The new parameter for MOND and the MOND cosmic structure formation entropic degree of freedom

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Abstract

In a previous paper I showed how a new parameter added to MOND, the entropic degree of freedom N, exactly solved the MOND galaxy cluster mass discrepancy problem. In this paper I show that the same entropic degree of freedom produces an exact interpretation of Milgrom's approximate $5a_0 \approx cH_0$. The new relation gives $N^2 a_0 = cH_0$. With present day values, $N = 2.13$, the cosmic degree of freedom of the entropic force in relation to cosmic structure formation.

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I. THE ENTROPIC FORCE OF GRAVITY AS THE MOND FORCE

In a previous paper I derived the entropic force of gravity [1] and then in a subsequent paper I introduced the entropic degree of freedom in that force [2]. This new parameter N allowed a simple solution for the MOND galaxy cluster mass discrepancy problem.

In my elementary particle Dark Matter halo model, see [1] for further information and references, I start with the gravitational source mass

$$m_g = m_0 + m_{\text{DM}} = m_0 + \frac{r}{r_{\text{DM}}} m_0 = m_0 \left(1 + \frac{r}{r_{\text{DM}}}\right) \tag{1}$$

and I get gravitational potential at r as

$$\phi = -\frac{GM_0}{r} - \frac{GM_0}{r_{\text{DM}}} = \phi_0 + \phi_{\text{DM}}. \tag{2}$$

For the resulting force of gravity on a classical charge mass m we get the unchanged Newtonian result

$$\mathbf{F} = -m\nabla\phi = -m\nabla\phi_0 + -m\nabla\phi_{\text{DM}} = -m\nabla\phi_0 = -\frac{GM_0 m}{r^2}\hat{r}. \tag{3}$$

The gravitational energy is however affected by the Dark Matter potential as

$$U_g = m\phi = m\phi_0 + m\phi_{\text{DM}} = -\frac{GM_0 m}{r} - \frac{GM_0 m}{r_{\text{DM}}}. \tag{4}$$

I assume that the virial theorem is still valid. Using $2U_k = -U_g$ I get $v^2 = -\phi$ for orbiting satellites and

$$v^2 = -\phi = \frac{GM_0}{r} + \frac{GM_0}{r_{\text{DM}}}. \tag{5}$$

The total force of gravity in my model is derived from the energy by application of the virial theorem

$$F_c = \frac{m_0 v^2}{r} = \frac{GM_0 m_0}{r^2} + \frac{GM_0 m_0}{r\ r_{\text{DM}}} = -F_N - F_{\text{DM}} \tag{6}$$

and so the entropic Dark Matter force must result in

$$F_{\text{DM}} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}} \tag{7}$$

This force is derived from a potential ϕ connected to an entropy S and a number of microstates W using the first law of thermodynamics. I define the Dark Matter entropy on the outer flat rotation curve parts of the galactic disks as

$$S = k_B \ln W = k_B \left(\frac{U_{\text{DM}}}{k_B T}\right) \ln\left(\frac{r}{r_m}\right) \tag{8}$$

From the entropy we can derive the entropic DM force using the first law of thermodynamics

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_{U_{\text{DM}}} = T\frac{d}{dr}k_B \ln\left(\frac{r}{r_m}\right)^{\frac{U_{\text{DM}}}{k_B T}} = U_{\text{DM}}\frac{d}{dr}\ln\left(\frac{r}{r_m}\right) = \frac{U_{\text{DM}}}{r} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}}. \tag{9}$$

For $r \gg r_{\text{DM}}$, $v = v_f$ and $F_c \approx -F_{\text{DM}}$ so

$$F_c = \frac{m_0 v_f^2}{r} \approx \frac{GM_0 m_0}{r\ r_{\text{DM}}} = -F_{\text{DM}} \tag{10}$$

leading to

$$v_f^2 = \frac{GM_0}{r_{\text{DM}}} \tag{11}$$

We can define a special Dark Matter centripetal acceleration given by

$$a_{\text{DM}} \equiv \frac{v_f^2}{r_{\text{DM}}} \tag{12}$$

so r_{DM} can be given by

$$r_{\text{DM}} \equiv \frac{v_f^2}{a_{\text{DM}}} \tag{13}$$

which, inserted into Eqn.(11) gives

$$v_f^2 = \frac{Ga_{\text{DM}}M_0}{v_f^2} \tag{14}$$

and this leads to

$$v_f^4 = Ga_{\text{DM}}M_0, \tag{15}$$

a relation that we recognize as Milgrom's form of the Baryonic Tully-Fisher relation. In 2005 McGaugh determined the baryonic version of the LT relation as $M_d = 50v_f^4$, see [3]. In this form, M_d is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30}\ kg$ units and the final velocity of the galactic rotation velocity curve v_f is expressed in km/s. If we express the galactic mass in kg and the velocity in m/s we get the total baryonic mass, final velocity relations in SI unit values as $M_b = 1,0 \cdot 10^{20} v_f^4$.

