DARK MATTER DENSITY FUNCTION DEPENDING ON INTENSITY OF GRAVITATIONAL FIELD AS UNIVERSAL LAW

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1.   ABSTRAT

The purpose this work is to study density function of Dark Matter (D.M.) depending on E, total intensity of gravitational field, in a group of six big spiral galaxies and try to explore a possible Universal law between both magnitudes.

Galaxies selected are the following ones: Milky Way, M31, NGC 3031, NGC 3992, NGC 7331, NGC 2841

The reason why it has been made this work is to check the model of DM proposed by the author in his previous paper [5] Abarca,M.2014 viXra. *Dark Matter model by quantum vacuum.*

Briefly, in that work was exposed a original theory of DM. This new theory defends that DM is generated by the own gravitational field according an unknown quantum gravitational mechanism. In other words, Density of Dark Matter in a point P of space, depend on E, according a Universal law. Therefore if two different points P and Q belonging different galaxies had the same E, intensity of gravitational field, then they should have the same density of DM. In short, If E(P) = E(Q) then $D_{DM}(P) = D_{DM}(Q)$.

In the present paper, for each galaxy it will be calculate E, total intensity of gravitational field, depending on R. This calculus is carry out through the Rotational Curve and the Virial Theorem.

In addition, for each galaxy it is considered a Dark matter density function, which is tabulated depending on R. After, it is plotted   DM density depending on E.

Data of eight galactic set are studied statistically and it is concluded that its coefficient correlation is enough high to fit a potential function to DM density depending on E. Statistical process has been carefully detailed through the paper, so I have found plausible to postulate this function as Universal for big galaxies . I have considered a big galaxy if its velocity inside its flat region of rotation curve is up to 200 Km/s.

Dark matter function Density as Universal law for big galaxies

$Density_{DM} = 1,26 \cdot 10^{-6} \cdot E^{B}$ being $B = 1,74$ and its dominion $100 < E < 6500$.

Where Unit for Density $_{D.M.}$ is $10^{-3} M_{\Theta}/pc^{3}$ or $m M_{\Theta}/pc^{3}$ and Unit for E is $Km^{2} / s^{2} / Kpc$.

The same function with other suitable system of units is $Density_{DM} = 3,7323 \cdot 10^{17} \cdot E^{B}$ being $B = 1,74$ Where Unit for Density $_{D.M.}$ is $m M_{\Theta}/pc^{3}$ and Unit for E is m/s^{2}.

In chapter 12 it is calculated Local DM density. Recent measures of Sun are $R_{\Theta} = 8,3\, Kpc$ $V_{\Theta} = 244\, Km/s$

With such values, $E_{\Theta} = 7173\, Km^{2}/s^{2}/ Kpc$ and Density $_{DM\text{-}SUN} = 6,45 \cdot 10^{-3} M_{\Theta}/pc^{3} = 6,45\, m M_{\Theta}/pc^{3} = 0,245$ GeV/cm^{3} . This value agree with direct measures of Local DM density.

In chapter 14, DM density potential function has been checked with NFW profile in Milky Way and it has been found that agreement between both curves are mainly under 10% inside the whole dominion up to 375 Kpc which is an excellent agreement in astrophysics¡.

In chapter 15, DM density potential function has been checked with NFW profile in M31.Although the agreement between both functions is not so good as in Milky Way case, relative error is enough small to accept that M31 supports DM density potential function.

In chapter 16, density potential function is written depending on radius x, and it is clearly showed that this function is equivalent to NFW, relative error inside the whole dominion are enough small that they could be considered come from error in astronomical measures.

Finally in chapter 17 has been got a Bernoulli differential equation for gravitational field which allows to calculate potential density function depending on radius. In addition potential function has been compared with NFW density function through halo region and it has been found that relative differences are below 13% in Milky Way and below 16% in M31 galaxy. This is a superb result as can understand any astrophysicist ¡

I think that it has been justified properly that results could confirm DM density as universal law, especially if it is considered experimental error in rotation curves and error in calculus of DM density functions.

Unfortunately this law is wrong for intermediate galaxies, whose flat region in rotation curve is under 150 Km/s. Specifically have been studied NGC 3198 (Ursa Major), NGC 2403 (M81 group of galaxies) and M33 (Local group). In these galaxies density of dark matter is more than four times bigger that density of dark matter inside big galaxies although this result has not been published in this paper.

However, I think that these results can not discard this model because DM density function inside an intermediate galaxy may be influenced by gravitational field of a nearby giant galaxy. In other words, gravitational field of a giant neighbour galaxy might generate DM inside an intermediate galaxy placed near a big galaxy.

In addition, a nearby giant galaxy may break significantly the spherical symmetry of gravitational field inside an intermediate galaxy and therefore calculus made through Virial Theorem become wrong.

The author think that it may be worth to study a bigger number of galaxies in order to check or reject results got in this paper because Dark matter nature is one of the most important challenges for astrophysics science nowadays.

2. INTRODUCTION

Method followed to calculate E(P), total gravitational field at point P, has been through   Virial theorem. It is a well known method but the problem is that   galaxies do not have perfect spherical symmetry and their stars do not have perfect gravitational equilibrium, so this method give a approximate measure of E and his error it will depend on degree of spherical symmetry and gravitational equilibrium of stars inside each galaxy.

Inside a galaxy with spherical symmetry, E(P) at a point P at R distance from galactic centre depend on spin speed of stars at R distance from galactic centre.
According Virial theorem $V^2 = GM(<R)/R$ where M(<R) is total mass inside a sphere with radius R, supposing a mass distribution with spherical symmetry.
As $E = GM(<R)/R^2$ it is right conclude that $E = V^2/R$.
 As measures of rotation curve are in Km/s and radii are in Kpc, unit of E in this work has been $Km^2/s^2/Kpc$ which is a bit weird as unit of E but it allows to simplify calculus.

In relation to Dark matter calculus, astrophysical researchers measure baryonic mass inside a sphere R, $M_{BARIONIC}(R)$. After they calculate density Dark Matter function which is necessary to reach total mass M(R) to get the speed given by $V^2 = GM(<R)/R$.

As it is known there are several model of Dark matter density. For Milky Way and M31, author has chosen NFW profile $D_{NFW}(R) = \dfrac{D_0}{x \cdot (1+x)^2}$ where x= R/h. The other four galaxies authors have chosen pseudo-isothermal profile

$D_{ISO}(R) = \dfrac{D_0}{1+x^2}$ where x= R/R_C.

Authors of rotation curves of galaxies and his data parameter of Dark matter density function will be quoted specifically for each galaxy.

Remember that for the whole paper, his graphs and tables, unit for E is $Km^2/s^2/Kpc$ and unit for Density of dark matter is $10^{-3} Msolar/pc^3 = m\ Msolar/pc^3$.

3. GALACTIC SAMPLE
After a meticulous research in arXiv, it has been selected six big galaxies from three different papers. Two galaxies have data from two different papers, because authors give different dark matter parameters for the same galaxy. Unfortunately, I have not found any more papers about rotation curves of big galaxies, because it is a hard work measuring rotation curves. It is a great human and technological achievement to be able to measure rotation curves of galaxies which are lot of Mpc away¡.

Galactic sample are the following ones:

NGC 2841
It is a spiral galaxy in the Ursa Major galaxy cluster. Its distance is 14 Mpc. Aprox. Calculus for this galaxy has been made two times according data provided by two different papers: [2] Randriamampandry & T. Carignan. and [3] Bottema,R.B. & Pestaña,J.LG.

NGC 7331
It is a spiral galaxy about 12 Mpc away in the Constelation Pegasus. This galaxy is similar in size and structure to Milky Way. Calculus for this galaxy has been made two times according data provided by two different papers: [2] Randriamampandry & T. Carignan. and [3] Bottema,R.B. & Pestaña,J.LG.

NGC 3992 It is one of the most prominent members of the Ursa Major cluster of galaxies. Its approximate distance is 18 Mpc. Calculus has been made according data from [3] Bottema,R.B. & Pestaña,J.LG.

NGC 3031
This galaxy also is known as M81 or Bode´s galaxy. It is the main galaxy in M81 group of galaxies. It is 3 Mpc away. Calculus has been made according data from [2] Randriamampandry & T. Carignan.

Milky Way
Thousand of stars are possible to see with naked eye. However measures toward galactic centre are difficult because of gas and cosmic dust. Calculus has been made according data from [1] Sofue,Y.

M31 or Andromeda Galaxy
It is the nearest major spiral galaxy to the Milky Way. M31 is 780 Kpc away to our Galaxy. it is the largest galaxy of the Local Group, which also contains the Milky Way, the Triangulum Galaxy (M33), and about 44 other smaller galaxies. Calculus has been made according data from [1] Sofue,Y.

Apart from these six galaxies, I have studied NGC 2998 and NGC 2903 as well. However, I have not considered NGC 2998 because it is 67 Mpc away, and it has an anomalous high density of D.M. I think that a distance so huge may originate lot of errors in measures of rotation curve.

In addition NGC 2903 has been rejected of the galactic sample because it has an outstanding high density of dark matter by unknown reasons. This galaxy has a anomalous behaviour to explain its rotation curve by MOND theory as well.

Milky Way, M31, NGC 3031, NGC 3992, NGC 7331, NGC 2841   are big spiral galaxies. Their flat region rotation curve have speeds from 200 km/s up to 300 Km/s.

Considering their distances:
NGC 2841, NGC 3992, belong to Ursa Major Cluster which is 18 Mpc away.
NGC 7331 is 12 Mpc away in constellation Pegasus.
NGC 3031 is 3 Mpc away and belong to M81 group of galaxies.
M31 is only 770 Kpc away and Milky Way no need presentation.
Therefore it has been covered a wide interval of distances. In addition, galaxies have been chosen from four different galaxy clusters which do not have gravitational interaction each other.

4.GALACTIC CALCULUS

Each galaxy is identified with his name, his cluster and his approximate distance. In addition it is quoted the author

who has measured the rotation curve and has calculate density of dark matter through NFW or pseudo-isothermal profiles.

Through the rotation curve is tabulated the spin speed for a wide number of radii and E, intensity of gravitational field according the formula $E = v^2/R$

In the second column, it is tabulated $X=R/R_c$ where R_c is a parameter of dark matter density of pseudo-isothermal profile. Similarly $X= R/h$ where h is a parameter of dark matter density of NFW profile. Finally, in the last column it

is tabulated dark matter density using   a   pseudo-isothermal profile $D_{ISO}(R) = \dfrac{D_0}{1+x^2}$ or   NFW profile

$D_{NFW}(R) = \dfrac{D_0}{x \cdot (1+x)^2}$ In the graphic is plotted density of dark matter depending E, through table data. The same

process has been followed for the six galaxies.

Graph of rotation curve which come from paper quoted show several curves. Reader may consult original paper to know directly meaning each curve. In general all graphs have similar curves. Briefly is explained bellow.

Dots are experimental data and they have a fitted function. The curves with relative maximum correspond to field generated by baryonic mass from bulge and from disk. The curve which is always increasing correspond to dark matter. It is important to emphasise that Dark matter contribution is not important for distances below 10 Kpc. I mean that calculus of Dark matter density function might have too much error for distances below 10 Kpc. This is the reason why it will be considered gravitational field E below 6000 to do statistical analysis in this paper.

4.1 NGC 2841   Bottema, R.B.

1 Name	Cluster	Distance	Author
NGC 2841	Ursa Major	14 Mpc	[3] Bottema,R.B.

Dark matter density function profile   Isothermal
Rc = 6,39 $\pm$ 0,52   Kpc
Do = 32 $\pm$ 5 10^{-3} Msolar/pc^3

Radii	X	Speed	E gravt.	Density
60	9,38967136	280	1306,66667	0,35888148
50	7,82472613	280	1568	0,51425169
45	7,04225352	280	1742,22222	0,63249438
40	6,25978091	275	1890,625	0,79631987
35	5,47730829	285	2320,71429	1,03222789
30	4,69483568	290	2803,33333	1,38879955
25	3,91236307	300	3600	1,96239743
20	3,12989045	300	4500	2,96400194
15	2,34741784	315	6615	4,91523484
10	1,56494523	325	10562,5	9,27790752
6	0,93896714	310	16016,6667	17,0062669

6

NGC 2841 E grav- Detail
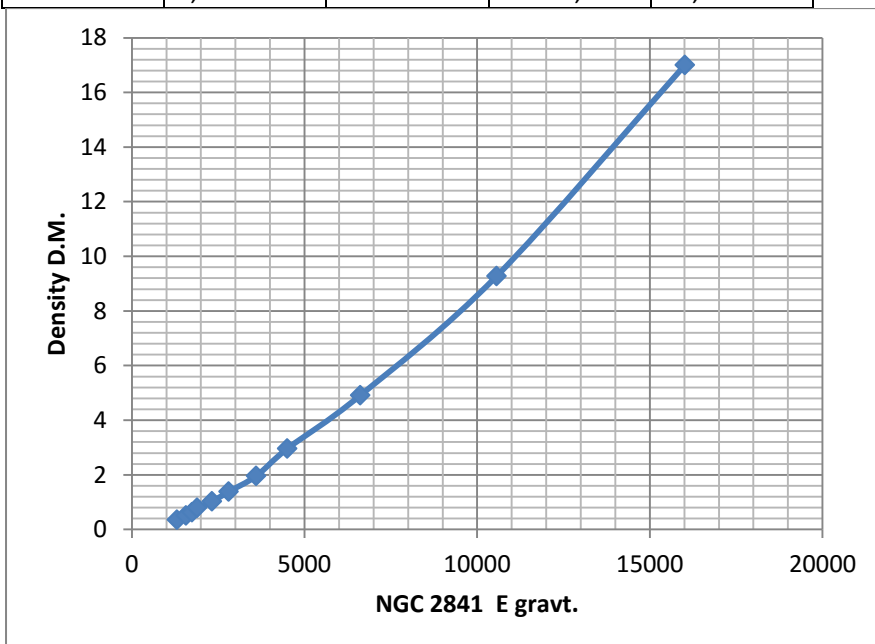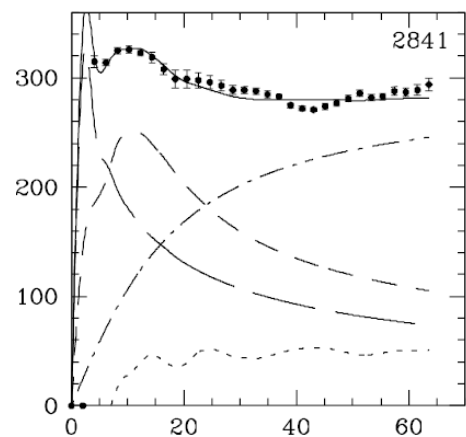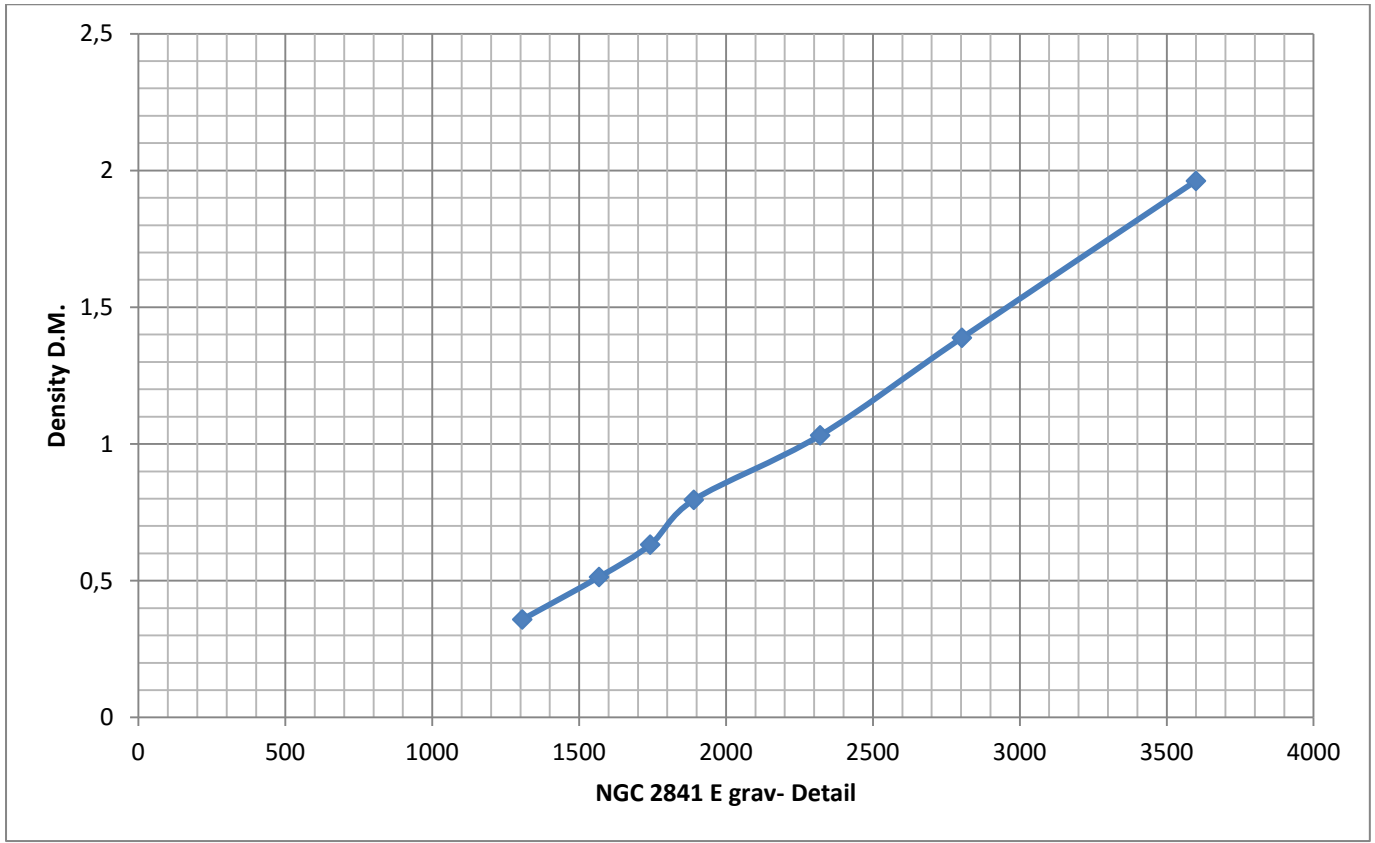
4.2 NGC 2841 Randriamampandry, T.

2 Name	Cluster	Distance	Author
NGC 2841	Ursa Major	14 Mpc	[2] Randriamampandry, T.

Dark matter density function profile   Isothermal
Rc = 5,08 $\pm$ 0,23   Kpc
Do = 49,06 $\pm$ 3,61$\cdot$ 10^{-3} Msolar/pc^{3}
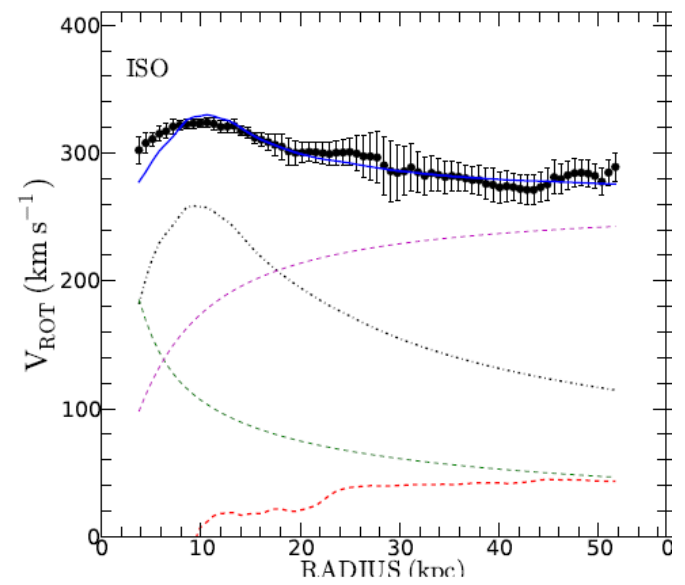
Radii	X	Speed	E gravt.	Density
6	1,18110236	315	16537,5	20,4843185
8	1,57480315	320	12800	14,0976811
10	1,96850394	325	10562,5	10,0635737
12	2,36220472	320	8533,33333	7,45591441
14	2,75590551	315	7087,5	5,70795966
16	3,1496063	310	6006,25	4,49266583
18	3,54330709	305	5168,05556	3,61932196
20	3,93700787	300	4500	2,97332775
22	4,33070866	300	4090,90909	2,4834172
24	4,72440945	300	3750	2,10376956
26	5,11811024	300	3461,53846	1,80400461
28	5,51181102	295	3108,03571	1,56341316
30	5,90551181	290	2803,33333	1,36752347
32	6,2992126	285	2538,28125	1,20599568
34	6,69291339	283	2355,55882	1,0712939
36	7,08661417	280	2177,77778	0,95782709
38	7,48031496	278	2033,78947	0,8613801
40	7,87401575	274	1876,9	0,77872863
42	8,26771654	272	1761,52381	0,7073737
44	8,66141732	280	1781,81818	0,64535521
46	9,05511811	280	1704,34783	0,59111878
48	9,4488189	290	1752,08333	0,54341939
50	9,84251969	280	1568	0,5012506
52	10,2362205	290	1617,30769	0,46379186
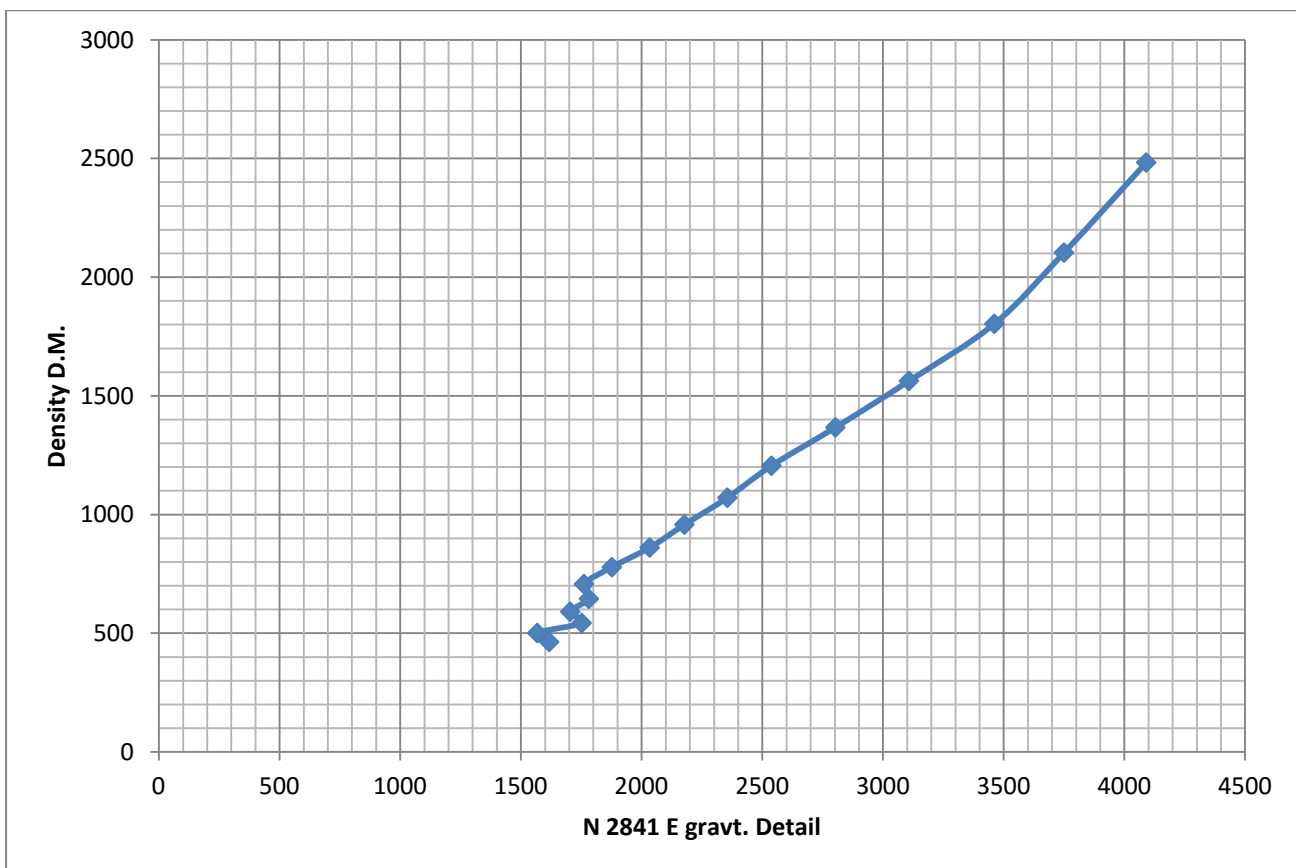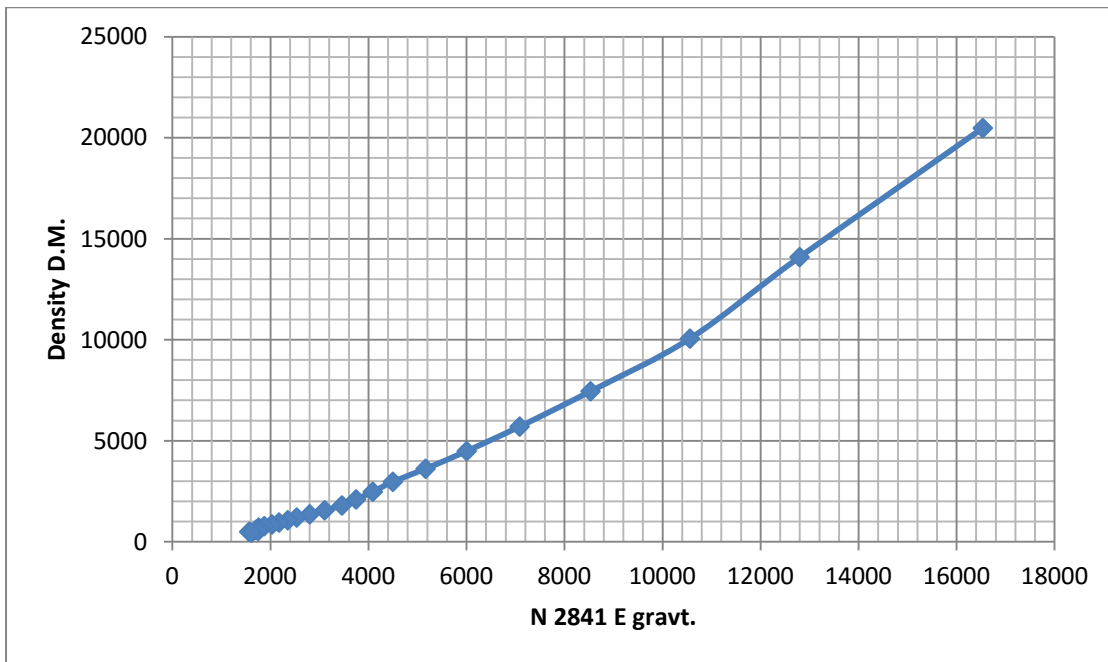
4.3 NGC 7331 Bottema, R.B.

3 Name	Constelation	Distance	Author
NGC 7331	Pegasus	12 Mpc	[3] Bottema,R.B.

Dark matter density function profile   Isothermal
Rc = 19,46 $\pm$ 1,79 Kpc
Do = 3,5 $\pm$ 0,7 $\cdot$ 10^{-3} Msolar/pc^3
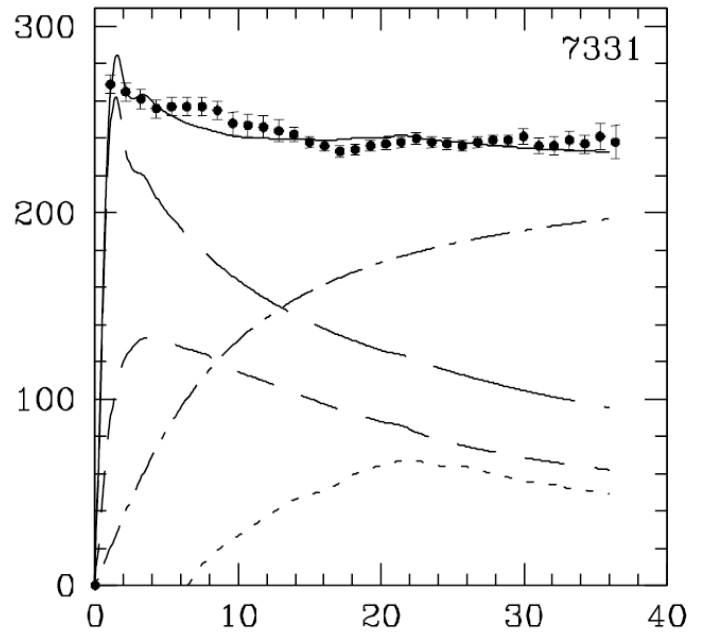
Radii	X	Spin speed	E gravt.	Density
4	0,20554985	260	16900	3,35811707
6	0,30832477	260	11266,6667	3,19615975
8	0,41109969	260	8450	2,9940044
10	0,51387461	250	6250	2,76884031
12	0,61664954	246	5043	2,53576028
14	0,71942446	244	4252,57143	2,30631629
16	0,82219938	238	3540,25	2,08829075
18	0,92497431	235	3068,05556	1,88620527
20	1,02774923	238	2832,2	1,70211236
22	1,13052415	240	2618,18182	1,53637824
24	1,23329908	236	2320,66667	1,38832331
26	1,336074	235	2124,03846	1,2566902
28	1,43884892	240	2057,14286	1,13995887
30	1,54162384	240	1920	1,03654439
32	1,64439877	235	1725,78125	0,94491234
34	1,74717369	238	1666	0,8636397
36	1,84994861	239	1586,69444	0,7914416
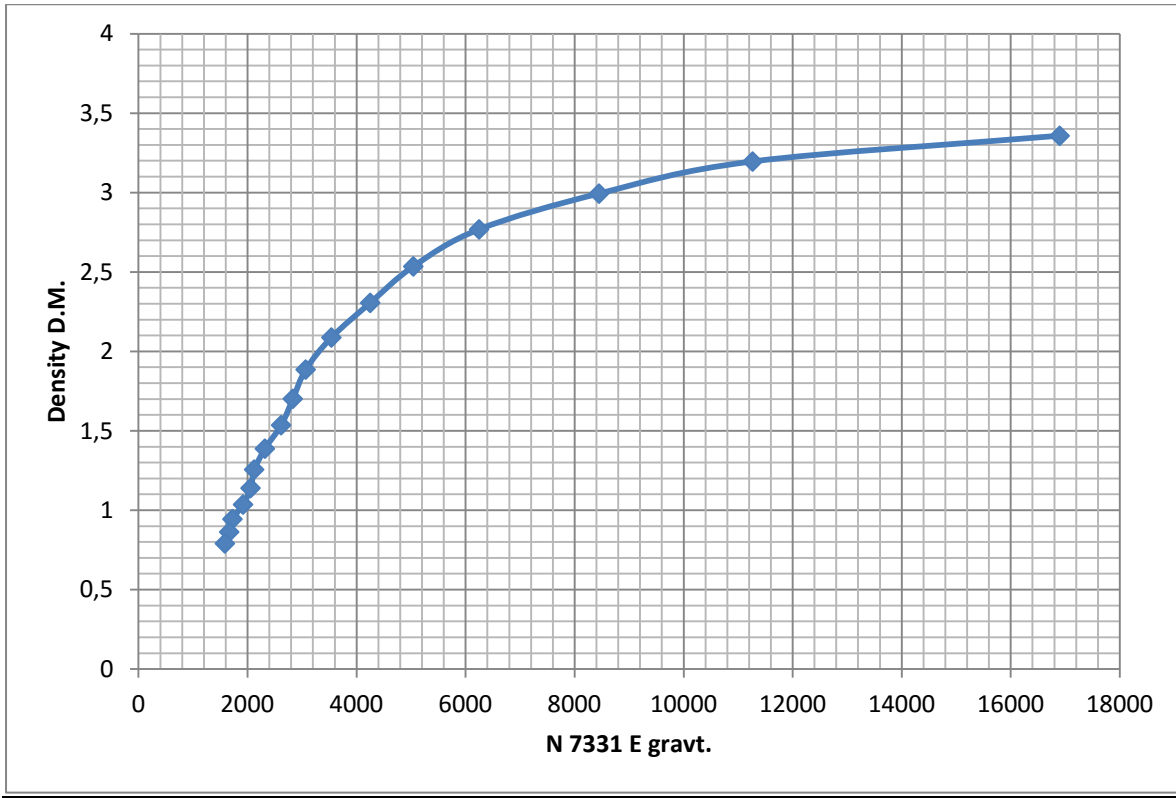
In the whole galactic sample, NGC 7331 is the only one which has a convex curve as it is shown above.
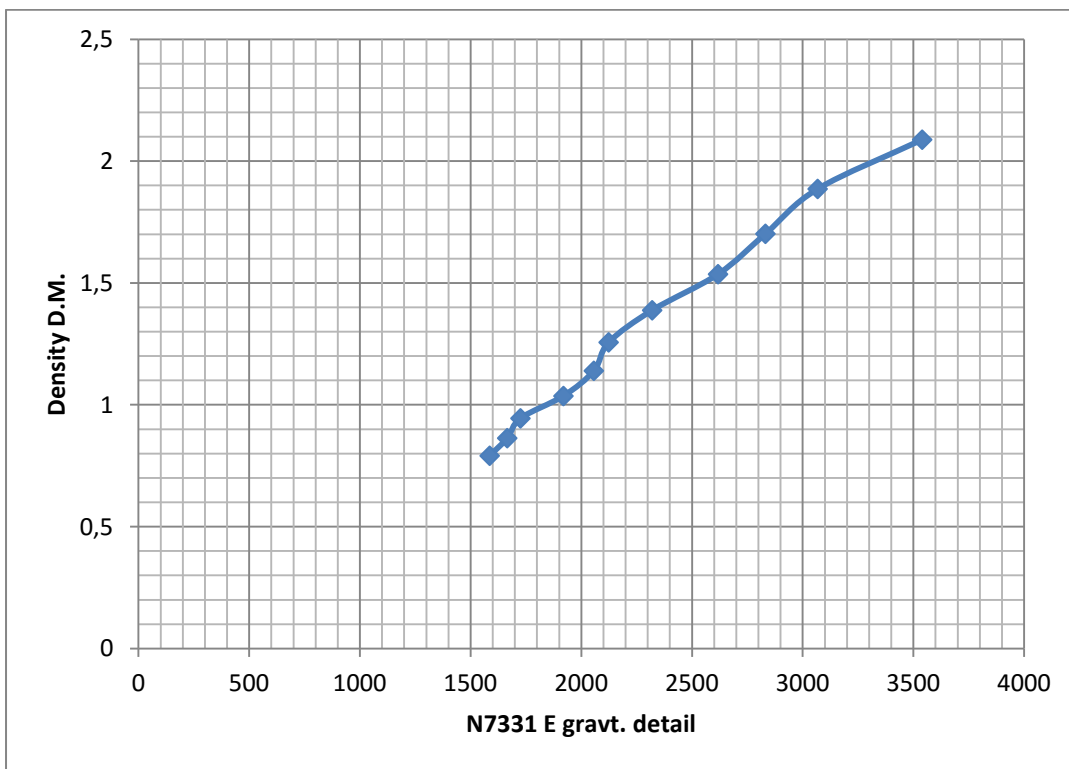
4.4 NGC 7331 Randriamampandry, T.

