Introducing Marxian Productivity Development Economics

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Abstract: In the first part of this paper, a neoclassical framework is proposed which places the Marxian conceptions of both Constant Capital and Variable Capital into a Cobb-Douglas production function like model in order to obtain the mathematical formulations of Marx labour value function $Q = B_0 e^{ft} C^{\beta} V^{1-\beta}$ and Marx surplus value function $M = b_0 e^{pt} C^{\beta} V^{1-\beta}$ as well as Marx production function $Y = a_0 e^{Ft} C^{\alpha} V^{1-\alpha}$, which leads to the Marxian 1st theorem about technical progress: $m = \alpha \dot{n} + (1-\alpha)\dot{w} + p\alpha/\beta$. In the second part, the general equilibrium properties of the quantitative Marxian productivity theories are investigated by using variation method. The Marxian 2nd theorem about dynamic equilibrium asserts, there is a input-output equilibrium existed in the reproduction process between Two Departments $\dot{Y}^* = \dot{C}^*$; The Marxian 3rd theorem states that only equilibrium growth leads to the positive value of the productivity parameter which is defined as the product of the change rate of the organic composite of capital with the labor output elasticity of Cobb-Douglas production function[$F = (1-\alpha)\dot{g}$], as well as the rising rate of profit. The present paper is also a

generalization of the precise conditions under which the profit rate rises or falls. Only when an economic system achieves the Marxian equilibrium including its each production Department, there would be no business cycle; otherwise there exists some potential crisis. At last, an econo-sociological Marxism model is proposed as a criterion for a regional optimal economic growth.

Key words: transformation problem, roundabout production, Solow residue, Okishio theorem， competitive equilibrium

JEL: E11, O47

I. Marx Productivity Economics[1]

Marx's labor theory of value[2] pointed out that the value of each commodity (Q) contained three sources: the first part is “constant capital (C)”, representing the value transferred from raw materials and machinery used up, the second part is “variable capital (V)” replacing the value of the labor power, and the third part is the surplus value (M) including net profit (P) and taxation (T). Therefore, the total value Q is expressed as a linear production function:

$$Q = C + V + M = C + V + p'(C + V) = P'(C + V) = P'Cv$$

$$M = P + T = p'(C + V) = p'Cv = m'V$$

$$Cv = C + V, P' = \frac{Q}{C + V}, p' = \frac{M}{Cv} = \frac{m'}{1 + g}$$

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[1] 曾尔曼、王萲祥.《马克思生产力经济学导引》[M]，厦门大学出版社，2013.09.

[2] 马克思.《资本论》I [M]. 北京：人民出版社，1975.

Where

C = nK (K: capital, n: capital turnover rate), constant capital;

V = wL (L: labor, w: per-capita wages), variable capital

P': the productivity rate, $P' = Q/(C+V) = p'+1$

M: the surplus value

p': the rate of profit, $p' = M/(C+V) = m'/(g+1) = P'-1$

m': the rate of surplus value, m'= M/V

g: the organic composition of capital (OCC): $g = C/V = nK/(wL) = nk/w$

Differentiating Q with respect to time t yields:

$$\frac{dQ}{Q} = \frac{dP'}{P'} + \frac{dCv}{Cv} = \frac{dP'}{P'} + \frac{C}{Cv}\frac{dC}{C} + \frac{V}{Cv}\frac{dV}{V}$$

$$\frac{dQ}{Qdt} = \frac{dP'}{P'dt} + \beta\frac{dC}{Cdt} + (1-\beta)\frac{dV}{Vdt} = f + \beta\frac{dC}{Cdt} + (1-\beta)\frac{dV}{Vdt}$$

$$\beta \equiv \frac{C}{Cv} = \frac{g}{g+1}, 1-\beta = \frac{V}{Cv} = \frac{1}{g+1}, \frac{dP'}{P'dt} \equiv f$$

re-integration gives the Labor Value Function[3] Q as:

$$Q = B_0 e^{ft} C^{\beta} V^{1-\beta}$$

f: the productivity growth rate, and the cost function is:

$$Cv = C + V = c_0 C^{\beta} V^{1-\beta}.$$

Similarly, differentiates M with time t: $M = p'Cv = p'(C+V)$

$$\frac{dM}{M} = \frac{dp'}{p'} + \frac{dCv}{Cv} = \frac{dp'}{p'} + \frac{C}{Cv}\frac{dC}{C} + \frac{V}{Cv}\frac{dV}{V}$$

$$\frac{dM}{Mdt} = \frac{dp'}{p'dt} + \beta\frac{dC}{Cdt} + (1-\beta)\frac{dV}{Vdt} = p + \beta\frac{dC}{Cdt} + (1-\beta)\frac{dV}{Vdt}, \frac{dp'}{p'dt} \equiv p = f\gamma$$

After integration, Marx surplus value function[4] M is obtained as:

3 曾尔曼.《厦门科技》2014（3）31-35.

4 曾尔曼.《厦门科技》2015（2）27-29;第四届中国经济学年会会议论文.

$$M = b_0 e^{pt} C^{\beta} V^{1-\beta},$$

$$m' = b_0 e^{pt} g^{\beta},$$

$$p' = \frac{M}{Cv} = \frac{m'}{g+1} = (1-\beta)m' = b_0 e^{pt} \frac{g^{\beta}}{g+1} = b\frac{g^{\beta}}{g+1}$$

As well as the function of the rate of surplus value m', p is the growth rate of profit.

Using the similar mathematical process, the Cobb-Douglas production function[5] can be rewritten as:

$$Y = V + M = V + p'Cv = p'(C + \frac{P'}{p'}V) \equiv p'(C + V') = p'Cv'$$

$$\frac{dY}{dtY} = \frac{dp'}{p'dt} + \frac{dCv'}{Cv'dt} = \frac{dp'}{p'dt} + \frac{C}{Cv'}\frac{dC}{Cdt} + \frac{V'}{Cv'}\frac{dV'}{V'dt} = p + \alpha\frac{dC}{Cdt} + (1-\alpha)\frac{dV'}{V'dt},$$

$$V' = \gamma wL \text{ , } \gamma = \frac{P'}{p'} = 1 + \frac{1}{p'} \text{ , } \alpha \equiv \frac{C}{Cv'} = \frac{g}{g+\gamma} < \beta \text{ , } 1-\alpha = \frac{\gamma}{g+\gamma} > 1-\beta$$

After integration, the production function became:

$$Y = a_0 e^{pt} C^{\alpha} V'^{1-\alpha} = a_0 e^{pt} C^{\alpha} V^{1-\alpha} \gamma^{1-\alpha} = a_0 e^{pt} (nK)^{\alpha} (\gamma wL)^{1-\alpha},$$

$$Y = AK^{\alpha} L^{1-\alpha} = A_0 e^{mt} K^{\alpha} L^{1-\alpha}$$

$$\Rightarrow p = m - \alpha\dot{n} - (1-\alpha)\dot{w}' = m - \alpha\dot{n} - (1-\alpha)(\dot{w} + \dot{\gamma}),$$

$$p = f\gamma, \dot{\gamma} = \dot{P}' - \dot{p}' = f - p$$

$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + [1 - (1-\alpha)(1+p')^{-1}]p = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{p'+\alpha}{p'+1}p$$

Therefore, it's obvious that the rate of technical change (m) could be characterized as the linear combination of the growth rate of profit (p'), the wage(w), and the capital circulation (n) with respect to the labor output elasticity of C-D production function(1-(). Technological progress ought to improve the rate of profit, wage rates and cash flow. Technical change stems from the division of labor, exacerbated by the division of

[5] Cobb C.W., Douglas P.H. "A Theory of Production", Amer. Econ. Rev. 1928,8(1), Spp1.139-165.

labor. The rate of technological progress (Solow residue[6]) is proportional to the growth rate of profit p (thus productivity growth rate f).

