.19E-02	1.16E-01	4.19E-02	6.60E-03	4.97E-04
Eighth	9.95E-02	3.45E-01	4.85E-02	7.60E-03	4.20E-04
Ninth	4.96E-02	1.47E-01	5.24E-02	7.60E-03	3.50E-04
Tenth	9.56E-02	1.66E-01	5.87E-02	8.30E-03	2.36E-04

From Table 1, it is seen that the higher moments of the solitary chirplet tend toward zero. From this trend, it is seen that even the Meyer wavelet moments increase after a certain order (sixth). This gives the solitary chirplet the exclusive advantages of smoothness, compactness and effective detection of bursts as explained earlier.

3. Analysis using the Solitary Chirplet

The solitary chirplet analysis of a function f(t) is formally defined as $F(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)dt$, where a and b denote the scales and shifts respectively [9]. The Solitary Chirplet analysis of sinusoidal and FM modulated signals are shown in Fig. 3 and 4, in comparison with analysis using a non-chirped solitary wavelet. The analysis shows a contour plot of F(a,b) corresponding to various values of a and b.

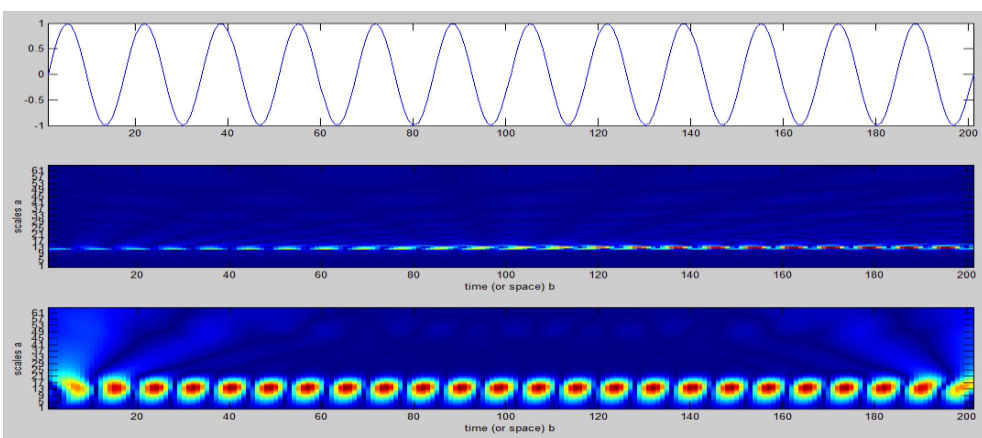

Figure 3 Solitary Chirplet (above) and Wavelet (below) analysis of a Sinusoidal Signal

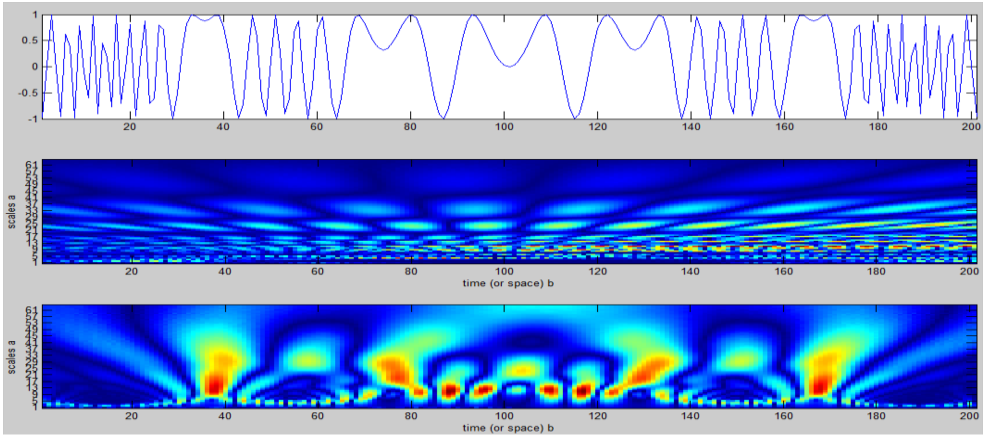

Figure 4 Solitary Chirplet (above) and Wavelet (below) analysis of a Frequency Modulated Signal

It is seen from the plots that the chirplet analysis provides better localization of the sinusoidal signal, corresponding to its narrow bandwidth. For the FM case, the frequency variations are shown as angular sweeps in shift-scale plot, rather than as scale dominances seen in the wavelet case, thus pertaining to an indication of a more continuous variation in the frequencies, as is the case in FM.

The solitary chirplet analysis of a signal with frequency discontinuity is shown in Fig. 5.

