A thermodynamic degree of freedom solution to the galaxy cluster problem of MOND

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Abstract

In this paper I discus the degree of freedom parameter of the emergent Dark Matter force. I show how this degree of freedom parameter N results in a possible difference between observed mass and apparent mass, in cases where N is larger than one. This might solve the galaxy cluster mass discrepancy of MOND. In my model the degree of freedom of galaxies in clusters influences their number of microstates inside the cluster and thus the entropy. And then it also influences the apparent baryonic mass in the emergent Dark Matter force that will appear a factor in between two or three bigger than can be directly observed.

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I. THE ENTROPIC DARK MATTER FORCE AS DERIVED FROM THE FIRST LAW OF THERMODYNAMICS

In my elementary particle Dark Matter halo model, see [1] for further information and references, I start with the gravitational source mass

$$m_g = m_0 + m_{\rm DM} = m_0 + \frac{r}{r_{\rm DM}} m_0 = m_0 \left(1 + \frac{r}{r_{\rm DM}}\right) \quad (1)$$

and I get gravitational potential at r as

$$\phi = -\frac{GM_0}{r} - \frac{GM_0}{r_{\rm DM}} = \phi_0 + \phi_{\rm DM}. \quad (2)$$

For the resulting force of gravity on a classical charge mass m we get the unchanged Newtonian result

$$\mathbf{F} = -m\nabla\phi = -m\nabla\phi_0 + -m\nabla\phi_{\rm DM} = -m\nabla\phi_0 = -\frac{GM_0 m}{r^2}\hat{r}. \quad (3)$$

The gravitational energy is however affected by the Dark Matter potential as

$$U_g = m\phi = m\phi_0 + m\phi_{\rm DM} = -\frac{GM_0 m}{r} - \frac{GM_0 m}{r_{\rm DM}}. \quad (4)$$

I assume that the virial theorem is still valid. Using $2U_k = -U_g$ I get $v^2 = -\phi$ for orbiting satellites and

$$v^2 = -\phi = \frac{GM_0}{r} + \frac{GM_0}{r_{\rm DM}}. \quad (5)$$

At all times in the galactic disk, the centripetal force F_c must match with the virial theorem, so $F_g = F_c$. The difference between the needed F_c and the Newtonian force of gravity F_N must be delivered by the emergent Dark Matter force $F_{\rm DM}$. Assuming the orbits to be circular, we can insert Eqn.(5) in the formula for F_c to get

$$F_c = \frac{m_0 v^2}{r} = \frac{GM_0 m_0}{r^2} + \frac{GM_0 m_0}{r\ r_{\rm DM}} = -F_N - F_{\rm DM} \quad (6)$$

and so the entropic Dark Matter force must result in

$$F_{\rm DM} = -\frac{GM_0 m_0}{r\ r_{\rm DM}} \quad (7)$$

This force cannot be derived from the divergence of the potential energy. To resolve this problem I go to the first law of thermodynamics $dU = TdS - Fdr$, written as

$$F_g = -\left(\frac{dU}{dr}\right)_{S_{\rm DM}} + T\left(\frac{dS}{dr}\right)_{U_{\rm DM}} = F_N + F_{\rm DM} \quad (8)$$

Newtonian gravity is derived from

$$F_N = -\left(\frac{dU_N}{dr}\right)_S \tag{9}$$

and I will derive the entropic Dark Matter force from

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_U. \tag{10}$$

I define the Dark Matter entropy on the outer flat rotation curve parts of the galactic disks as

$$S = k_B \ln W = k_B \left(\frac{U_{\text{DM}}}{k_B T}\right) \ln\left(\frac{r}{r_m}\right) \tag{11}$$

From the entropy we can derive the entropic DM force using

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_{U_{\text{DM}}} = T\frac{d}{dr}k_B \ln\left(\frac{r}{r_m}\right)^{\frac{U_{\text{DM}}}{k_B T}} = U_{\text{DM}}\frac{d}{dr}\ln\left(\frac{r}{r_m}\right) = \frac{U_{\text{DM}}}{r} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}}. \tag{12}$$

Because the Dark Matter halo energy is negative, the entropy decreases outwards, creating an entropic force inwards.