4 Name	Constelation	Distance	Author
NGC 7331	Pegasus	12 Mpc	[2] Randriamampandry, T.

Dark matter density function profile   Isotermal
Rc = 17,38 ± 2,75   Kpc
Do = 4,75 ± 0,6 10^{-3} Msolar/pc^3

Radii	X	Speed	E gravt.	Density
4	0,2301496	250	15625	4,51105468
6	0,3452244	268	11970,6667	4,24417922
8	0,46029919	262	8580,5	3,91954503
10	0,57537399	250	6250	3,56859722
12	0,69044879	250	5208,33333	3,21658913
14	0,80552359	245	4287,5	2,88076381
16	0,92059839	240	3600	2,57104001
18	1,03567319	240	3200	2,29178644
20	1,15074799	240	2880	2,04369556
22	1,26582278	242	2662	1,82530324
24	1,38089758	245	2501,04167	1,63405543

NGC 7331 Egravt. Detail
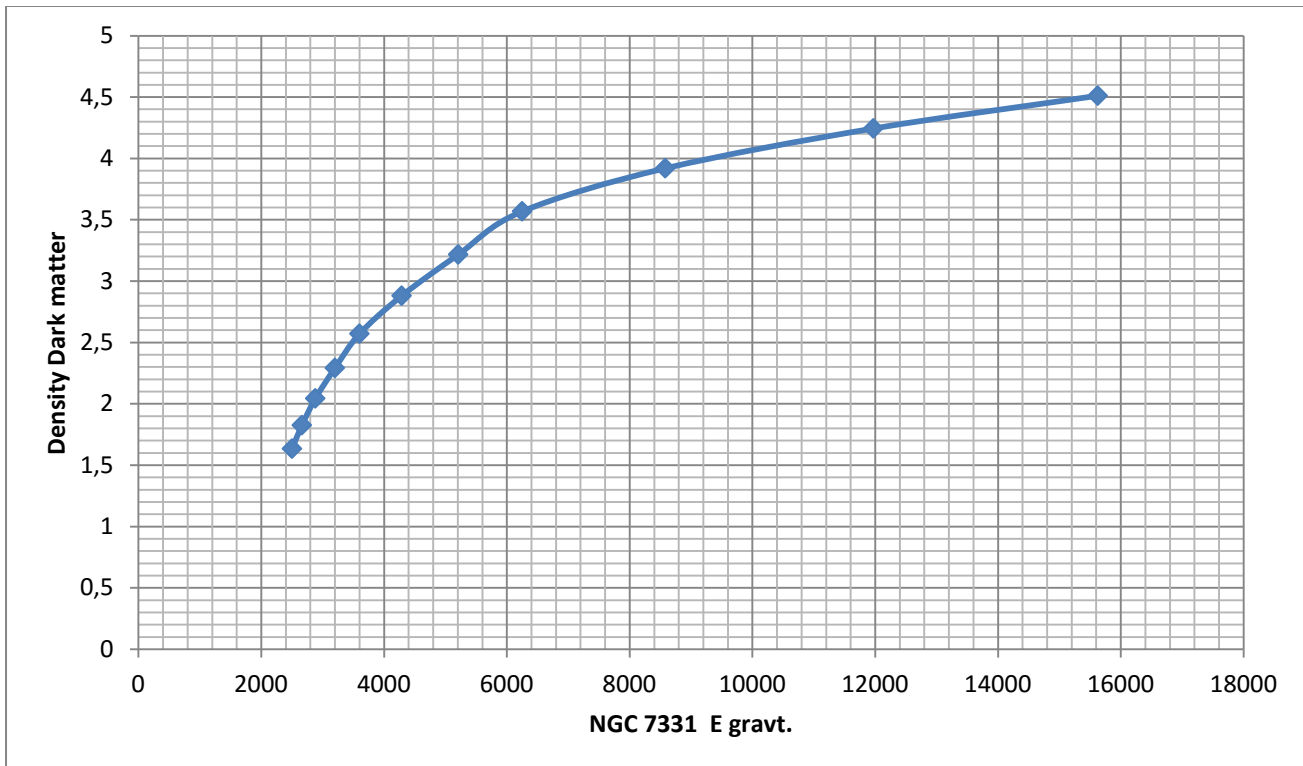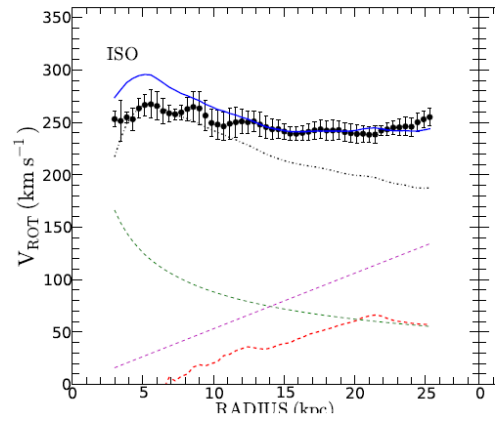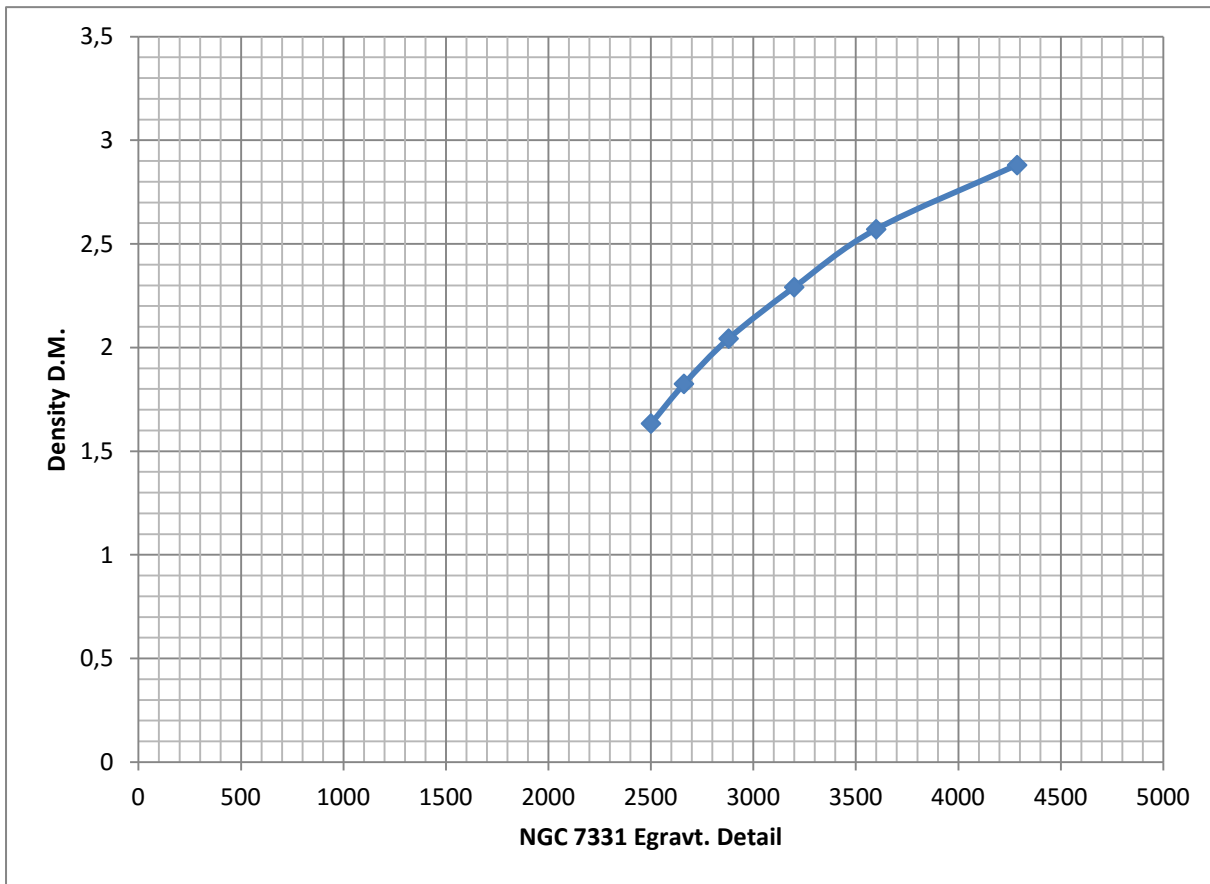
4.5 NGC 3992 [3] Bottema,R.B.

5 Name	Cluster	Distance	Author
NGC 3992	Ursa Major	18,6 Mpc	[3] Bottema,R.B.

Dark matter density function profile   Isothermal
Rc = 3,89 ± 0,34   Kpc
Do = 66 ± 11 $\cdot 10^{-3}$ Msolar/pc^3

Dark Matter Density Function Depending on Gravitational Field as Universal Law - M. Abarca
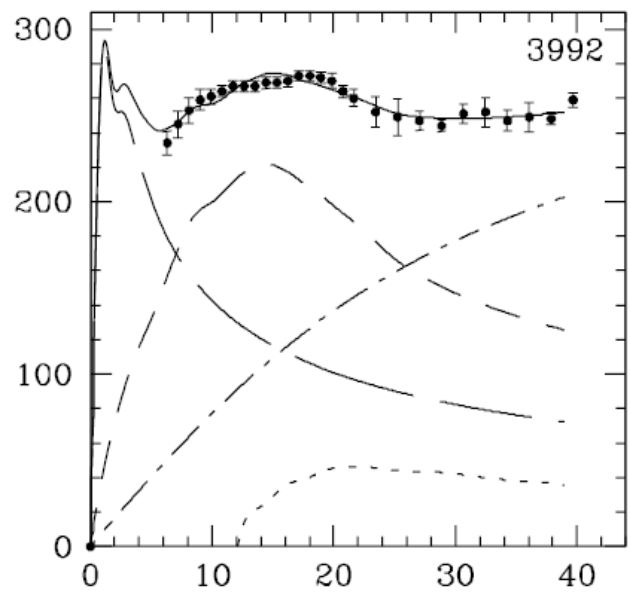
Radii	X	Speed	E gravt.	Density
38	9,76863753	250	1644,73684	0,68446071
36	9,25449871	250	1736,11111	0,76172233
34	8,7403599	250	1838,23529	0,85278048
32	8,22622108	250	1953,125	0,96110841
30	7,71208226	247	2033,63333	1,09133818
28	7,19794344	245	2143,75	1,24975408
26	6,68380463	250	2403,84615	1,44504734
24	6,16966581	250	2604,16667	1,68950155
22	5,65552699	260	3072,72727	2,00091038
20	5,14138817	265	3511,25	2,40578505
18	4,62724936	268	3990,22222	2,944925
16	4,11311054	272	4624	3,68351294
14	3,59897172	270	5207,14286	4,73030202
12	3,0848329	265	5852,08333	6,27603482
10	2,57069409	260	6760	8,67454515
8	2,05655527	250	7812,5	12,6209035
6	1,54241645	235	9204,16667	19,5321256
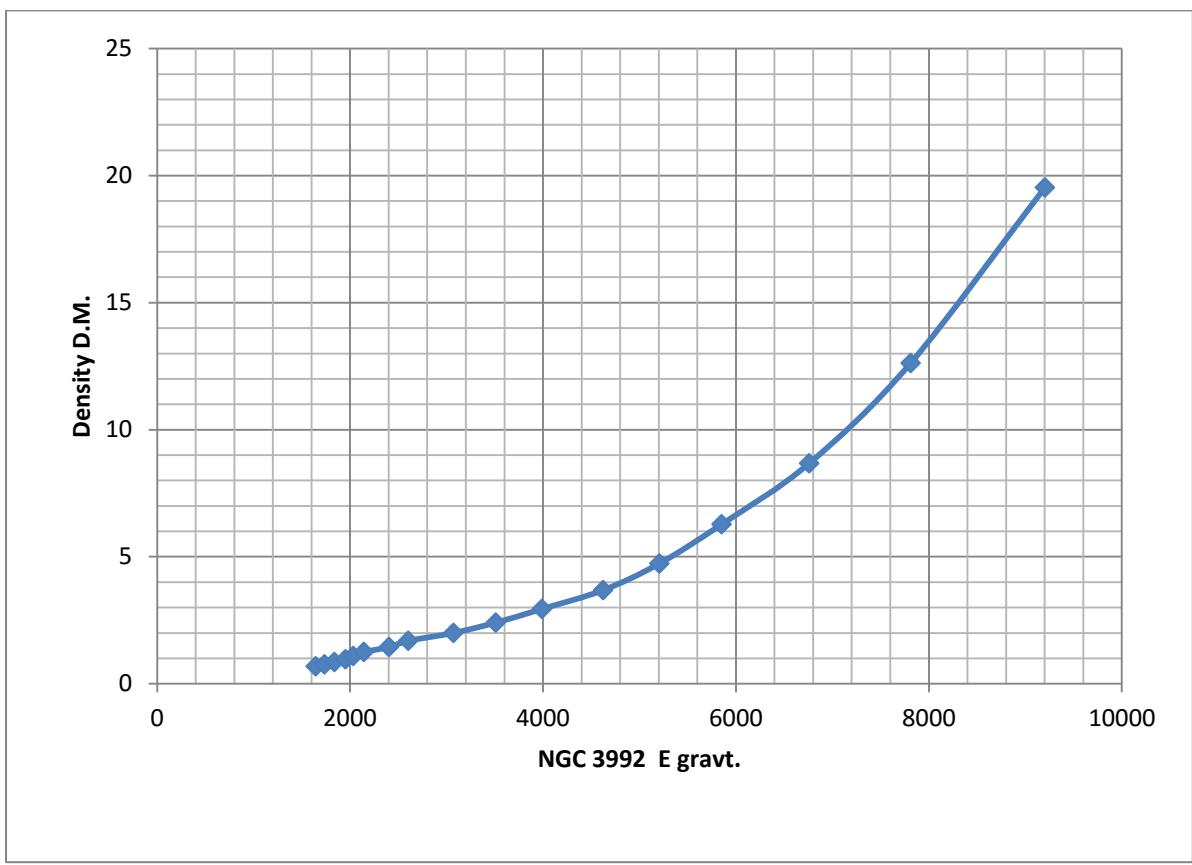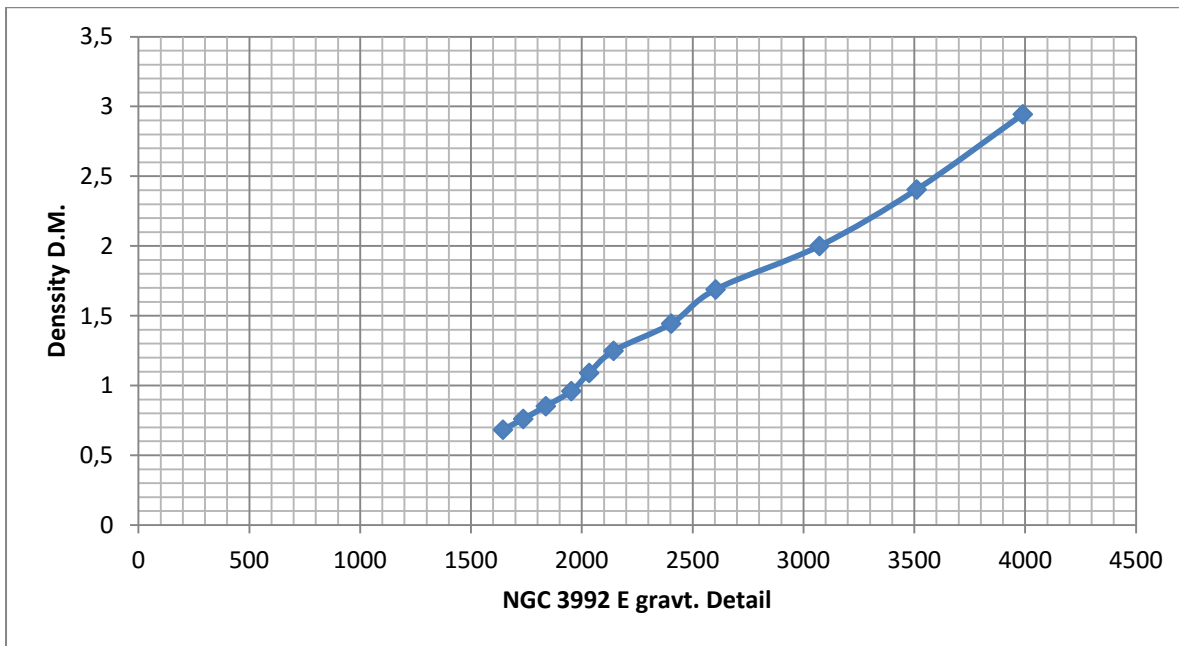
As it will be shown in paragraph 4. Dark matter density has an anomalous increasing behaviour for values bigger than E=9000 by comparison with another galaxies.

4.6 NGC 3031 Randriamampandry, T.

6 Name	Group / Constelation	Distance	Author
NGC 3031 or Bode	Group M81	2,5 Mpc	[2] Randriamampandry, T.

Dark matter density function profile   Isothermal
Rc = 5,34 $\pm$ 1,97   Kpc
Do = 14, 55 $\pm$ 5,87 10^{-3} Msolar/pc^3
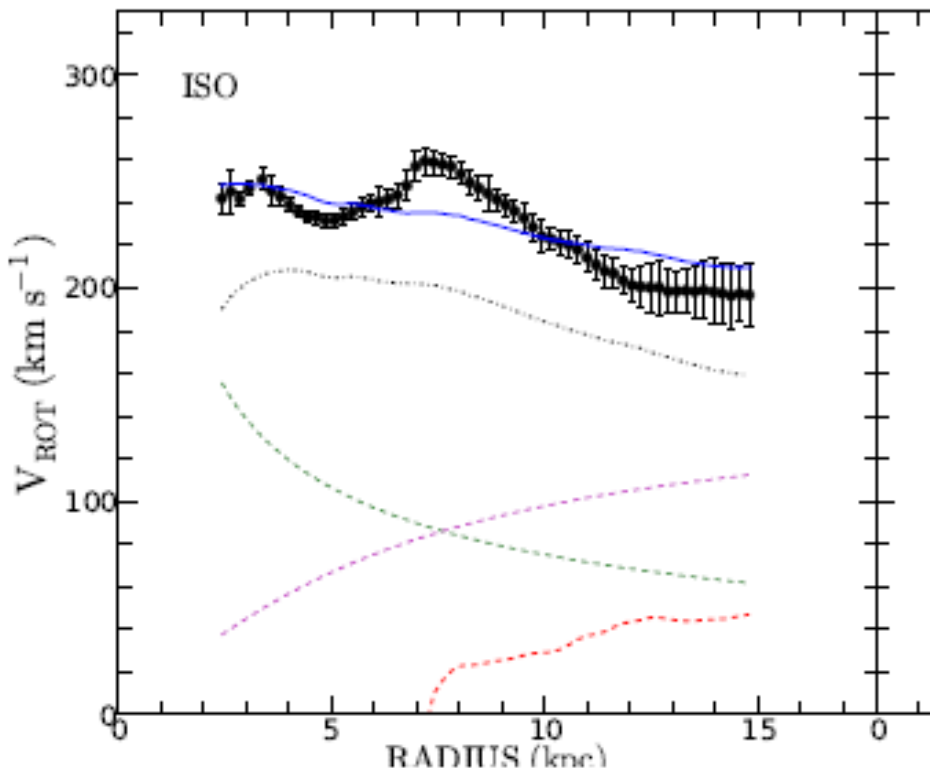
Radii	X	Speed	E gravt.	Density
15	2,80898876	200	2666,66667	1,63096946
14	2,62172285	200	2857,14286	1,84163684
13	2,43445693	200	3076,92308	2,09338503
12	2,24719101	200	3333,33333	2,39674673
11	2,05992509	220	4400	2,76543852
10	1,87265918	225	5062,5	3,21732303
9	1,68539326	240	6400	3,77550048
8	1,49812734	260	8450	4,46925924
7	1,31086142	250	8928,57143	5,33410307
6	1,12359551	240	9600	6,40893365
5	0,93632959	230	10580	7,72627421
4	0,74906367	240	14400	9,28834386
3	0,56179775	240	19200	11,0214471

NGC 3031 E gravt.

NGC3031 E gravt. Detail
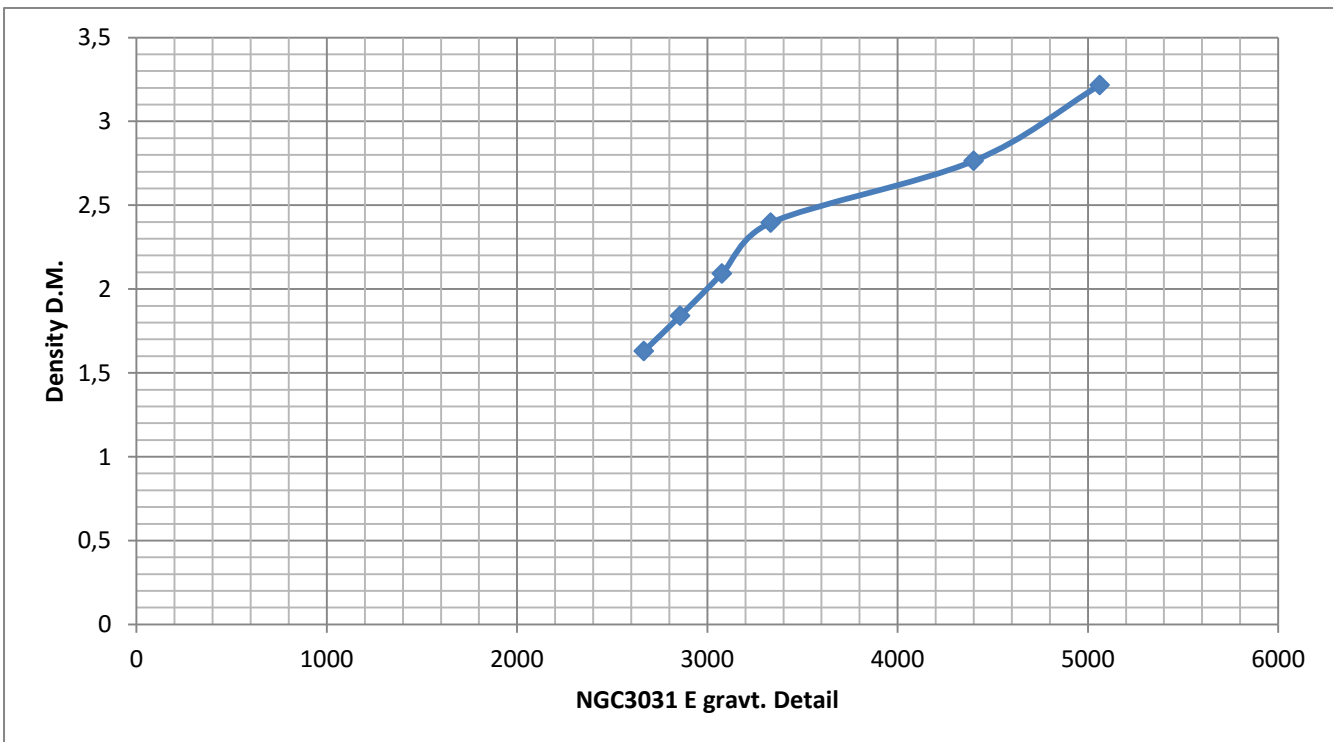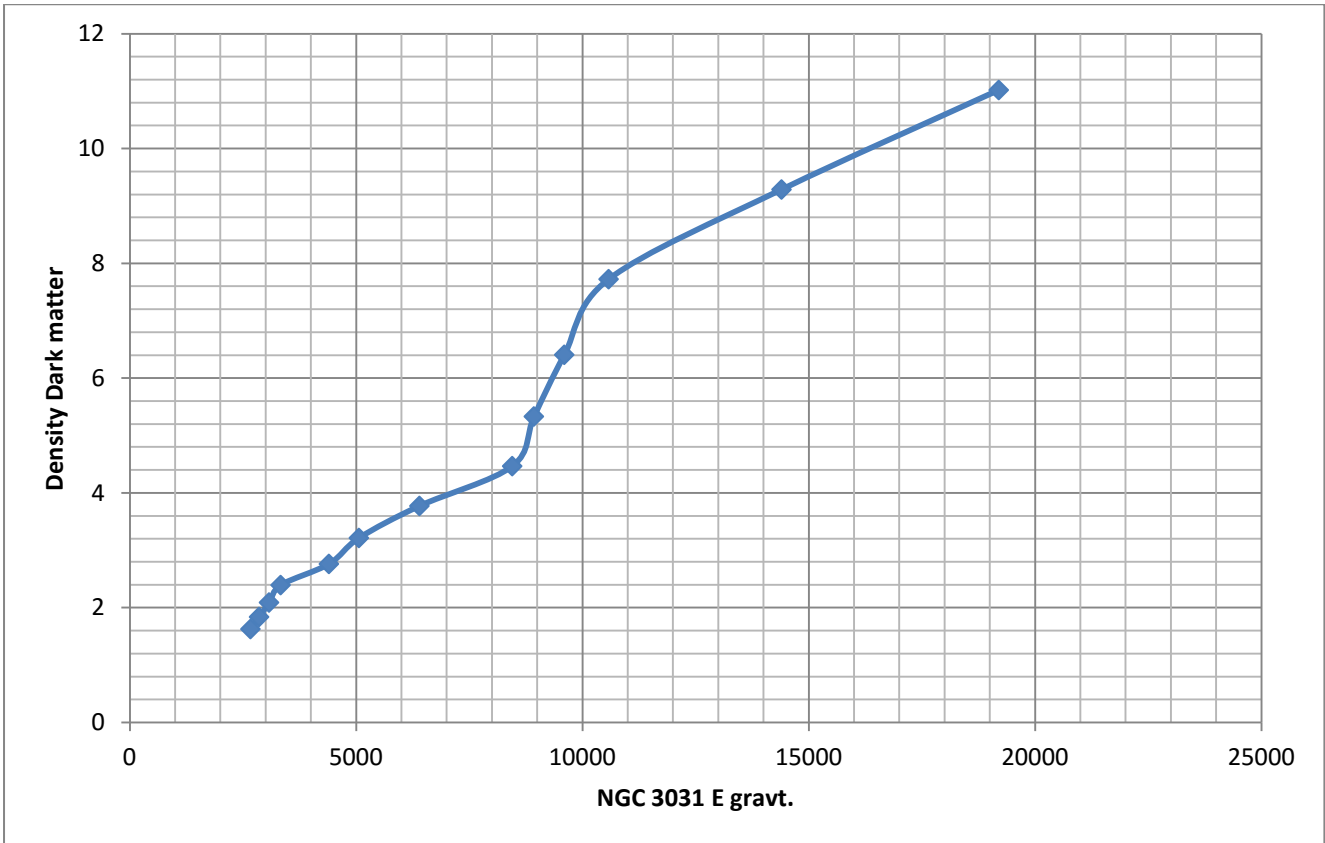
4.7 MILKY WAY [1] Sofue,Y.

7 Name	Group of galaxies	Stellar Radius	Author
Milky Way	Local group	15 Kpc	[1] Sofue,Y. 2015.

Dark matter density function profile NFW
h = 10,7 $\pm$ 2,9 Kpc
Do = 18,2 $\pm$ 7,4 10^{-3} Msolar/pc^3 **Error 40% ¡¡¡**

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2} \text{ where } x = R/h$$

As it will be show in epigraph 5. Density of dark matter in Milky Way is outstandingly high. Considering that Do = 18,2 $\pm$ 7,4 10^{-3} Msolar/pc^3 is a parameter with error = 40% , perhaps this error so high might explain its atypical D.M. density calculated.

Dark Matter Density Function Depending on Gravitational Field as Universal Law - M. Abarca
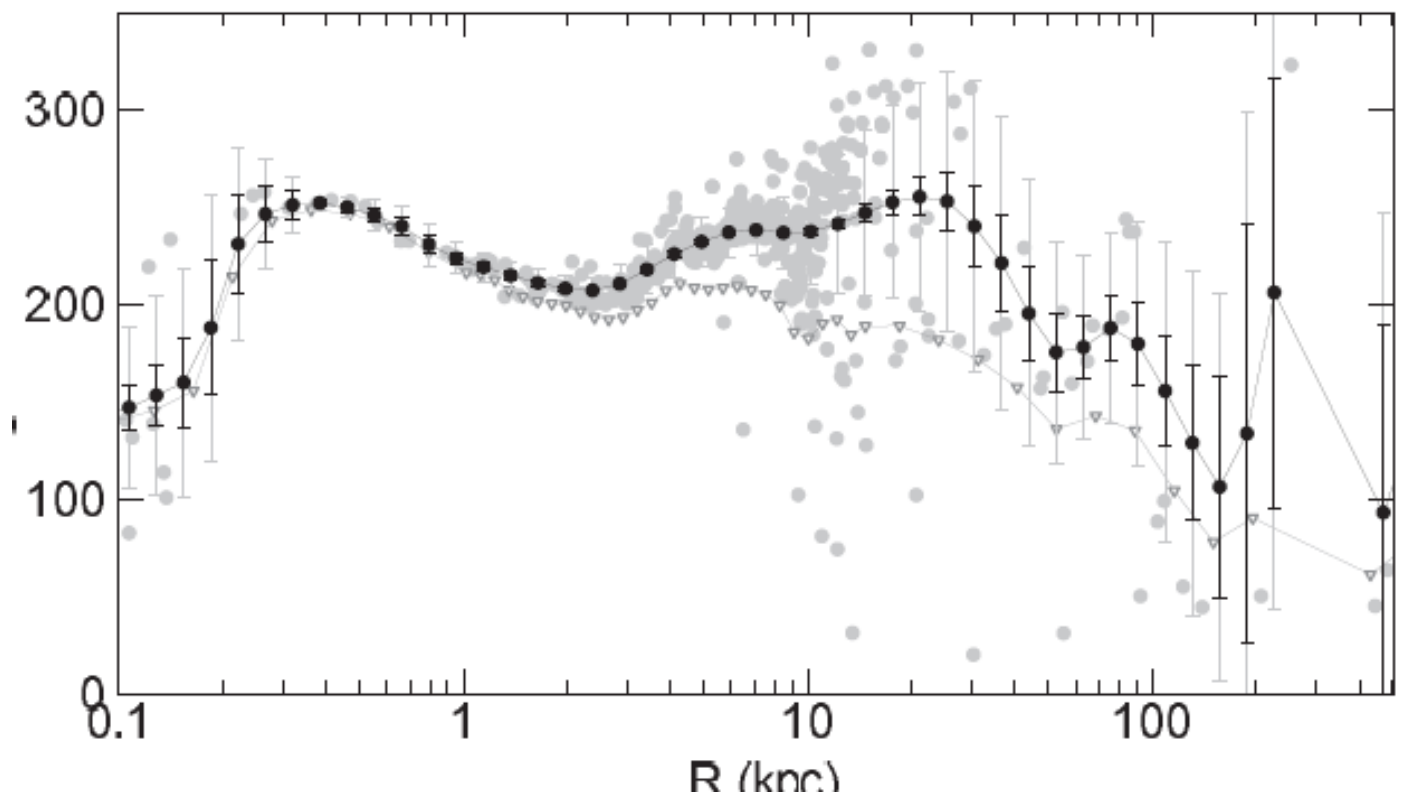
Radii	X	Speed	E gravt.	Density
3	0,28037383	210	14700	39,5968221
4	0,37383178	225	12656,25	25,7945562
5	0,46728972	240	11520	18,0906184
6	0,56074766	240	9600	13,3241198
7	0,65420561	240	8228,57143	10,1666565
8	0,74766355	240	7200	7,9698385
9	0,8411215	240	6400	6,3833368
10	0,93457944	240	5760	5,20333791
20	1,86915888	255	3251,25	1,18281269
30	2,80373832	240	1920	0,44865514
40	3,73831776	220	1210	0,2168437
50	4,6728972	180	648	0,12102507
60	5,60747664	180	540	0,07434173
70	6,54205607	190	515,714286	0,04890774
80	7,47663551	190	451,25	0,03387803
90	8,41121495	180	360	0,02442988
100	9,34579439	160	256	0,01819397
200	18,6915888	140	98	0,0025111
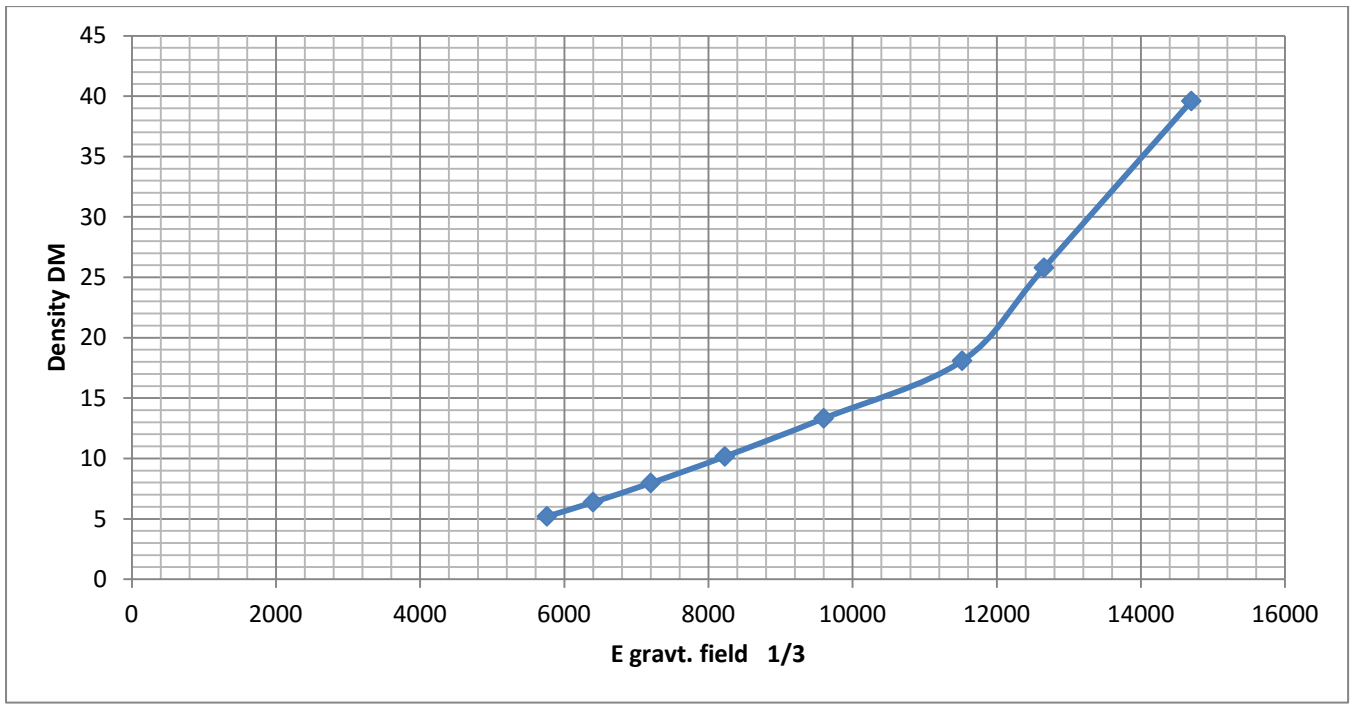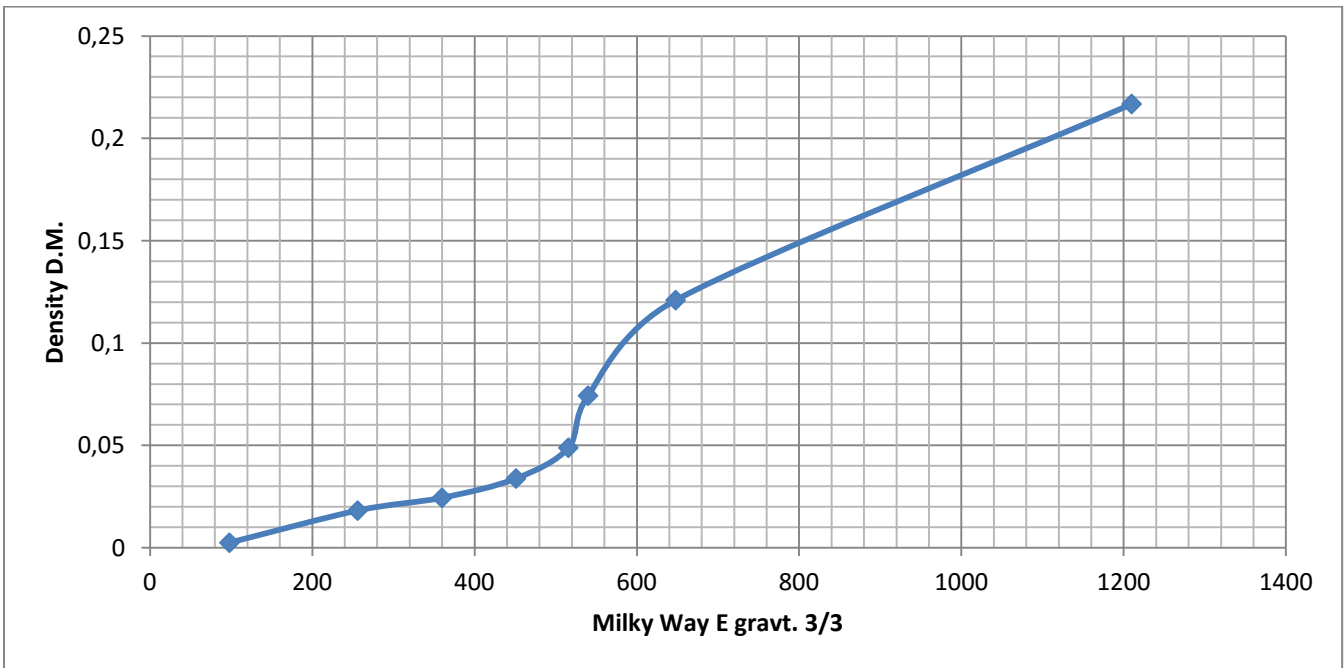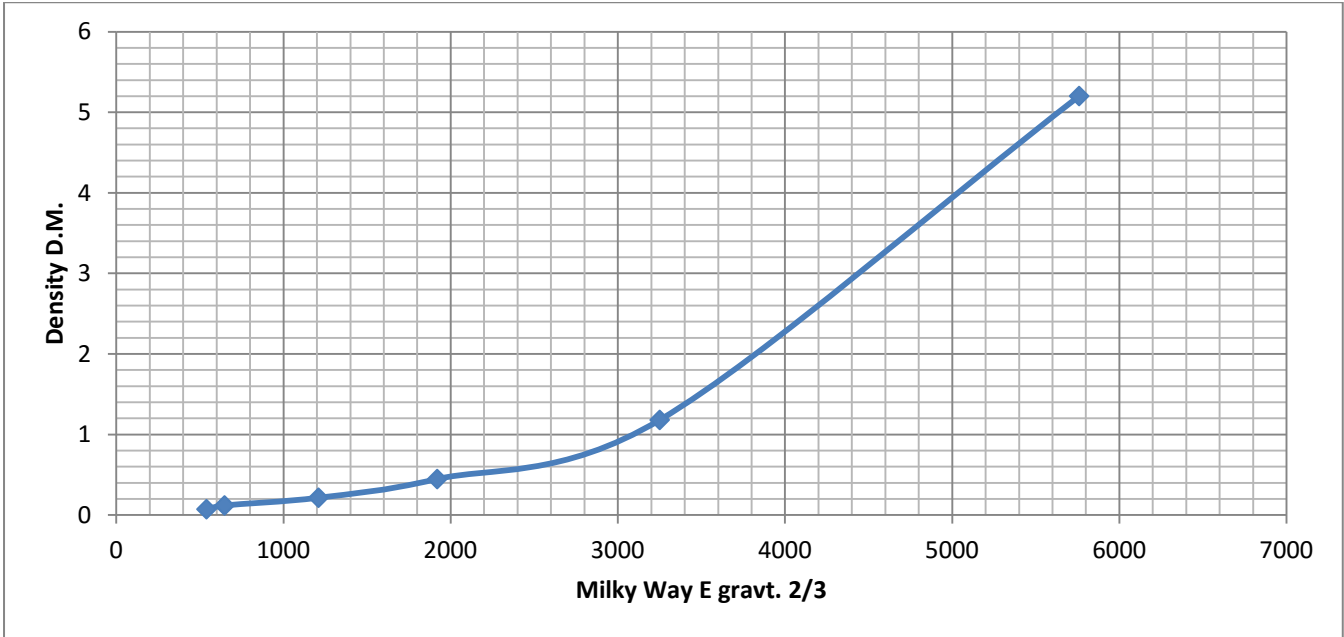
4.8 M31-ANDROMEDA [1] Sofue,Y.