In 1983, Milgrom interpreted the BTF relation as an indication of a deviation from Newtonian gravity, making a modification of Newtonian dynamics or MOND necessary [4]. Using McGaug's 2005 values in SI units, Milgrom presented the BTF relation in the form

$$v_f^4 = 1,0 \cdot 10^{-20} M_b = Ga_0 M_b, \tag{16}$$

resulting in an acceleration $a_0 = 1,5 \cdot 10^{-10}\ m/s^2$ in McGaug's values. According to Milgrom, this relation should hold exactly, thus interpreting it as an inductive law of nature instead of looking at it as just an empirical relation [5]. The resulting acceleration can be written as $5 \cdot a_0 \approx cH_0$, with the velocity of light c and the Hubble constant H_0. According to Milgrom, the deeper significance of this relation between the galactic critical acceleration and the Hubble acceleration should be revealed by future cosmological insights [4]. I identify a_{DM} with Milgrom's a_0. So in my model, $a_{\text{DM}} = v_f^2/r_{\text{DM}}$ is the galactic Dark Matter constant. In MOND terminology, the entropic DM force is given by

$$F_{\text{DM}} = -\frac{Ga_0 M_0 m_0}{r\ v_f^2}, \tag{17}$$

so with $F_{\text{DM}} \propto r^{-1}$. And with $v_f^4 = Ga_0 M_b$ this reduces to

$$F_{\text{DM}} = -\frac{m_0}{r}\sqrt{Ga_0 M_0}, \tag{18}$$

so to Milgrom's 1983 formulation [5].

II. FROM THE GALAXY CLUSTER PROBLEM TO THE COSMIC INTERPRETATION OF MOND'S ACCELERATION CONSTANT

In my model, objects on a disk have one degree of freedom, objects moving freely on a sphere have two degree's of freedom and objects that behave as in a mono-atomic gas have three degrees of freedom. For the entropic Dark Matter force this degree of freedom parameter N with value between 1 and 3 can be inserted to give

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_{U_{\text{DM}}} = U_{\text{DM}}\frac{d}{dr}\ln\left(\frac{r}{r_m}\right)^N = \frac{NU_{\text{DM}}}{r} = -\frac{G(NM_0)m_0}{r\ r_{\text{DM}}}. \tag{19}$$

Without this number of microstates degree of freedom related factor N, in certain situations the needed baryonic mass might be overestimated by a factor between 2 and 3. The parameter N will never be exactly three because such systems behave as a free gas and do not display gravitational attraction phenomena.

In the case of galaxy clusters, the degree of freedom cannot be 2 or smaller because then the cluster should have been shaped like a disk or a recognizable sphere. Neither can it be 3 because then the cluster would disperse like a free gas. So its degree of freedom should be somewhere in between 2 and 3, giving it an apparent baryonic mass $2M_b < M_a < 3M_b$, see [2].

The same reasoning can be applied to the cosmos as a whole, so because cluster formation takes place in the universe, its entropic degree of freedom should be somewhere in between the values $N = 2$ and $N = 3$. If we add the degree of freedom to Milgrom's MOND form of the entropic force we get

$$F_{\text{DM}} = -\frac{G(NM_0)m_0}{r\ r_{\text{DM}}} = F_{\text{DM}} = -\frac{m_0}{r}\sqrt{GN^2 a_0 M_0}. \tag{20}$$

If we adopt Milgrom's interpretation but now include the entropic degree of freedom parameter N we get

$$N^2 a_0 = cH_0 \tag{21}$$

so

$$N = \sqrt{\frac{cH_0}{a_0}} = 2,13. \tag{22}$$

In our model, the cosmic entropic degree of freedom is given by this value $N = 2,13$. In the calculation of N we used the value of $Ga_0 = 1,0 \cdot 10^{-20}$ from McGaugh [3]. This parameter should be important in understanding cosmic structure formation.

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