Okishio Theorem[7] asserts that if real wages remain unchanged, the rate of profit necessarily rises in consequence of an cost-saving technology innovation. Then after transformation, the relationship between the profit rate and the organic composition of capital can be obtained:

$$p = \frac{m - \alpha\dot{n} - (1-\alpha)\dot{w}}{(1-\alpha+\alpha\gamma)/\gamma} = \frac{\dot{y} - \alpha\dot{g} - \dot{w}}{(1-\alpha+\alpha\gamma)/\gamma} = (\dot{y} - \alpha\dot{g} - \dot{w})\frac{p'+1}{p'+\alpha}$$

the growth rate of profit is determined by the growth rate of labor productivity (y) positively only; it seems that Marx was right about that the rate of profit tends to fall due to the rise of the OCC (g).

Differentiates p' with t:

$$p' = \frac{M}{Cv} = \frac{m'}{g+1} = \frac{Y-V}{C+V} = \frac{y-w}{nk+w} = \frac{\frac{y}{w}-1}{g+1}$$

$$p \equiv \frac{dp'}{p'\,dt} = \frac{d(y/w-1)}{(y/w-1)\,dt} - \frac{d(1+g)}{(1+g)\,dt} = \frac{\dot{y}-\dot{w}}{1-w/y} - \beta\dot{g} = (\dot{y}-\dot{w}-\alpha\dot{g})\frac{p'+1}{p'+\alpha}$$

$$\Rightarrow \frac{\beta}{\alpha} = \frac{p'+1}{p'+\alpha} = \frac{1}{1-w/y} = \frac{Y}{M}, \frac{1-\alpha}{1-\beta} = \frac{Q}{Y}, 1-\beta = \frac{p'}{m'}, 1-\alpha = \frac{p'+1}{m'+1}$$

$$p' = \frac{\alpha(1-\beta)}{\beta-\alpha} = \frac{\frac{1}{\beta}-1}{\frac{1}{\alpha}-\frac{1}{\beta}}, P' = p'+1 = \frac{\beta(1-\alpha)}{\beta-\alpha} = \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-\frac{1}{\beta}}$$

[6] Solow RM. Technical Change and the Aggregate Production Function [J]. The Review of Economics and Statistics, 1957, 39(3): 312

[7] Okishio, N. "Technical Change and the Rate of Profit", Kobe Univ. Econ. Review, 7, 1961, pp. 85–99.

$$m = \alpha \dot{n} + (1-\alpha)\dot{w} + \frac{p'+\alpha}{p'+1} p \Leftrightarrow$$

$$m = \alpha \dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta} p$$

And the following relationships about the equilibrium state are obtained:

$$m = \alpha \dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta} p = \alpha \dot{n} + (1-\alpha)\dot{w} + \frac{1-\alpha}{1-\beta} f$$

$$= (1-\alpha)(\frac{f}{1-\beta} + \dot{w} - \dot{n}) + \dot{n}$$

$$= (1-\alpha)\dot{k} + (\dot{y} - \dot{k}) \Rightarrow$$

$$\frac{f}{1-\beta} + \dot{w} - \dot{n} = \dot{k}$$

$$\dot{y}^* = \dot{n}^* + \dot{k}^*,$$

$$f = (1-\beta)(\dot{k} + \dot{n} - \dot{w}) = (1-\beta)\dot{g}^*$$

$$m = \dot{n}^* + (1-\alpha)\dot{k}^*$$

Therefore, the Marx production function is obtained as:

$$Y = A_0 e^{mt} K^{\alpha} L^{1-\alpha} = \frac{A_0}{n_0^{\alpha} w_0^{1-\alpha}} e^{[\alpha \dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta} p]t} K^{\alpha} n_0^{\alpha} w_0^{1-\alpha} L^{1-\alpha}$$

$$= a_0 e^{\frac{\alpha}{\beta} pt} C^{\alpha} V^{1-\alpha} = a_0 e^{\frac{1-\alpha}{1-\beta} ft} C^{\alpha} V^{1-\alpha} (a_0 \equiv \frac{A_0}{n_0^{\alpha} w_0^{1-\alpha}})$$

$$= a_0 e^{(1-\alpha)\dot{g}^* t} C^{\alpha} V^{1-\alpha} \equiv a_0 e^{Ft} C^{\alpha} V^{1-\alpha}$$

Since the above derivation is based on the equilibrium condition, the symbol "*" is put as a label, and the productivity development parameter (F) is characterized as:

$$\underline{F = (1-\alpha)\dot{g}^*}$$

Thus , the Solow residue combined with Okishio theorem could be rewritten as Marxian 1st theorem about technical change, which depends upon the combination of the growth rates of capital circulating, the wage,

and the profit rate: $m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta}p = \alpha\dot{n} + (1-\alpha)\dot{w} + F$ **.**

The division of labor by Adam Smith[8] can be characterized as the labor output elasticity $(1-\alpha)$ of the Cobb-Douglas production function: $d_L := 1-\alpha = \frac{\gamma}{g+\gamma} = \frac{p'+1}{m'+1}$ **; similarly, the degree of the roundabout production by Allyn Young[9] can be characterized as the variable capital output elasticity $(1-\beta)$ of the Marx production function:**

$$d_R := 1-\beta = \frac{1}{1+g} = \frac{p'}{m'}, \gamma_{\downarrow} = 1 + \frac{1}{p'} = \frac{\beta}{1-\beta}\frac{1-\alpha}{\alpha} = \frac{\frac{1}{(1-\beta)} - 1}{\frac{1}{1-\alpha} - 1},$$

$$\dot{\gamma} = \frac{d(P'/p')}{(P'/p')dt} = f - p = f(1-\gamma) = (1-\beta)\dot{g}[1 - \frac{\beta(1-\alpha)}{(1-\beta)\alpha}] = \dot{g}(1 - \frac{\beta}{\alpha}) < 0$$

$$\dot{d}_L = \dot{\gamma} - \frac{d(g+\gamma)}{(g+\gamma)dt} = \dot{\gamma} - \frac{g}{g+\gamma}\dot{g} - \frac{\gamma}{g+\gamma}\dot{\gamma} = \frac{g}{g+\gamma}(\dot{\gamma} - \dot{g}) = \alpha(\dot{\gamma} - \dot{g}) = -\beta\dot{g}$$

$$\dot{d}_R = -\frac{d(1+g)}{(1+g)dt} = -\frac{g}{1+g}\dot{g} = -\beta\dot{g} = \dot{d}_L$$