8 Name	Group of galaxies	Distance	Author
M31	Local group	770 Kpc	[1] Sofue,Y. 2015.

Dark matter density function profile   NFW
h = 34,6 $\pm$ 2,1 Kpc
Do = 2,23 $\pm$ 0,24 10^{-3} Msolar/pc^3
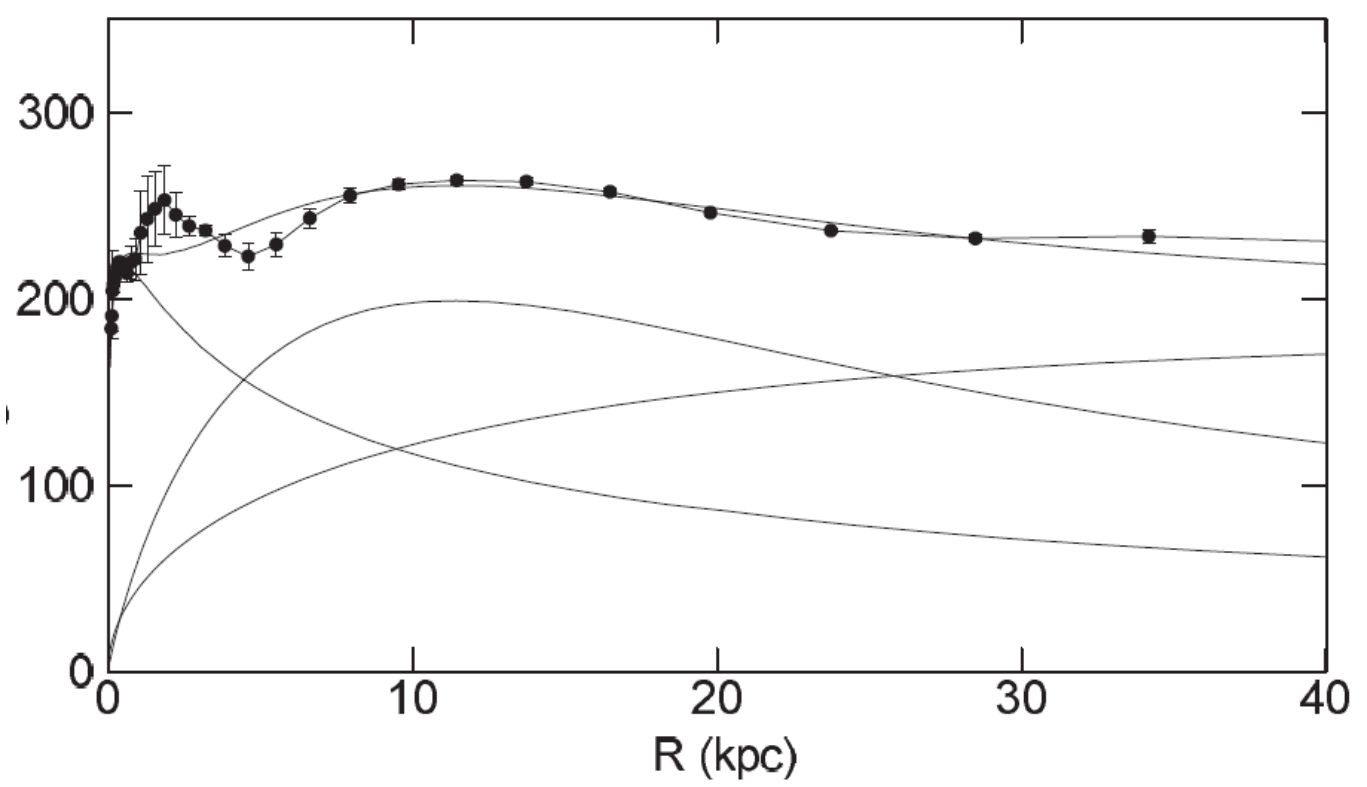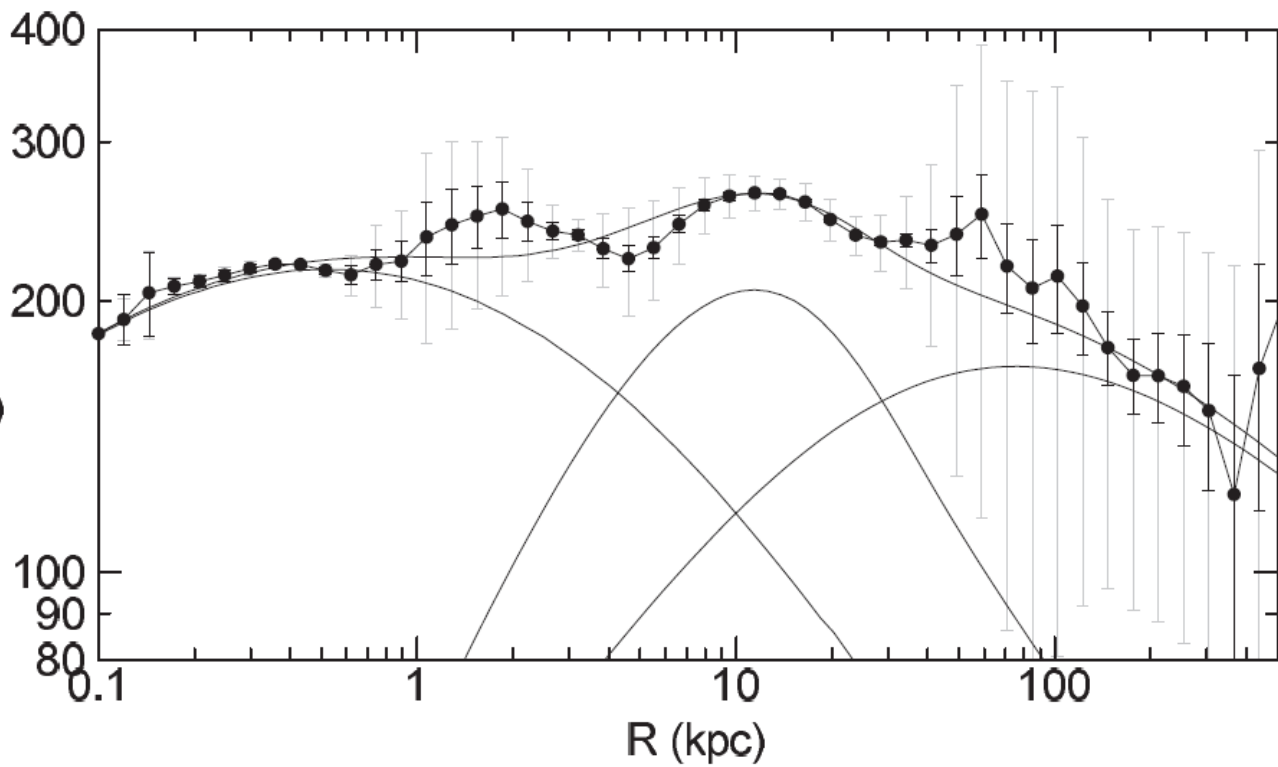
Rotation curve up 400 Kpc. Logarithmic scale for distances and speed.

Radii	X	Speed	E gravt.	Density
3	0,0867052	220	16133,3333	21,7789137
4	0,11560694	225	12656,25	15,4988173
5	0,14450867	225	10125	11,7807458
7,5	0,21676301	255	8670	6,94876628
10	0,28901734	260	6760	4,64369238
12,5	0,36127168	260	5408	3,33105138
15	0,43352601	260	4506,66667	2,503102
17,5	0,50578035	255	3715,71429	1,94455246
20	0,57803468	250	3125	1,54923706
22,5	0,65028902	242	2602,84444	1,25915277
25	0,72254335	240	2304	1,04016116
27,5	0,79479769	235	2008,18182	0,87099822
30	0,86705202	235	1840,83333	0,73781396
32,5	0,93930636	235	1699,23077	0,63125489
35	1,01156069	235	1577,85714	0,54481196
37,5	1,08381503	253	1706,90667	0,47383961
40	1,15606936	235	1380,625	0,41494976
50	1,44508671	229	1048,82	0,25812046
60	1,73410405	250	1041,66667	0,1720282
70	2,02312139	211	636,014286	0,12060679
100	2,89017341	200	400	0,05098507
200	5,78034682	165	136,125	0,00839165
300	8,67052023	152	77,0133333	0,00275017
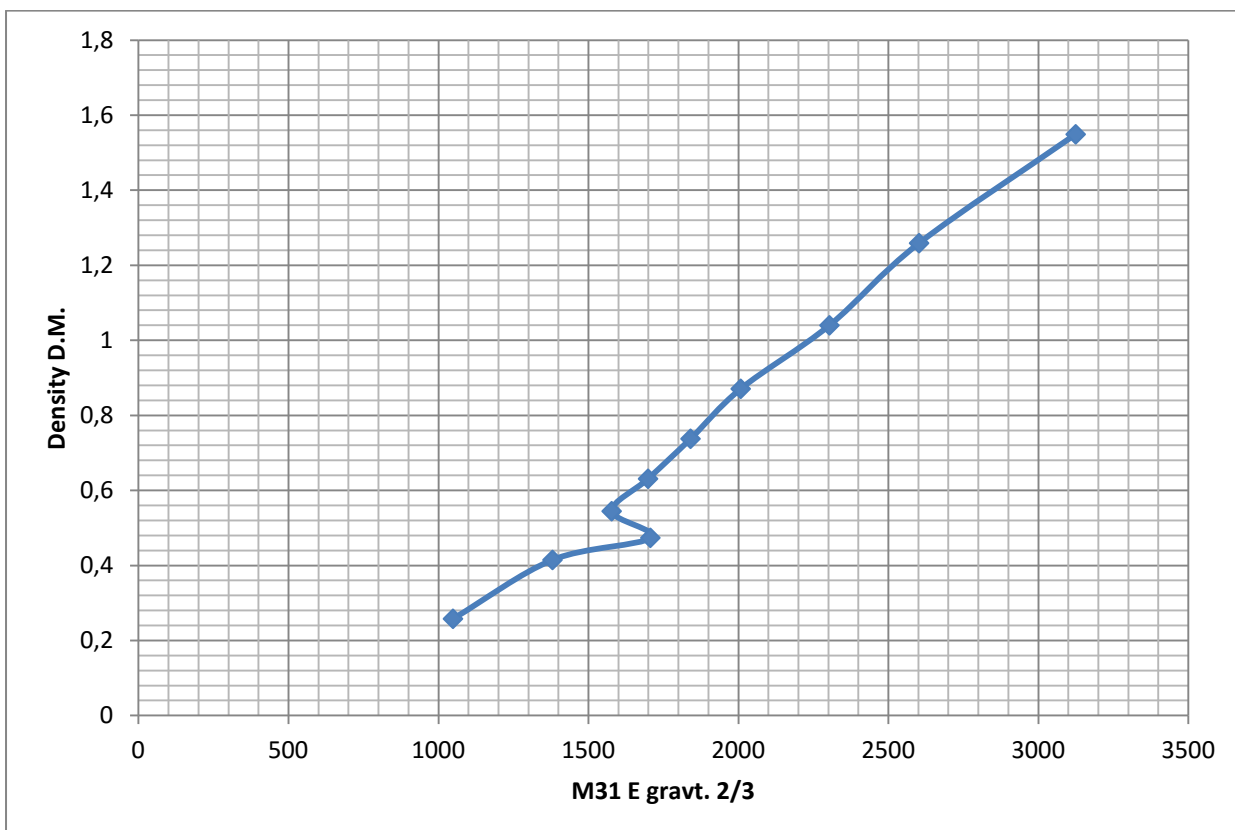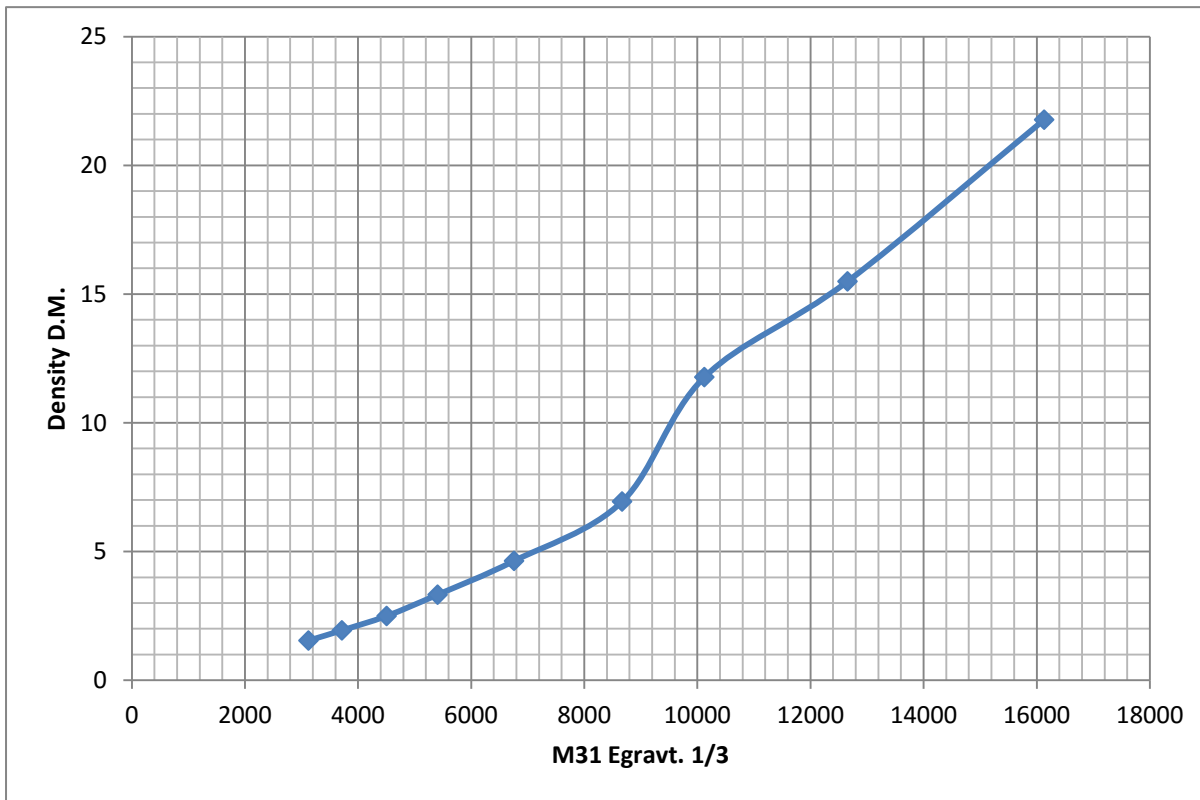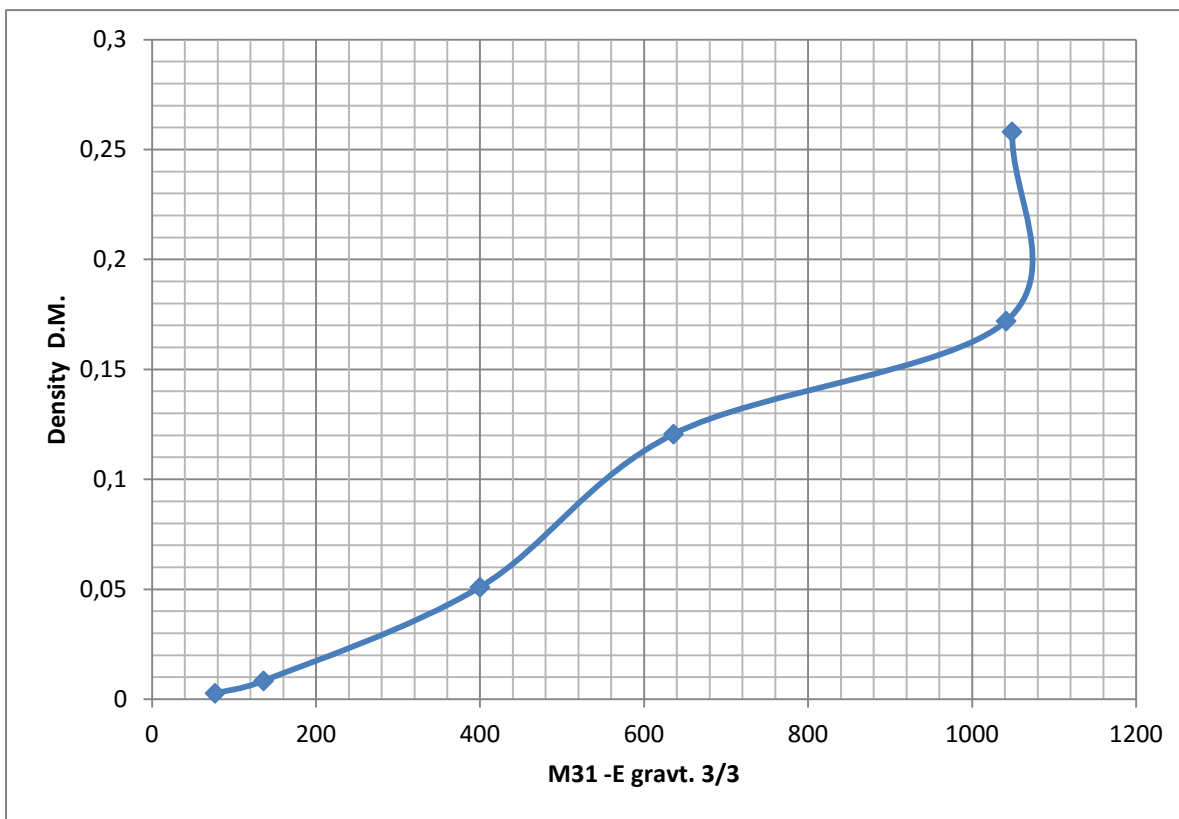
5. STATISTICAL ANALYSIS OF EIGHT GALACTIC DATA SET - 8GDS –

As eight galactic data set it will be used frequently some times it will be called 8GDS.
Below are collected data from eight galactic set got in previous epigraph. Dominion extend from 1500 < E < 15000 because curve rotation data extend from inner region of bulge to outskirt region of galactic disks. Number beside each galaxy shows author who come from rotation curve data.

E gravt.	NGC 3031[2]	NGC 7331[2]	NGC 2841[3]	Milky Way [1]	M31[1]	NGC 2841[2]	NGC 7331[3]	NGC 3992[3]
1500			0,47	0,3	0,5	0,5	0,72	0,6
2000			0,85	0,5	0,85	0,86	1,1	1
3000	2	2,15	1,5	0,95	1,48	1,5	1,84	1,95
4000	2,6	2,75	2,4	2,3	2,2	2,4	2,2	2,95
5000	3,2	3,2	3,4	3,9	2,95	3,5	2,5	4,5
6000	3,6	3,5	4,3	5,5	3,9	4,5	2,7	6,5
7000	4	3,75	5,3	7,5	5	5,6	2,85	9,5
8000	4,3	3,85	6,4	9,5	6	7	2,93	13,5
9000	5,5	4	7,4	12	8	8	3,03	18,5
10000	7,2	4,1	8,6	14	11,5	9	3,1	
11000	8	4,17	9,8	16,5	13,3	10,6	3,15	
12000	8,4	4,25	11,2	20,5	14,5	12,6	3,2	
13000	8,8	4,333	12,6	28	16	14,5	3,24	
14000	9,2	4,42	14	35	18	16,1	3,28	
15000	9,6	4,5	15,4	41	19,5	17,9	3,32	

E gravt.	$\overline{D}_{DM}$	Standard Deviation	Variation coefficient
1500	0,515	0,12776932	0,24809577
2000	0,86	0,18574176	0,21597879
3000	1,67125	0,36381443	0,21769001
4000	2,475	0,25248762	0,1020152
5000	3,39375	0,56592706	0,16675567
6000	4,3125	1,12742794	0,26143256
7000	5,4375	2,01877902	0,37126971
8000	6,685	3,2179885	0,4813745
9000	8,30375	4,64788645	0,55973343
10000	8,21428571	3,56868466	0,43444857
11000	9,36	4,40104208	0,4701968
12000	10,6642857	5,57383648	0,52266384
13000	12,4961429	7,77034091	0,62181915
14000	14,2857143	9,93243871	0,69527071
15000	15,8885714	11,8026963	0,74284188

Results shown above from statistical analysis of galactic data allow conclude quickly that Density of DM not depend on E, when gravitational field is increasing because standard deviation and variation coefficient are too high.
Studying rotational curves I have concluded that Density of DM has too much error for radius under 10 Kpc which correspond E= 6000 aproximately. So I have decided to restrict dominion of E in order to get reliable results in statistical analysis.

6. FUNCTION FITTED TO EIGHT GALACTIC DATA SET - 8GDS -   FOR E BELOW 6000

It is clearly shown in previous figure of data cloud that point dispersion increase with E. I think, the reason why this happen is that the more near the bulge the more is dark matter negligible versus baryonic matter and error in calculus of dark matter density is bigger.

According the model proposed by the author, [4] Abarca,M.2014 dispersion of point cloud only is due to experimental error because the author defend a Universal law for Density of dark matter. In other words D.M. density does not depend on the galaxy but the E. Therefore point dispersion must decrease when contribution of dark matter increase because the more DM proportion is not negligible the less experimental error will be produced. That is precisely what happen in figures of cloud points.

I have decided to consider E< 6000 because point dispersion is a great deal lower that dispersion point at above table.

Looking for data table of galaxies studied, it is right to see that if radius is bigger than radius tabulated below, then E < 6000 for each galaxy.

Galaxy	NGC 2841	NGC 7331	NGC 3031	Milky Way	M31	NGC 3992
Radius	R > 15 Kpc	R >10 Kpc	R > 9 Kpc	R > 9 Kpc	R > 10 Kpc	R > 12

And looking for in original papers [1], [2], [3] it is possible to check that for R bigger than values showed in table above for each galaxy, contribution of dark matter is about 50% . So at bigger distances errors in calculus of dark matter decrease. Therefore point dispersion decreases when DM proportion increasing when it is considered bigger distances. As it is possible to see in graphic.
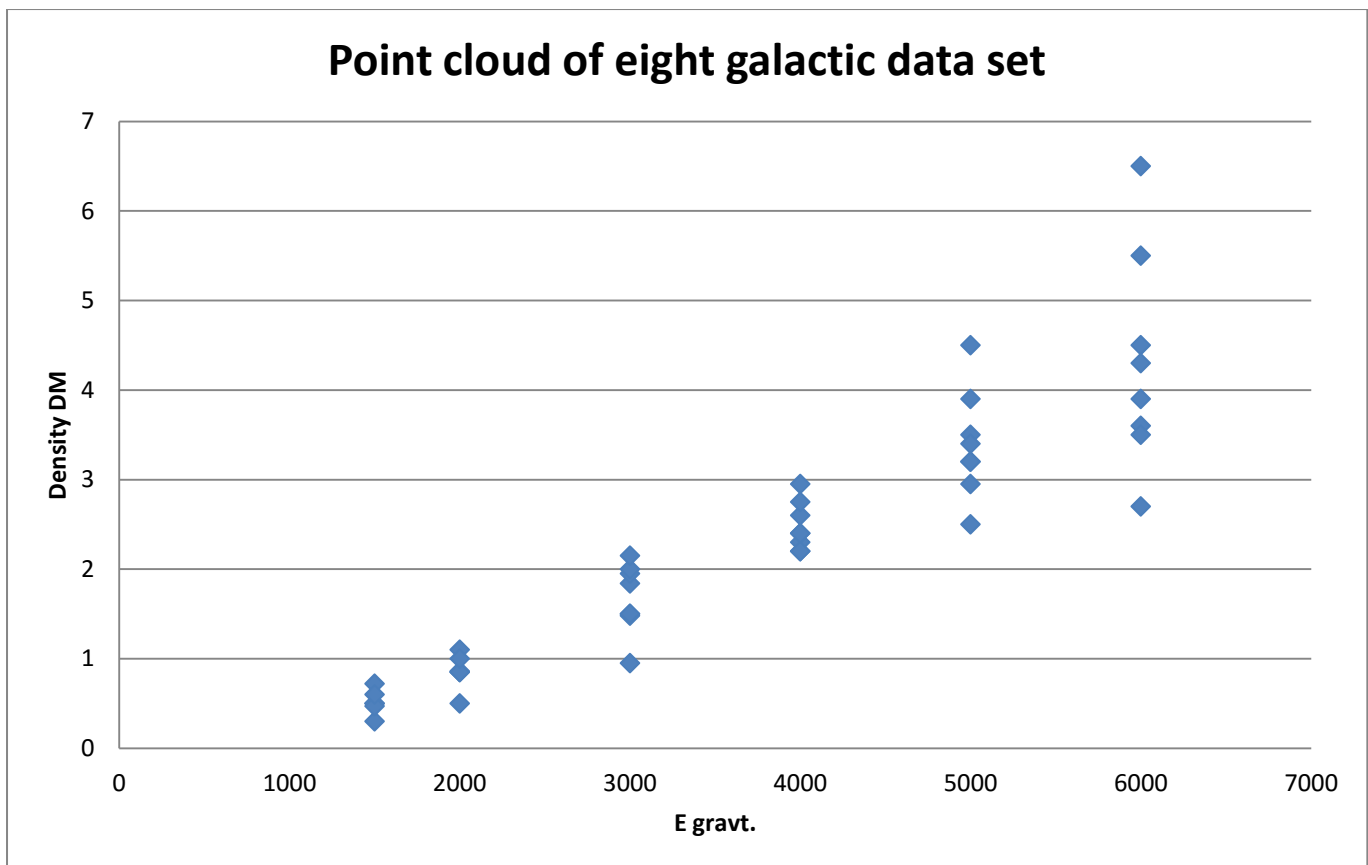
Point cloud of eight galactic data set

Below is table of data set when E < 6000

E gravt.	NGC 3031[2]	NGC 7331[2]	NGC 2841[3]	Milky Way [1]	M31[1]	NGC 2841[2]	NGC 7331[3]	NGC 3992[3]
	Density of Dark Matter in different galaxies							
1500			0,47	0,3	0,5	0,5	0,72	0,6
2000			0,85	0,5	0,85	0,86	1,1	1
3000	2	2,15	1,5	0,95	1,48	1,5	1,84	1,95
4000	2,6	2,75	2,4	2,3	2,2	2,4	2,2	2,95
5000	3,2	3,2	3,4	3,9	2,95	3,5	2,5	4,5
6000	3,6	3,5	4,3	5,5	3,9	4,5	2,7	6,5

Table below show maximum radius of each galaxy measuring its rotation curve. At that distance E is the lowest value.

Galaxies	N 3031 [2]	N 7331 [2]	N 2841 [3]	Milky Way	M31 [1]	N 2841 [2]	N 7331 [3]	N 3992 [3]
Lowest E	2700	3000	1500	1500	1500	1500	1500	1500
Radius	15 Kpc	25 Kpc	50 Kpc	35 Kpc	35 Kpc	50 Kpc	36 Kpc	40 Kpc

Table bellow shows statistical parameters of dark matter density: average mean, standard deviation and variation coefficient.

Average mean	Standart Deviation	Variation Coefficient	E gravt
0,515	0,12776932	0,24809577	1500
0,86	0,18574176	0,21597879	2000
1,67125	0,34300753	0,20524011	3000
2,475	0,23804761	0,09618085	4000
3,39375	0,53356115	0,15721875	5000
4,3125	1,06294925	0,24648099	6000

It is remarkable that variation coefficient is always under 0,25.

6.1 LINEAR FIT TO AVERAGE MEAN OF DM DENSITY FUNCTION

In graph bellow is plotted average mean of DM Density depending on E.

E gravt.	Avg mean DM density
1500	0,515
2000	0,86
3000	1,6713
4000	2,475
5000	3,394
6000	4,312

Average mean DM density function & fitted curve

Chart: x-axis "E gravt." ranging 0 to 7000, y-axis "Density DM" ranging 0 to 5.

Trendline equation: $y = 0{,}0008x - 0{,}8233$, $R^2 = 0{,}9984$
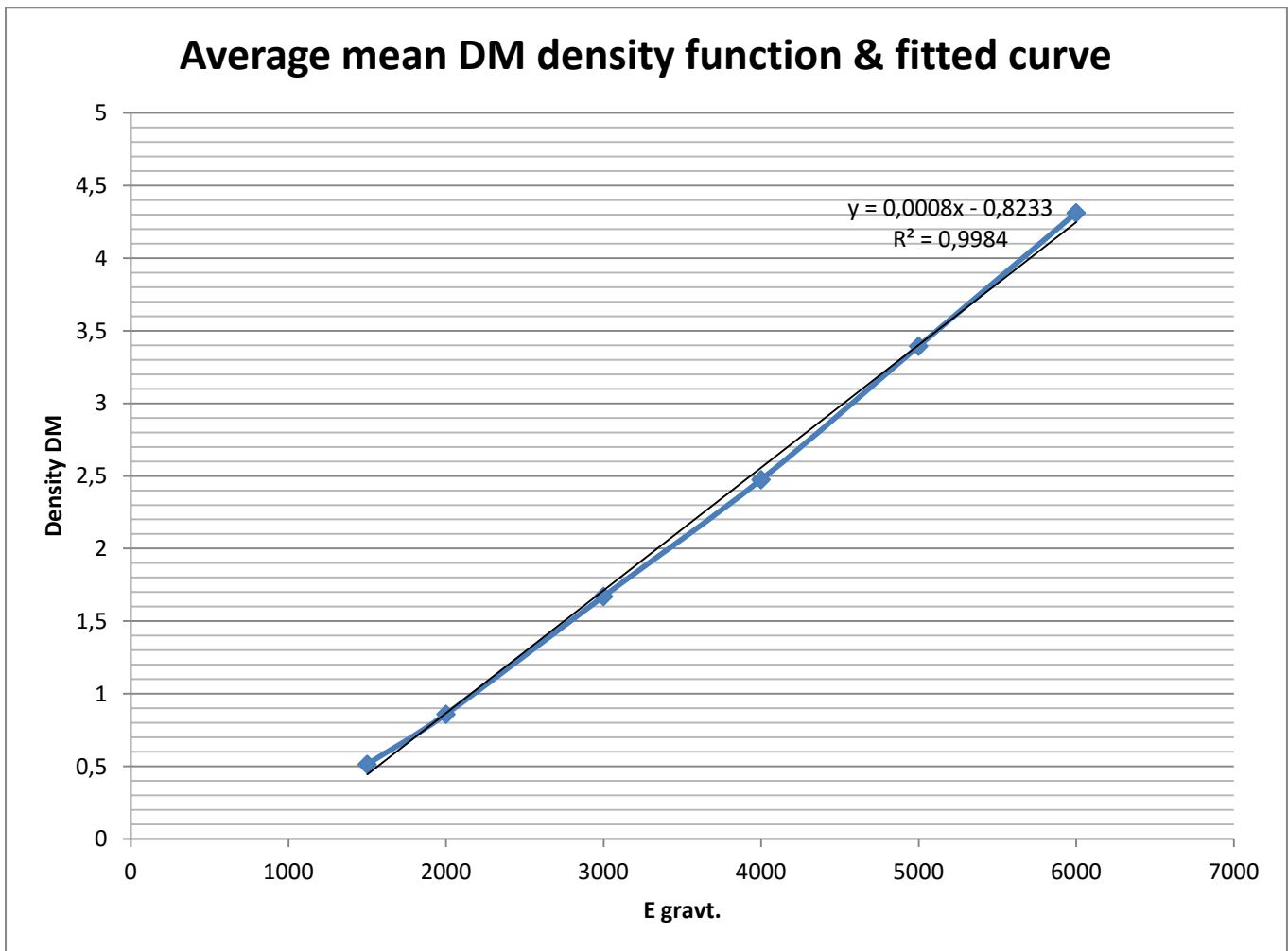
Linear function fitted to Average data of DM density $\overline{D}_{DM} = 0{,}000845 \cdot E -0{,}8236$ and R=0.99919						
E gravt.	1500	2000	3000	4000	5000	6000
Den.	0,44	0,87	1,71	2,56	3,40	4,25

This linear fit function is only right inside dominion 1500 < E < 6000. For example this function become negative when E= 974. It is needed to look for another more suitable fit function because it is necessary to wide dominion for E and DM density should go to cero when E goes to cero. Therefore potential fit is the most suitable function to fit DM density.

6.2 POTENTIAL FIT TO AVERAGE MEAN OF DM DENSITY FUNCTION

Physically it is right a potential fit because Density DM goes to 0 when E goes to 0, whereas linear fit above produces Density DM negative when E < 955 which has not physical meaning. Therefore a linear fit is only possible for reduced interval inside dominion.

Data table produces this potential fit function $\overline{D}_{DM} = A \cdot E^B$ where A= 7,686·10^{-6} and B= 1,5268 and R= 0,9985

E gravt.	Avg mean DM density	Potential fit
1500	0,515	0,54319594
2000	0,86	0,84277759
3000	1,6713	1,56519738
4000	2,475	2,42842992
5000	3,394	3,41419076
6000	4,312	4,51005366
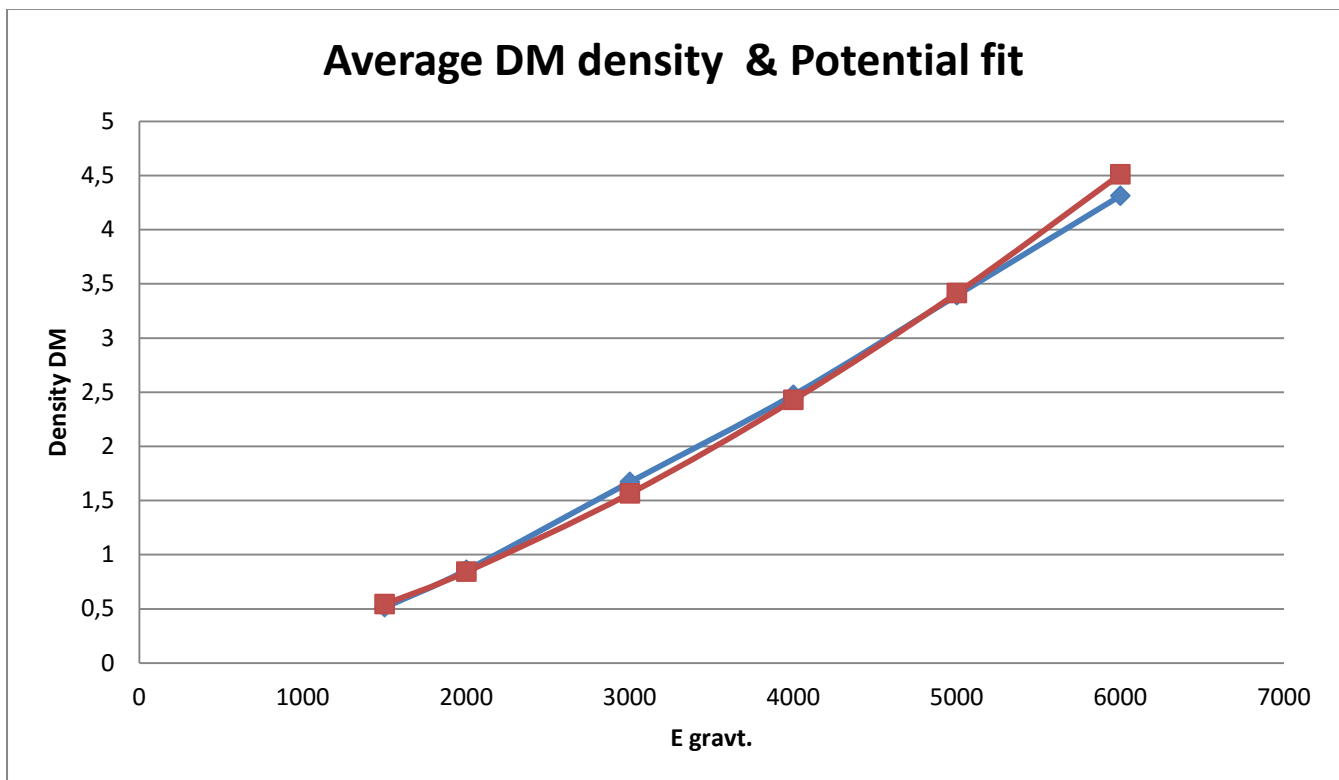
6.3 FUNCTIONAL FIT TO POINT CLOUD OF EIGHT GALACTIC DATA SET - 8GDS –

In this epigraph it will be fitted the point cloud by linear and potential function.