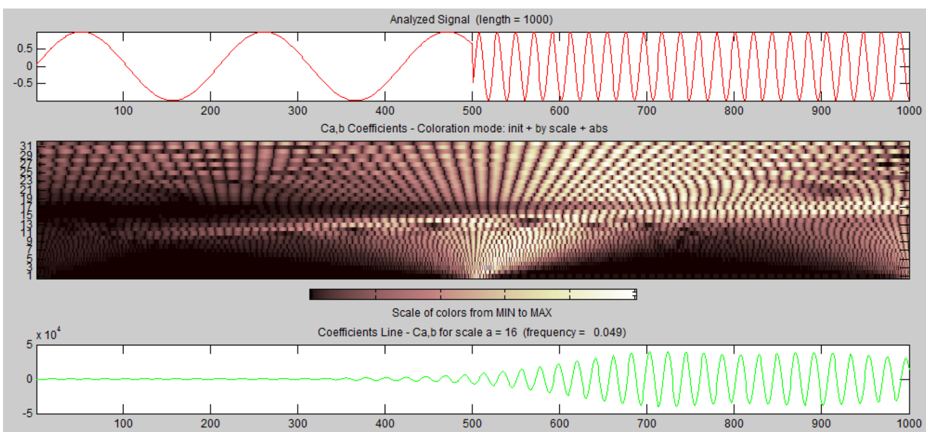

Figure 5 Solitary Chirplet analysis of a Signal with Discontinuity in frequency

It is seen that the solitary chirplet provides a clear distinction with a point at minimal scale corresponding to the break in frequency, and consequently, the coefficients line have amplitudes proportional to the frequency, thus proving the chirplet a useful tool in analyzing changes and breaks in frequencies.

Finally, the chirplet analysis of an earthquake signal, obtained from the Luxorion project website (http://www.astrosurf.com/luxorion/qsl-audiofiles.htm) is performed, and is plotted in Fig. 6.

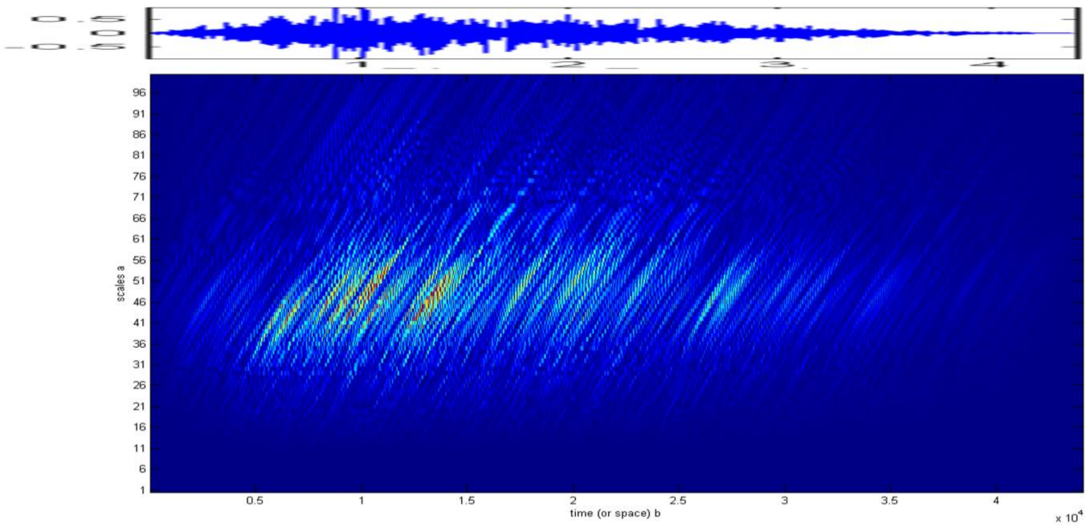

Figure 6 Solitary Chirplet analysis of an Earthquake Signal

The analysis shows remarkable localization in the 40-50 scale region, with a progressive increase in frequency with time, as seen from the angle of slant in the dominance patches. By proper normalization of the shifts and scales, it is possible to compute the rate of frequency range in the signal.

4. Conclusion

In the present work, a solitary chirplet, formed by modulating a FM signal with a hyperbolic secant (sech) envelope is proposed, and it is seen that this chirplet possesses a high number of vanishing moments, owing to the extreme smoothness and compactness of the sech function, thus enabling it to detect bursts and discontinuities effectively. Sample analyses of sinusoids, FM and frequency breaking signals reveal that the chirplet is indeed effective in detecting changes and discontinuities in frequency. Finally, a sample chirplet analysis of an earthquake signal is presented. It is seen that the proposed solitary chirplets come in handy while analyzing frequency variations and breaks in signals.

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