$$\dot{\alpha} = \dot{g} - \frac{g}{g+\gamma}\dot{g} - \frac{\gamma}{g+\gamma}\dot{\gamma} = \frac{\gamma}{g+\gamma}(\dot{g} - \dot{\gamma}) = \gamma f = p,$$

$$\dot{\beta} = \dot{g} - \frac{g}{g+1}\dot{g} = (1-\beta)\dot{g} = f$$

$$\frac{Y}{C+V} = \frac{A_0 e^{\frac{\alpha}{\beta}pt} C^{\alpha} V^{1-\alpha}}{a_0 C^{\beta} V^{1-\beta}} = \frac{Y}{Cv} = \frac{A_0}{a_0} e^{\frac{\alpha}{\beta}pt} g^{\alpha-\beta} = \frac{A_0}{a_0} e^{\frac{1-\alpha}{1-\beta}ft} g^{\alpha-\beta} \xrightarrow{f=(1-\beta)\dot{g}}$$

$$\dot{Y} - \dot{C}v = (1-\alpha)\dot{g} + (\alpha-\beta)\dot{g} = (1-\beta)\dot{g} = f \xrightarrow{p=\frac{1-\alpha}{\alpha}\beta\dot{g}} \beta = \frac{\dot{C}v - \dot{V}}{\dot{C} - \dot{V}}, \alpha = \frac{\dot{C}v - \dot{V}}{\dot{M} - \dot{V}}$$

Schumpeterian innovation[10] function/degree can be characterized by $\frac{\gamma}{g}$

[8] 亚当·斯密著，郭大力 王亚南译. 国民财富的性质和原因的研究[M]. 北京：商务印书馆，1972.

[9] Young AA. Increasing Returns and Economic Progress [J]. The Economic Journal, 1928, 38: 527-42.

[10] Schumpeter, J.A. *The theory of economic development: an inquiry into profits, capital, credit, interest, and the business cycl* translated from the German by Redvers Opie (1961) New York: OUP

since $\frac{1}{g} = \frac{1-\beta}{\beta}$ and the labor division coefficient d_L has the same change rate as the degree of the roundabout production d_R:

$$S := \frac{\gamma}{g} = \frac{1-\alpha}{\alpha} = \frac{Q}{M}\frac{V}{C} = \frac{B_0 e^{ft} g^{\beta-1}}{b_0 e^{pt} g^{\beta}} = \frac{B_0 e^{(f-p)t}}{b_0 g} = \frac{B_0}{b_0} e^{(1-\frac{\beta}{\alpha})\dot{g}^* t} g^{-1}$$

$$\dot{S} = \dot{\gamma} - \dot{g} = -\frac{\beta}{\alpha}\dot{g}^* = -\dot{m}'$$

$$S = \frac{Q/C}{M/V} = \frac{\mathrm{Y}+\mathrm{C}}{\mathrm{C}}\frac{\mathrm{Y}-\mathrm{M}}{\mathrm{M}} = \frac{(Y+C)/C}{(Y-V)/V} = \frac{\frac{Y}{C}+1}{\frac{Y}{V}-1} = \frac{\frac{1}{C_{\downarrow}}+\frac{1}{Y}}{\frac{1}{V}-\frac{1}{Y}} \geq 1 \Leftrightarrow \frac{1}{C}+\frac{2}{Y} \geq \frac{1}{V}$$

$$\dot{\mathrm{S}} = \dot{\mathrm{Q}} - \dot{\mathrm{M}}^{\downarrow} - \dot{\mathrm{g}}^{\downarrow} = \dot{\mathrm{Y}} + \dot{\mathrm{V}} - \dot{\mathrm{C}}^{\downarrow} - \dot{\mathrm{M}}$$

$$\frac{\partial S}{\partial C} = \frac{-C^{-2}}{\frac{1}{V}-\frac{1}{Y}} < 0,$$

$$\frac{\partial S}{\partial V} = (\frac{1}{C}+\frac{1}{Y})(-)(\frac{1}{V}-\frac{1}{Y})^{-2}(-)V^{-2} > 0,$$

$$\frac{\partial S}{\partial Y} = \frac{V}{CM}\frac{Q}{CM} > 0$$

All the equations are:

$$LTV:\ Q = C + \mathrm{V} + \mathrm{M} = \mathrm{C} + \mathrm{Y} = B_0 e^{ft} C^{\beta} V^{1-\beta} = B_0 e^{(1-\beta)\dot{g}^* t} C^{\beta} V^{1-\beta},$$

$$MPF:\ Y = M + V = a_0 e^{Ft} C^{\alpha} V^{1-\alpha} = a_0 e^{(1-\alpha)\dot{g}^* t} C^{\alpha} V^{1-\alpha}, \alpha p = \beta F = \alpha\gamma f,\ p > F > f$$

$$STV:\ M = b_0 e^{pt} C^{\beta} V^{1-\beta} = b_0 e^{\frac{1-\alpha}{\alpha}\beta\dot{g}^* t} C^{\beta} V^{1-\beta}$$

$$CostFun\ .:\ Cv = C + V = c_0 C^{\beta} V^{1-\beta},\ g+1 = c_0 g^{\beta}$$

$$\text{Productivity}:\ P' = Q / Cv = B_0 c_0^{-1} e^{ft} = B_0 c_0^{-1} e^{(1-\beta)\dot{g}^* t}$$

$$SuplusValueRate:\ m' = M / V = b_0 e^{pt} g^{\beta} = b_0 e^{\frac{\beta}{\alpha}Ft} g^{\beta} = b_0 e^{\frac{1-\alpha}{\alpha}\beta\dot{g}^* t} g^{\beta}$$

$$\text{Profit Rate}:\ p' = M / Cv = \frac{m'}{g+1} = m'(1-\beta) = b_0 e^{pt}\frac{g^{\beta}}{g+1}$$

II. Transformation Problem

If there is no currency inflation, and the values of commodities keep invariant, then we have:

C+V=Cv=N=const.,

dN=0=dCv=d(C+V);

dC=dV=0

Total value (C+V) equal total production price (P_1C+P_2V):

$$\begin{aligned}
dN &= d(P_1C + P_2V) = CdP_1 + P_1dC + P_2dV + VdP_2 = CdP_1 + VdP_2 \\
&= Cv[\beta dP_1 + (1-\beta)dP_2] = Cv[\beta\delta P_1 + (1-\beta)\delta P_2] \\
&\cong Cv[\beta\ln(1+\delta P_1) + (1-\beta)\ln(1+\delta P_2)] = Cv[\beta\ln P_1 + (1-\beta)\ln P_2] \\
&= Cv\ln(P_1^{\beta}P_2^{1-\beta}) = 0 \Rightarrow \\
& P_1^{\beta}P_2^{1-\beta} = 1 \Leftrightarrow P^{\beta}P_2 = 1 \Rightarrow \\
& \beta\dot{P}_1 + (1-\beta)\dot{P}_2 = 0, or: \\
& \beta\delta P_1 + (1-\beta)\delta P_2 = 0
\end{aligned}$$

let (I denotes the inflation index and

$$\delta P_1 := PPI - 1, \delta P_2 := CPI - 1$$

$$\beta = \frac{\delta P_2}{\delta P_2 - \delta P_1} = \frac{CPI - 1}{CPI - PPI},$$

$$\begin{aligned}
& Q' = C' + V' + M' = P_1C + P_2V + P_3M = B(P_1C)^{\beta}(P_2V)^{1-\beta} = QP_1^{\beta}P_2^{1-\beta} = Q \\
& M' = P_3M = b(P_1C)^{\beta}(P_2V)^{1-\beta} = MP_1^{\beta}P_2^{1-\beta} = M, P_3 = 1; \\
& P_1C + P_2V = C + V; \\
& M' = Q' - (P_1C + P_2V) = P_3M = M = r(C+V) = r'(P_1C + P_2V)
\end{aligned}$$

Total profit equals total surplus values.