II. THE PARAMETERS OF THE ENTROPIC MODEL

The number of microstates W of an ideal gas particle m with a microstate radius r_m in a volume with macrostate radius r is traditionally given by the times one can fit this small or microscopic volume in the large or macroscopic volume, so by the thermodynamic gas in a bottle number of microstates

$$\ln W = \ln\frac{V}{V_m} = 3\ln\frac{r}{r_m}. \tag{13}$$

But if we have such a particle moving with only one degree of freedom on a circular trajectory with radius r and this particle can be considered a quantum particle with a de Broglie wavelength $\lambda = h/p$, then the number of microstates can be defined as the number of microscopic wavelengths that fit onto the macroscopic circumference. This gives the number of microstates as

$$\ln W = \ln\frac{2\pi r}{\lambda} = \ln\frac{rp}{\hbar} = \ln\frac{S}{\hbar} \tag{14}$$

with S as the phase space of the particle. In the case of galactic neutral hydrogen gas particles, this phase space is huge. This approach also gives us a way to look at a relativistic generalization of the model. And it connects the Dark Matter halo of elementary particles to a de Broglie subquantum thermodynamics. This interpretation turns the model into Quantum Gravity.

If we look at the entropic force, derived from a system with N degrees of freedom

$$\frac{d}{dr}\ln W = \frac{d}{dr}\ln\left(\frac{r}{r_m}\right)^N = N\frac{d}{dr}\ln r - N\frac{d}{dr}\ln r_m = N\frac{d}{dr}\ln r = \frac{N}{r} \tag{15}$$

then it is clear that r_m is a free parameter of our theory from the perspective of the derived F_{DM}. This means that both a thermodynamic and a quantum interpretation of r_m are possible, in principle.

Practical considerations should determine the choice of model for r_m. The quantum interpretation has the advantage to look like a move towards a quantum theory of gravity, but as long as it remains a free parameter, this has no specific use. On the other hand, a star orbiting a galaxy at a large distance has an incredible small de Broglie wavelength but clearly a much smaller number of microstates relative to the length of its orbit, as compared to a neutral hydrogen atom. So for large objects the volume of classical mass radius interpretation seems to make the most sense.

The other free parameter of our theory is the temperature T, because its interpretation doesn't effect the resulting F_{DM} either, as can be seen in

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_{U_{\text{DM}}} = T\frac{d}{dr}k_B \ln\left(\frac{r}{r_m}\right)^{\frac{U_{\text{DM}}}{k_B T}} = T\frac{U_{\text{DM}}}{T}\frac{d}{dr}\ln\left(\frac{r}{r_m}\right) = \frac{U_{\text{DM}}}{r}. \tag{16}$$

The temperature as a free parameter of our theory only works as far the temperature of the orbiting objects is independent from the radius at which they orbit. The key non-free parameters of this entropic force derivation are the Dark Matter energy U_{DM}, the radius r and the degree of freedom N.

III. THE GALAXY CLUSTER PROBLEM OF MOND AS CAUSED BY THE DEGREE OF FREEDOM PARAMETER

In my model, objects on a disk have one degree of freedom, objects moving freely on a sphere have two degree's of freedom and objects that behave as in a mono-atomic gas have three degrees of freedom. For the entropic Dark Matter force this degree of freedom parameter N with value between 1 and 3 can be inserted to give

$$F_{\rm DM} = T\left(\frac{dS}{dr}\right)_{U_{\rm DM}} = U_{\rm DM}\frac{d}{dr}\ln\left(\frac{r}{r_m}\right)^N = \frac{NU_{\rm DM}}{r} = -\frac{G(NM_0)m_0}{r\ r_{\rm DM}}. \tag{17}$$

Without this number of microstates degree of freedom related factor N, in certain situations the needed baryonic mass might be overestimated by a factor between 2 and 3. The parameter N will never be exactly three because such systems behave as a free gas and do not display gravitational attraction phenomena.

In the case of galaxy clusters, the degree of freedom cannot be 2 or smaller because then the cluster should have been shaped like a disk or a recognizable sphere. Neither can it be 3 because then the cluster would disperse like a free gas. So its degree of freedom should be somewhere in between 2 and 3, giving it an apparent baryonic mass $2M_b < M_a < 3M_b$.

The fact that MOND has this problem, indication a distinct difference between MOND and my model. In my model this would not be a problem but a chance to measure the degree of freedom factor in galaxy clusters. This parameter should contain information regarding the process of formation of such clusters.

[1] E. P. J. de Haas, The Dark Matter Entropic Force and Newtons Energetic Force as a Complete First Law of Thermodynamics Set of Gravitational Forces, viXra:1510.0337 (2015).