E gravt.	NGC 3031[2]	NGC 7331[2]	NGC 2841[3]	Milky Way [1]	M31[1]	NGC 2841[2]	NGC 7331[3]	NGC 3992[3]
1500			0,47	0,3	0,5	0,5	0,72	0,6
2000			0,85	0,5	0,85	0,86	1,1	1
3000	2	2,15	1,5	0,95	1,48	1,5	1,84	1,95
4000	2,6	2,75	2,4	2,3	2,2	2,4	2,2	2,95
5000	3,2	3,2	3,4	3,9	2,95	3,5	2,5	4,5
6000	3,6	3,5	4,3	5,5	3,9	4,5	2,7	6,5

6.3.1 LINEAR FIT
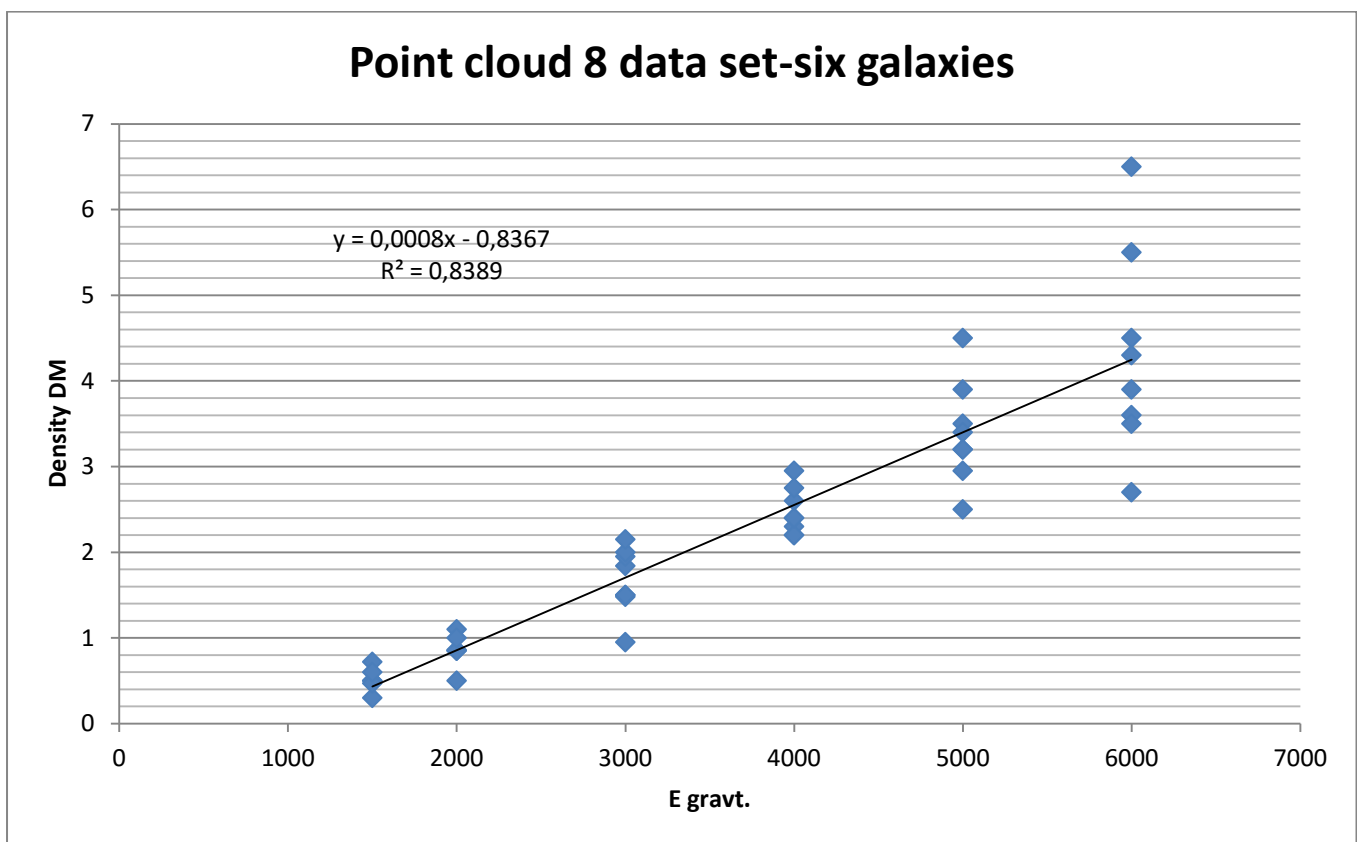
Linear function fitted to DM density DM Density = 0,0008 · E -0,8367 and R=0,916					
E gravt.	1500	2000	3000	4000	5000	6000
Den.	0,3633	0,76	1,56	2,36	3,16	3,96

As you can see this fitted function is very similar to done with average density DM. $\overline{D}_{DM}$ = 0,000845 · E -0,8236

6.3.2 POTENTIAL FIT

The point cloud is plotted in figure with diamond blue points and potential fitted results are plotted with square red points.
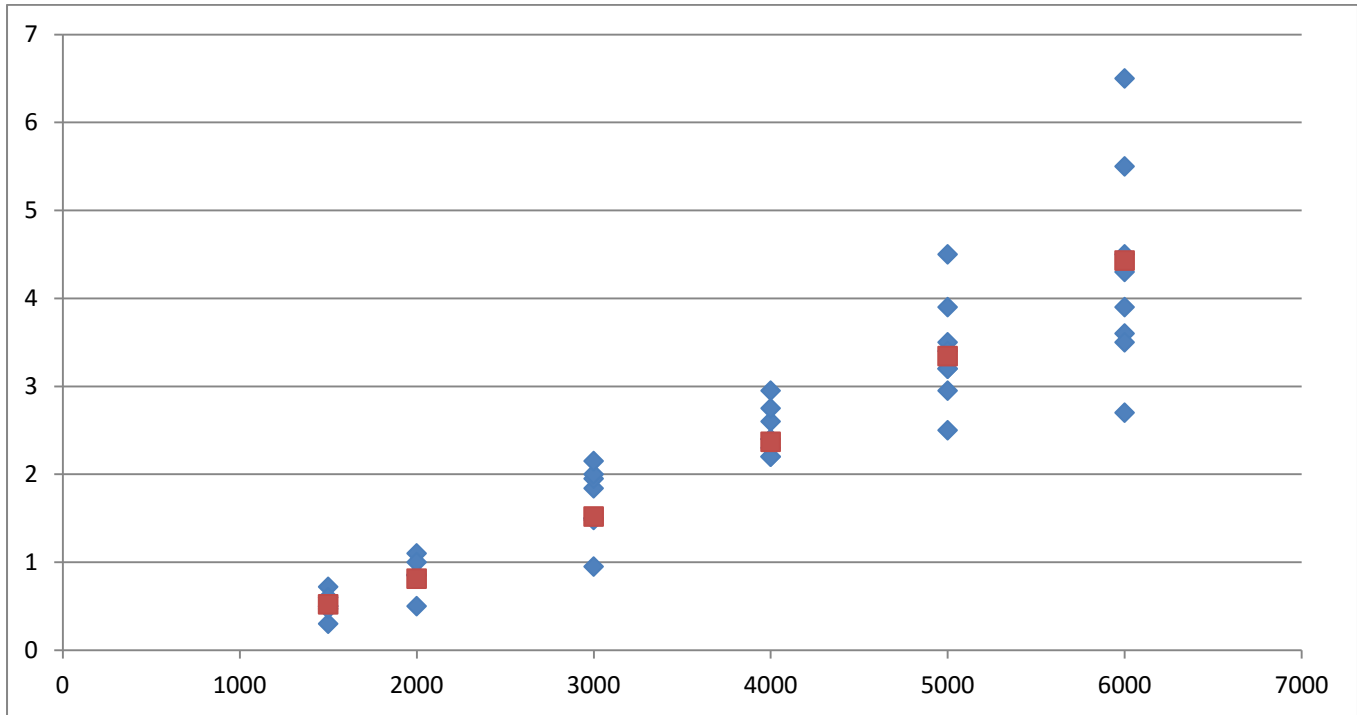
DM Density from eight galactic data set DM Density = $6{,}5{\cdot}10^{-6} \cdot E^{B}$ where B= 1,544 and R= 0,9562						
E gravt.	1500	2000	3000	4000	5000	6000
Density DM	0,521	0,81	1,52	2,37	3,34	4,43
Dominion 1500 < E < 6000						

Discussion

Firstly it could be seen that correlation coefficient is 0,04 higher than linear fit coefficient. Secondly, this fitted function is very similar to done in epigraph 6.2 for $\overline{D}_{DM}$ =A·E^{B} where A= 7,686·10^{-6} and B= 1,5268 and R= 0,9985.

Hyphotesis about   big galaxies guarantee that gravitational field is not disturbed significantly by galactic neighbour. I think that this law must be true everywhere. The problem is that inside either intermediate or dwarf galaxies rotational curve can not show the real gravitational field because neighbour big galaxies disturb field. I will explain this idea again in epigraph 11 which is dedicated to intermediate galaxies.
In the following epigraph it will be studied   Milky Way and M31 whose rotation curves extend beyond 200 Kpc. Thanks the remarkable paper [1] Sofue,Y.2015 it will be possible to check Density DM function inside a wide dominion of E. Particularly, dominion will be extend thus 100 < E < 6500.

7. FUNCTION FITTED TO GRAND ROTATION CURVE OF M31

8 Name	Group of galaxies	Distance
M31	Local group	770 Kpc

Dark matter density function profile   NFW - [1] Sofue,Y.
h = 34,6 $\pm$ 2,1 Kpc
Do = 2,23 $\pm$ 0,24 10^{-3} Msolar/pc^3

E gravt.	Density DM	Radii
16133,3333	21,7789137	3
12656,25	15,4988173	4
10125	11,7807458	5
8670	6,94876628	7,5
6760	4,64369238	10
5408	3,33105138	12,5
4506,66667	2,503102	15
3715,71429	1,94455246	17,5
3125	1,54923706	20
2602,84444	1,25915277	22,5
2304	1,04016116	25
2008,18182	0,87099822	27,5
1840,83333	0,73781396	30
1699,23077	0,63125489	32,5
1577,85714	0,54481196	35
1706,90667	0,47383961	37,5
1380,625	0,41494976	40
1048,82	0,25812046	50
1041,66667	0,1720282	60
636,014286	0,12060679	70
400	0,05098507	100
136,125	0,00839165	200
77,0133333	0,00275017	300
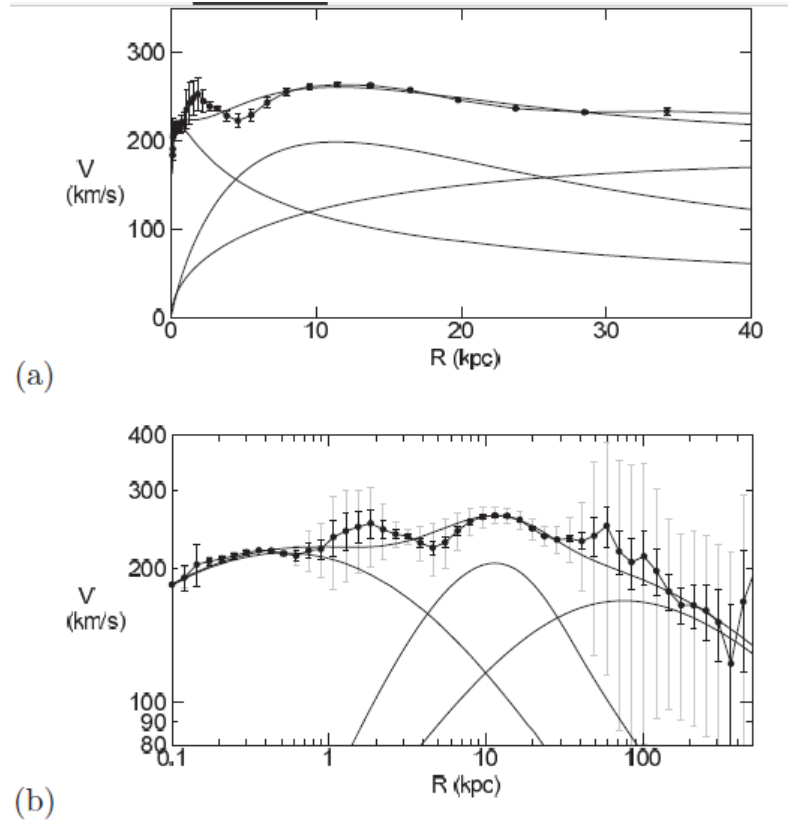
Fig. 5. Least-χ^2 fit of the GRC of M31 by the bulge, disk and dark halo components in (a) linear and (b) logarithmic scalings.

Above   Grand Rotation Curve of M31   [1] Sofue,Y. 2015.

Inside graphic (a) there are four curves: higher curve is the fitted to experimental data, the curve decreasing belong to bulge, the curve which has a maximum at 10 Kpc belong to disk and curve always increasing belong to dark matter. (b) they are the same curves but in logarithmic scale.

Looking the graph at 10 Kpc it is right that dark matter curve is so high as bulge curve, they get 100 Km/s whereas disk curve gets almost 200 Km/s. Therefore it is possible conclude that for R=10 Kpc dark matter is a not negligible proportion of total matter.
Looking for table   it is got E= 6760 when R = 10 Kpc. So it will be considered rotation curve for R> 10 Kpc . This way error associated calculus of DM density will be reduced.

Density DM versus E gravt.

Field 77< E< 6760 and Radius 10 < R<300
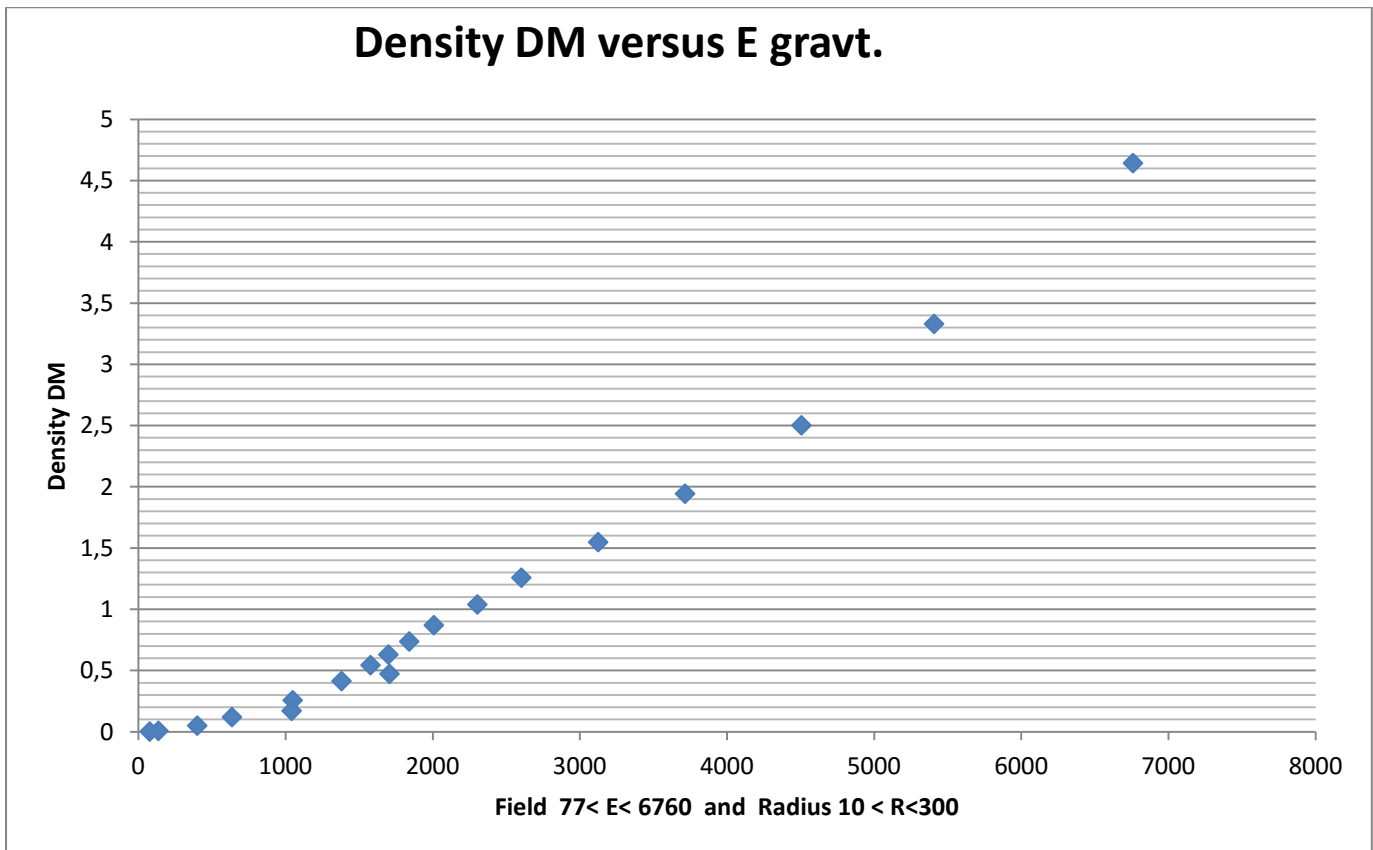
7.1 ANOMALOUS EXPERIMENTAL DATA

E gravt.	Density DM
2008,18182	0,87099822
1840,83333	0,73781396
1699,23077	0,63125489
1577,85714	0,54481196
~~1706,90667~~	~~0,47383961~~
1380,625	0,41494976
1048,82	0,25812046
~~1041,66667~~	~~0,1720282~~
636,014286	0,12060679
400	0,05098507
136,125	0,00839165
77,0133333	0,00275017

Looking graph it is obvious that points P(1041,8 ; 0,1720) and P(1706,9 ;0,4738) are anomalous so it will be ignored to fit a curve over the cloud points.

This anomalous values may be originated either experimental error or local perturbations of gravitational field.
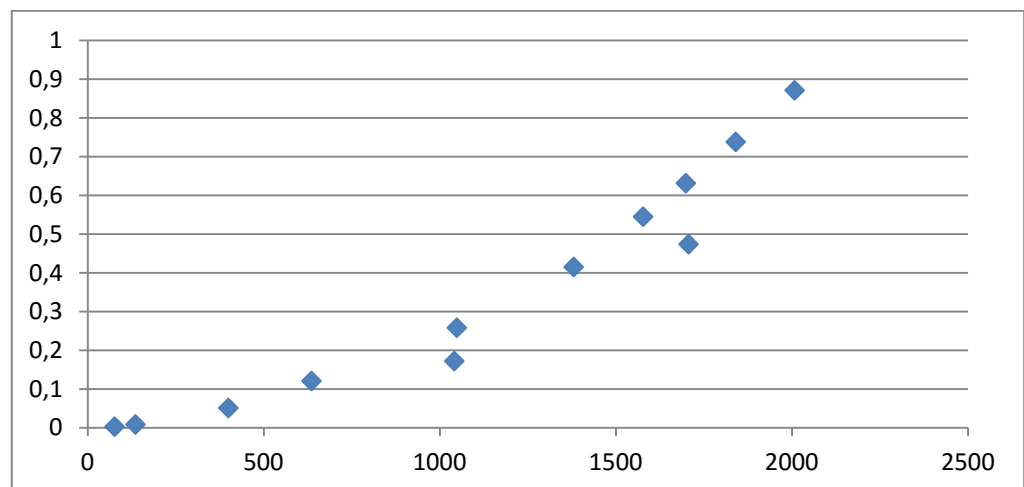
"Cleaned cloud of Points"		
E grav.	Density DM	Radii
6760	4,64369238	10
5408	3,33105138	12,5
4506,66667	2,503102	15
3715,71429	1,94455246	17,5
3125	1,54923706	20
2602,84444	1,25915277	22,5
2304	1,04016116	25
2008,18182	0,87099822	27,5
1840,83333	0,73781396	30
1699,23077	0,63125489	32,5
1577,85714	0,54481196	35
1380,625	0,41494976	40
1048,82	0,25812046	50
636,014286	0,12060679	70
400	0,05098507	100
136,125	0,00839165	200
77,0133333	0,00275017	300

As it is shown in graph this cloud of point allow fit a smooth curve with a high correlation coefficient as it will be seen.

Cleaned cloud of points
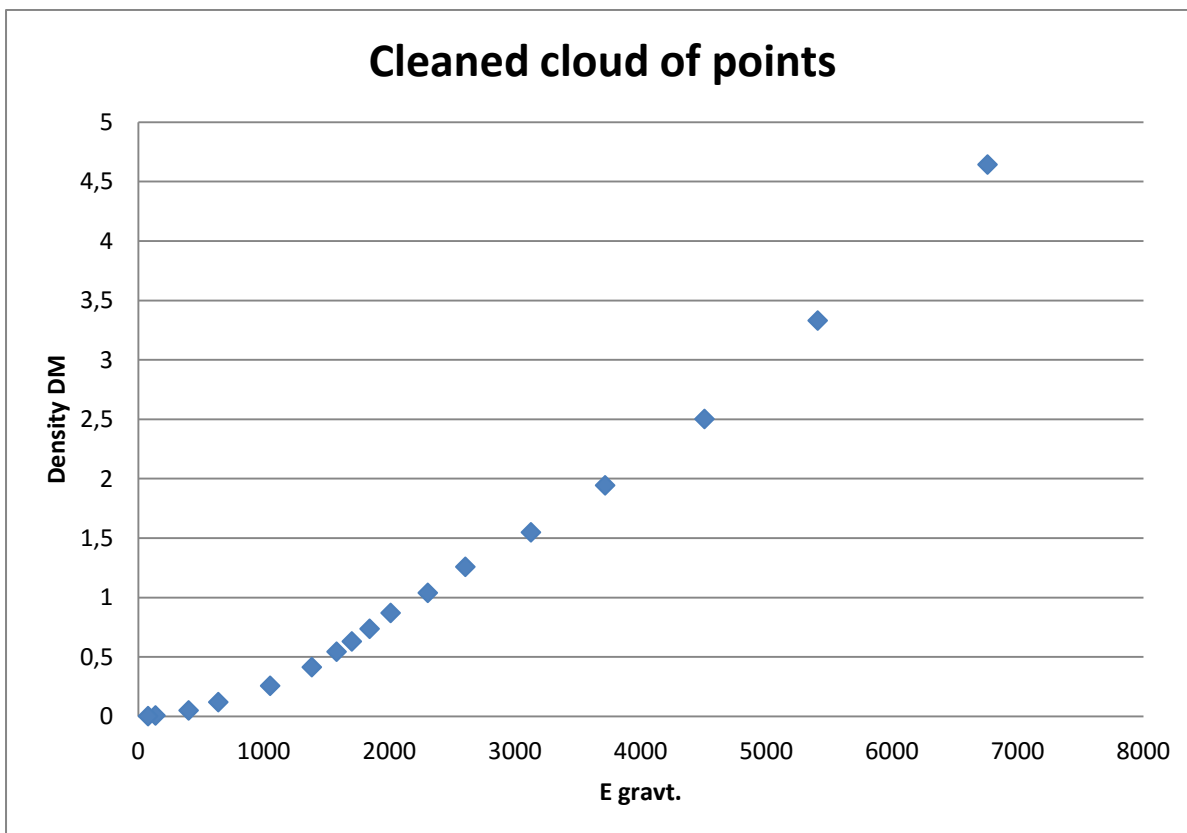
7.2 POTENTIAL FIT TO M31

E gravt.	Density DM	Fitted function
6760	4,64369238	5,59E+00
5408	3,33105138	3,86E+00
4506,66667	2,503102	2,85E+00
3715,71429	1,94455246	2,07E+00
3125	1,54923706	1,55E+00
2602,84444	1,25915277	1,14E+00
2304	1,04016116	9,35E-01
2008,18182	0,87099822	7,44E-01
1840,83333	0,73781396	6,44E-01
1699,23077	0,63125489	5,64E-01
1577,85714	0,54481196	4,98E-01
1380,625	0,41494976	3,99E-01
1048,82	0,25812046	2,53E-01
636,014286	0,12060679	1,10E-01
400	0,05098507	5,10E-02
136,125	0,00839165	8,51E-03
77,0133333	0,00275017	3,31E-03

As M31 and Milky Way are the biggest galaxies in Local group, according the theory I proposed in paper [4] Abarca,M.2014, it could be considered as a good approximation that radii of both galaxies are 385 Kpc. Which is half its distance. So 300 Kpc is a distance where field of M31 dominates fully over field of galactic neighbours. Therefore gravitational field of M31 would be the only main responsible of DM generation at 300 Kpc of distance because the intergalactic field would be negligible.

Graph above suggest that potential fit is suitable, in addition in epigraph 6.1 were pointed out a physics reason that justify potential fit.

Made statistical calculus through data from left table, it has been got the fitted function Density DM = $2,43 \cdot 10^{-6} \cdot E^B$ where B= 1,661 and R= 0,9984

Blue curve plot calculated data and red curve plot fitted function.

M31- data & potential regresion curves
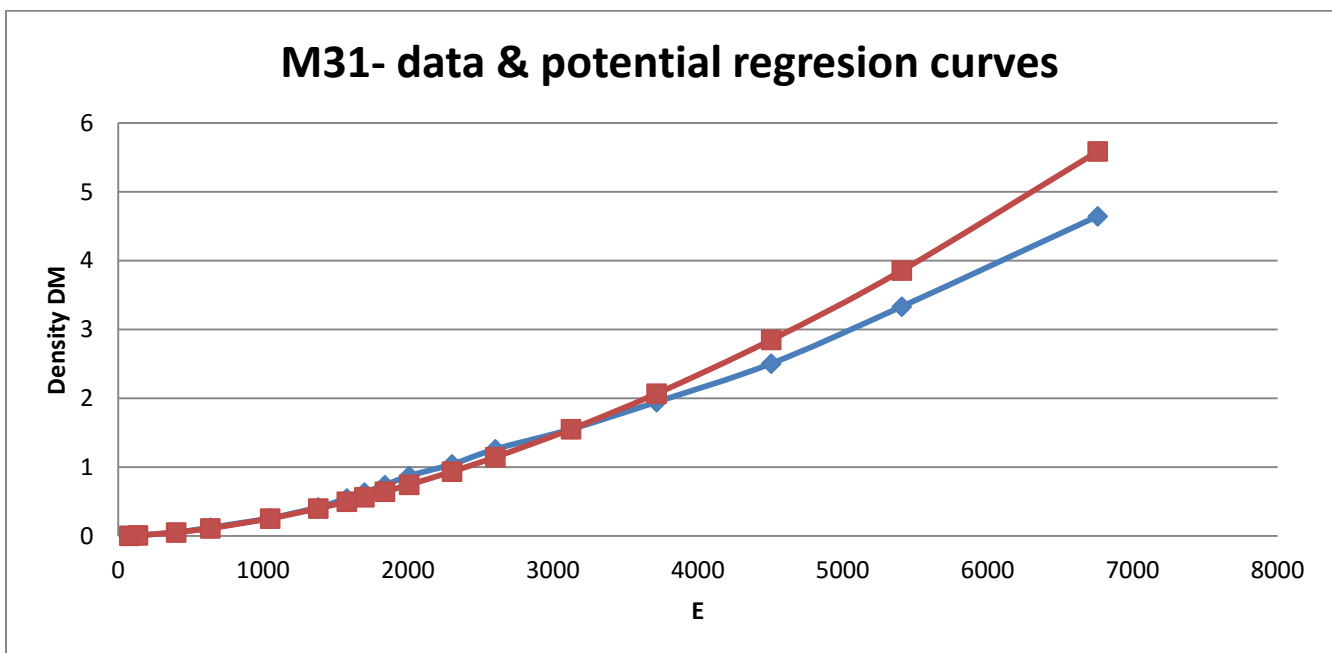
Bellow are tabulated a selected set of dominion which will be compared with Milky Way data in next epigraph.

DM Density from M31 DM Density = $2,43 \cdot 10^{-6} \cdot E^B$ where B= 1,661 and correlation coefficient R= 0,99847						
E gravt.	100	500	1500	3000	4500	6400
Den.	0,0051	0,074	0,46	1,45	2,84	5,10
Dominion 77 < E < 6760						

8. FUNCTION FITTED TO GRAND ROTATION CURVE OF MILKY WAY

7 Name	Group of galaxies	Stellar Radius
Milky Way	Local group	15 Kpc

Dark matter density function profile   NFW   [1] Sofue,Y. 2015.
h = 10,7$\pm$ 2,9 Kpc
Do = 18,2$\pm$7,4 10^{-3} Msolar/pc^3 **Error 40% ¡¡¡**

Graphs bellow Extended Rotation curve of Milky Way   [1] Sofue,Y. 2015.
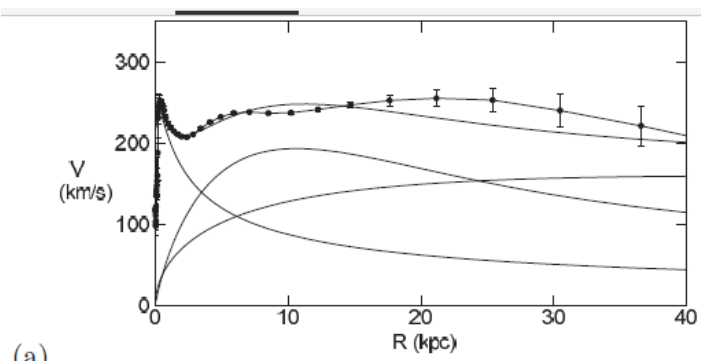
(a)
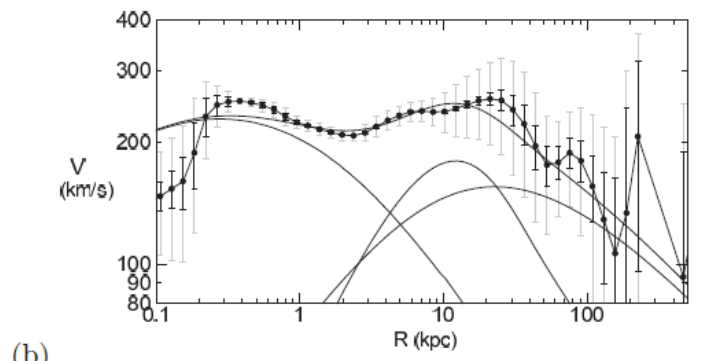
(b)

Inside graphic (a) there are four curves: higher curve is the fitted to experimental data, the curve decreasing belong to bulge, the curve which has a maximum at 10 Kpc belong to disk and curve always increasing belong to dark matter.

(b) they are the same curves but in logarithmic scale.

Looking the graph at 10 Kpc it is right that dark matter curve is 150 % higher than bulge curve, it gets 125 Km/s whereas disk curve gets almost 200 Km/s. Therefore it is possible conclude that for R=10 Kpc dark matter is a not negligible proportion of total matter.

Looking for table   it is got E= 6400 when R = 9 Kpc. So it will be considered rotation curve for R> 9Kpc . This way error associated calculus of DM density will be reduced.

Fig. 6. Same as figure 5, but for the revised new GRC of Milky Way using the same procedure as for M31.

Milky Way - detail
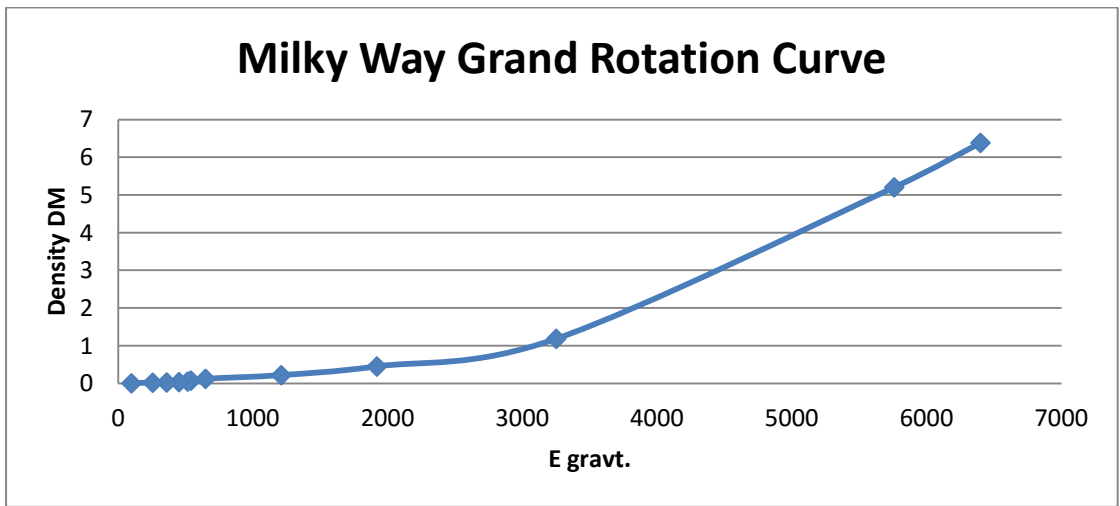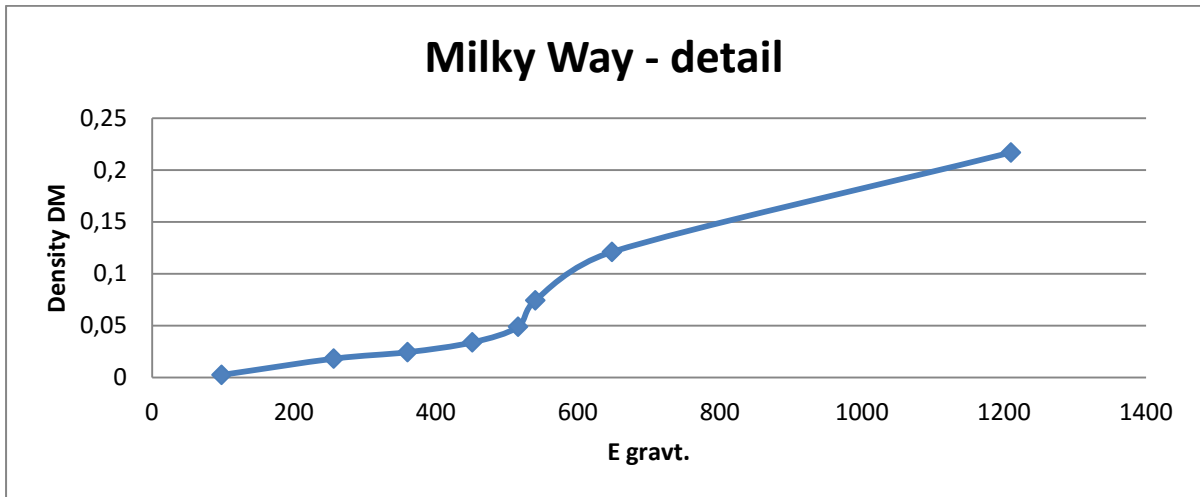
E gravt.	Density	Radii Kpc
~~14700~~	~~39,5968221~~	3
~~12656,25~~	~~25,7945562~~	4
~~11520~~	~~18,0906184~~	5
~~9600~~	~~13,3241198~~	6
~~8228,57143~~	~~10,1666565~~	7
~~7200~~	~~7,9698385~~	8
6400	6,3833368	9
5760	5,20333791	10
3251,25	1,18281269	20
1920	0,44865514	30
1210	0,2168437	40
648	0,12102507	50
540	0,07434173	60
515,714286	0,04890774	70
451,25	0,03387803	80
360	0,02442988	90
256	0,01819397	100
98	0,0025111	200

Some values have been crossed out because they belong to inner region of galaxy where there is too much error in DM density.

8.1 POTENTIAL FIT IN MILKY WAY

E gravt.	Density DM	Fitted Function
6400	6,3833368	5,30664603
5760	5,20333791	4,37911057
3251,25	1,18281269	1,54349132
1920	0,44865514	0,59074903
1210	0,2168437	0,25455545
648	0,12102507	0,08151873
540	0,07434173	0,05846263
515,714286	0,04890774	0,05375742
451,25	0,03387803	0,04214013
360	0,02442988	0,02791217
256	0,01819397	0,01499051
98	0,0025111	0,00260275

As dominion is a great deal wider than galaxies in 8GDS it is not right to consider linear fit method so it will be got only potential fit like M31.

In addition, as was explained in epigraph 6.1, by physics reason it is suitable a DM density function that goes to cero when E goes to cero.

In graph bellow, blue curve represents DM density calculated and red curve represents potential fit.

Milky Way- Potential fit

DM Density from Milky Way DM Density = $6,09 \cdot 10^{-7} \cdot E^B$ where B= 1,8234 and correlation coefficient R= 0,9953						
E gravt.	100	500	1500	3000	4500	6400
Den.	0,0027	0,0508	0,376	1,333	2,79	5,30
Dominion 100 < E < 6400						

It is clear that there is differences between results of fitted function of Milky Way and fitted function got from eight galactic data set which is written bellow.

DM Density from eight galactic data set - 8GDS-. DM. Density = $6,5 \cdot 10^{-6} \cdot E^B$ where B= 1,544 and R= 0,9562								
E gravt.	1500	2000	3000	4000	4500	5000	6000	6400
Den.	0,521	0,81	1,52	2,37	2,84	3,34	4,43	4,89
Dominion 1500 < E < 6000 Unit for $D_{D.M.}$ is $10^{-3} M_{\Theta}/pc^3$ and unit for E is $Km^2/s/Kpc$.								

There are several reason that could explain that difference.

Firstly [1] Sofue,Y. 2015. has shown in his paper $D_o = 18,2 \pm 7,4 \cdot 10^{-3}$ Msolar/pc^3 which is a error of 40% ¡¡¡

Other authors who study Milky Way state that measures of rotation curves inside Milky Way are particularly difficulty because they have to measure star speed from Earth, which orbits at 30 Km/s around the Sun which is a star whose vector velocity is not well known. Even scalar speed of Sun has an error about 10% that is 220 ± 20 km/s.