For a Marxian two production departments system:

$$\begin{aligned}
& (1+r_1)(P_1C_1 + P_2V_1) = Q_1' = P_1(C_1 + C_2) = P_1C, \\
& (1+r_2)(P_1C_2 + P_2V_2) = Q_2' = P_2V + M, \\
& r_1 = r_2 = r = \frac{PC_2 - V_1}{PC_1 + V_1} = \frac{M}{P_1C + P_2V} = \frac{M}{C+V} = \frac{M}{Cv}, P = \frac{(1+r)V_1}{C_2 - rC_1}
\end{aligned}$$

Or a three production departments system described by J. Winternitz[11]:

$$(1):\ P_1C_1 + P_2V_1 + P_3M_1 = (P_1C_1 + P_2V_1)(1+p') = P_1C = P_1(C_1 + C_2 + C_3)$$

$$(2):\ P_1C_2 + P_2V_2 + P_3M_2 = (P_1C_2 + P_2V_2)(1+p') = P_2V = P_2(V_1 + V_2 + V_3)$$

$$(3):\ P_1C_3 + P_2V_3 + P_3M_3 = (P_1C_3 + P_2V_3)(1+p') = P_3M = M = M_1 + M_2 + M_3$$

$$p' = \frac{M}{P_1C + P_2V} = \frac{M}{Cv},\ P = \frac{(1+p')V_1}{C_2 + C_3 - p'C_1},\ P_1 = \frac{Cv - P_2V}{C} = \frac{PCv}{PC + V},\ P_2 = \frac{Cv}{PC + V}.$$

$$P_1^{\beta}P_2^{1-\beta} = I \Leftrightarrow \ln P_2 = \ln I + \beta\ln\frac{P_2}{P_1},\ \beta\dot{P}_1 + (1-\beta)\dot{P}_2 = \dot{I},\ or:\ \beta\delta P_1 + (1-\beta)\delta P_2 = \delta I$$

$$P^{\beta} = \frac{1}{P_2} \xrightarrow{P_2 = \frac{P_1}{p} = \frac{C+V}{PC+V}} P^{\beta} = P\beta + 1 - \beta$$

$$1.\ e^{\beta\ln P} = 1 + \beta\ln P + \frac{\beta^2\ln^2 P}{2} + \ldots = 1 + \beta(P-1) \xrightarrow{\ln P \equiv y}$$

$$(a)\ \frac{\beta}{2}y^2 = e^y - 1 - y = \frac{y^2}{2} + \frac{y^3}{6},\ y = 3(\beta - 1) = \ln P,\ P = e^{-3(1-\beta)},$$

$$(b)\ \frac{\beta}{2}y^2 + \frac{\beta^2y^3}{6} = \frac{y^2}{2} + \frac{y^3}{6},\ y = \frac{-3}{1+\beta} \Rightarrow P = e^{\frac{-3}{1+\beta}},$$

$$(c)\ \frac{\beta}{2}y^2 + \frac{\beta^2y^3}{6} + \frac{\beta^3y^4}{24} = \frac{y^2}{2} + \frac{y^3}{6} \Rightarrow P = e^{\frac{2}{\beta^3}(1-\beta^2 \pm \sqrt{(1-\beta)(1+\beta-\beta^2+2\beta^3)})}$$

$$(d)\ \frac{\beta}{2}y^2 + \frac{\beta^2y^3}{6} + \frac{\beta^3y^4}{24} = \frac{y^2}{2} + \frac{y^3}{6} + \frac{y^4}{24} \Rightarrow P = e^{\frac{-2(1+\beta)\pm i2\sqrt{2+\beta+2\beta^2}}{1+\beta+\beta^2}}$$

$$\Leftrightarrow P = e^{\frac{-2(1+\beta)}{1+\beta+\beta^2}}\left(\cos\frac{2\sqrt{2+\beta+2\beta^2}}{1+\beta+\beta^2} \pm i\sin\frac{2\sqrt{2+\beta+2\beta^2}}{1+\beta+\beta^2}\right), \cdots$$

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[11] 'Values and Prices: a solution to the so-called transformation problem', Econ. Jour. 1948, 58, 276-280.

$$2.\ \frac{d}{d\beta}(P^{\beta}) = \frac{d}{d\beta}(e^{\beta\ln P}) = e^{\beta\ln P}\ln P = \frac{d}{d\beta}(P\beta + 1 - \beta) = P - 1$$

$$\xrightarrow{\ln P \equiv x} \beta = \frac{\ln(\frac{e^x - 1}{x})}{x} \approx \frac{1}{x}(\frac{e^x - 1}{x} - 1) = \frac{e^x - 1 - x}{x^2} \approx \frac{1}{2} + \frac{x}{6} + \frac{x^2}{24} \Rightarrow$$

$$x = -2 \pm 2\sqrt{6\beta - 2},\ \beta > 1/3,\ P = e^{-2\pm 2\sqrt{6\beta-2}}$$

Taking into consideration of the prices, the change rate of OCC would be:

$$g_P = \frac{P_1 C}{P_2 V} = Pg \Rightarrow \dot{g}_P = \dot{P} + \dot{g}$$

$$\dot{P} = \frac{dP}{Pdt} = \frac{d \ln Pd\beta}{d\beta dt} = \pm\frac{6\beta}{\sqrt{6\beta - 2}}\dot{\beta} = \pm\frac{6\beta(1-\beta)}{\sqrt{6\beta - 2}}\dot{g};\ \dot{g}_P = (1 \pm \frac{6\beta(1-\beta)}{\sqrt{6\beta - 2}})\dot{g}$$

III. General Equilibrium

In *Das Kapital,* Marx[12] defined the reproduction schemes as abstract, two-sector models of the production and circulation of capital. Department one produces means of production, the value of its output(Q_1) is made up of $C_1 + V_1 + M_1 = Q_1$; where C_1 is the constant capital and V_1 the variable capital used up in production, M_1 is the surplus value produced. Department two produces means of consumption and the value of its output(Q_2) is likewise made up of $C_2 + V_2 + M_2 = Q_2$. Simple reproduction requires that capitalists in Department two acquire means of production to the value C_2 from Department one in order to be able to

12 马克思.《资本论》I [M]. 北京：人民出版社，1975.

produce again, namely : $C_2 = V_1 + M_1 = Y_1$ **, therefor---the Marxian 1st theorem:** $\dot{C}_2 = \dot{Y}_1$**; Then, Marx's theory about crisis is:**

$$Y_1 = Y_1^0 e^{\dot{Y}_1 t} = C_2 = C_2^0 e^{\dot{C}_2 t}$$

$$\Rightarrow t = \frac{\ln Y_1^0 - \ln C_2^0}{\dot{C}_2 - \dot{Y}_1} = \begin{cases} \infty, \dot{Y}_1 = \dot{C}_2; \\ 0, Y_1^0 = C_2^0 \end{cases}$$

Namely, if an economic system achieves the Marxian equilibrium including its each production Department ($\dot{Y} = \dot{Y}_1 = \dot{C}_1 = \dot{Y}_2 = \dot{C}_2 = \dot{C}$ **), there would be no business cycle; otherwise there exists some potential crisis:**

$$\dot{\beta} = f = \frac{\beta'}{\beta} \Rightarrow \dot{f} = \ddot{\beta} = \frac{d\beta'}{\beta' dt} - \dot{\beta} = \frac{\beta''}{\beta f} - f = \dot{d}_R \Rightarrow$$

$$\beta'' = (f - \beta\dot{g})\beta f = \beta(1-\beta)(1-2\beta)\dot{g}^2 = -\beta\frac{(g-1)\dot{g}^2}{(1+g)^2} = -\beta\omega^2,$$

$$\omega \equiv \frac{\sqrt{g-1}\,\dot{g}}{1+g} = \sqrt{(2\beta-1)(1-\beta)}\,\dot{g}$$

$$\Rightarrow \beta_t = A_\beta \sin(\omega t) + B_\beta \cos(\omega t) = \beta_0 \sin(\omega t + \theta),$$

$$\beta_0 \equiv \sqrt{A_\beta^2 + B_\beta^2} \le 1, \theta \equiv arctg \frac{A_\beta}{B_\beta}$$

$$\Rightarrow T = \frac{2\pi}{\dot{g}}\frac{g+1}{\sqrt{g-1}} = \frac{2\pi}{\dot{g}}(\sqrt{g-1} + \frac{2}{\sqrt{g-1}}) \ge \frac{4\sqrt{2}\pi}{\dot{g}};$$

$$\dot{\beta}_t = \frac{d\beta_t}{\beta_t dt} = \frac{\omega}{tg(\omega t + \theta)} = f = (1-\beta_t)\dot{g} = \frac{\sqrt{(2\beta-1)(1-\beta)}\,\dot{g}}{tg(\omega t + \theta)}$$

$$\Rightarrow tg(\omega t + \theta) = \sqrt{\frac{2\beta-1}{1-\beta}} = \sqrt{g-1}, \sin(\omega t + \theta) = \sqrt{\frac{g-1}{g}} = \sqrt{\frac{2\beta-1}{\beta}}$$

$$p'=\frac{\frac{1}{\beta}-1}{\frac{1}{\alpha}-\frac{1}{\beta}}=\frac{\frac{1}{1-\alpha}-1}{\frac{1}{1-\beta}-\frac{1}{1-\alpha}}$$

$$\dot{\alpha}=p=\frac{\alpha'}{\alpha}\Rightarrow\dot{p}=\ddot{\alpha}=\frac{d\alpha'}{\alpha' dt}-\dot{\alpha}=\frac{\alpha''}{\alpha p}-p=\dot{S}+\dot{\beta}+\ddot{g},$$

$$\Rightarrow\alpha''=(p-\frac{\beta}{\alpha}+1-\beta)\dot{g}p\alpha=-(2\beta-1)S\beta\dot{g}^2\alpha=-(2\beta-1)\gamma(1-\beta)\dot{g}^2\alpha=-\varpi^2\alpha,$$

$$\varpi\equiv\sqrt{(2\beta-1)S\beta}\dot{g}=\dot{g}\sqrt{(2\beta-1)\gamma(1-\beta)}=\omega\sqrt{\gamma}$$

$$\Rightarrow\alpha_t=\alpha_0\sin(\varpi t+\vartheta)<\beta_t,\alpha_0\equiv\sqrt{A_\alpha^2+B_\alpha^2}\le 1,\vartheta\equiv arctg\ \frac{A_\alpha}{B_\alpha}$$

$$\Rightarrow T'=\frac{2\pi}{\dot{g}\sqrt{(2\beta-1)S\beta}}=\frac{2\pi}{\dot{g}\sqrt{(2\beta-1)\gamma(1-\beta)}}\ge\frac{4\sqrt{2}\pi}{\dot{g}\sqrt{\gamma}}$$

$$\dot{Y}=\frac{Y'}{Y},\ddot{Y}=\frac{dY'}{Y' dt}-\dot{Y},Y''=(\ddot{Y}+\dot{Y})Y'=0$$

$$\Rightarrow\ddot{Y}+\dot{Y}=0\Rightarrow\ddot{Y}=-\dot{Y}=-\dot{C}$$

$$Y''=(\dot{Y}-\dot{C})Y'=-Y\dot{Y}(\dot{C}-\dot{Y})$$

$$\Rightarrow Y=A_0\sin(\omega t)+B_0\cos(\omega t),\omega=\sqrt{\dot{Y}(\dot{C}-\dot{Y})};$$

$$or\ :Y''=(\dot{C}-\dot{Y})Y',\omega=\sqrt{\dot{Y}(\dot{Y}-\dot{C})}$$

Moreover,

$$Q_1=C_1+V_1+M_1=C_1+Y_1=C_1+C_2=C$$

$$Q_2=C_2+V_2+M_2=C_2+Y_2=Y_2+Y_1=Y$$

$$Q=Q_1+Q_2=C+Y\Leftrightarrow Q\dot{Q}=C\dot{C}+Y\dot{Y}\Leftrightarrow(1-\alpha)\dot{Q}=(\beta-\alpha)\dot{C}+(1-\beta)\dot{Y}$$

according to Leontief's input-output theory[13] and the steady state analysis of the CD production function[14], there should be a dynamic input-output equilibrium between Department I (input: C) and Department II (output: Y=V+M)

$$\dot{y}^{ss}=\dot{k}^{ss},\dot{n}^{ss}=0\Leftrightarrow\dot{Y}^{ss}=\dot{k}^{ss}+\dot{L}^{ss}+\dot{n}^{ss}=\dot{C}^{ss}=\dot{Q}^{ss}$$ **;**

13 Wassily Leontief.Conference on Research in Income and Wealth, 1955. "Input-Output Analysis: An Appraisal," NBER Books, National Bureau of Economic Research, Inc, number 2864.