This is the Milky Way paradox. Despite the fact we live inside Milky Way, errors in rotation curve measures are too big¡

9. COMPARISON BETWEEN D.M. DENSITY FITTED TO MILKY WAY TO M31 AND TO THE EIGHT GALACTIC DATA SET

9.1 COMPARISON BETWEEN D.M. DENSITY FITTED TO MILKY WAY AND TO M31

| DM Density from Milky Way DM Density = $6,09 \cdot 10^{-7} \cdot E^B$ where B= 1,8234 and correlation coefficient R= 0,9953						
E gravt.	100	500	1500	3000	4500	6400
Den.	0,0027	0,0508	0,376	1,333	2,79	5,30
Dominion $100 < E < 6400$						

DM Density from M31 DM Density = $2,43 \cdot 10^{-6} \cdot E^B$ where B= 1,661 and correlation coefficient R= 0,99847						
E gravt.	100	500	1500	3000	4500	6400
Den.	0,0051	0,074	0,46	1,45	2,84	5,10
Dominion $77 < E < 6760$						

E gravt.	Fit MW	Fit M31	(M31-MW)*100/M31
100	0,00270042	5,10E-03	4,71E+01
300	0,02001771	0,0316304	3,67E+01
600	0,07084554	0,1000266	2,92E+01
900	0,14838751	0,19615667	2,44E+01
1200	0,2507325	0,31631976	2,07E+01
1500	0,37663125	0,45824091	1,78E+01
2000	0,63639924	0,73895348	1,39E+01
3000	1,33295182	1,44912109	8,02E+00
4000	2,25230781	2,3368343	3,62E+00
5000	3,38324509	3,38528667	6,03E-02
6000	4,71750691	4,58263746	-2,94E+00
6500	5,45880737	5,23426081	-4,29E+00

As table shows, relative difference between both curves increase when field decrease.

It is obvious that a 47 % of relative difference when E=100 is too high. However it is necessary to remember that NFW model in Milky Way has parameter $D_o = 18,2 \pm 7,4 \cdot 10^{-3}$ Msolar/pc^3 which has an error of 40%.

In conclusion, perhaps differences between Milky Way and M31 might be explained by experimental error in calculus of Dark matter density.

MW-M31 Detail
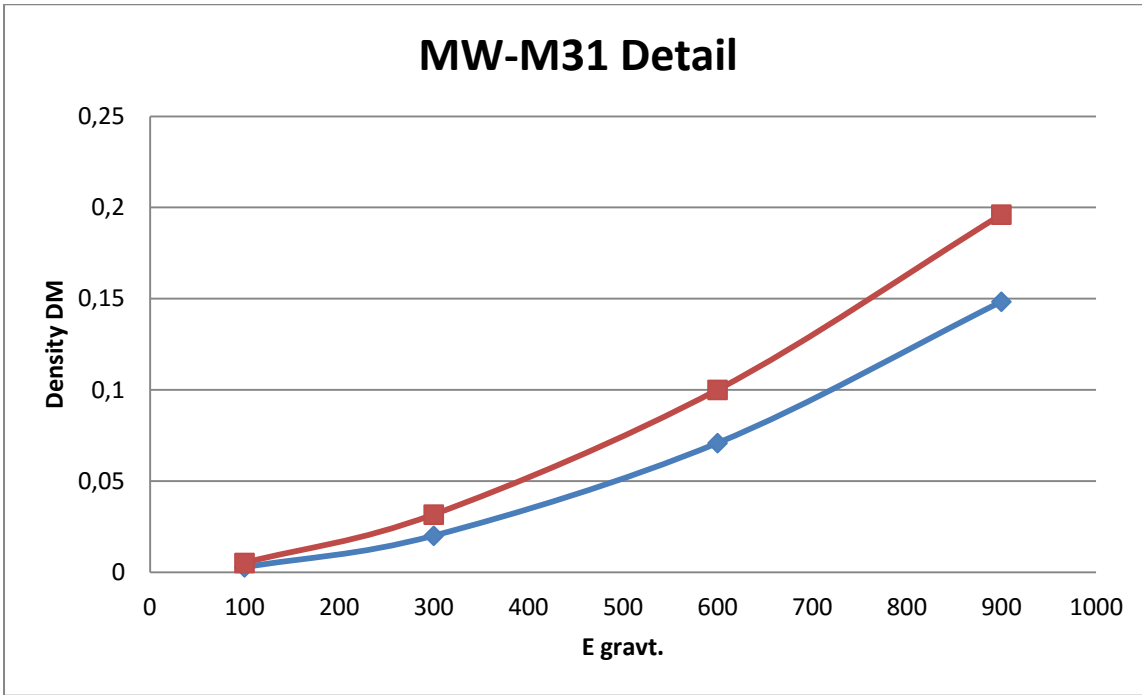
Blue curve belong to Milky Way and Red curve belong to M31.

MW-M31
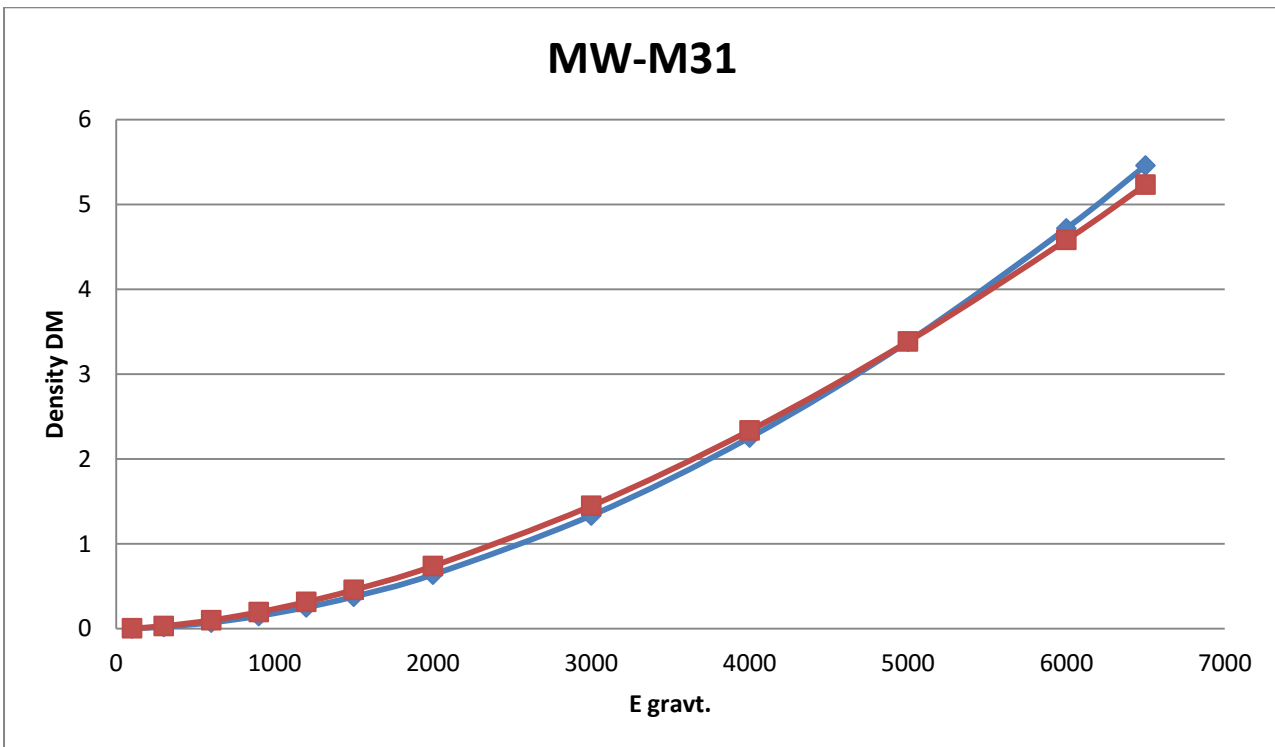
Despite the fact that relative difference is high for low values of E, globally both curves are very similar ¡¡

9.2 COMPARISON BETWEEN DM DENSITY   FITTED TO MILKY WAY TO M31 AND TO THE 8GDS

Bellow are shown the three functions fitted.

DM Density from Milky Way DM Density = $6{,}09 \cdot 10^{-7} \cdot E^B$ where B= 1,8234 and correlation coefficient R= 0,9953					
E gravt.	100	500	1500	3000	4500	6400
Den.	0,0027	0,0508	0,376	1,333	2,79	5,30
Dominion 100 < E < 6400						

DM Density function from M31 DM Density = $2{,}43 \cdot 10^{-6} \cdot E^B$ where B= 1,661 and correlation coefficient R= 0,99847					
E gravt.	100	500	1500	3000	4500	6400
Den.	0,0051	0,074	0,46	1,45	2,84	5,10
Dominion 77 < E < 6760						

DM Density function from eight galactic data set Density DM = $6{,}5 \cdot 10^{-6} \cdot E^B$ where B=1,544 and R= 0,9562					
E gravt.	1500	2000	3000	4000	5000	6000
Den.	0,521	0,81	1,52	2,37	3,34	4,43
Dominion 1500 < E < 6000						

Although experimental data on eight galactic data set belong to dominion $1500 < E < 6000$. I am going to extend his dominion to compare the three fitted function over dominion $100 < E < 6500$ and cross out data of 8GDS for E< 1500 because these data have not experimental background.

E gravt.	Fit MW	Fit M31	Fit 8GDS	Rel. Diff. in % MW& M31	Rel. Diff. in % MW& 8GDS	Rel. Diff. in % M31&8GDS
100	0,00270042	5,10E-03	~~7,96E-03~~	4,71E+01	~~6,61E+01~~	~~3,59E+01~~
300	0,02001771	0,0316304	~~4,34E-02~~	3,67E+01	~~5,39E+01~~	~~2,71E+01~~
600	0,07084554	0,1000266	~~1,27E-01~~	2,92E+01	~~4,40E+01~~	~~2,10E+01~~
900	0,14838751	0,19615667	~~2,37E-01~~	2,44E+01	~~3,73E+01~~	~~1,71E+01~~
1200	0,2507325	0,31631976	~~3,69E-01~~	2,07E+01	~~3,21E+01~~	~~1,43E+01~~
1500	0,37663125	0,45824091	5,21E-01	1,78E+01	2,77E+01	1,20E+01
2000	0,63639924	0,73895348	8,12E-01	1,39E+01	2,17E+01	9,03E+00
3000	1,33295182	1,44912109	1,52E+00	8,02E+00	1,23E+01	4,61E+00
4000	2,25230781	2,3368343	2,37E+00	3,62E+00	4,91E+00	1,34E+00
5000	3,38324509	3,38528667	3,34E+00	6,03E-02	-1,21E+00	-1,27E+00
6000	4,71750691	4,58263746	4,43E+00	-2,94E+00	-6,50E+00	-3,45E+00
6500	5,45880737	5,23426081	5,01E+00	-4,29E+00	-8,90E+00	-4,42E+00

Discussion of results

As table shows, relative differences increase when field decrease. It is important to remark that relative differences between MW and M31 or 8GDS are bigger that ones between M31 and 8GDS. I think that this fact might be explained by experimental error in calculus of DM inside Milky Way. Remember that Do = $18{,}2 \pm 7{,}4$ 10^{-3} Msolar/pc^3 so error is 40%.   Also it is important to remark that inside dominion $1500 < E < 6500$ relative difference between M31 and 8GDS are bellow 12 % which is even lower that error of parameters of DM model calculated.

MW-M31-8GDS -Detail

Density DM vs E gravt.

MW-M31-8GDS

Density DM vs E gravt.
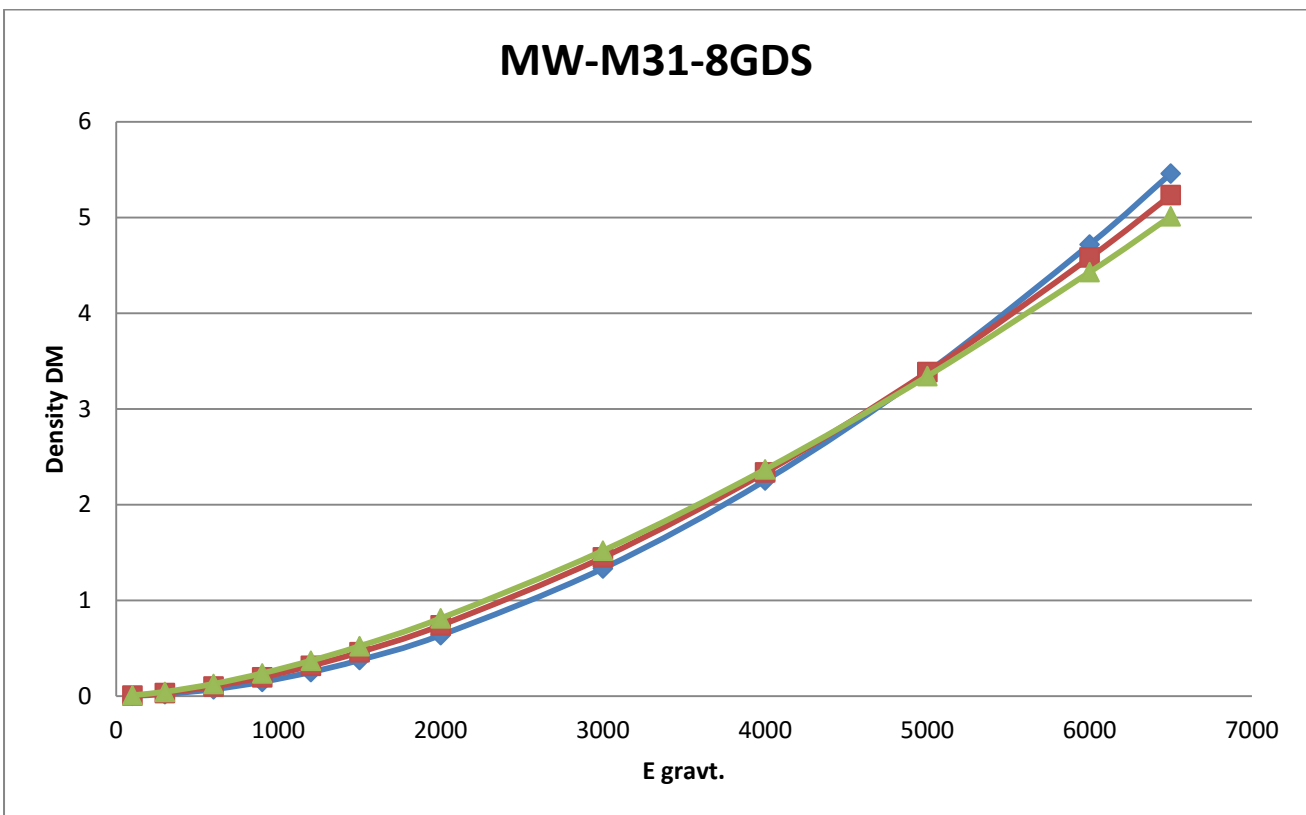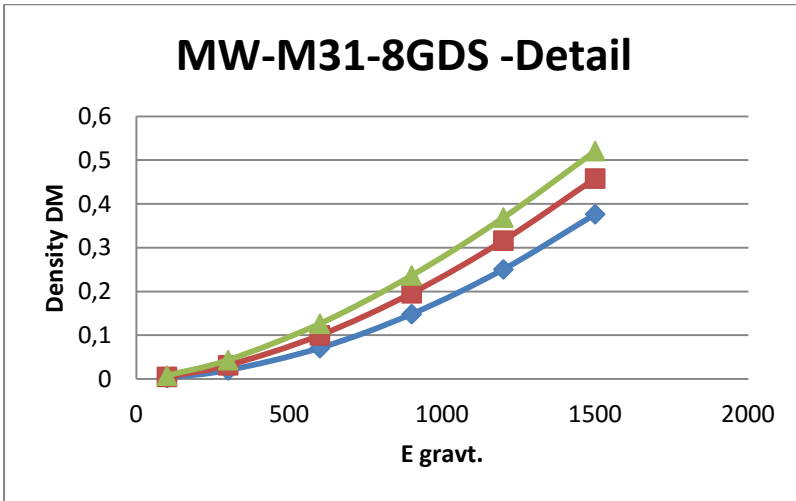
Blue curve belong to Milky Way. Red curve belong to M31. Green curve belong to 8GDS.

Globally it is possible to state that three curve are very similar.

10. DARK M. DENSITY FUNCTION DEPENDING ON E AS UNIVERSAL LAW FOR BIG GALAXIES

Differences found between curves fitted to Milky Way, M31 and 8GDS are enough small to think that it could be caused by experimental errors in galactic rotation curves and dark matter density function depend on E according a Universal law. It is known that experimental errors in rotation curves of galaxies are important because of difficult measures.

According data collected in this paper, the simplest way to calculate an average curve would be to fit a function to fitted functions calculated to Milky Way, to M31 and to 8GDS. Bellow is shown the function found which abbreviate is called Fto3F. Fto3F means Fit curve which fit to 3 fitted curves.

Function Fto3F		Density $_{DM}$ = 1,257·10^{-6} ·E^{B} where B= 1,739 and R= 0,9975				
E gravt.	100	500	1500	3000	4500	6500
Den.	0,003787	0,06226	0,42093	1,4056	2,8456	5,3948
Dominion 100 < E < 6500						

E gravt.	Fit MW	Fit M31	Fit 8GDS	Fit to 3 Fit = Fto3F
100	0,00270042	5,10E-03	7,96E-03	3,79E-03
300	0,02001771	0,0316304	4,34E-02	2,56E-02
600	0,07084554	0,1000266	1,27E-01	8,55E-02
900	0,14838751	0,19615667	2,37E-01	1,73E-01
1200	0,2507325	0,31631976	3,69E-01	2,86E-01
1500	0,37663125	0,45824091	5,21E-01	4,21E-01
2000	0,63639924	0,73895348	8,12E-01	6,94E-01
3000	1,33295182	1,44912109	1,52E+00	1,41E+00
4000	2,25230781	2,3368343	2,37E+00	2,32E+00
5000	3,38324509	3,38528667	3,34E+00	3,42E+00
6000	4,71750691	4,58263746	4,43E+00	4,69E+00
6500	5,45880737	5,23426081	5,01E+00	5,39E+00

Fitted curve to three fitted functions= Fto3F
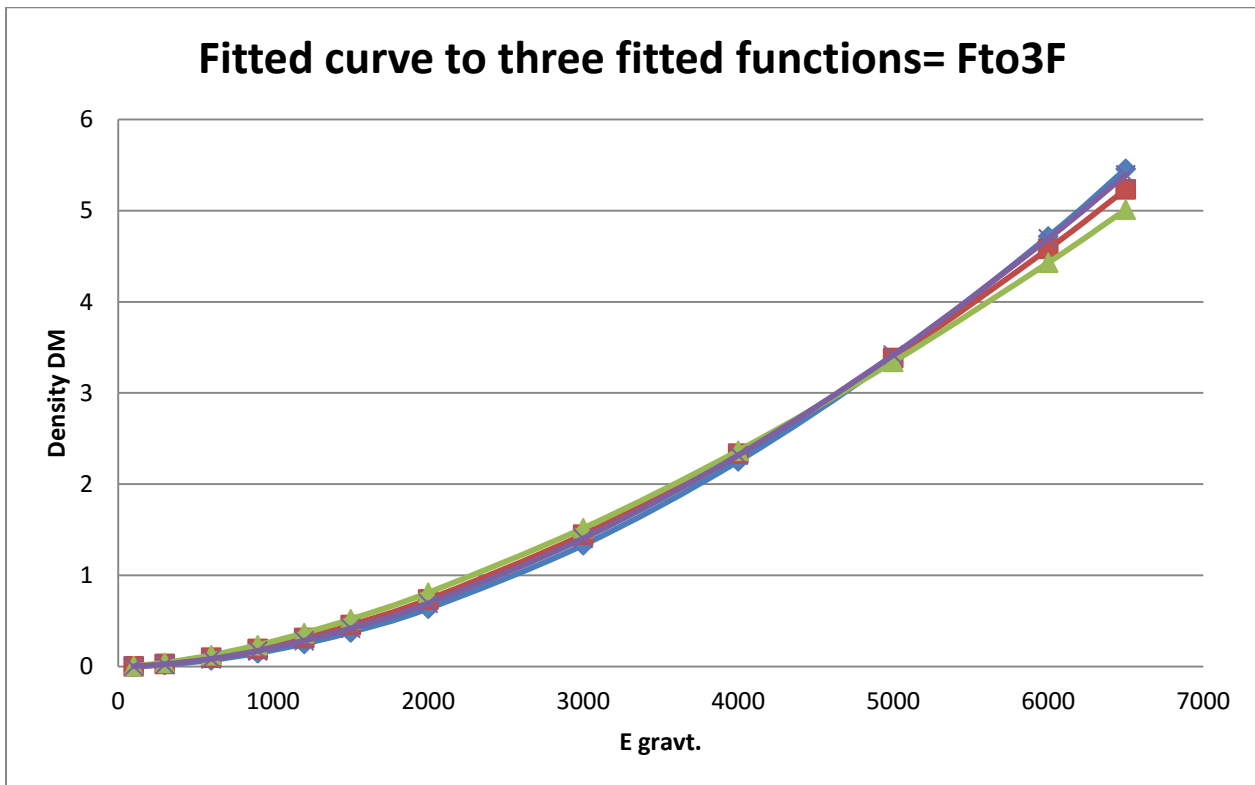
Blue curve belong to Milky Way. Red curve belong to M31. Green curve belong to 8GDS.

Magenta curve belong to fitted function which fit the fitted functions to Milky Way, to M31 and to 8GDS.

As reader can check it is not possible to differentiate magenta curve of blue curve because both are very similar.

Differences relatives between curves of MW, M31, 8GDS and Fto3F are tabulated bellow in order to check it.

Relative Differences between fit curves to MW, M31, 8GDS and Fto3F which is the "average curve"						
E gravt.	Fit MW	Fit M31	Fit 8GDS	Fit to 3 Fit	Rel. Diff. % MW & Fto3F	Rel. Diff. % M31 & Fto3F	Rel. Diff. % 8GDS & Fto3F
100	0,00270042	5,10E-03	7,96E-03	3,79E-03	2,87E+01	-3,47E+01	-1,10E+02
300	0,02001771	0,0316304	4,34E-02	2,56E-02	2,18E+01	-2,35E+01	-6,95E+01
600	0,07084554	0,1000266	1,27E-01	8,55E-02	1,71E+01	-1,70E+01	-4,80E+01
900	0,14838751	0,19615667	2,37E-01	1,73E-01	1,43E+01	-1,33E+01	-3,68E+01
1200	0,2507325	0,31631976	3,69E-01	2,86E-01	1,22E+01	-1,08E+01	-2,93E+01
1500	0,37663125	0,45824091	5,21E-01	4,21E-01	1,05E+01	-8,86E+00	-2,38E+01
2000	0,63639924	0,73895348	8,12E-01	6,94E-01	8,34E+00	-6,43E+00	-1,70E+01
3000	1,33295182	1,44912109	1,52E+00	1,41E+00	5,17E+00	-3,10E+00	-8,08E+00
4000	2,25230781	2,3368343	2,37E+00	2,32E+00	2,85E+00	-7,94E-01	-2,16E+00
5000	3,38324509	3,38528667	3,34E+00	3,42E+00	1,02E+00	9,57E-01	2,20E+00
6000	4,71750691	4,58263746	4,43E+00	4,69E+00	-5,08E-01	2,37E+00	5,62E+00
6500	5,45880737	5,23426081	5,01E+00	5,39E+00	-1,19E+00	2,98E+00	7,09E+00

Discussion of data table

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In the last and fourth columns some data have been crossed out because curve of 8GDS were obtained inside dominion 1500< E < 6000 so data E < 1500 have not experimental background.

Except data belong to E=100 and E= 300 the relative differences are bellow 17 % and relative differences inside 1500 < E < 6500 interval are mainly bellow 10%.

Results suggest to think that dark matter density depending on E could be a Universal law because relative differences between functions fitted to Milky Way, M31 and 8GDS are similar to experimental error to measure the galactic rotation curves. Therefore relative difference might be explained by experimental error in measures rotation curves.

Bellow have been collected DM density function, its characteristic parameters and its calculated errors.

Parameters of DM density function fitted by the authors quoted	
1 - NGC 2841 - Bottema Dark matter density function profile Isothermal $Rc = 6,39 \pm 0,52$ Kpc $Do = 32 \pm 5$ 10^{-3} Msolar/pc^3	5 - NGC 3992 - Bottema Dark matter density function profile Isothermal $Rc = 3,89 \pm 0,34$ Kpc $Do = 66 \pm 11$ $\cdot 10^{-3}$ Msolar/pc^3
2 - NGC 2841 - Randriamampandry Dark matter density function profile Isothermal $Rc = 5,08 \pm 0,23$ Kpc $Do = 49,06 \pm 3,61 \cdot 10^{-3}$ Msolar/pc^3	6 - NGC 3031 - Randriamampandry Dark matter density function profile Isothermal $Rc = 5,34 \pm 1,97$ Kpc $Do = 14,55 \pm 5,87$ 10^{-3} Msolar/pc^3
3 - NGC 7331 - Bottema Dark matter density function profile Isothermal $Rc = 19,46 \pm 1,79$ Kpc $Do = 3,5 \pm 0,7 \cdot 10^{-3}$ Msolar/pc^3	7 -Milky Way - Sofue. Dark matter density function profile NFW $h = 10,7 \pm 2,9$ Kpc $Do = 18,2 \pm 7,4$ 10^{-3} Msolar/pc^3 Error 40% ¡¡
4 - NGC 7331 - Randriamampandry Dark matter density function profile Isothermal $Rc = 17,38 \pm 2,75$ Kpc $Do = 4,75 \pm 0,6$ 10^{-3} Msolar/pc^3	8 - M31 - Sofue Dark matter density function profile NFW $h = 34,6 \pm 2,1$ Kpc $Do = 2,23 \pm 0,24$ 10^{-3} Msolar/pc^3

As it is right to see, most of the times parameters Do has 15% of error or even higher. Except Milky Way whose Do has 40% of error¡.

In addition, another way to explain differences in DM density function in different galaxies might be that galaxies does not verify symmetry hypothesis rigorously so Virial theorem cannot calculate real gravitational field with total accuracy.

Another way to explain those difference might be that according my previous paper [4] Abarca,M.2014, dark matter is a quantum gravity effect so a classical treatment only could aspire to be a good approximation of a deeper quantum gravity theory.

Taking in consideration this ideas I dare to postulate dark matter density function depending on E as universal law.

At this point, it is obvious that candidate to function as Universal law should be the "average" curve between MW, M31 and 8GDS.

Dark matter density function as Universal law for big galaxies					
Density $_{DM}$ = 1,26 · 10^{-6} · E^{B} where B= 1,74					
E gravt.	100	500	1500	3100	4500	6500
Den. ($m\,M_{\Theta}$/pc^3)	0,00381	0,0626	0,423	1,50	2,86	5,43
Dominion 100 < E < 6500 Unit for E is Km^2/s/Kpc					

DM Density Function as Universal Law	
E gravt.	DM Density
100	3,81E-03
300	2,57E-02
500	6,26E-02
700	1,12E-01
900	1,74E-01
1100	2,47E-01
1300	3,30E-01
1500	4,23E-01
1700	5,26E-01
1900	6,39E-01
2100	7,60E-01
2300	8,91E-01
2500	1,03E+00
2700	1,18E+00
2900	1,33E+00
3100	1,50E+00
3300	1,67E+00
3500	1,85E+00
3700	2,04E+00
3900	2,23E+00
4100	2,44E+00
4300	2,65E+00
4500	2,86E+00
4700	3,09E+00
4900	3,32E+00
5100	3,56E+00
5300	3,81E+00
5500	4,06E+00
5700	4,32E+00
5900	4,59E+00
6100	4,86E+00
6300	5,14E+00
6500	5,43E+00

DM Density Function as Universal law

Density DM (y-axis) vs E gravt. field (x-axis)

DM Density Function - Detail

Density DM (y-axis) vs E gravt. (x-axis)
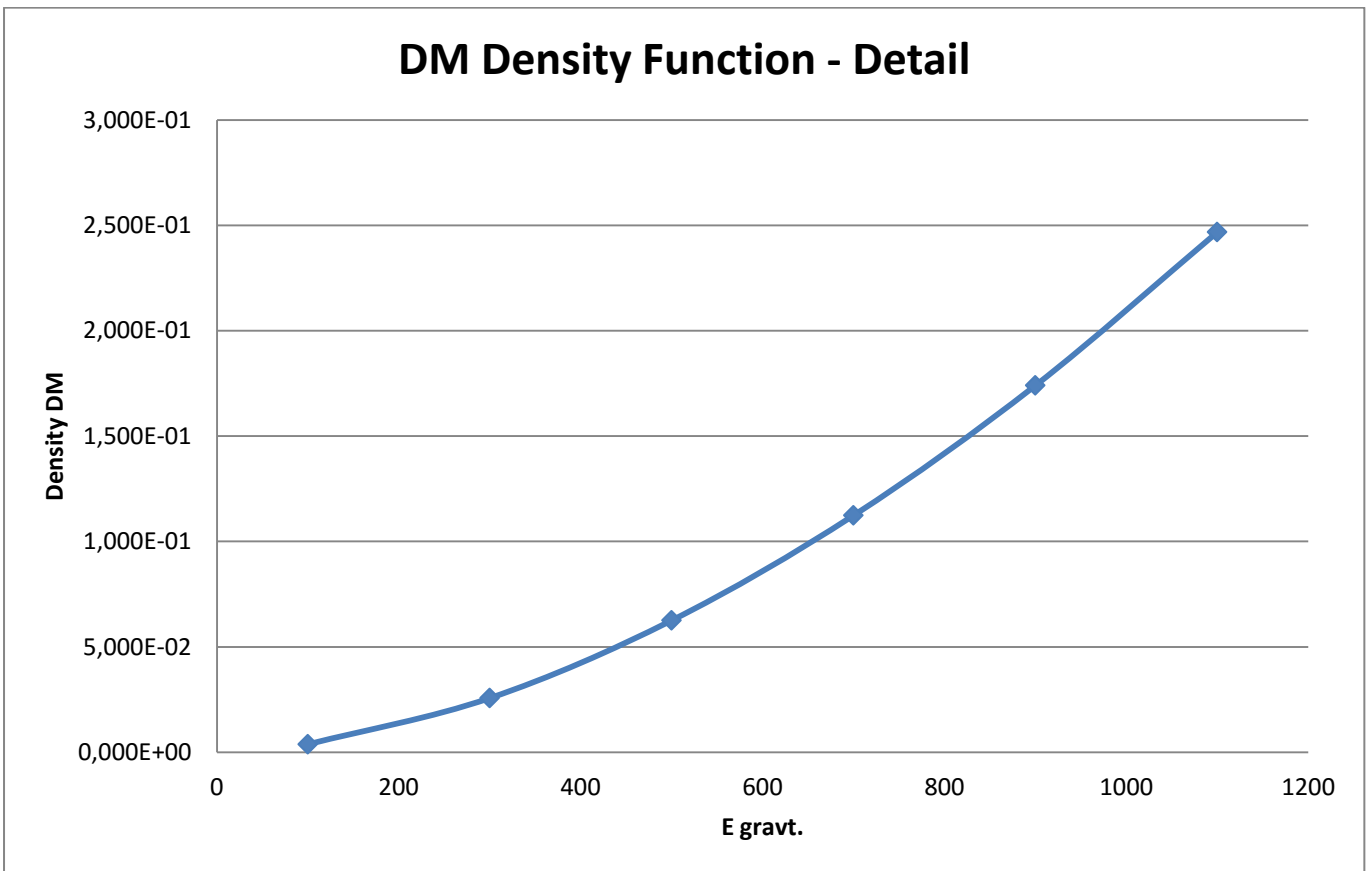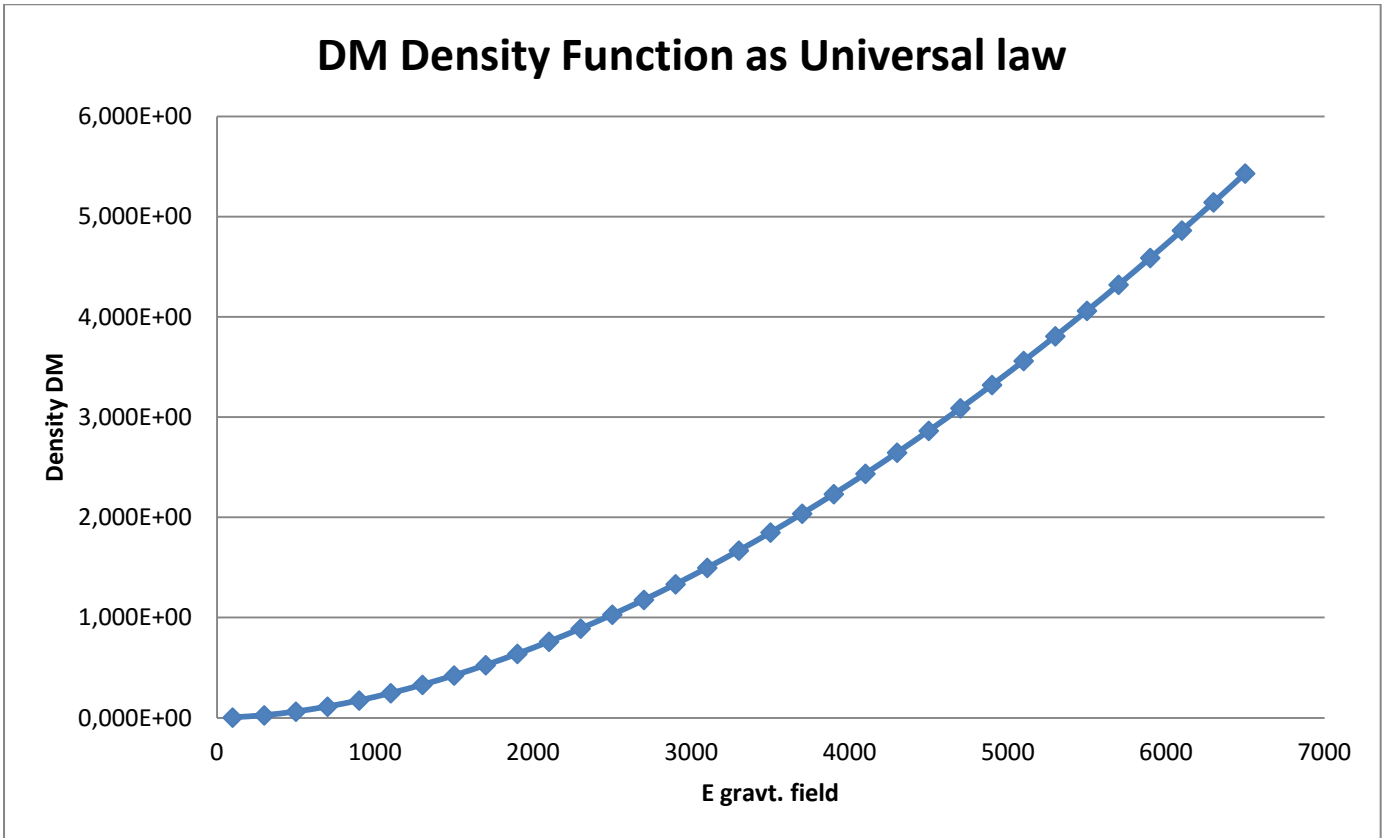
11. EXPRESSION OF DM DENSITY FUNCTION INTO INTERNATIONAL SYSTEM OF UNITS AND ANOTHER UNITS

In epigraph 10 has been got Density D.M. Density function using a particular system of units. In this epigraph it will be got the same function expressing its magnitudes through International System of Units.

Dark matter density function as Universal law for big galaxies – Paper Units					
Density $_{DM}$ = 1,26 · 10^{-6} · E^B where B= 1,74					
E gravt.	100	500	1500	3100	4500	6500
Den. ($m M_\Theta$ /pc^3)	0,00381	0,0626	0,423	1,50	2,86	5,43
Dominion 100 < E < 6500 Unit for E is Km^2/s/Kpc					

11.1 CONVERSION OF PAPER UNITS INTO INTERNATIONAL SYSTEM OF UNITS

Astronomical data

M_Θ = 1,989·10^{30} Kg

1 pc = 3,086 · 10^{16} m → 1 Kpc = 3,086 · 10^{19} m

In this paper unit for D$_{D.M.}$ is $10^{-3} M_\Theta$ /pc^3 = 6,768 ·10^{-23} Kg/m^3 using astronomical data above.

Unit for E is Km^2/s^2/Kpc = 3,24 · 10^{-14} m/s^2 using astronomical data above.

As a consequence Dominion go from   100 < E < 6500 (paper Units) to   3,24 · 10^{-12} < E < 2,11·10^{-10} (I.S.)

11.2 DM DENSITY FUNCTION INTO INTERNATIONAL SYSTEM OF UNITS

Density $_{DM}$ = A$_{PAPER\ UNITS}$ · E^B where A$_{PAPER\ UNITS}$ = 1,26 · 10^{-6} and B= 1,74   is adimensional

Density $_{DM}$ (I.S. of Units) = A$_{I.S.}$ · E^B where $A_{I.S.} = \dfrac{1,26·10^{-6} · 6,768·10^{-23}}{\left(3,24·10^{-14}\right)^{1,74}} = 2,526·10^{-5}$ is constant A in I.S.

of units.