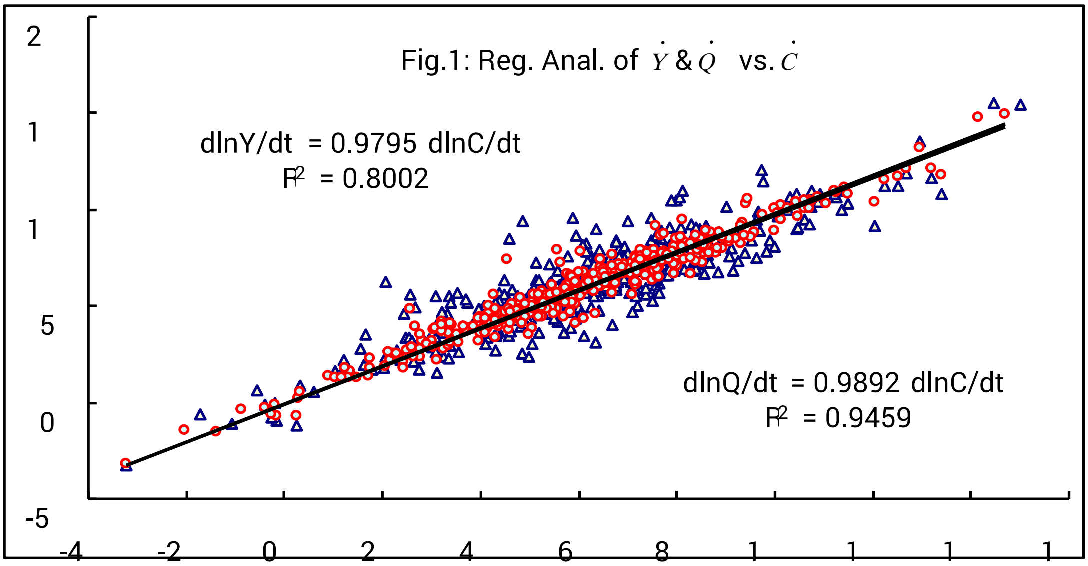
14 Solow R.M. A Contribution to the Theory of Economic Growth [J]. The Quarterly Journal of Economics, 1956, (1), 65-94

$$Y/C = b_0 e^{(1-\alpha)\dot{g}^* t} g^{\alpha-1}, \dot{Y} - \dot{C} = (1-\alpha)\dot{g}^* + (\alpha-1)\dot{g} = (1-\alpha)(\dot{g}^* - \dot{g})$$

$$Q/C = B_0 e^{(1-\beta)\dot{g}^* t} g^{\beta-1}, \dot{Q} - \dot{C} \doteq (1-\beta)\dot{g} + (\beta-1)\dot{g}^* = (1-\beta)(\dot{g}^* - \dot{g})$$

$$\therefore \dot{Y}^* = \dot{C}^* = \dot{Q}^* \Leftrightarrow \dot{g}^* = \dot{g}$$

Marxian 2nd theorem about reproducibility states: there is a dynamic input-output equilibrium inside an economic system. The regression analysis of the USA manufacture industry data[15] from 1958-1996 supported the above results:



The Marxian 3rd theorem about productivity development asserts: only Marxian equilibrium leads to productivity development and a rising profit rate.

$$F = \dot{Y} - \alpha\dot{C} - (1-\alpha)\dot{V} = (1-\alpha)\dot{g}^* > 0 \Leftrightarrow \dot{Y} = \dot{C}, \dot{g}^* > 0;$$

$$f = \dot{Q} - \beta\dot{C} - (1-\beta)\dot{V} = (1-\beta)\dot{g}^* > 0 \Leftrightarrow \dot{Q} = \dot{C}, \dot{g}^* > 0$$

$$\because \dot{g}^* = \dot{g} = \frac{f}{1-\beta} = \frac{\dot{\beta}}{1-\beta}, \beta = \frac{g}{g+1} = \frac{C}{Cv}, \dot{\beta} = \dot{C} - \dot{C}v = \dot{Q} - \dot{C}v$$

$$Q = C + V + M = Cv + M$$

$$\dot{Q} - \dot{C}v = \frac{M(\dot{M} - \dot{Q})}{Cv} = p'(-\dot{\gamma}) > 0$$

$$\therefore \dot{g}^* = \dot{g} = \frac{f}{1-\beta} = \frac{\dot{\beta}}{1-\beta} > 0$$

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[15] NBER-CES Manufacturing Industry Database [EB/OL].(2011-02-02)[2012-10-11]. www.nber.org.

$$p=\frac{1-\alpha}{\alpha}\beta\dot{g}*=\frac{\beta}{\alpha}F=\gamma f>0\Leftrightarrow F>0\Leftrightarrow f>0$$

IV. Marxian Optimal Growth

The tendency of the rate of profit depends on the OCC(g) and the output elasticity of the constant capital((), similar to the conclusion obtained by D.H. Dickinson[16]:

$$p'=b_0e^{pt}\frac{g^{\beta}}{g+1}=b\frac{g^{\beta}}{g+1}$$

$$\frac{\partial p'}{\partial g}=\frac{p'}{g}(\beta-\frac{g}{g+1})\geq 0$$

$$\Leftrightarrow\beta\geq\frac{g}{g+1},$$

$$\frac{\partial^2 p'}{\partial g^2}=-\frac{p'\beta}{g^2(g+1)}<0$$

$$p'_{\max}=b\beta^{\beta}(1-\beta)^{1-\beta}\Rightarrow(\beta\to 0\;or\;1)b>p'_{\max}\geq\frac{b}{2}(\beta=\frac{1}{2})$$

By means of variation, Marx was right about the falling rate of the profit under the competitive equilibrium situation:

$$p'=\frac{M}{Cv}=\frac{b_0e^{pt}C^{\beta}V^{1-\beta}}{C+V}=\frac{b_0e^{(1-\alpha)\frac{\beta}{\alpha}\dot{g}t}C^{\beta}V^{1-\beta}}{a_0C^{\beta}V^{1-\beta}}=\frac{b_0}{a_0}e^{\frac{\beta}{\alpha}(1-\alpha)\frac{g'}{g}t}$$

$$\frac{\partial p'}{\partial g}=p'\frac{-\beta g't}{\alpha g^2}(1-\alpha)=-\frac{p'}{g}\frac{1-\alpha}{\alpha}\beta\dot{g}t$$

$$\frac{d}{dt}(\frac{\partial p'}{\partial g'})=\frac{d}{dt}[p'\frac{\beta t}{\alpha g}(1-\alpha)]=\frac{p'}{g}\frac{1-\alpha}{\alpha}\beta(tp+1-t\dot{g})$$

$$\Rightarrow pt=-1<0$$

$$\Rightarrow p'=p'_0t^{-1}$$

[16] "The falling rate of profit in Marxian economics", Rev. Econ. Stud., 1975, 24, 120-130

$$if:\frac{d}{dt}(\frac{\partial p'}{\partial g'})=\frac{d}{dt}(p'\frac{\beta t}{g}S)=\frac{p'}{g}S\beta(tp+1+\dot{S}t+\dot{\beta}t-t\dot{g})=-\frac{p'}{g}S\beta\dot{g}t\Rightarrow$$

$$tp+\dot{S}t+\dot{\beta}t=-1=tp+\dot{S}t+\ ft=t(\frac{1-\alpha}{\alpha}\beta-\frac{\beta}{\alpha}+1-\beta)\dot{g}=t(1-2\beta)\dot{g}\Rightarrow$$

$$\dot{g}t=\frac{1}{2\beta-1}>0\Leftrightarrow\beta>\frac{1}{2};$$

$$or:$$

$$pt=\frac{1-\alpha}{\alpha}\beta\dot{g}t=\frac{S\beta}{2\beta-1}=\frac{\gamma}{g-1}>0\Leftrightarrow g>1\Leftrightarrow C>V$$

The exploitation rate will decrease under the equilibrium state:

$$m'=\frac{M}{V}=a_0e^{pt}g^{\beta}=a_0e^{\beta\frac{1-\alpha}{\alpha}\dot{g}t}g^{\beta}=a_0g^{\beta+\frac{\beta(1-\alpha)\dot{g}t}{\alpha\ln g}},\ \dot{g}=\frac{dg}{gdt}=\frac{g'}{g},\ S:=\frac{1-\alpha}{\alpha}$$

$$\frac{\partial m'}{\partial g}=a_0e^{\frac{1-\alpha}{\alpha}\beta\frac{g'}{g}t}[\frac{1-\alpha}{\alpha}\beta\frac{(-)g'}{g^2}tg^{\beta}+\beta g^{\beta-1}]=m'\frac{\beta}{g}(1-\frac{1-\alpha}{\alpha}\frac{g'}{g}t)=\frac{M}{C}\beta(1-S\dot{g}t)$$

$$\frac{\partial m'}{\partial g'}=a_0e^{\frac{1-\alpha}{\alpha}\beta\frac{g'}{g}t}(\frac{1-\alpha}{\alpha}\beta\frac{t}{g})g^{\beta}=m'\frac{\beta}{g}t\frac{1-\alpha}{\alpha}=\beta\frac{1-\alpha}{\alpha}\frac{M}{C}t=\beta S\frac{m'}{g}t$$

$$(1):\frac{d}{dt}(\frac{\partial m'}{\partial g'})=\beta\frac{1-\alpha}{\alpha}(\frac{M'}{C}t-\frac{Mt}{C^2}C'+\frac{M}{C})=\beta\frac{1-\alpha}{\alpha}\frac{M}{C}(\dot{M}t-\dot{C}t+1)$$

$$=\frac{\partial m'}{\partial g}=\frac{M}{C}\beta(1-\frac{1-\alpha}{\alpha}\dot{g}t)\Rightarrow\dot{M}t-\dot{C}t+1=\frac{\alpha}{1-\alpha}-\dot{g}t$$

$$\Leftrightarrow\dot{m}'t=\frac{\alpha}{1-\alpha}-1=S^{-1}-1=-\dot{S}t=\frac{\beta}{\alpha}\dot{g}t<0\Rightarrow m'=m'_0t^{S^{-1}-1},\alpha<\frac{1}{2}$$

$$(2):\frac{d}{dt}(\frac{\partial m'}{\partial g'})=\beta S\frac{m'}{g}[(\dot{\beta}+\dot{S}+\dot{m}'-\dot{g})t+1]=\frac{M}{C}\beta(1-\frac{1-\alpha}{\alpha}\dot{g}t)\Rightarrow\ ft=S^{-1}-1$$

$$\Rightarrow\dot{m}'t=\frac{\beta}{\alpha}\dot{g}t=\frac{\beta(S^{-1}-1)}{(1-\beta)\alpha}=(\frac{1}{1-\alpha}-\frac{1}{\alpha})\frac{\beta}{1-\beta}<0\Leftrightarrow\alpha<\frac{1}{2}$$

Moreover，the rate of variable capital accumulation will increase under equilibrium state：

$$V = \frac{b_0}{B_0} e^{(\frac{\beta}{\alpha}-1)\frac{g'}{g}t} CS$$

$$\frac{\partial V}{\partial g} = -(\frac{\beta}{\alpha}-1)\frac{g'}{g^2} tV = (1-\frac{\beta}{\alpha})\frac{\dot{g}}{g} tV$$

$$\frac{d}{dt}(\frac{\partial V}{\partial g'}) = \frac{d}{dt}[V(\frac{\beta}{\alpha}-1)\frac{t}{g}] = (\frac{\beta}{\alpha}-1)(\dot{V}\frac{tV}{g} - \frac{tV\beta}{g\alpha}\dot{g} + \frac{V}{g} - V\frac{t\dot{g}}{g}) \Rightarrow$$

$$\dot{V}t = \frac{\beta}{\alpha}\dot{g}t - 1 = \dot{m}'t - 1 > 0 \xrightarrow{ft=S^{-1}-1} \frac{1}{1-\alpha} - \frac{1}{\alpha} > \frac{1-\beta}{\beta} \Leftrightarrow \frac{g+\gamma}{\gamma} - \frac{g+\gamma}{g} > \frac{1}{g}$$

$$\Leftrightarrow g^2 > (1+\gamma)\gamma > 2 \Rightarrow g > \sqrt{2} \approx 1.414 \ > 1;$$

$$1-\beta = \frac{1}{1+g} < \sqrt{2} - 1 \approx 0.414 \ ,(2\alpha >)\beta > 0.586 \ ;$$

$$\alpha^2 + (2g-1)\alpha - g > 0 \Rightarrow (\beta >)\alpha > \frac{1+\sqrt{4g^2+1}-2g}{2} > 0.5$$

A maximum production would achieve under equilibrium state together with a minimum input requirement under equilibrium state:

$$Y = A_0 e^{\frac{\alpha}{\beta}pt} C^{\alpha} V^{1-\alpha} = A_0 e^{\frac{1-\alpha}{1-\beta}ft} g^{\alpha} V = A_0 e^{\frac{1-\alpha}{g}g't+\alpha\ln g} V$$

$$\frac{\partial Y}{\partial g} = Y(\frac{\alpha}{g} - \frac{1-\alpha}{g^2}g't) = \frac{Y}{g}[\alpha - (1-\alpha)\dot{g}t]$$

$$\frac{d}{dt}(\frac{\partial Y}{\partial g'}) = \frac{d}{dt}(Y\frac{1-\alpha}{g}t) = Y\frac{1-\alpha}{g}(\dot{Y}t + \dot{d}_L t + 1 - \dot{g}t)$$

$$\Rightarrow \dot{Y}t = S^{-1} - 1 > 0 \Leftrightarrow \alpha > 0.5, S < 1, Y = Y_0 t^{S^{-1}-1};$$

$$or: (\dot{Y} - \beta\dot{g})t = S^{-1} - 1 \equiv b \xrightarrow{\beta\dot{g}\equiv a} \ln Y = at + b\ln t + c \geq 0$$

$$\Leftrightarrow \dot{Y}t = \beta\dot{g}t + S^{-1} - 1 \geq 0 \Leftrightarrow S^{-1} = \frac{\alpha}{1-\alpha} \geq 1 - \beta\dot{g}t \Leftrightarrow \alpha \geq \frac{1-\beta\dot{g}t}{2-\beta\dot{g}t}$$

$$Cv = a_0 C^{\beta} V^{1-\beta} = a_0 g^{\beta} V \xrightarrow{1-\frac{f}{\dot{g}}=\beta} Cv = a_0 e^{(1-\frac{f}{\dot{g}})\ln g} V = a_0 e^{(1-\frac{fg}{g'})\ln g} V$$

$$\frac{\partial Cv}{\partial g} = Cv[(-\frac{f}{g'})\ln g + \frac{1}{g} - \frac{f}{g'}] = \frac{Cv}{g}[1-(1-\beta)(1+\ln g)] = \frac{V}{g}(g - \ln g)$$