So A$_{I.S.}$ is approximately 20 times bigger than A in Units of paper, exactly A$_{I.S.}$ = 20,05 · A$_{PAPER\ UNITS}$

Dark matter density function as Universal law for big galaxies – International System of Units					
Density $_{DM}$ = 2,526 · 10^{-5} · E^B where B= 1,74					
Where Unit for Density $_{D.M.}$ is Kg / m^3 and Unit for E is m/s^2					
E paper Units	100	500	1500	3000	4500	6500
E gravt. I.S.	3,24·10^{-12}	1,62·10^{-11}	4,86·10^{-11}	9,72·10^{-11}	1,458·10^{-10}	2,106·10^{-10}
Density$_{DM}$ I.S.	2,58·10^{-25}	4,24·10^{-24}	2,86·10^{-23}	9,57·10^{-23}	1,94·10^{-22}	3,67·10^{-22}
Dominion	3,24 · 10^{-12} < E < 2,11·10^{-10}				

It is clear that International System Units is not suitable to work with because measures are too tiny.

For example Excel cannot plot properly these numbers so tiny.

However I.S. Units are needed to check results this paper with results which come from other sources.

In table below I have inserted a raw which shows equivalence of DM density in numbers of electrons/m^3.

Dark matter density function as Universal law for big galaxies – International System of Units						
Density$_{DM}$ = 2,526 $\cdot$ 10^{-5} $\cdot$ E^B where B= 1,74						
Where Unit for Density $_{D.M.}$ is Kg / m^3 and Unit for E is m/s^2						
E paper Unit	100	500	1500	3000	4500	6500
E gravt. I.S.	3,24$\cdot$10^{-12}	1,62$\cdot$10^{-11}	4,86$\cdot$10^{-11}	9,72$\cdot$10^{-11}	1,458$\cdot$10^{-10}	2,106$\cdot$10^{-10}
Density$_{DM}$ I.S.	2,58$\cdot$10^{-25}	4,24$\cdot$10^{-24}	2,86$\cdot$10^{-23}	9,57$\cdot$10^{-23}	1,94$\cdot$10^{-22}	3,67$\cdot$10^{-22}
N° electrons/m^3	2,8$\cdot$10^5	4,65$\cdot$10^6	3,1 $\cdot$10^7	1.05$\cdot$10^8	2,1 $\cdot$10^8	4$\cdot$10^8
Dominion 3,24 $\cdot$ 10^{-12} < E < 2,11$\cdot$10^{-10}						

11.3 EXPRESSION OF DM DENSITY FUNCTION INTO GeV / cm^3

As GeV / cm^3 is a typical unit for Density of D.M. I have made this conversion of units. Reader can check this conversion knowing that   10^{-3} M_Θ /pc^3 = 0,038 GeV/cm^3

Dark matter density function as Universal law for big galaxies – Unit DM Density GeV/cm^3						
Density$_{DM}$ = 4,788 $\cdot$ 10^{-8} $\cdot$ E^B where B= 1,74						
Where Unit for D$_{D.M.}$ is GeV/cm^3 and Unit for E is Km^2/s^2/Kpc						
E gravt.	100	500	1500	3000	4500	6500
Den.	1,44$\cdot$10^{-4}	2,38$\cdot$10^{-3}	0,0161	0,0537	0,1088	0,2064
Dominion 100 < E < 6500						

11.4 DM DENSITY FUNCTION INTO A HYBRID SYSTEM OF UNITS

In further chapter it will be needed DM density formula with E in I.S. of units and $m\,M_\Theta$ /pc^3 as unit for DM Density.

Below constant of DM Density potential function are calculated in $m\,M_\Theta$ /pc^3 for Density and m/s^2 for E.

Dark matter density function as Universal law for big galaxies - Hybrid System of Units- $m\,M_\Theta$ /pc^3						
Density$_{DM}$ = 3,7323 $\cdot$ 10^{17} $\cdot$ E^B where B= 1,74						
Where Unit for Density $_{D.M.}$ is $m\,M_\Theta$ /pc^3= **10^{-3}** M_Θ /pc^3 and Unit for E is m/s^2						
E paper Units	100	500	1500	3000	4500	6500
E gravt. I.S.	3,24$\cdot$10^{-12}	1,62$\cdot$10^{-11}	4,86$\cdot$10^{-11}	9,72$\cdot$10^{-11}	1,458$\cdot$10^{-10}	2,106$\cdot$10^{-10}
Density$_{DM}$ I.S.	2,58$\cdot$10^{-25}	4,24$\cdot$10^{-24}	2,86$\cdot$10^{-23}	9,57$\cdot$10^{-23}	1,94$\cdot$10^{-22}	3,67$\cdot$10^{-22}
Den.$_{DM}$ $m\,M_\Theta$ /pc^3	3,81$\cdot$10^{-3}	0,0626	0,422	1,414	2,866	5,42
Dominion 3,24 $\cdot$ 10^{-12} < E < 2,11$\cdot$10^{-10}						

Taking into account that $m\,M_\Theta$ /pc^3 = 0,038 GeV/cm^3 also it is very easy a new change of units for density.

Dark matter density function as Universal law for big galaxies – Hybrid System of Units-GeV/cm^3						
Density$_{DM}$ = 1,4183 $\cdot$ 10^{16} $\cdot$ E^B where B= 1,74						
Where Unit for Density $_{D.M.}$ is GeV/cm^3 and Unit for E is m/s^2						
E paper Units	100	500	1500	3000	4500	6500
E gravt. I.S.	3,24$\cdot$10^{-12}	1,62$\cdot$10^{-11}	4,86$\cdot$10^{-11}	9,72$\cdot$10^{-11}	1,458$\cdot$10^{-10}	2,106$\cdot$10^{-10}
Density$_{DM}$ I.S.	2,58$\cdot$10^{-25}	4,24$\cdot$10^{-24}	2,86$\cdot$10^{-23}	9,57$\cdot$10^{-23}	1,94$\cdot$10^{-22}	3,67$\cdot$10^{-22}
Den.$_{DM}$ $m\,M_\Theta$ /pc^3	3,81$\cdot$10^{-3}	0,0626	0,422	1,414	2,866	5,42
Den.$_{DM}$ *GeV*/cm^3	1,45$\cdot$10^{-4}	2,38$\cdot$10^{-3}	0,016	0,0537	0,109	0.206
Dominion 3,24 $\cdot$ 10^{-12} < E < 2,11$\cdot$10^{-10}						

12. CHECKING DM DENSITY IN SOLAR REGION WITH RECENTS MEASURES OF LOCAL DM DENSITY

Most common values of solar parameters are $R_\Theta = 8\,Kpc$ and $V_\Theta = 220\,Km/s$. As $E = V^2/R$ it is right to get $E_\Theta = 6050\,Km^2/s^2/\,Kpc$.

Using Density $_{DM-SUN} = 1,26 \cdot 10^{-6} \cdot E^{1,74}$ in paper units it is right to get

Density $_{DM-SUN} = 4,8 \cdot 10^{-3}\,M_\Theta/pc^3 = 4,8\,m\,M_\Theta/pc^3 = 0,182\,GeV/cm^3$

According this author [9] Pijushpani Bhattacharjee.2014 values for Sun are $R_\Theta = 8,3\,Kpc$ $V_\Theta = 244\,Km/s$

These results show that currently R_Θ and V_Θ are known with a wide experimental error.

Using such values, $E_\Theta = 7173\,Km^2/s^2/\,Kpc$ so Density $_{DM-SUN} = 6,45 \cdot 10^{-3}\,M_\Theta/pc^3 = 6,45\,m\,M_\Theta/pc^3 = 0,245\,GeV/cm^3$

These values of DM density are very close to experimental measures.

For example, bellow there are three recent paper about Local DM density measures.

[6] Qiran Xia et al.2015 $\varphi_{DM-LOCAL} = 0.0159 \pm 0.005\,M_\Theta/pc^3$

[7] McKee, C.F.2015 $\varphi_{DM-LOCAL} = 0.013 \pm 0.003\,M_\Theta/pc^3$

6   Q. Xia et al.

Table 1. The best fit values of the parameters.

Parameter	Value
$\Sigma_\star$ ($M_\odot$ pc^{-2})	$40.5^{+7.1}_{-6.6}$
ρ_{DM} ($M_\odot$ pc^{-3})	$0.0159^{+0.0047}_{-0.0057}$
z_h (pc)	588^{+151}_{-192}
h (pc)	293.8 ± 0.5

In his remarkable paper the author - [8] J. I. Read.2014- presents a review of ancient and recent measures of local DM density which is summarized in table below.

Label	Reference	Description	Sampling	ρ_{dm} [$M_\odot$ pc^{-3}]	ρ_{dm} [GeV cm^{-3}]
a) Local measures (ρ_{dm})					
Kapteyn	Kapteyn (1922)	–	–	0.0076	0.285
Jeans	Jeans (1922)	–	–	0.051	1.935
Oort	Oort (1932)	–	–	0.0006 ± 0.0184	0.0225 ± 0.69
Hill	Hill (1960)	–	–	-0.0054	-0.202
Oort	Oort (1960)	–	–	0.0586 ± 0.015	2.2 ± 0.56
Bahcall	Bahcall (1984a)	–	–	0.033 ± 0.025	1.24 ± 0.94
Bienayme†	Bienayme et al. (1987)	–	–	0.006 ± 0.005	0.22 ± 0.187
KG†	Kuijken & Gilmore (1991)	–	–	0.0072 ± 0.0027	0.27 ± 0.102
Bahcall	Bahcall et al. (1992)	–	–	0.033 ± 0.025	1.24 ± 0.94
Creze	Creze et al. (1998)	–	–	-0.015 ± 0.015	-0.58 ± 0.56
HF†	Holmberg & Flynn (2000b)	–	–	0.011 ± 0.01	0.4 ± 0.375
HF†	Holmberg & Flynn (2004)	–	–	0.0086 ± 0.0027	0.324 ± 0.1
Bienayme	Bienaymé et al. (2006)	–	–	0.0059 ± 0.005	0.51 ± 0.56
Latest measurements					
MB12	Moni Bidin et al. (2012)	CSF	412	0.00062 ± 0.001 [0 ± 0.001]	0.023 ± 0.042 [0 ± 0.042]
BT12	Bovy & Tremaine (2012)	CSF	412	0.008 ± 0.003	0.3 ± 0.11
G12	Garbari et al. (2012)	VC	2×10^3	$0.022^{+0.015}_{-0.013}$	$0.85^{+0.57}_{-0.5}$
G12*	Garbari et al. (2012)	VC + Σ_b	2×10^3	$0.0087^{+0.007}_{-0.002}$	$0.33^{+0.26}_{-0.075}$
S12	Smith et al. (2012)	CSF	10^4	0.005 [no error] [0.015]	0.19 [0.57]
Z13	Zhang et al. (2013)	CSF	10^4	0.0065 ± 0.0023	0.25 ± 0.09
BR13	Bovy & Rix (2013)	CSF + MAP	10^4	0.006 ± 0.0018 [0.008 ± 0.0025]	0.22 ± 0.07 [0.3 ± 0.094]
b) Global measures assuming spherical symmetry ($\rho_{dm,ext}$)					
S10	Salucci et al. (2010)	NP	–	0.011 ± 0.004	0.43 ± 0.15
CU10	Catena & Ullio (2010)	NFW; SP	–	0.0103 ± 0.00072	0.385 ± 0.027
WB10	Weber & de Boer (2010)	NFW/ISO; WP	–	0.005 - 0.01	0.2 - 0.4
I11	Iocco et al. (2011)	gNFW; WP; ML	–	0.005 - 0.015	0.2 - 0.56
M11	McMillan (2011)	NFW; SP	–	0.011 ± 0.0011	0.4 ± 0.04

Although I recommend to read the paper, briefly I comment that measures of group a) are made through local methods which involve measures of stars few hundred parsecs away. Local measures use the vertical kinematics of stars near the Sun called `tracers'.

Group of measures b) involve global measures which extrapolate $\varphi_{DM-EXTR}$ from the rotation curve of Galaxy

DISCUSSION

Despite the fact that currently there are lot of groups of astrophysicist working in Local DM with sophisticated instruments, it is clear that differences between different measures are important.

By other side result of DM density got through the DM density function is similar to low values of DM density measures.

Therefore it can be concluded that DM density function is compatible with current measures of Local DM density.

13.DARK MATTER DENSITY FUNCTION INSIDE INTERMEDIATE GALAXIES

The same method used by big spiral galaxies was used for intermediate galaxies which spin speed in flat region is under 150 km/s

Particularly were studied rotation curves of NGC 3198 (Ursa Major), NGC 2403 (M81 group of galaxies) published by [2] Randriamampandry, T. and [3] Bottema,R.B. In adition was studied M33 -Triangle galaxy- using rotation curve published   by [5] Corbelli,E which is a satellite galaxy of M31 and its distance to the Earth is under 2 Mpc.

In these galaxies density of dark matter is more than four times bigger that density of dark matter inside big galaxies although these results has not been published in this paper.

The reason to not publish these results is that the author is looking for a Universal law for Density of dark matter therefore apparently these results destroy the paper objective.

However, I think that these results can not discard this model because the gravitational field inside an intermediate galaxy may be influenced by its nearby giant galaxy. In addition, a nearby giant galaxy may break significantly the spherical symmetry of gravitational field inside an intermediate galaxy and therefore calculus made through Virial Theorem become wrong.

I encourage the reader to consult the previous paper the author [4] Abarca,M.2014 to understand better the reasons why behaviour of density of D.M. inside intermediate galaxies cannot reject density of D.M. depending on E as a Universal law.

According the theory exposed in that paper could be better understood the following example of DM inside intermediate galaxies.

M33 -The triangle galaxy- is 220 Kpc away to M31. As I defend in [4] Abarca,M.2014 that M31 halo radius is 375 Kpc, it is obvious that M33 is placed inside M31 halo. As it is known M31 is a big galaxy twin of Milky Way. Therefore, total amount of DM in M33 come not only from DM generated by gravitational field of M33 but also from Density of DM   generates by M31. Therefore is logical that Density of DM measured in M33 is bigger that density DM calculated by the law got from eight data set because this law is for big galaxies and inside this galaxies, dark matter only come from mechanism of generation of its own gravitational field.

For this reason it should be a Universal law the following statement:

Density of DM on intermediate galaxies, which are inside halo a big galaxy   should be bigger that density of DM on big galaxies.

14.TEST OF DM DENSITY POTENTIAL FUNCTION IN MILKY WAY HALO

14.1 TEST OF DM DENSITY POTENTIAL FUNCTION IN MILKY WAY HALO UP TO 200 Kpc

In this chapter it will be compared DM density potential function with DM density NFW according experimental data

Published by [1] Sofue,Y.2015 for Milky Way.

Data from [1] Sofue,Y.2015 for Milky Way
Baryonic mass $M_{BA}= 13,7 \cdot 10^{10}\ M_{\Theta}$ Parameters of NFW profile of DM model   h= 10,7 $\pm$ 2,9 Kpc $D_0 = 18,2 \pm 7,4\ m\,M_{\Theta}/pc^3$
Astronomical data
$M_{\Theta} = 1,989 \cdot 10^{30}$ Kg   and $m\,M_{\Theta} = 1,989 \cdot 10^{27}$ Kg   1 pc = $3,085 \cdot 10^{16}$ m
Density conversion units
1 m M_{Θ}/pc^3 = $6,768 \cdot 10^{-23}$ Kg/m^3

Mr. Sofue in a previous paper [10] Sofue,Y.2013 published that baryonic mass in Milky Way was $8,3 \cdot 10^{10}\ M_{\Theta}$ almost a half of current value ¡. As Mr. Sofue commented in his last work this huge increase of mass is due to recent measures which give a larger rotation velocity of the Sun. For example [9] Pijushpani Bhattacharjee.2014 published these Solar parameters $R_{\Theta} = 8,3\,Kpc$ and $V_{\Theta} = 244\,Km/s$

Supposing a good approximation that baryonic mass in Milky Way is enclosed inside a radius of 20 Kpc, it is possible to calculate $M_{TOTAL}(<R)$ for radius bigger than 20 Kpc adding Dark matter to baryonic mass of Milky Way.

According NFW DM density model it is known that $M_{DM}(<R)=\ 4 \cdot \pi \cdot D_0 \cdot h^3 \left(\ln(1+x) - \dfrac{x}{1+x} \right)$ where   x= R/h

and $D_0\ and\ h$ are parameters calculated by Mr. Sofue for Milky Way.

Therefore $M_{TOTAL}(<R) =\ M_{BA} + M_{DM}(<R)$ when $20 < R < 200$ and it is right to calculate gravitational field $E = \dfrac{GM(<R)}{R^2}$.

Knowing E in I.S. of units it is right to calculate DM density through potential function postulated in this paper.

| *Dark matter density function as Universal law for big galaxies - Hybrid System of Units-* $m\,M_{\Theta}/pc^3$					
Density $_{DM}$ = 3,7323 · 10^{17} · E^B where B= 1,74					
Where Unit for Density $_{D.M.}$ is $m\,M_{\Theta}/pc^3$= $10^{-3}\ M_{\Theta}/pc^3$ and Unit for E is m/s^2					
E paper Units	100	500	1500	3000	4500	6500
E gravt. I.S.	$3,24 \cdot 10^{-12}$	$1,62 \cdot 10^{-11}$	$4,86 \cdot 10^{-11}$	$9,72 \cdot 10^{-11}$	$1,458 \cdot 10^{-10}$	$2,106 \cdot 10^{-10}$
Density$_{DM}$ I.S.	$2,58 \cdot 10^{-25}$	$4,24 \cdot 10^{-24}$	$2,86 \cdot 10^{-23}$	$9,57 \cdot 10^{-23}$	$1,94 \cdot 10^{-22}$	$3,67 \cdot 10^{-22}$
Den.$_{DM}$ $m\,M_{\Theta}/pc^3$	$3,81 \cdot 10^{-3}$	0,0626	0,422	1,414	2,866	5,42
Dominion	$3,24 \cdot 10^{-12} < E < 2,11 \cdot 10^{-10}$					

Dark Matter Density Function Depending on Gravitational Field as Universal Law - M. Abarca

This DM density function calculated may be compared with DM density calculated by NFW profile which is given by

formula $D_{NFW}(R) = \dfrac{D_0}{X \cdot (1 + X)^2}$ .Calculus has been made in Excel through I.S. of units although finally densities

has been converted to $m\,M_\Theta/pc^3$ as you can see in table below.

Radius	Dens. DM Potential fit	Dens. DM NFW profile	Relative Error %	E gravt.
Kpc	$m\,M_\Theta/pc^3$	$m\,M_\Theta/pc^3$		m/s^2
2,00E+01	1,16E+00	1,18E+00	2,21E+00	8,66E-11
2,50E+01	6,43E-01	7,00E-01	8,18E+00	6,18E-11
3,00E+01	3,99E-01	4,49E-01	1,11E+01	4,70E-11
3,50E+01	2,67E-01	3,05E-01	1,26E+01	3,73E-11
4,00E+01	1,88E-01	2,17E-01	1,33E+01	3,05E-11
4,50E+01	1,38E-01	1,60E-01	1,35E+01	2,55E-11
5,00E+01	1,05E-01	1,21E-01	1,35E+01	2,18E-11
5,50E+01	8,14E-02	9,39E-02	1,34E+01	1,88E-11
6,00E+01	6,46E-02	7,43E-02	1,31E+01	1,65E-11
6,50E+01	5,22E-02	5,99E-02	1,28E+01	1,46E-11
7,00E+01	4,28E-02	4,89E-02	1,25E+01	1,30E-11
7,50E+01	3,55E-02	4,05E-02	1,22E+01	1,17E-11
8,00E+01	2,99E-02	3,39E-02	1,19E+01	1,06E-11
8,50E+01	2,53E-02	2,86E-02	1,15E+01	9,63E-12
9,00E+01	2,17E-02	2,44E-02	1,12E+01	8,81E-12
9,50E+01	1,87E-02	2,10E-02	1,09E+01	8,09E-12
1,00E+02	1,63E-02	1,82E-02	1,06E+01	7,47E-12
1,05E+02	1,42E-02	1,59E-02	1,03E+01	6,91E-12
1,10E+02	1,25E-02	1,39E-02	1,00E+01	6,42E-12
1,15E+02	1,11E-02	1,23E-02	9,78E+00	5,99E-12
1,20E+02	9,84E-03	1,09E-02	9,53E+00	5,59E-12
1,25E+02	8,79E-03	9,69E-03	9,30E+00	5,24E-12
1,30E+02	7,88E-03	8,66E-03	9,08E+00	4,92E-12
1,35E+02	7,09E-03	7,78E-03	8,87E+00	4,63E-12
1,40E+02	6,40E-03	7,01E-03	8,67E+00	4,37E-12
1,45E+02	5,80E-03	6,34E-03	8,49E+00	4,13E-12
1,50E+02	5,28E-03	5,76E-03	8,31E+00	3,91E-12
1,55E+02	4,81E-03	5,24E-03	8,14E+00	3,71E-12
1,60E+02	4,40E-03	4,78E-03	7,99E+00	3,52E-12
1,65E+02	4,03E-03	4,38E-03	7,84E+00	3,35E-12
1,70E+02	3,71E-03	4,02E-03	7,70E+00	3,19E-12
1,75E+02	3,42E-03	3,69E-03	7,57E+00	3,04E-12
1,80E+02	3,15E-03	3,41E-03	7,44E+00	2,91E-12
1,85E+02	2,92E-03	3,15E-03	7,32E+00	2,78E-12
1,90E+02	2,70E-03	2,91E-03	7,21E+00	2,66E-12
1,95E+02	2,51E-03	2,70E-03	7,11E+00	2,55E-12
2,00E+02	2,33E-03	2,51E-03	7,01E+00	2,45E-12

In my opinion results are a true success for DM density potential function because relative error is mainly bellow 10% despite the experimental error, and other sources of errors such as spherical symmetry hypothesis for Virial Theorem.

I congratulate Mr. Sofue for his excellent measures in Milky Way.
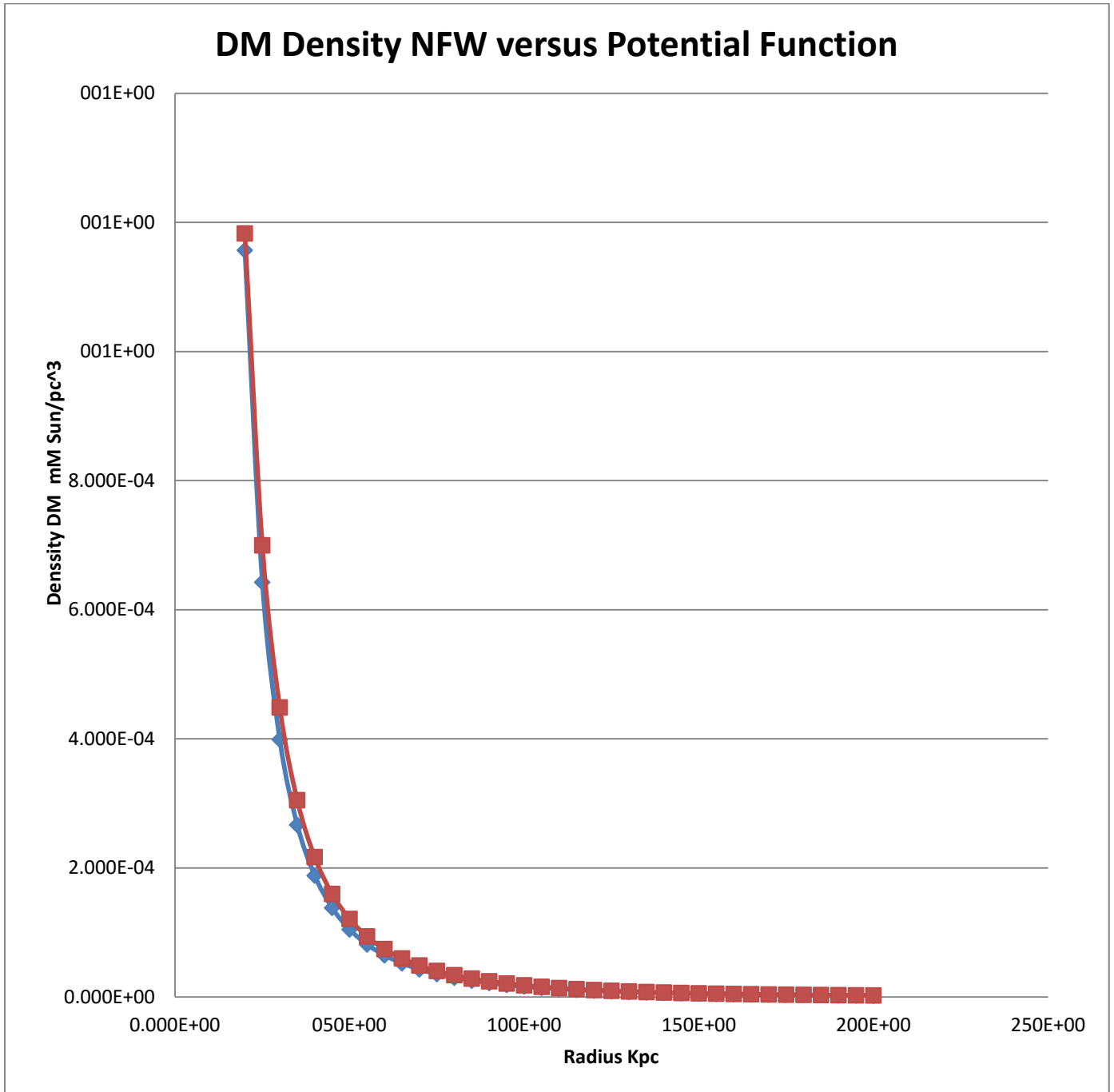
Blue curve belong to potential function and red curve belong to NFW function.

Above are plotted results and I suppose reader is astonished as me because agreement between curves is almost perfecti

DM Density Functions - Detail

DM Density Functions - Detail
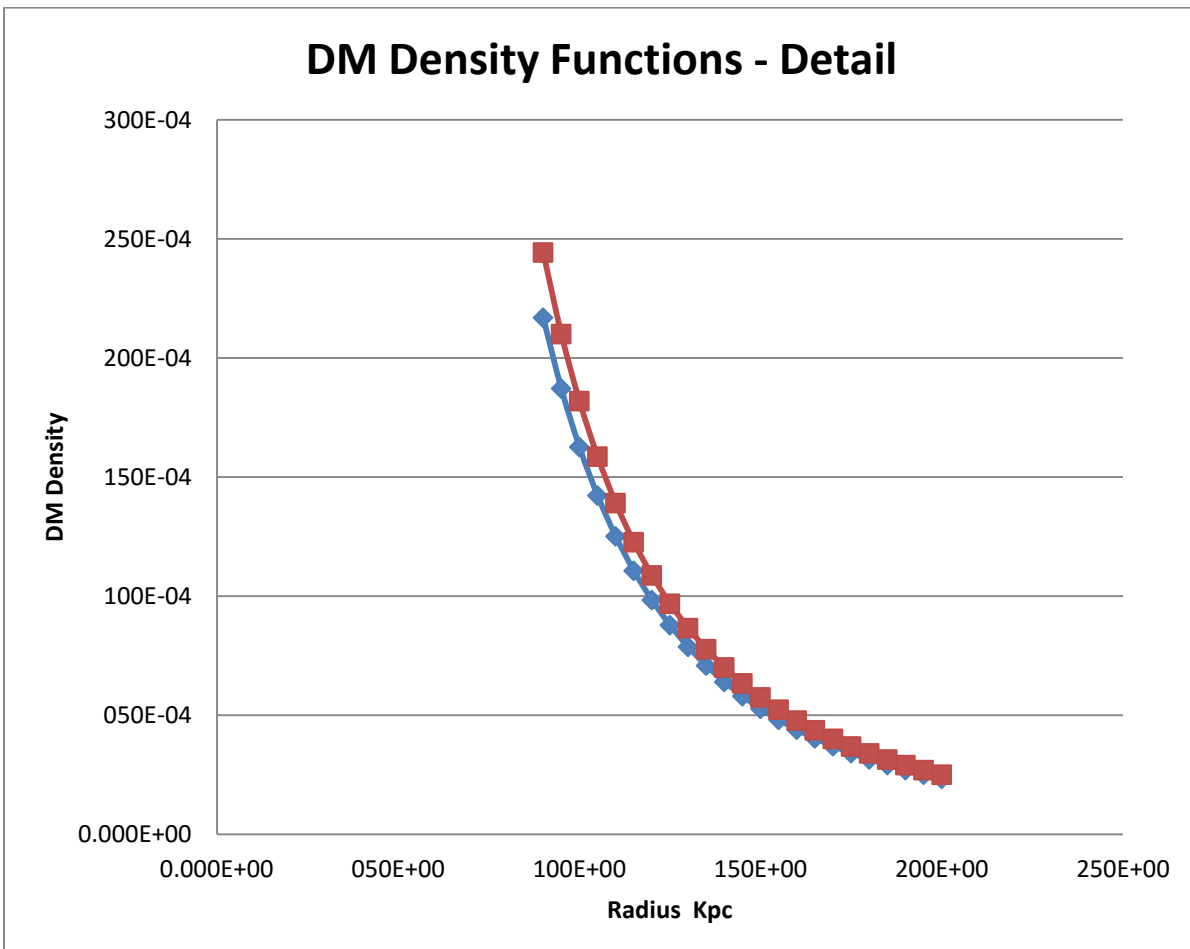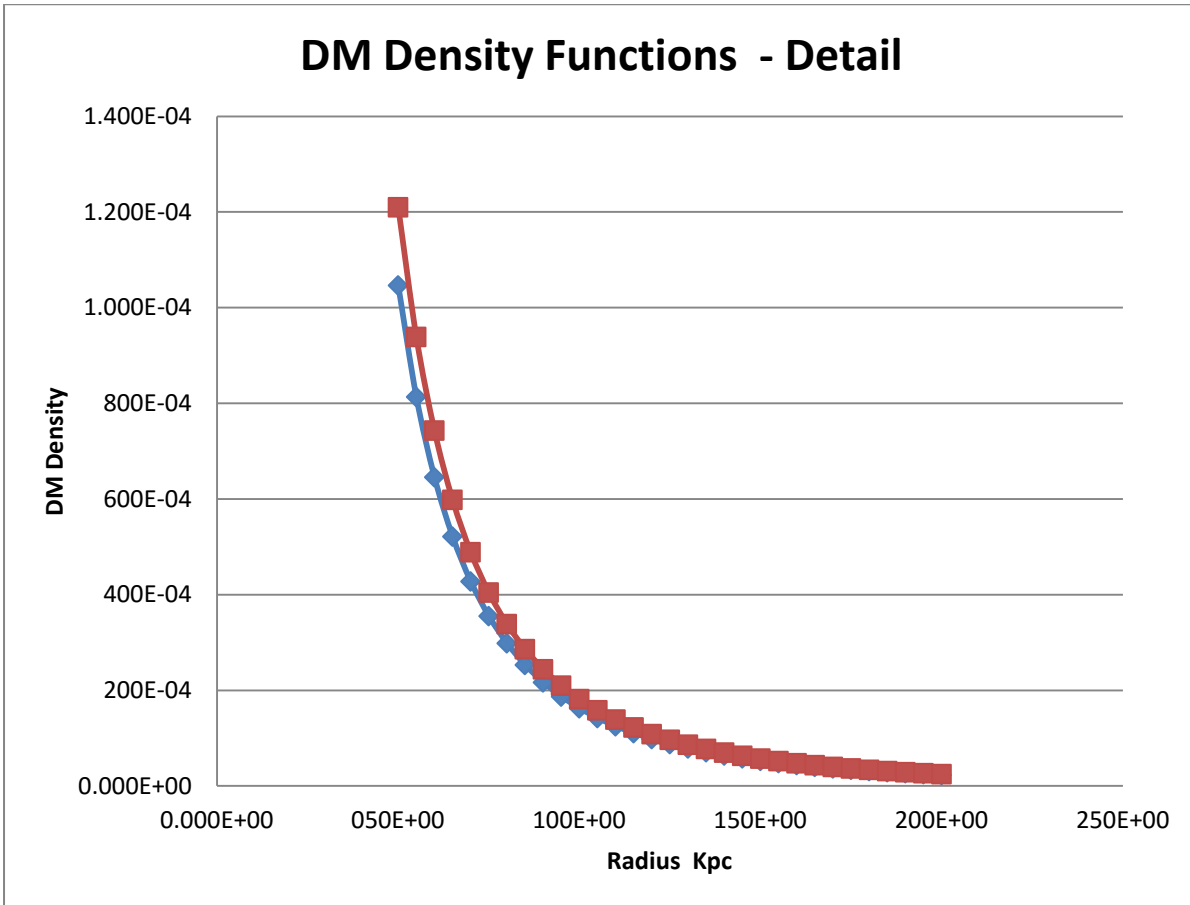
14.2 TEST OF DM DENSITY POTENTIAL FUNCTION IN MILKY WAY HALO UP TO 375 Kpc

Radius	Den. DM Potential fit	Dens. DM NFW profile	Relative Error %	E gravt.
Kpc	$m\,M_\Theta/pc^3$	$m\,M_\Theta/pc^3$		m/s^2
2,00E+01	1,16E+00	1,18E+00	2,21E+00	8,66E-11
3,00E+01	3,99E-01	4,49E-01	1,11E+01	4,70E-11
4,00E+01	1,88E-01	2,17E-01	1,33E+01	3,05E-11
5,00E+01	1,05E-01	1,21E-01	1,35E+01	2,18E-11
6,00E+01	6,46E-02	7,43E-02	1,31E+01	1,65E-11
7,00E+01	4,28E-02	4,89E-02	1,25E+01	1,30E-11
8,00E+01	2,99E-02	3,39E-02	1,19E+01	1,06E-11
9,00E+01	2,17E-02	2,44E-02	1,12E+01	8,81E-12
1,00E+02	1,63E-02	1,82E-02	1,06E+01	7,47E-12
1,10E+02	1,25E-02	1,39E-02	1,00E+01	6,42E-12
1,20E+02	9,84E-03	1,09E-02	9,53E+00	5,59E-12
1,30E+02	7,88E-03	8,66E-03	9,08E+00	4,92E-12
1,40E+02	6,40E-03	7,01E-03	8,67E+00	4,37E-12
1,50E+02	5,28E-03	5,76E-03	8,31E+00	3,91E-12
1,60E+02	4,40E-03	4,78E-03	7,99E+00	3,52E-12
1,70E+02	3,71E-03	4,02E-03	7,70E+00	3,19E-12
1,80E+02	3,15E-03	3,41E-03	7,44E+00	2,91E-12
1,90E+02	2,70E-03	2,91E-03	7,21E+00	2,66E-12
2,00E+02	2,33E-03	2,51E-03	7,01E+00	2,45E-12
2,10E+02	2,03E-03	2,18E-03	6,84E+00	2,26E-12
2,20E+02	1,78E-03	1,90E-03	6,68E+00	2,09E-12
2,30E+02	1,56E-03	1,67E-03	6,55E+00	1,94E-12
2,40E+02	1,38E-03	1,48E-03	6,43E+00	1,81E-12
2,50E+02	1,23E-03	1,31E-03	6,32E+00	1,69E-12
2,60E+02	1,10E-03	1,17E-03	6,24E+00	1,59E-12
2,70E+02	9,83E-04	1,05E-03	6,16E+00	1,49E-12
2,80E+02	8,85E-04	9,42E-04	6,10E+00	1,40E-12
2,90E+02	7,99E-04	8,50E-04	6,04E+00	1,32E-12
3,00E+02	7,24E-04	7,70E-04	6,00E+00	1,25E-12
3,10E+02	6,58E-04	6,99E-04	5,97E+00	1,18E-12
3,20E+02	5,99E-04	6,37E-04	5,94E+00	1,12E-12
3,30E+02	5,48E-04	5,82E-04	5,93E+00	1,06E-12
3,40E+02	5,02E-04	5,33E-04	5,92E+00	1,01E-12
3,50E+02	4,61E-04	4,90E-04	5,91E+00	9,63E-13
3,60E+02	4,24E-04	4,51E-04	5,92E+00	9,18E-13
3,70E+02	3,91E-04	4,16E-04	5,92E+00	8,76E-13
3,80E+02	3,62E-04	3,84E-04	5,94E+00	8,38E-13

According theory proposed by the author, Milky Way halo extend to half distance to M31 which is 375 Kpc.
Below are tabulated and plotted results up to 375 Kpc.

As reader can check relative error goes continuously decreasing up to 5% .

Particularly relative error goes from 7% at R= 200 Kpc to 5% at R = 375 Kpc which is an excellent result because potential function was fitted for E inside dominion   100 < E < 6500 which correspond to radius lower than 170 Kpc and potential function fitted has an excellent agreement with NFW profile out of dominion where that function was fitted to a point cloud. See table in page 33.

As reader can check in chapter 8 dedicated to Milky Way experimental data of rotational curve for radius bigger than 100 Kpc have an enormous error, therefore relative error under 10% is a fantastic result ¡

DM Density NFW versus Potential function
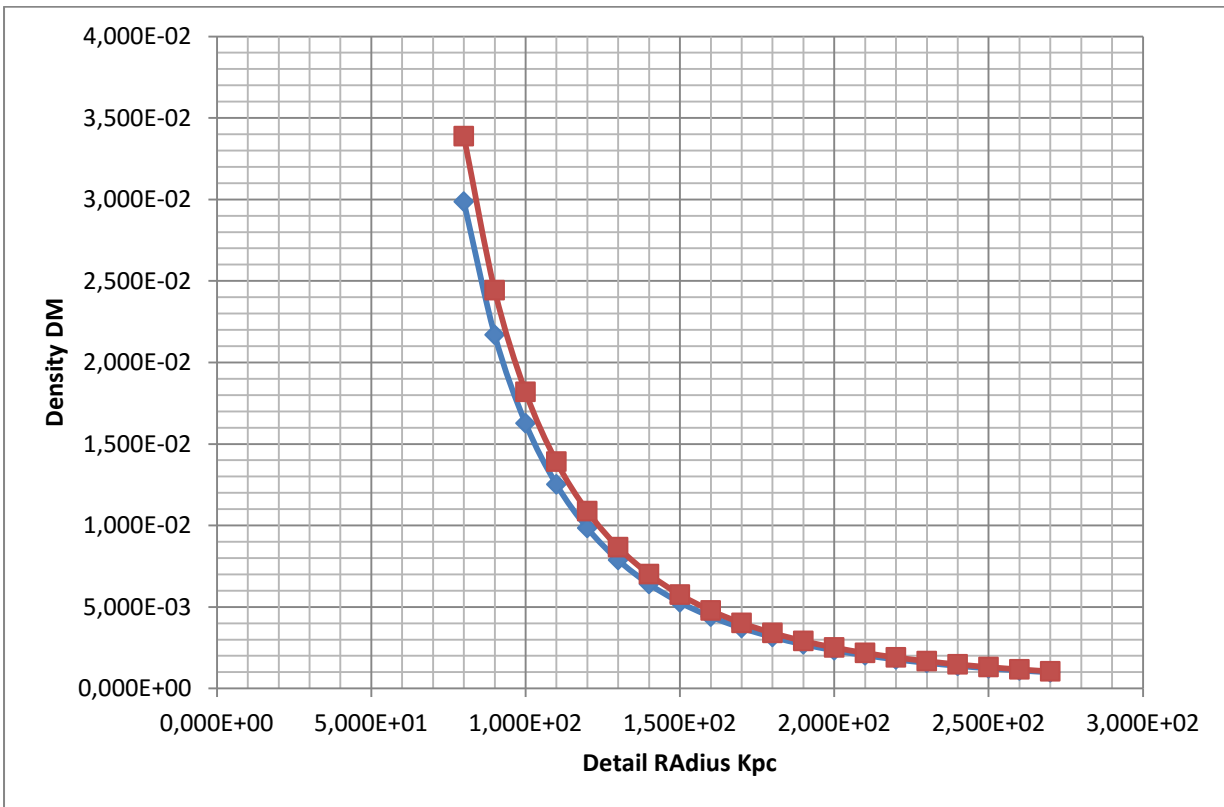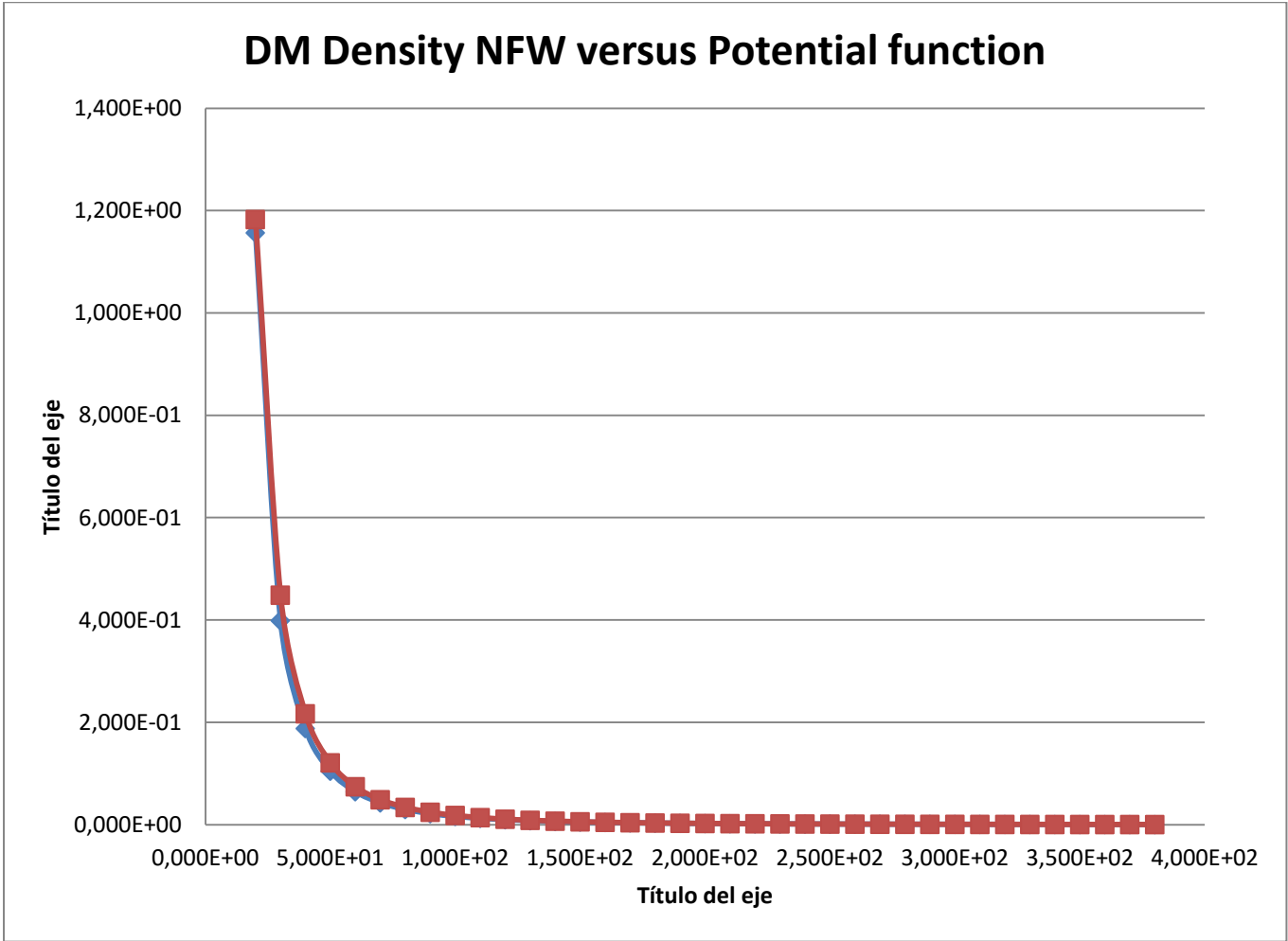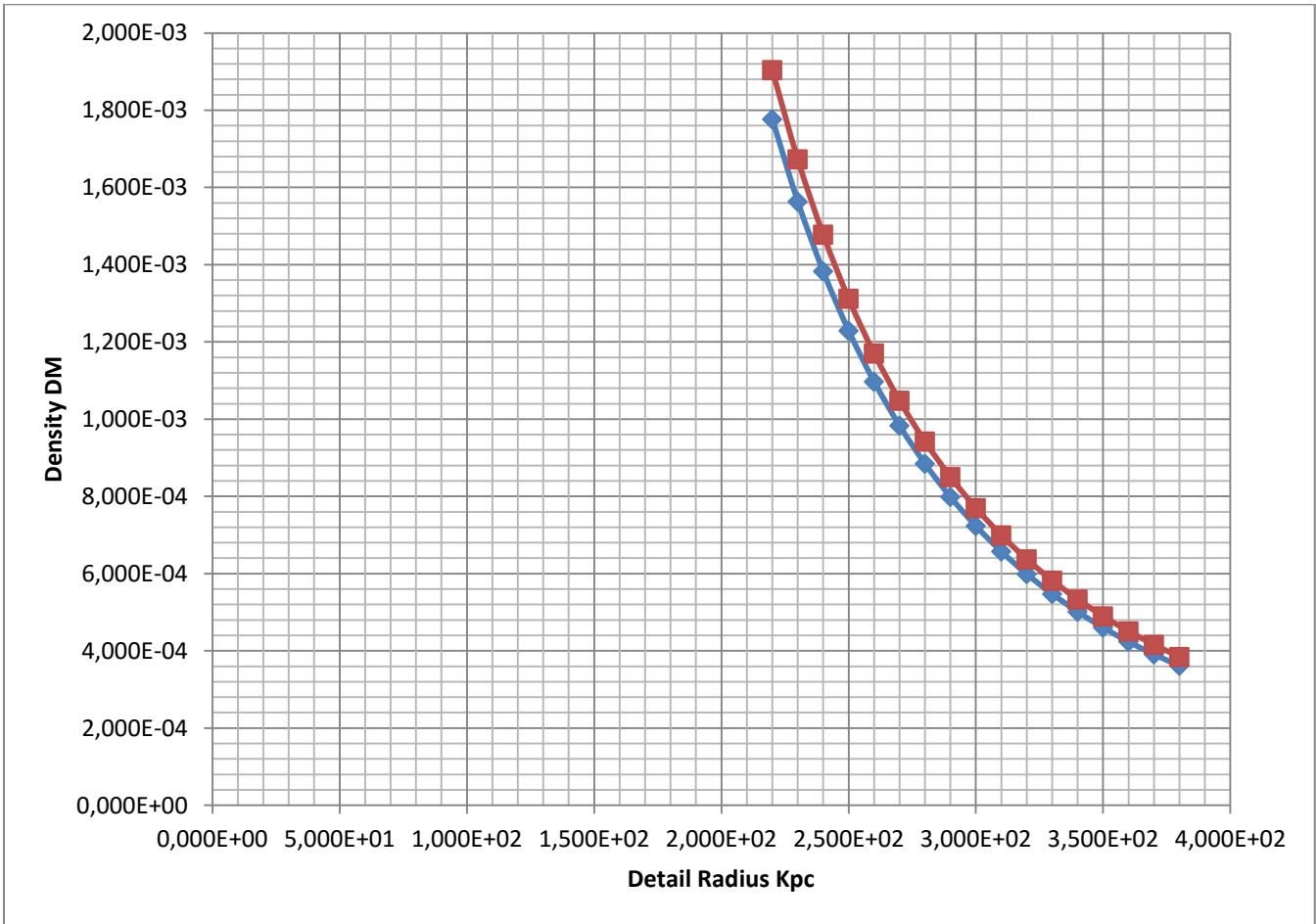
15. TEST OF DM DENSITY POTENTIAL FUNCTION IN M31 HALO

In this chapter it will be compared DM density potential function with DM density NFW according experimental data

Published by [1] Sofue,Y.2015 for M31.

Data from [1] Sofue,Y.2015 for M31

Baryonic mass $M_{BA}= 16,1 \cdot 10^{10}$ M_{Θ}

Parameters of NFW profile of DM model   h= 34,6 $\pm$ 2,1 Kpc

$D_0 = 2,23 \pm 0,24$ m M_{Θ}/pc^3

Supposing a good approximation that baryonic mass in M31 is enclosed inside a radius of 35 Kpc, it is possible to calculate M_{TOTAL}(<R) for radius bigger than 35 Kpc adding Dark matter to baryonic mass of M31.

According NFW- DM density model it is known that M_{DM}(<R)= $4 \cdot \pi \cdot D_0 \cdot h^3 \left(\ln(1+x) - \dfrac{x}{1+x} \right)$ where   x= R/h

and D_0 and h are parameters calculated by Mr. Sofue for M31.

Therefore M_{TOTAL}(<R) = M_{BA} + M_{DM}(<R)   when   35 < R < 375 Kpc and it is right to calculate gravitational

field $E = \dfrac{GM(<R)}{R^2}$.

Knowing E in I.S. of units it is right to calculate DM density through potential function postulated in this paper.

Dark matter density function as Universal law for big galaxies - Hybrid System of Units- $m M_{\Theta}$/pc^3						
Density$_{DM}$ = 3,7323 $\cdot$ 10^{17} $\cdot$ E^B where B= 1,74						
Where Unit for Density $_{D.M.}$ is $m M_{\Theta}$/pc^3= 10^{-3} M_{Θ}/pc^3 and Unit for E is m/s^2						
E paper Units	100	500	1500	3000	4500	6500
E gravt. I.S.	3,24·10^{-12}	1,62·10^{-11}	4,86·10^{-11}	9,72·10^{-11}	1,458·10^{-10}	2,106·10^{-10}
Density$_{DM}$ I.S.	2,58·10^{-25}	4,24·10^{-24}	2,86·10^{-23}	9,57·10^{-23}	1,94·10^{-22}	3,67·10^{-22}
Den.$_{DM}$ $m M_{\Theta}$/pc^3	3,81·10^{-3}	0,0626	0,422	1,414	2,866	5,42
Dominion	3,24 $\cdot$ 10^{-12} < E < 2,11·10^{-10}					

This DM density function calculated may be compared with DM density calculated by NFW profile which is given by

formula $D_{NFW}(R) = \dfrac{D_0}{X \cdot (1+X)^2}$

Calculus has been made in Excel through I.S. of units although finally densities has been converted to $m M_{\Theta}$/pc^3 as you can see in table below.

The last column shows relative error, which at the begin is 34%, too high although it is going decreasing progressively. When radius is upper 200 Kpc relative error is under 10%.

However in Milky Way relative error was under 10% almost in the whole dominion.

I have checked that increasing baryonic mass relative error decrease in the whole dominion. Perhaps futures measures of M31 could support a higher baryonic mass than current value.

By other side perhaps little variation of parameters A= 2,526 · 10^{-5} (I.S.) and B= 1,74 could improve the agreement between NFW density profile and Density as a potential function of field.

In addition $D_0 = 2,23 \pm 0,24$ m M_Θ/pc^3 has a 10% of error and h= 34,6 $\pm 2,1$ Kpc has an error of 6% therefore in my opinion, the agreement between both curves is enough good to confirm that potential function postulated in this paper is supported by experimental data in M31.

M31 Galaxy Radius Kpc	Dens. DM Potential fit $m\,M_\Theta/pc^3$	Dens. DM NFW profile $m\,M_\Theta/pc^3$	Relative Error %	E gravt. m/s^2
3,50E+01	3,56E-01	5,45E-01	3,47E+01	4,40E-11
4,50E+01	2,08E-01	3,24E-01	3,58E+01	3,23E-11
5,50E+01	1,36E-01	2,09E-01	3,48E+01	2,53E-11
6,50E+01	9,59E-02	1,43E-01	3,31E+01	2,07E-11
7,50E+01	7,07E-02	1,03E-01	3,10E+01	1,74E-11
8,50E+01	5,40E-02	7,60E-02	2,89E+01	1,49E-11
9,50E+01	4,24E-02	5,79E-02	2,68E+01	1,30E-11
1,05E+02	3,40E-02	4,51E-02	2,47E+01	1,14E-11
1,15E+02	2,77E-02	3,59E-02	2,27E+01	1,02E-11
1,25E+02	2,30E-02	2,90E-02	2,08E+01	9,11E-12
1,35E+02	1,93E-02	2,38E-02	1,90E+01	8,23E-12
1,45E+02	1,63E-02	1,97E-02	1,72E+01	7,49E-12
1,55E+02	1,40E-02	1,66E-02	1,56E+01	6,85E-12
1,65E+02	1,21E-02	1,41E-02	1,41E+01	6,29E-12
1,75E+02	1,05E-02	1,20E-02	1,26E+01	5,81E-12
1,85E+02	9,20E-03	1,04E-02	1,12E+01	5,38E-12
1,95E+02	8,10E-03	8,99E-03	9,86E+00	5,00E-12
2,05E+02	7,17E-03	7,85E-03	8,59E+00	4,67E-12
2,15E+02	6,39E-03	6,90E-03	7,39E+00	4,36E-12
2,25E+02	5,71E-03	6,09E-03	6,24E+00	4,09E-12
2,35E+02	5,13E-03	5,41E-03	5,15E+00	3,85E-12
2,45E+02	4,62E-03	4,82E-03	4,10E+00	3,62E-12
2,55E+02	4,19E-03	4,32E-03	3,10E+00	3,42E-12
2,65E+02	3,80E-03	3,88E-03	2,15E+00	3,24E-12
2,75E+02	3,46E-03	3,50E-03	1,24E+00	3,07E-12
2,85E+02	3,16E-03	3,17E-03	3,64E-01	2,91E-12
2,95E+02	2,90E-03	2,88E-03	-4,73E-01	2,77E-12
3,05E+02	2,66E-03	2,63E-03	-1,28E+00	2,64E-12
3,15E+02	2,45E-03	2,40E-03	-2,05E+00	2,51E-12
3,25E+02	2,26E-03	2,20E-03	-2,79E+00	2,40E-12
3,35E+02	2,09E-03	2,02E-03	-3,50E+00	2,30E-12
3,45E+02	1,94E-03	1,86E-03	-4,18E+00	2,20E-12
3,55E+02	1,80E-03	1,71E-03	-4,84E+00	2,11E-12
3,65E+02	1,67E-03	1,58E-03	-5,47E+00	2,02E-12
3,75E+02	1,56E-03	1,47E-03	-6,08E+00	1,94E-12

As reader can check, in table page 30 data belonging to M31 has radius lower than 300 Kpc. In other words potential function was fitted for a point cloud with radius under 300 Kpc.

However this table show that relative error is under 6% for radius up to 375 Kpc which is an excellent result ¡.

DM Density NFW versus Potential Function in M31
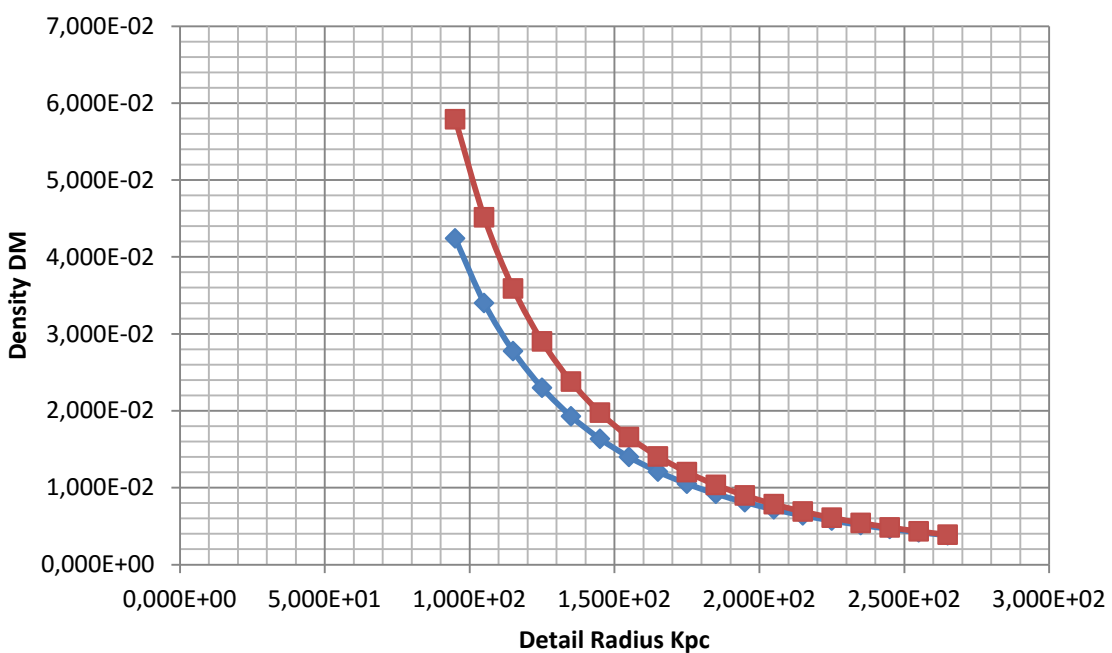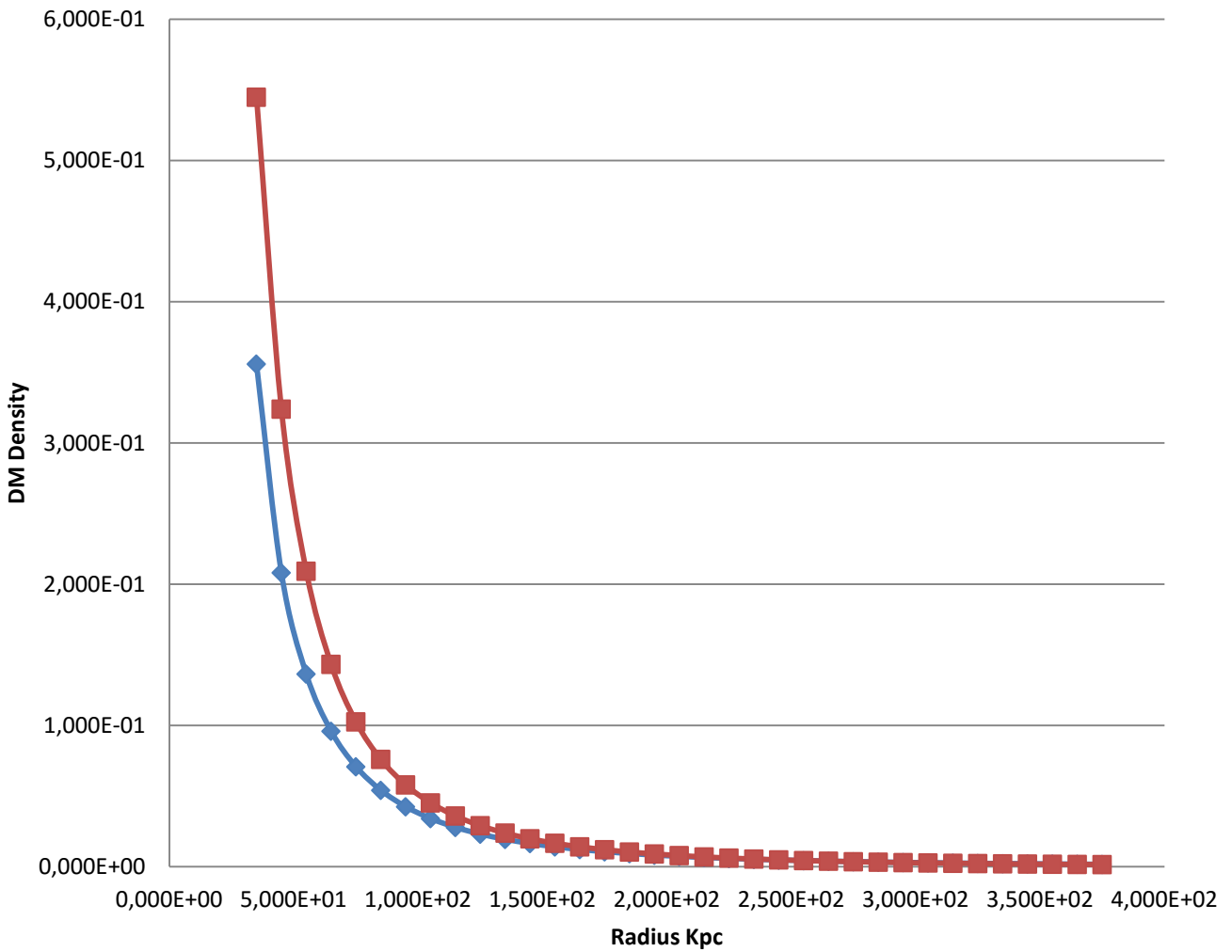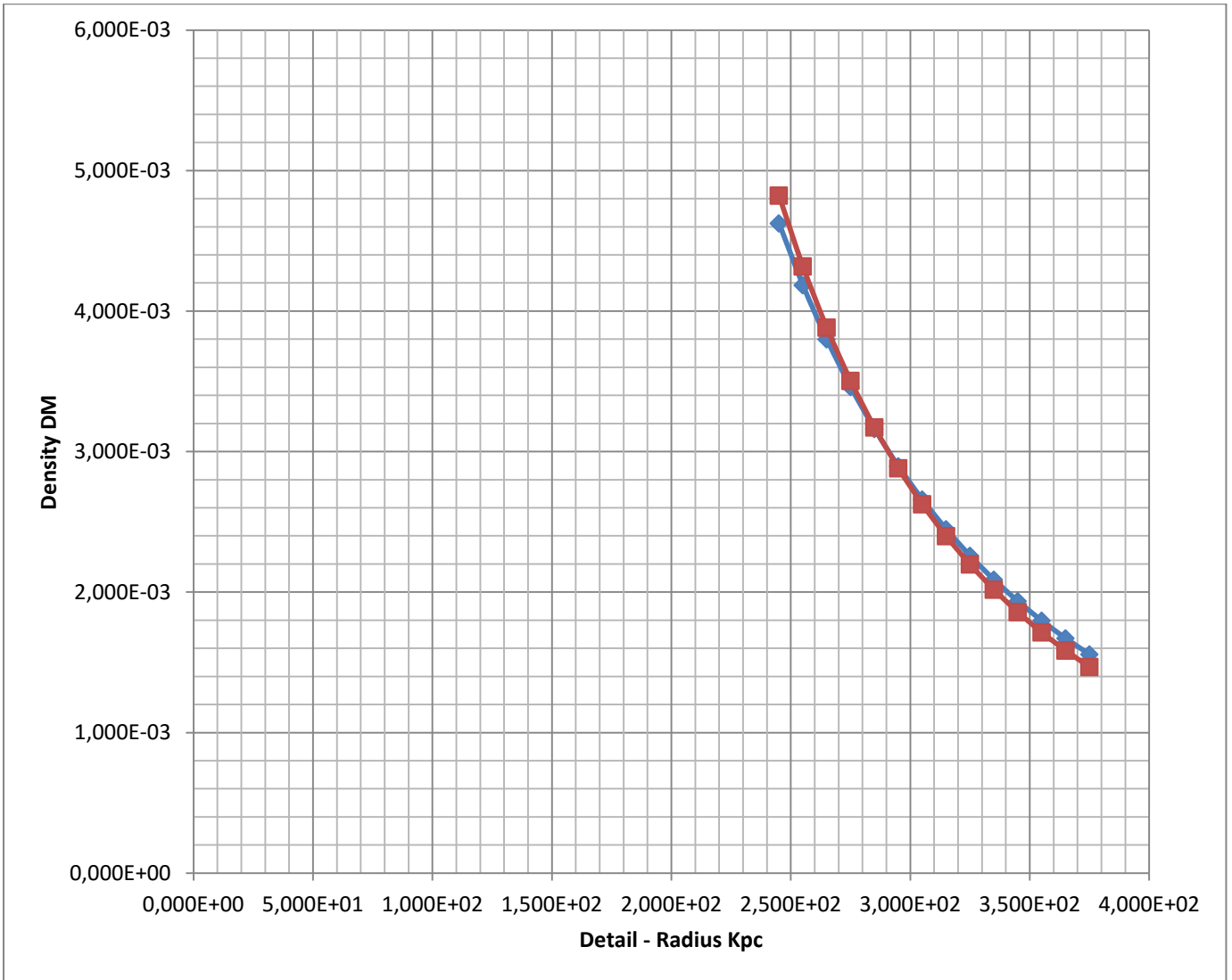
16. EQUIVALENCE BETWEEN POTENTIAL DENSITY FUNCTION AND DENSITY NFW

This epigraph is going to demonstrate that potential density is equivalent to density NFW. Results are showed in tables for Milky Way and M31. The method followed to demonstrate the equivalence is:

- To calculate total gravitational field depending on radius, and taking into account baryonic mass.

- To substitute E in formula Density $_{DM}$ = A $\cdot$ E^B by its function depending on x = Radius/ h

- To tabulate both functions depending on x and calculate relative error between both formulas for each radius.

16.1 EQUIVALENCE BETWEEN POTENTIAL DENSITY FUNCTION AND DENSITY NFW IN MILKY WAY HALO

In this chapter it will be compared DM density potential function with DM density NFW according experimental data

Published by [1] Sofue,Y.2015 for Milky Way.

Data from [1] Sofue,Y.2015 for Milky Way
Baryonic mass M_{BA}= 13,7 $\cdot 10^{10}$ M_{Θ} Parameters of NFW profile of DM model   h= 10,7 $\pm$ 2,9 Kpc   and D_0 = 18,2 $\pm$ 7,4 m M_{Θ} /pc^3
Astronomical data
M_{Θ} = 1,989 $\cdot 10^{30}$ Kg   and   m M_{Θ} = 1,989 $\cdot 10^{27}$ Kg   1 pc = 3,085$\cdot 10^{16}$ m
Density conversion units
1 m M_{Θ} /pc^3 = 6,768$\cdot 10^{-23}$ Kg/m^3

Supposing as good that baryonic mass in Milky Way is enclosed inside a radius of 21,4 Kpc. As such distance is equivalent to   x = 21,4/ h = 2 this study will be made inside dominion 21,4 / h < x < 385Kpc /h → 2< x < 36

According NFW DM density model it is known that $M_{DM}(<R)$= $4 \cdot \pi \cdot D_0 \cdot h^3 \cdot f(x)$ where

$$f(x) = \left(\ln(1 + x) - \frac{x}{1 + x} \right) \text{ and } x= \text{Radius}/ h.$$

$$E_{TOTAL} = \frac{GM_{BARYONIC}(< R) + GM_{DARK\ M}(< R)}{R^2} = E_{BARYONIC} + E_{DARK\ M}$$

$$E_{DARK\ M} = \frac{4\pi G D_0 \cdot h^3 / h^2}{R^2 / h^2} \cdot f(x) = K_{DARK\ M} \frac{f(x)}{x^2} \text{ Where } K_{DARK\ M} = 4\pi G D_0 h$$

for Milky Way $K_{DARK\ M} = 3{,}407 \cdot 10^{-10} m/s^2$

$$E_{BARYONIC} = \frac{GM_B / h^2}{R^2 / h^2} = \frac{K_B}{x^2} \text{ where } K_B = GM_B / h^2 \text{ so } K_B = 1{,}6688 \cdot 10^{-10} \text{ m/s}^2 \text{ and so } K_B = 0{,}4898\ K_{DARK\ M}$$

Therefore $E_{TOTAL} = K_{DARK\ M} \dfrac{f(x)}{x^2} + \dfrac{K_B}{x^2} = K_{DARK\ M} \dfrac{0{,}4898 + f(x)}{x^2} = K_{DM} \cdot g(x)$ where $g(x) = \dfrac{0{,}4898 + f(x)}{x^2}$.

Using formula got in epigraph 11.2

Dark matter density function as Universal law for big galaxies – International System of Units
Density $_{DM}$ = A $\cdot$ E^B where A=2,526 $\cdot$ 10^{-5} and B= 1,74
Where Unit for Density $_{D.M.}$ is Kg / m^3 and Unit for E is m/s^2

Density $_{DM}$ (R) = A · E^B = $A·K_{DM}^{1,74}·[g(x)]^{1,74}$ Where $= A·K_{DM}^{1,74} = \aleph_{MW} = 8,500·10^{-22}\ Kg/m^3$

This DM density function calculated may be compared with DM density calculated by NFW profile which is given by

formula $D_{NFW}(R) = \dfrac{D_0}{X·(1+X)^2}$. Where $D_0 = 18,2$ m$M_\Theta/pc^3 = 12,3·10^{-22}$ Kg/m^3

$A·K_{DM}^{1,74} = \aleph_{MW} ==8,500·10^{-22}$ Kg/m^3 . $\aleph_{MW}$ is similar to $D_0 = 18,2$ m$M_\Theta/pc^3 = 12,3·10^{-22}$ Kg/m^3.
Exactly $D_0 = 1,4797· \aleph_{MW}$

In order to compare both Density functions Density $_{DM}$ (R) = A · E^B may written this way

Density $_{DM}$ (R) $= \aleph_{MW} \dfrac{1,4797}{1,4797}[g(x)]^{1,74} = D_0·\dfrac{1}{1,4797}·[g(x)]^{1,74} = D_0·S(x)$ where $S(x) = \dfrac{1}{1,4797}·[g(x)]^{1,74}$

Therefore S(x) should be equivalent to $R(x) = \dfrac{1}{X·(1+X)^2}$.

In table below will be compared both functions, R(x) and S(x), but previously are written the auxiliary functions needed to calculate densities.

$f(x) = \left(\ln(1+x) - \dfrac{x}{1+x}\right)$; $g(x) = \dfrac{0,4898+ f(x)}{x^2}$; $[g(x)]^{1,74}$; $R(x) = \dfrac{1}{X·(1+X)^2}$; $S(x) = \dfrac{1}{1,4797}·[g(x)]^{1,74}$

Density $_{DM}$ = A · E^B written as Density $_{DM}$ =D_0· S(x) is almost $D_{NFW}(R) = \dfrac{D_0}{X·(1+X)^2}$ as you can see in table

below. This table is very similar to table showed in 14.1.

Radius	x	R(x)	S(x)	Relt. Error	D - NFW	D-Pot E
Kpc				%	mM_Θ/pc^3	mM_Θ/pc^3
2,14E+01	2,00E+00	5,56E-02	0,05555556	7,31E+00	1,01E+00	9,37E-01
2,35E+01	2,20E+00	4,44E-02	0,0443892	9,67E+00	8,08E-01	7,30E-01
2,57E+01	2,40E+00	3,60E-02	0,03604383	1,14E+01	6,56E-01	5,81E-01
2,78E+01	2,60E+00	2,97E-02	0,02967711	1,27E+01	5,40E-01	4,72E-01
3,00E+01	2,80E+00	2,47E-02	0,02473288	1,36E+01	4,50E-01	3,89E-01
3,21E+01	3,00E+00	2,08E-02	0,02083333	1,43E+01	3,79E-01	3,25E-01
5,35E+01	5,00E+00	5,56E-03	0,00555556	1,55E+01	1,01E-01	8,54E-02
7,49E+01	7,00E+00	2,23E-03	0,00223214	1,42E+01	4,06E-02	3,49E-02
9,63E+01	9,00E+00	1,11E-03	0,00111111	1,27E+01	2,02E-02	1,77E-02
1,18E+02	1,10E+01	6,31E-04	0,00063131	1,15E+01	1,15E-02	1,02E-02
1,39E+02	1,30E+01	3,92E-04	0,00039246	1,05E+01	7,14E-03	6,39E-03
1,61E+02	1,50E+01	2,60E-04	0,00026042	9,77E+00	4,74E-03	4,28E-03
1,82E+02	1,70E+01	1,82E-04	0,00018155	9,17E+00	3,30E-03	3,00E-03
2,03E+02	1,90E+01	1,32E-04	0,00013158	8,71E+00	2,39E-03	2,19E-03
2,25E+02	2,10E+01	9,84E-05	9,8386E-05	8,36E+00	1,79E-03	1,64E-03
2,46E+02	2,30E+01	7,55E-05	7,5483E-05	8,09E+00	1,37E-03	1,26E-03
2,68E+02	2,50E+01	5,92E-05	5,9172E-05	7,89E+00	1,08E-03	9,92E-04

2,89E+02	2,70E+01	4,72E-05	4,7241E-05	7,75E+00	8,60E-04	7,93E-04
3,10E+02	2,90E+01	3,83E-05	3,8314E-05	7,66E+00	6,97E-04	6,44E-04
3,32E+02	3,10E+01	3,15E-05	3,1502E-05	7,60E+00	5,73E-04	5,30E-04
3,53E+02	3,30E+01	2,62E-05	2,6214E-05	7,58E+00	4,77E-04	4,41E-04
3,75E+02	3,50E+01	2,20E-05	2,2046E-05	7,59E+00	4,01E-04	3,71E-04
3,85E+02	3,60E+01	2,03E-05	2,0291E-05	7,60E+00	3,69E-04	3,41E-04

DISCUSSION

This table is very similar to table in epigraph 14.1. especially two last rows where is showed Density $_{DM}$.
However there is an important difference, in this epigraph potential density function which depend on E has suffered
a variable change, and it has been written depending on distance x.
This way has been demonstrated that values of density given by both functions are very similar because relative error
is mainly bellow 10% despite the experimental error, and other sources of errors such as spherical symmetry
hypothesis for Virial Theorem.
I congratulate Mr. Sofue for his excellent measures in Milky Way.

As reader can see maximum error,15%, is at 53 Kpc and error is decreasing under 8% at 385 Kpc. In my opinion this
happen why NFW profile is a better approximation to Dark matter when are considered big radius inside galactic halo.
Supposing that Density dark matter depend on gravitational field is better than NFW, formula Density $_{DM}$ = A · E^{B}
should give more precise results than NFW profile when are considered disk region or halo region near galactic disk.

16.2 EQUIVALENCE BETWEEN POTENTIAL DENSITY FUNCTION AND DENSITY NFW IN M31

Data from [1] Sofue,Y.2015 for M31

Baryonic mass M_{BA}= 16,1·10^{10} M_{Θ}
Parameters of NFW profile of DM model   h= 34,6 $\pm$ 2,1 Kpc
D_0 = 2,23 $\pm$ 0,24 mM_{Θ}/pc^3

Supposing as good that baryonic mass in M31 is enclosed inside a radius of 34,6 Kpc. As such distance is equivalent
to   x = 1 this study will be made inside dominion 34,6 / h < x < 385Kpc /h → 1< x < 11

According NFW DM density model it is known that $M_{DM}(<R)$= $4 \cdot \pi \cdot D_0 \cdot h^3 \cdot f(x)$ where

$$f(x) = \left(\ln(1+x) - \frac{x}{1+x} \right) \text{ and } x= \text{Radius/ h.}$$

$$E_{TOTAL} = \frac{GM_{BARYONIC}(<R) + GM_{DARK\,M}(<R)}{R^2} = E_{BARYONIC} + E_{DARK\,M}$$

$$E_{DARK\,M} = \frac{4\pi GD_0 \cdot h^3/h^2}{R^2/h^2} \cdot f(x) = K_{DARK\,M} \frac{f(x)}{x^2} \text{ Where } K_{DARK\,M} = 4\pi GD_0 h \text{ so } K_{DARK\,M} = 1,35 \cdot 10^{-10} m/s^2$$

$$E_{BARYONIC} = \frac{GM_B/h^2}{R^2/h^2} = \frac{K_B}{x^2} \text{ where } K_B = GM_B/h^2 \text{ so } K_B = 0,1875 \cdot 10^{-10} \text{ m/s}^2 \text{ and so } K_B = 0,13888 \ K_{DARK\,M}$$

Therefore $E_{TOTAL} = K_{DARK\,M} \dfrac{f(x)}{x^2} + \dfrac{K_B}{x^2} = K_{DARK\,M} \cdot \dfrac{0,1388+f(x)}{x^2} = K_{DM} \cdot g(x) \text{ where } g(x) = \dfrac{0,1388+f(x)}{x^2}$.