$$\frac{d}{dt}(\frac{\partial Cv}{\partial g'}) = \frac{d}{dt}[Cv \ln g(\frac{fg}{g'^2})] = \frac{d}{dt}(Cv\frac{1-\beta}{g\dot{g}}\ln g) = \frac{d}{dt}(\frac{V}{g\dot{g}}\ln g) = \frac{V}{g}(\frac{\ln g \dot{V}}{\dot{g}} + 1 - \ln g)$$

$$\Rightarrow \frac{\dot{V}}{\dot{g}} = \frac{g-1}{\ln g} \Leftrightarrow \frac{\dot{V}}{\dot{C}} = \frac{g-1}{g-1+\ln g} \Leftrightarrow \dot{g} = \frac{\ln g}{g-1+\ln g}\dot{C} = \frac{1}{1+\frac{g-1}{\ln g}}\dot{Y} \approx \frac{\dot{Y}}{2} (g \approx 1)$$

$$f = (1-\beta)\dot{g} = \frac{1-\beta}{1+\frac{g-1}{\ln g}}\dot{Y} \propto (1-\beta)\dot{Y} = (1-\beta)(\dot{y} + \dot{l} + \dot{E})$$

$$or: \frac{d}{dt}(\frac{\dot{V}}{\dot{g}}\ln g) = \frac{d}{dt}(g-1) \Rightarrow \frac{\dot{V}}{\dot{g}} = g, \frac{\dot{V}}{\dot{C}} = \frac{g}{g+1} = \beta, \dot{g} = (1-\beta)\dot{C}, f = (1-\beta)^2\dot{C}$$

$$\dot{V}^* = \beta\dot{C}^* \Leftrightarrow \dot{w}^* = \beta\dot{Y}^* - \dot{L}^*$$

$$\dot{g}^* = (1-\beta)\dot{C}^* = (1-\beta)\dot{Y}^*$$

$$f = (1-\beta)\dot{g}^* = (1-\beta)^2\dot{C}^* = (1-\beta)^2\dot{Y}^*$$

$$p = \frac{1-\alpha}{\alpha}\beta\dot{g}^* = S\beta\dot{g}^* = S\beta(1-\beta)\dot{Y}^* \le \frac{S\dot{Y}^*}{4} \approx 1\% : S = 0.5, \dot{Y} = 8\%;\ S = 2, \dot{Y} = 2\%$$

$$\dot{S} = -\frac{\beta}{\alpha}\dot{g}^* = -\frac{\beta}{\alpha}(1-\beta)\dot{Y}^*$$

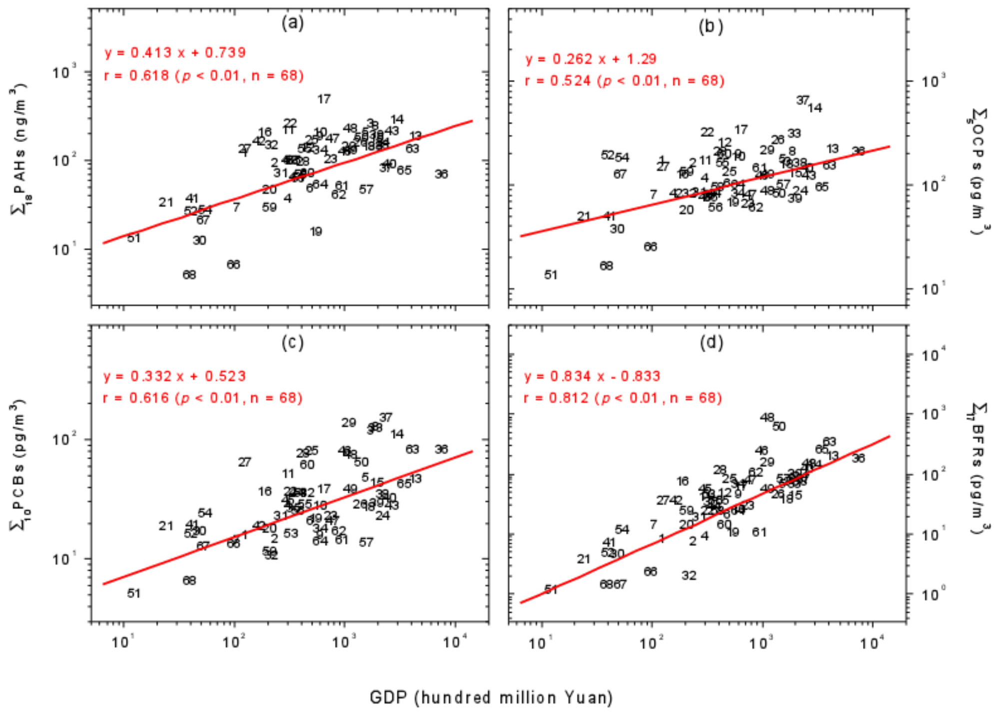
$$\dot{f} = -\beta\dot{g}^* = \dot{d}_L^* = \dot{d}_R^* = \dot{F} = -\frac{g^*}{g^*+1}\frac{dg^*}{g^* dt} = -\frac{d(g^*+1)}{(g8+1)dt} = -(\dot{C}v^* - \dot{V}^*) = \dot{V}^* - \dot{C}v^*$$

$$F = (1-\alpha)\dot{g}^* = (1-\alpha)(1-\beta)\dot{Y}^* = d_L d_R \dot{Y}^*$$

V. The econo-sociological Marxism

The first-rate reaction of natural decomposition of some Persistent Organic Pollutants is： $C = C_0 e^{-k_i t}$ **, k: reaction rate, C: concentration of certain POPs；the economic growth could be related to the amount of the POPs[17] as：** $\ln Y = a + b\ln C$ **，**

[17] Private communication with Prof. QQ WANG (Chem. Coll., Xiamen Univ.)



a、b are both positive coefficients ; therefore, the total change of the POPs is :

$$\dot{C} = \frac{d[(\ln Y - a)/b]}{dt} - k = \frac{d \ln Y}{bdt} - k = \frac{\dot{Y}}{b} - k$$

The environment couldn't be worse meaning the amount of POPs wouldn't increase:

$$\dot{C} \leq 0 \Leftrightarrow \frac{\dot{Y}}{b} \leq k \Leftrightarrow \dot{Y} \leq bk < k(if : b < 1)$$

namely, the growth rate of GDP should not be over a critic value.

VI. Conclusion

In short, this study is aimed to obtain a quantitative description of Marxian capital theory including Marx labour value function and Marx surplus value function as well as Marx production function. The labor output elasticity ($1-\alpha$) of Cobb-Douglas production function is defined as the parameter for the division of labor. The productivity parameter in Marx production function is defined as the product of the change rate of the organic composite of capital with the coefficient of the division of labor. Furthermore, three Marxian theorems are proposed, which assert that there is a dynamic equilibrium existed in reproduction between the Two Departments, only equilibrium growth leads to the positive value of the productivity parameter (Productivity Development Theorem) and also the rate of profit, of which the change rate combined that of the wage and capital circulating with respect to the capital output elasticity of Cobb-Douglas production function characterizes the technological progress rate or as called the Solow residue. By means of variation, the tendency of the profit rate to fall is proved under the situation that the degree of the labor division remains unchanged.