Using formula got in epigraph 11.2

Dark matter density function as Universal law for big galaxies – International System of Units
Density $_{DM}$ = A · E^B where A=2,526 · 10^{-5} and B= 1,74
Where Unit for Density $_{D.M.}$ is Kg / m^3 and Unit for E is m/s^2

Density $_{DM}$ = A · E^B = $A·K_{DM}^{1,74}·[g(x)]^{1,74}$ Where = $A·K_{DM}^{1,74} = \aleph_{M31} = 1,695 10^{-22}\ Kg/m^3$

$A·K_{DM}^{1,74} = \aleph_{M31} == 1,695 ·10^{-22}$ Kg/m^3. $\aleph_{M31}$ is similar to $D_0 = 2,23$ mM_Θ/pc^3 = $1,509 ·10^{-22}$ Kg/m^3.
Exactly $\aleph_{M31} = 1,123·$ D$_0$ and D$_0$= $\aleph_{M31}/1,123$

This DM density function calculated may be compared with DM density calculated by NFW profile which is given by

formula $D_{NFW}(R) = \dfrac{D_0}{X·(1+X)^2}$

In order to compare both Density functions Density $_{DM}$ = A · E^B may written this way

Density $_{DM} = \aleph_{M31}·\dfrac{1,123}{1,123}[g(x)]^{1,74} = D_0·1,123[g(x)]^{1,74} = D_0·S(x)$ where $S(x) = 1,123[g(x)]^{1,74}$

Therefore S(x) should be equivalent to $R(x) = \dfrac{1}{X·(1+X)^2}$. In order to compare both Density functions it is written

functions needed to calculate densities.

$f(x) = \left(\ln(1+x) - \dfrac{x}{1+x}\right)$;$g(x) = \dfrac{0,1388+f(x)}{x^2}$; [g(x)]^{1,74} ; $S(x) = 1,123[g(x)]^{1,74}$; $R(x) = \dfrac{1}{X·(1+X)^2}$

Radius	x	S(x)	R(x)	Relt. Error	D-NFW	D-Pot E
Kpc				%	mM_Θ/pc^3	mM_Θ/pc^3
3,46E+01	1,00E+00	1,65E-01	0,25	3,41E+01	5,58E-01	3,68E-01
4,33E+01	1,25E+00	1,02E-01	0,15802469	3,53E+01	3,52E-01	2,28E-01
5,19E+01	1,50E+00	6,96E-02	0,10666667	3,47E+01	2,38E-01	1,55E-01
6,06E+01	1,75E+00	5,03E-02	0,0755608	3,34E+01	1,69E-01	1,12E-01
6,92E+01	2,00E+00	3,79E-02	0,05555556	3,17E+01	1,24E-01	8,46E-02
7,79E+01	2,25E+00	2,95E-02	0,04207758	2,99E+01	9,38E-02	6,58E-02
8,65E+01	2,50E+00	2,35E-02	0,03265306	2,80E+01	7,28E-02	5,24E-02
9,52E+01	2,75E+00	1,91E-02	0,02585859	2,62E+01	5,77E-02	4,26E-02
1,04E+02	3,00E+00	1,58E-02	0,02083333	2,44E+01	4,65E-02	3,51E-02
1,12E+02	3,25E+00	1,32E-02	0,01703487	2,26E+01	3,80E-02	2,94E-02
1,21E+02	3,50E+00	1,12E-02	0,01410935	2,09E+01	3,15E-02	2,49E-02
1,30E+02	3,75E+00	9,54E-03	0,01181902	1,93E+01	2,64E-02	2,13E-02
1,38E+02	4,00E+00	8,22E-03	0,01	1,78E+01	2,23E-02	1,83E-02
1,47E+02	4,25E+00	7,15E-03	0,00853675	1,63E+01	1,90E-02	1,59E-02
1,56E+02	4,50E+00	6,25E-03	0,00734619	1,49E+01	1,64E-02	1,39E-02
1,64E+02	4,75E+00	5,51E-03	0,00636753	1,35E+01	1,42E-02	1,23E-02
1,73E+02	5,00E+00	4,88E-03	0,00555556	1,22E+01	1,24E-02	1,09E-02
1,90E+02	5,50E+00	3,88E-03	0,00430339	9,81E+00	9,60E-03	8,66E-03

2,08E+02	6,00E+00	3,14E-03	0,00340136	7,59E+00	7,59E-03	7,01E-03
2,25E+02	6,50E+00	2,58E-03	0,00273504	5,56E+00	6,10E-03	5,76E-03
2,42E+02	7,00E+00	2,15E-03	0,00223214	3,68E+00	4,98E-03	4,79E-03
2,60E+02	7,50E+00	1,81E-03	0,00184544	1,95E+00	4,12E-03	4,04E-03
2,77E+02	8,00E+00	1,54E-03	0,00154321	3,48E-01	3,44E-03	3,43E-03
2,94E+02	8,50E+00	1,32E-03	0,00130357	-1,14E+00	2,91E-03	2,94E-03
3,11E+02	9,00E+00	1,14E-03	0,00111111	-2,52E+00	2,48E-03	2,54E-03
3,29E+02	9,50E+00	9,91E-04	0,00095477	-3,81E+00	2,13E-03	2,21E-03
3,46E+02	1,00E+01	8,68E-04	0,00082645	-5,02E+00	1,84E-03	1,94E-03
3,63E+02	1,05E+01	7,64E-04	0,00072014	-6,14E+00	1,61E-03	1,70E-03
3,81E+02	1,10E+01	6,77E-04	0,00063131	-7,20E+00	1,41E-03	1,51E-03

DISCUSSION

This table is very similar to table in epigraph 15, especially two last rows where is showed Density $_{DM}$.
However there is an important difference, in this epigraph potential function which depend on E has suffered a variable change, and it has been written depending on distance x.

Relative error, 35%, is maximum at 43 Kpc but decreases continuously until 275 Kpc where is almost cero. For bigger distances error increase up to 7 % at 385 Kpc.

It is obvious that relative error in M31 is bigger than Milky Way. However it is important to consider that error decrease with radius. Error is under 25% for radius bigger than 100 Kpc and is under 15% for radius bigger than 150 Kpc. Taking into account that NFW is more precise for big radius it is right to conclude that results got in this epigraph support that Dark matter Density depending on gravitational field agree with NFW profile in M31 in region where NFW is more reliable.

It is important to remember that Potential Density has Universal parameters whereas NFW has specific parameters each galaxy.

Taking into account that Potential density parameters have been got as an average parameters from six galaxies, these parameters should be considered as "approximate" parameters.

I think that potential density should be checked in other galaxies in order to declare Potential Density of Dark matter a true Universal law.

17. BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN MILKY WAY AND M31 HALOS

Formula got in chapter 11 allow   to get quickly a Bernoulli differential equation for gravitational field in halo region of a galaxy.

Dark matter density function as Universal law for big galaxies – International System of Units
Density $_{DM}$ = 2,526 · 10^{-5} · E^B where B= 1,74
Where Unit for Density $_{D.M.}$ is Kg / m^3 and Unit for E is m/s^2
Dominion   3,24 · 10 $^{-12}$ < E < 2,11·10^{-10}

In halo region it is a good approximation to consider cero to density of baryonic matter so it is very easy differentiates gravitational field. In the following paragraphs it will been got Bernoulli equation for Milky Way and M31.

Also it will been solved to get the gravitational field and so to calculate density of dark matter function depending on radius. Thanks to get formula of density of dark matter depending on radius it will be possible to compare with NFW density of dark matter. As it will be shown at the end of paragraphs, relative differences between both function are below 13% in Milky Way and below 16% in M31 galaxy through their dominions.

17.1 BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN MILKY WAY HALO

It will be considered outskirt of Milky way, the region where density of baryonic matter is negligible. 21 Kpc as radius may be a good approximation for baryonic size our galaxy. So for radius bigger than 21 Kpc, Dark matter is the only kind of matter which exits in a significant amount.

As it is known in this formula $E = G\dfrac{M(r)}{r^2}$, M(r) represents mass enclosed by a sphere with radius r. Now it will

differentiated E(r) when r > 21 Kpc

If $E = G\dfrac{M(r)}{r^2}$ is differentiated it is got $E'(r) = G\dfrac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

As $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ due to baryonic matter is negligible in this region, it is right to get

$E'(r) = 4\pi G\varphi(r) - 2G\dfrac{M(r)}{r3}$ As $\varphi_{DM}(r) = A\cdot E^{1,74}(r)$ it is right to get $E'(r) = 4\pi\cdot G\cdot A\cdot E^{1,74}(r) - 2\dfrac{E(r)}{r}$ which
is a Bernoulli differential equation.

As parameter $A = 2,526\,10^{-5}$ .I.S.   parameter $K = 4\pi\cdot G\cdot A = 2,147\cdot 10^{-14}$ I.S.

Calling *y* to E, the differential equation is written this way $y` = K\cdot y^{1,74} - \dfrac{2\cdot y}{x}$

Bernoulli family equations $y` = K\cdot y^n - \dfrac{2\cdot y}{x}$ may be converted into a differential linear equation with this

variable change $u = y^{1-n}$.

This is the general solution $E(r) = \left(Cr^{2n-2} + \dfrac{Kr(1-n)}{3-2n} \right)^{\frac{1}{1-n}}$ $with\ n \neq 1\ and\ n \neq 3/2$

For n= 1,74 formula above is reduced to $E(r) = \left(Cr^{1,48} + Dr\right)^{-1,35135}$ $where$ $D = 3,3099410^{-14}$ and C may be calculated by initial condition of gravitational field at a specific radius.

Calculus of parameter C

Calculus will be made four times for R=21,4 Kpc. R= 50 Kpc. R= 200 Kpc. and R= 385 Kpc. Data come from Sofue 2015, which use NFW to calculate density of dark matter and total masses.

According this author total baryonic mass is $M_B = 1,37 \cdot 10^{11}$ M_Θ and dark matter inside a sphere with radius R is calculated by this formula $M_{DM}(<R) = K \cdot (\ln(1+x) - x / (1+x))$ Where K= $2,8 \cdot 10^{11}$ M_Θ. x = R/ h . h= 10,7 Kpc.

Total mass inside a sphere with radius R > 21,4 Kpc is got adding total baryonic mass to Dark matter included in the sphere whose radius is R

Calculus has been made carefully and all magnitudes has been measured into International System (I.S.).

For R= 21,4 Kpc. $M_{TOTAL}(R< 21,4$ Kpc$) = 2,58 \cdot 10^{11}$ M_Θ .Gravitational field is E(r=21,4 Kpc) = $7,857 \cdot 10^{-11}$ m/s^2 and parameter is C= $1,2571 \cdot 10^{-24}$ (I.S.).

For R= 50 Kpc. $M_{TOTAL}(R< 50$ Kpc$) = 3,92 \cdot 10^{11}$ M_Θ.Gravitational field is E(r=50 Kpc) = $2,1867 \cdot 10^{-11}$ m/s^2 and parameter is   C= $1,1523 \cdot 10^{-24}$ (I.S.)

For R= 200 Kpc. $M_{TOTAL}(R< 200$ Kpc$) = 7,05 \cdot 10^{11}$ M_Θ.Gravitational field is E(r=200 Kpc) = $2,458 \cdot 10^{-12}$ m/s^2 and parameter is   C= $1,0450 \cdot 10^{-24}$ (I.S.)

For R= 385 Kpc. $M_{TOTAL}(R< 385$ Kpc$) = 8,76 \cdot 10^{11}$ M_Θ.Gravitational field is E(r=385 Kpc) = $8,242 \cdot 10^{-13}$ m/s^2 and parameter is   C= $1,0295 \cdot 10^{-24}$ (I.S.)

As it was said at the beginning of chapter, Bernouilli differential equation is wrong inside galactic disk. As a pure curiosity it was calculated parameter C in Sun region.

For R_{SUN}= 8,3 Kpc $M_{TOTAL}(R< 8,3$ Kpc$) = 1,11 \cdot 10^{11}$ M_Θ. Gravitational field is E(r= 8,3 Kpc) = $2,25 \cdot 10^{-10}$ m/s^2

E_{SUN} may be calculated through this formula E= V^2_{SUN} / R_{SUN} as well.   Where V_{SUN} = 240 Km/s. Parameter is

C= $3,32 \cdot 10^{-24}$ (I.S.) which is very different from values calculated in halo region.

Discussion of results of parameter C

As reader can understand four results are surprisingly close, especially if it is considered distances so different in four cases. R= 21,4 Kpc is is twice times 10,7 which is a parameter of milky Way in NFW model of Sofue 2015 and approximately radius of baryonic mass in milky Way. R= 50 Kpc and R= 200 Kpc are intermediate radius inside galactic halo. R= 385 kpc is half distance to M31 galaxy and maximum radius of Milky Way halo according theory defended in this paper.
Taking in consideration four results for C it is reasonable to assign $1,1 \cdot 10^{-24}$ (I.S.) as an average value.

Conclusion

This simple formula is gravitational field in Milky Way halo.

$$E_{MW}(r) = \left(Cr^{1,48} + Dr\right)^{-1,35135} \ where \ D = 3,30994 \cdot 10^{-14} \ (I.S.) \ and \ C = 1,1 \cdot 10^{-24} \ (I.S.)$$

$$inside \, halo \, region \ \ 21 kpc < Radius < 385 kpc$$

This formula allows to write rightly density of dark matter function in halo region.

$$D_{DM}(r) = A \cdot E^{1,74}{}_{MW}(r) = A \cdot \left(Cr^{1,48} + Dr\right)^{-2,35135}$$

$$where \ A = 2,526 10^{-5} \ and \ D = 3,30994 \cdot 10^{-14} \ are \ Universal \ parameters$$

$$and \ C = 1,1 \cdot 10^{-24} \ (I.S.) \ is \ a \ parameter \ especific \ for \ MilkyWay$$

$$halo \, region \ of \ MilkyWay \ \ \ 21 kpc < Radius < 385 kpc$$

NFW density dark matter function is a well known function for astrophysicist community, so comparison between both function is a crucial test for validity of potential density dark matter function.

17.1.1 COMPARISON BETWEEN POTENTIAL DENSITY AND NFW DENSITY FUNCTIONS IN MILKY WAY HALO

Dark matter density function profile   NFW   [1] Sofue,Y. 2015.
h = 10,7 $\pm$ 2,9 Kpc
Do =   18,2 $\pm$ 7,4 10^{-3} Msolar/pc^3 = mMsolar/pc^3

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2} \ Where \ x = R \, / \, h$$

Potential D.M. density function depending on radius D_{PDM} Parameters and magnitudes measured by I.S.
$D_{PDM}(r) = A \cdot E^{1,74}(r) = A \cdot (C \cdot r^{1,48} + D \cdot r)^{-2,35135}$ Where A= 2,526$\cdot 10^{-5}$ D=3,301$\cdot 10^{-14}$ C= 1,1$\cdot 10^{-24}$
Dominion is the halo region of Milky Way   21 kpc < radius < 385 Kpc

As formula of potential D.M. calculates Density in Kg/m^3 it is convenient to change units to mMsolar/pc^3 in order to compare with NFW density which is in mMsolar/pc^3 units. Conversion is right knowing that mMsolar/pc^3 = 6,768$\cdot 10^{-23}$ Kg/m^3

Radius	X = R(Kpc)/ 10,7	Radius	Density Potential DM	Density NFW	Relative error
Kpc	Adimensional	meters	mMsun/pc^3	mMsun/pc^3	%
2,14E+01	2,00E+00	6,6E+20	1,06E+00	1,01E+00	-4,79E+00
2,45E+01	2,29E+00	7,56E+20	7,41E-01	7,34E-01	-9,69E-01
3,26E+01	3,05E+00	1,01E+21	3,47E-01	3,65E-01	4,87E+00
4,07E+01	3,80E+00	1,26E+21	1,91E-01	2,07E-01	7,80E+00
4,88E+01	4,56E+00	1,51E+21	1,17E-01	1,29E-01	9,42E+00
5,69E+01	5,32E+00	1,76E+21	7,68E-02	8,57E-02	1,04E+01
6,50E+01	6,08E+00	2,01E+21	5,33E-02	5,98E-02	1,09E+01
7,31E+01	6,83E+00	2,26E+21	3,85E-02	4,34E-02	1,12E+01
8,12E+01	7,59E+00	2,51E+21	2,88E-02	3,25E-02	1,14E+01
9,43E+01	8,82E+00	2,91E+21	1,89E-02	2,14E-02	1,16E+01
1,07E+02	1,00E+01	3,31E+21	1,31E-02	1,49E-02	1,16E+01
1,21E+02	1,13E+01	3,72E+21	9,50E-03	1,07E-02	1,16E+01
1,34E+02	1,25E+01	4,12E+21	7,08E-03	8,01E-03	1,16E+01

1,47E+02	1,37E+01	4,53E+21	5,42E-03	6,13E-03	1,16E+01
1,60E+02	1,49E+01	4,93E+21	4,24E-03	4,80E-03	1,16E+01
1,73E+02	1,62E+01	5,33E+21	3,38E-03	3,83E-03	1,16E+01
1,86E+02	1,74E+01	5,74E+21	2,74E-03	3,10E-03	1,16E+01
1,99E+02	1,86E+01	6,14E+21	2,25E-03	2,54E-03	1,16E+01
2,12E+02	1,98E+01	6,55E+21	1,87E-03	2,12E-03	1,16E+01
2,25E+02	2,11E+01	6,95E+21	1,57E-03	1,78E-03	1,17E+01
2,38E+02	2,23E+01	7,35E+21	1,33E-03	1,51E-03	1,17E+01
2,51E+02	2,35E+01	7,76E+21	1,14E-03	1,29E-03	1,18E+01
2,65E+02	2,47E+01	8,16E+21	9,80E-04	1,11E-03	1,19E+01
2,78E+02	2,60E+01	8,57E+21	8,50E-04	9,66E-04	1,20E+01
2,91E+02	2,72E+01	8,97E+21	7,42E-04	8,44E-04	1,20E+01
3,04E+02	2,84E+01	9,37E+21	6,51E-04	7,42E-04	1,21E+01
3,17E+02	2,96E+01	9,78E+21	5,75E-04	6,55E-04	1,23E+01
3,30E+02	3,08E+01	1,02E+22	5,10E-04	5,82E-04	1,24E+01
3,43E+02	3,21E+01	1,06E+22	4,54E-04	5,19E-04	1,25E+01
3,56E+02	3,33E+01	1,10E+22	4,06E-04	4,65E-04	1,26E+01
3,69E+02	3,45E+01	1,14E+22	3,65E-04	4,18E-04	1,27E+01
3,82E+02	3,57E+01	1,18E+22	3,29E-04	3,77E-04	1,29E+01

Relative error are always under 13% which is a superb results, as can understand any scientist who used to work in astrophysics.

17.2 BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN HALO OF M31 GALAXY

It will be considered outskirt of M31, the region where density of baryonic matter is negligible. 34,6 Kpc as radius may be a guaranty that density of baryonic matter is negligible when radius is bigger.

Process to get the Bernoulli equation is the same because formula for $\varphi_{DM}(r) = A \cdot E^{1,74}(r)$ claim to be Universal.

So formula for gravitational field is the same except in parameter C which depend of total matter considered at a

specific radius. $E(r) = \left(Cr^{1,48} + Dr\right)^{-1,35135}$ *where* $D = 3{,}3099410^{-14}$

Calculus of parameter C

Calculus will be made five times for R=34,6 Kpc. R= 69,2 Kpc. R= 103,8 Kpc. R= 200 Kpc and R= 385 Kpc. Data come from Sofue 2015, which uses NFW to calculate density of dark matter and total masses.

According this author total baryonic mass is $M_B = 1{,}6 \cdot 10^{11} M_\Theta$ and dark matter inside a sphere with radius R is calculated by this formula $M_{DM}(<R) = K \cdot (\ln(1+x) - x / (1+x))$ Where K= $1{,}1602 \cdot 10^{12} M_\Theta$. x = R/ h . h= 34,6 Kpc.

Total mass inside a sphere with radius R > 34,6 Kpc is got adding total baryonic mass to Dark matter included in the sphere whose radius is R.
Calculus has been made carefully and all magnitudes has been measured into International System (I.S.).

For R= 34,6 Kpc. M_{TOTAL} (R< 34,6 Kpc) = 3,84 $\cdot$ 10^{11} M_Θ .Gravitational field is E(r=34,6 Kpc) = $4{,}47 \cdot 10^{-11}$ m/s^2 and parameter is C= $7{,}7 \cdot 10^{-25}$ (I.S.).

For R= 69,2 Kpc. M_{TOTAL} (R< 69,2 Kpc) = 6,6 $\cdot 10^{11}$ M_Θ .Gravitational field is E(r=69,2 Kpc) = $1{,}93 \cdot 10^{-11}$ m/s^2 and parameter is   C= $3{,}88 \cdot 10^{-25}$ (I.S.)

For R= 103,8 Kpc. M_{TOTAL} (R< 103,8 Kpc) = $8,98 \cdot 10^{11}$ M_Θ .Gravitational field is E(r=103,8 Kpc) = $1,16 \cdot 10^{-11}$ m/s^2 and parameter is C=$2,59 \cdot 10^{-25}$ (I.S.)

For R= 200 Kpc. M_{TOTAL} (R< 200 Kpc) = $13,92 \cdot 10^{11}$ M_Θ .Gravitational field is E(r=385 Kpc) = $4,85 \cdot 10^{-12}$ m/s^2 and parameter is C= $1,77 \cdot 10^{-25}$ (I.S.)

For R= 385 Kpc. M_{TOTAL} (R< 385 Kpc) = $19,9 \cdot 10^{11}$ M_Θ .Gravitational field is E(r=385 Kpc) = $1,87 \cdot 10^{-12}$ m/s^2 and parameter is C= $1,787 \cdot 10^{-25}$ (I.S.)

Discussion of results of parameter C

Results of parameter C are unexpectedly different except value for R= 200 Kpc and R = 385 Kpc. Calculus have been made carefully so perhaps experimental data are not so good as Milky Way case or NFW parameters are not so good as Miky Way case.
Taking in consideration five results for C it is reasonable to assign $1,77 \cdot 10^{-25}$ (I.S.) which correspond to 200 kpc.

Conclusion

This simple formula is gravitational field in M31 galaxy halo.

$$E_{M31}(r) = \left(Cr^{1,48} + Dr \right)^{-1,35135} \; where \; D = 3,30994 \cdot 10^{-14} \; (I.S.) \; and \; C = 1,77 \cdot 10^{-25} \; (I.S.)$$
$$inside\,halo\,region \quad 34,6\,kpc < Radius < 385\,kpc$$

This formula allow to write rightly density of dark matter function depending on radius in halo region.

$$D_{DM}(r) = A \cdot E^{1,74}{}_{M31}(r) = A \cdot \left(Cr^{1,48} + Dr \right)^{-2,35135}$$

$$where \; A = 2,526 \cdot 10^{-5} \; and \; D = 3,30994 \cdot 10^{-14} \; are \; Universal \; parameters$$

$$and \; C = 1,77 \cdot 10^{-25} \; (I.S.) \; is \, a \; parameter \, especific \, for \, M31$$

$$halo\,region \; of \, M31 \quad 34,6\,kpc < Radius < 385\,kpc$$

NFW density dark matter function is a well known function for astrophysicist community, so comparison between both function is a crucial test for validity of potential density dark matter function.

17.2.2 COMPARISON BETWEEN POTENTIAL DENSITY AND NFW DENSITY FUNCTIONS IN M31 HALO REGION

NFW Dark matter density function   [1] Sofue,Y. 2015.
h = 34,6 $\pm$ 2,1 Kpc
Do = 2,23 $\pm$ 0,24 10^{-3} Msolar/pc^3

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2} \quad Where \; x=R/h$$

Potential D.M. density function depending on radius D_{PDM} Parameters and magnitudes measured by I.S.
$D_{PDM}(r) = A \cdot E^{1,74}(r) = A \cdot (C \cdot r^{1,48} + D \cdot r)^{-2,35135}$ Where A= $2,526 \cdot 10^{-5}$ D=$3,301 \cdot 10^{-14}$ C= $1,77 \cdot 10^{-25}$
Dominion is the halo region of M31   34,6 kpc < radius < 385 Kpc

As formula of potential D.M. calculates Density in Kg/m^3 it is convenient to change units to $mMsolar/pc^3$ in order to compare with NFW density which is in $mMsolar/pc^3$ units. Conversion is right knowing that
$mMsolar/pc^3$ = $6,768 \cdot 10^{-23}$ Kg/m^3

Radius Kpc	X Adimensional	Radius m	E gravt. m/s^2	D. potential mMsun/pc^3	D NFW mMsun/pc^3	Relative error %
3,46E+01	1,00E+00	1,07E+21	5,77799E-11	5,72E-01	5,57E-01	-2,73E+00
4,12E+01	1,19E+00	1,27E+21	4,53893E-11	3,76E-01	3,91E-01	3,83E+00
4,77E+01	1,38E+00	1,47E+21	3,69348E-11	2,63E-01	2,86E-01	8,09E+00
5,43E+01	1,57E+00	1,67E+21	3,08523E-11	1,92E-01	2,16E-01	1,09E+01
6,08E+01	1,76E+00	1,88E+21	2,6298E-11	1,45E-01	1,67E-01	1,28E+01
6,74E+01	1,95E+00	2,08E+21	2,27797E-11	1,13E-01	1,32E-01	1,41E+01
7,39E+01	2,14E+00	2,28E+21	1,99927E-11	9,03E-02	1,06E-01	1,50E+01
8,05E+01	2,33E+00	2,48E+21	1,77389E-11	7,33E-02	8,67E-02	1,55E+01
8,70E+01	2,51E+00	2,68E+21	1,58847E-11	6,05E-02	7,18E-02	1,58E+01
9,35E+01	2,70E+00	2,89E+21	1,43367E-11	5,06E-02	6,01E-02	1,58E+01
1,00E+02	2,89E+00	3,09E+21	1,3028E-11	4,28E-02	5,09E-02	1,58E+01
1,07E+02	3,08E+00	3,29E+21	1,19094E-11	3,66E-02	4,34E-02	1,56E+01
1,13E+02	3,27E+00	3,49E+21	1,09441E-11	3,16E-02	3,74E-02	1,53E+01
1,20E+02	3,46E+00	3,69E+21	1,0104E-11	2,75E-02	3,24E-02	1,50E+01
1,26E+02	3,65E+00	3,90E+21	9,36739E-12	2,41E-02	2,83E-02	1,46E+01
1,33E+02	3,84E+00	4,10E+21	8,71706E-12	2,13E-02	2,48E-02	1,42E+01
1,39E+02	4,03E+00	4,30E+21	8,13942E-12	1,89E-02	2,19E-02	1,37E+01
1,46E+02	4,22E+00	4,50E+21	7,6235E-12	1,69E-02	1,94E-02	1,32E+01
1,52E+02	4,41E+00	4,70E+21	7,16039E-12	1,51E-02	1,73E-02	1,26E+01
1,59E+02	4,60E+00	4,91E+21	6,74276E-12	1,36E-02	1,55E-02	1,21E+01
1,66E+02	4,79E+00	5,11E+21	6,36455E-12	1,23E-02	1,39E-02	1,15E+01
1,72E+02	4,97E+00	5,31E+21	6,02071E-12	1,12E-02	1,26E-02	1,09E+01
1,79E+02	5,16E+00	5,51E+21	5,707E-12	1,02E-02	1,14E-02	1,04E+01
1,85E+02	5,35E+00	5,71E+21	5,41981E-12	9,31E-03	1,03E-02	9,76E+00
1,92E+02	5,54E+00	5,92E+21	5,1561E-12	8,54E-03	9,40E-03	9,16E+00
1,98E+02	5,73E+00	6,12E+21	4,91324E-12	7,85E-03	8,59E-03	8,55E+00
2,05E+02	5,92E+00	6,32E+21	4,68898E-12	7,24E-03	7,86E-03	7,94E+00
2,11E+02	6,11E+00	6,52E+21	4,48138E-12	6,69E-03	7,22E-03	7,33E+00
2,18E+02	6,30E+00	6,72E+21	4,28874E-12	6,20E-03	6,64E-03	6,72E+00
2,25E+02	6,49E+00	6,93E+21	4,10959E-12	5,75E-03	6,13E-03	6,11E+00
2,31E+02	6,68E+00	7,13E+21	3,94263E-12	5,35E-03	5,66E-03	5,50E+00
2,38E+02	6,87E+00	7,33E+21	3,78672E-12	4,99E-03	5,25E-03	4,89E+00
2,44E+02	7,06E+00	7,53E+21	3,64087E-12	4,66E-03	4,87E-03	4,28E+00
2,51E+02	7,25E+00	7,73E+21	3,50419E-12	4,36E-03	4,53E-03	3,67E+00
2,57E+02	7,43E+00	7,94E+21	3,37587E-12	4,09E-03	4,22E-03	3,06E+00
2,64E+02	7,62E+00	8,14E+21	3,25522E-12	3,84E-03	3,93E-03	2,46E+00
2,70E+02	7,81E+00	8,34E+21	3,14162E-12	3,61E-03	3,67E-03	1,86E+00
2,77E+02	8,00E+00	8,54E+21	3,03448E-12	3,39E-03	3,44E-03	1,27E+00
2,83E+02	8,19E+00	8,74E+21	2,93331E-12	3,20E-03	3,22E-03	6,77E-01
2,90E+02	8,38E+00	8,95E+21	2,83765E-12	3,02E-03	3,02E-03	8,84E-02
2,97E+02	8,57E+00	9,15E+21	2,74709E-12	2,86E-03	2,84E-03	-4,96E-01
3,03E+02	8,76E+00	9,35E+21	2,66124E-12	2,70E-03	2,67E-03	-1,08E+00

3,10E+02	8,95E+00	9,55E+21	2,57979E-12	2,56E-03	2,52E-03	-1,65E+00
3,16E+02	9,14E+00	9,75E+21	2,5024E-12	2,43E-03	2,37E-03	-2,23E+00
3,23E+02	9,33E+00	9,96E+21	2,42881E-12	2,30E-03	2,24E-03	-2,80E+00
3,29E+02	9,52E+00	1,02E+22	2,35876E-12	2,19E-03	2,12E-03	-3,36E+00
3,36E+02	9,71E+00	1,04E+22	2,29201E-12	2,08E-03	2,00E-03	-3,92E+00
3,42E+02	9,89E+00	1,06E+22	2,22835E-12	1,98E-03	1,90E-03	-4,48E+00
3,49E+02	1,01E+01	1,08E+22	2,16757E-12	1,89E-03	1,80E-03	-5,03E+00
3,55E+02	1,03E+01	1,10E+22	2,10951E-12	1,80E-03	1,71E-03	-5,58E+00
3,62E+02	1,05E+01	1,12E+22	2,05399E-12	1,72E-03	1,62E-03	-6,12E+00
3,69E+02	1,07E+01	1,14E+22	2,00086E-12	1,64E-03	1,54E-03	-6,66E+00
3,75E+02	1,08E+01	1,16E+22	1,94998E-12	1,57E-03	1,47E-03	-7,20E+00
3,82E+02	1,10E+01	1,18E+22	1,90121E-12	1,50E-03	1,40E-03	-7,73E+00

As table shows, relative error between both density functions are below 16% which is a superb result as can understand any astrophysicist ¡

18. CONCLUSION

The objective the paper has been try to justify that DM density function depend on E according a Universal law.

According my previous paper [4] Abarca,M.2014, DM is generated by gravitational field, so I thought that it was needed to study big galaxies because inside their disks, gravitational field may be measured through rotational curve of a galaxy. In other words, inside a big galaxy it is known that gravitational field is generated by its own mass and gravitational influence of galactic neighbours is negligible.

After a long search in arXiv looking for papers published during last five years only I found four papers with rotational curves published.

Apart from dwarf an intermediate galaxies only there is eight big galaxies, whose spin speed in flat region is higher than 200 Km/s. By reasons explained in epigraph 3, I had to reject two galaxies although I found two galaxies studied by two authors simultaneously, so finally I have worked with eight galactic data set -8GDS-. Specifically the galaxies are Milky Way, M31, NGC 3031, NGC 3992, NGC 7331, NGC 2841.

After a long statistical study carefully detailed through the paper I have calculated an "average" curve which fit all data set studied whose correlation coefficient is bigger than 0,997, so I have found plausible to postulate this function as Universal:

Dark matter density function as Universal law for big galaxies – flat speed region in rot. curve above 200 Km/s -						
Density $_{DM}$ = 1,26 · 10^{-6} · E^{B} where B= 1,74						
Where Unit for D$_{D.M.}$ is $m\,M_{\Theta}/pc^3 = \boldsymbol{10^{-3}}\ M_{\Theta}/pc^3$ and Unit for E is Km^2/s^2/Kpc						
E gravt.	100	500	1500	3000	4500	6500
Den.	0,003787	0,06226	0,42093	1,4056	2,8456	5,3948
Dominion 100 < E < 6500						

Below DM density function is expressed with a more suitable system of units.

Dark matter density function as Universal law for big galaxies - Hybrid System of Units- $m\,M_\Theta$/pc^3						
Density $_{DM}$ = 3,7323 · 10^{17} · E^B where B= 1,74						
Where Unit for Density $_{D.M.}$ is $m\,M_\Theta$/pc^3 and Unit for E is m/s^2						
E paper Units	100	500	1500	3000	4500	6500
E gravt. I.S.	3,24·10^{-12}	1,62·10^{-11}	4,86·10^{-11}	9,72·10^{-11}	1,458·10^{-10}	2,106·10^{-10}
Density$_{DM}$ I.S.	2,58·10^{-25}	4,24·10^{-24}	2,86·10^{-23}	9,57·10^{-23}	1,94·10^{-22}	3,67·10^{-22}
Den.$_{DM}$ $m\,M_\Theta$/pc^3	3,81·10^{-3}	0,0626	0,422	1,414	2,866	5,42
Dominion	$3,24 \cdot 10^{-12} < E < 2,11 \cdot 10^{-10}$					

Although statistical data of galaxies has been studied by linear regression inside a restricted region of dominion 1500 < E < 6000, according theory proposed by the author in [4] Abarca,M.2014, DM is generated by the gravitational field so when E goes to cero DM density should go to cero as well. Therefore potential fit is the most suitable function. In addition potential fit allows study data of E in the whole dominion $0 \le E < 6500$ although statistical data for E in this paper has been 100< E.

In chapter 12 it is calculated Local DM density in vicinity of Sun. From Solar parameters $R_\Theta = 8,3\,Kpc$ and $V_\Theta = 244\,Km/s$ is calculated $E_\Theta = 7173$ Km^2/s^2/ Kpc and   Density $_{DM-SUN}$ = 6,45·10^{-3} M_Θ/pc^3 = 6,45 mM_Θ/pc^3 = 0,245 GeV/cm^3.

This value has been compared with data which come from recent Local DM density measures and it has been concluded that $\varphi_{LOCAL-DM}$ calculated in this paper is included inside interval defined by $\varphi_{LOCAL-DM}$ measures published in recent papers.   Therefore it can be concluded that DM density function is compatible with current measures of Local DM density.

Apart from the six big galaxies I have studied rotation curves of NGC 3198 (Ursa Major), NGC 2403 (M81 group of galaxies) and M33 -Triangle galaxy which are galaxies whose spin speed in flat region are under 150 Km/s and they belong to cluster of galaxies dominates by gravitational field of big galaxies.

In chapter 13 were pointed out that experimental results for intermediate an dwarf galaxies show that DM density is a great deal bigger than inside big galaxies. However also were pointed out reasons that in opinion the author disallow this results as an ultimate counter-example to reject that Density of DM depend on gravitational field E according a Universal law. In addition, reasons developed in this epigraph lead to postulate a new Universal law:

DM density on intermediate galaxies near a big galaxy should have a bigger DM density than DM density in big galaxies because DM density inside intermediate galaxies it is not only generated by its own gravitational field but also by its nearby giant galaxy.

In chapter 14, DM density potential function has been checked with DM Density NFW profile got by Mr. Sofue in Milky Way and it has been found that agreement between both curves are mainly under 10% of relative error inside the whole dominion, which is an excellent agreement in astrophysics¡.

In chapter 15, DM density potential function has been checked with DM Density NFW profile got by Mr. Sofue in M31. Although the agreement between both functions is not so good as Milky Way case, relative error is enough small to accept that M31 supports DM density potential function as Universal law.

In chapter 16, density potential function is written depending on radius x, and it is clearly showed that this function is equivalent to NFW, relative error inside the whole dominion are enough small that they could be considered come from error in astronomical measures.

Finally in chapter 17 has been got a Bernoulli differential equation for gravitational field which allows to calculate potential density function depending on radius. In addition potential function has been compared with NFW density function through halo region and it has been found that relative differences are below 13% in Milky Way and below 16% in M31 galaxy. This is a superb result as can understand any astrophysicist ¡

The objective this paper has been to check the model of dark matter proposed by the author in [4] Abarca,M.2014. I think that it has been justified properly that results could confirm DM density as universal law because statistical differences in different galaxies might be explained by experimental error in rotation curves and error in calculus of DM density functions.

The author think that it may be worth to study a bigger number of galaxies in order to check if new results could confirm or reject theory of Dark matter proposed by the author   because Dark matter nature is one of the most important challenge in astrophysics science nowadays.

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