Smarandache Multi-Space Theory(IV)

-Applications to theoretical physics

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Abstract. A Smarandache multi-space is a union of n different spaces equipped with some different structures for an integer $n \geq 2$, which can be both used for discrete or connected spaces, particularly for geometries and spacetimes in theoretical physics. This monograph concentrates on characterizing various multi-spaces including three parts altogether. The first part is on *algebraic multi-spaces with structures*, such as those of multi-groups, multi-rings, multi-vector spaces, multi-metric spaces, multi-operation systems and multi-manifolds, also multi-voltage graphs, multi-embedding of a graph in an n-manifold,$\cdots$, etc.. The second discusses *Smarandache geometries*, including those of map geometries, planar map geometries and pseudo-plane geometries, in which the *Finsler geometry*, particularly the *Riemann geometry* appears as a special case of these Smarandache geometries. The third part of this book considers the *applications of multi-spaces to theoretical physics*, including the relativity theory, the M-theory and the cosmology. Multi-space models for p-branes and cosmos are constructed and some questions in cosmology are clarified by multi-spaces. The first two parts are relative independence for reading and in each part open problems are included for further research of interested readers.

Key words: multi-space, relativity theory, M-theory, p-brane, multi-space model of cosmos.

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6. Applications to theoretical physics

Whether are there finite, or infinite cosmoses? Is there just one? What is the dimension of our cosmos? Those simpler but more puzzling problems have confused the eyes of human beings thousands years and one does not know the answer even until today. The dimension of the cosmos in the eyes of the ancient Greeks is 3, but Einstein's is 4. In recent decades, 10 or 11 is the dimension of our cosmos in superstring theory or M-theory. All these assumptions acknowledge that there is just one cosmos. Which one is the correct and whether can human beings realize the cosmos or cosmoses? By applying results gotten in Chapters 3-5, we tentatively answer those problems and explain the Einstein's or Hawking's model for cosmos in this chapter.

§6.1 Pseudo-Faces of Spaces

Throughout this chapter, $\mathbf{R}^n$ denotes an Euclid space of dimensional n. In this section, we consider a problem related to how to represent an Euclid space in another. First, we introduce the conception of pseudo-faces of Euclid spaces in the following.

Definition 6.1.1 *Let $\mathbf{R}^m$ and $(\mathbf{R}^n, \omega)$ be an Euclid space and a pseudo-metric space. If there is a continuous mapping $p : \mathbf{R}^m \to (\mathbf{R}^n, \omega)$, then the pseudo-metric space $(\mathbf{R}^n, \omega(p(\mathbf{R}^m)))$ is called a pseudo-face of $\mathbf{R}^m$ in $(\mathbf{R}^n, \omega)$.*

Notice that these pseudo-faces of $\mathbf{R}^3$ in $\mathbf{R}^2$ have been considered in Chapter 5. For the existence of a pseudo-face of an Euclid space $\mathbf{R}^m$ in $\mathbf{R}^n$, we have a result as in the following.

Theorem 6.1.1 *Let $\mathbf{R}^m$ and $(\mathbf{R}^n, \omega)$ be an Euclid space and a pseudo-metric space. Then there exists a pseudo-face of $\mathbf{R}^m$ in $(\mathbf{R}^n, \omega)$ if and only if for any number $\epsilon > 0$, there exists a number $\delta > 0$ such that for $\forall \overline{u}, \overline{v} \in \mathbf{R}^m$ with $\|\overline{u} - \overline{v}\| < \delta$,*

$$\|\omega(p(\overline{u})) - \omega(p(\overline{v}))\| < \epsilon,$$

where $\|\overline{u}\|$ denotes the norm of a vector $\overline{u}$ in the Euclid space.

Proof We only need to prove that there exists a continuous mapping $p : \mathbf{R}^m \to (\mathbf{R}^n, \omega)$ if and only if all of these conditions in this theorem hold. By the definition of a pseudo-space $(\mathbf{R}^n, \omega)$, since ω is continuous, we know that for any number $\epsilon > 0$,

$\|\omega(\overline{x}) - \omega(\overline{y})\| < \epsilon$ for $\forall \overline{x}, \overline{y} \in \mathbf{R}^n$ if and only if there exists a number $\delta_1 > 0$ such that $\|\overline{x} - \overline{y})\| < \delta_1$.

By definition, a mapping $q : \mathbf{R}^m \to \mathbf{R}^n$ is continuous between Euclid spaces if and only if for any number $\delta_1 > 0$, there exists a number $\delta_2 > 0$ such that $\|q(\overline{x}) - q(\overline{y})\| < \delta_1$ for $\forall \overline{u}, \overline{v} \in \mathbf{R}^m$ with $\|\overline{u} - \overline{v})\| < \delta_2$.

Combining these assertions, we know that $p : \mathbf{R}^m \to (\mathbf{R}^n, \omega)$ is continuous if and only if for any number $\epsilon > 0$, there is number $\delta = \min\{\delta_1, \delta_2\}$ such that

$$\|\omega(p(\overline{u})) - \omega(p(\overline{v}))\| < \epsilon$$

for $\forall \overline{u}, \overline{v} \in \mathbf{R}^m$ with $\|\overline{u} - \overline{v})\| < \delta$. $\natural$

Corollary 6.1.1 *If $m \geq n+1$, let $\omega : \mathbf{R}^n \to \mathbf{R}^{m-n}$ be a continuous mapping, then $(\mathbf{R}^n, \omega(p(\mathbf{R}^m)))$ is a pseudo-face of $\mathbf{R}^m$ in $(\mathbf{R}^n, \omega)$ with*

$$p(x_1, x_2, \cdots, x_n, x_{n+1}, \cdots, x_m) = \omega(x_1, x_2, \cdots, x_n).$$

Particularly, if $m = 3, n = 2$ and ω is an angle function, then $(\mathbf{R}^n, \omega(p(\mathbf{R}^m)))$ is a pseudo-face with $p(x_1, x_2, x_3) = \omega(x_1, x_2)$.

There is a simple relation for a continuous mapping between Euclid spaces and that of between pseudo-faces established in the next result.

Theorem 6.1.2 *Let $g : \mathbf{R}^m \to \mathbf{R}^m$ and $p : \mathbf{R}^m \to (\mathbf{R}^n, \omega)$ be continuous mappings. Then $pgp^{-1} : (\mathbf{R}^n, \omega) \to (\mathbf{R}^n, \omega)$ is also a continuous mapping.*

Proof Because a composition of continuous mappings is a continuous mapping, we know that pgp^{-1} is continuous.

Now for $\forall \omega(x_1, x_2, \cdots, x_n) \in (\mathbf{R}^n, \omega)$, assume that $p(y_1, y_2, \cdots, y_m) = \omega(x_1, x_2, \cdots, x_n)$, $g(y_1, y_2, \cdots, y_m) = (z_1, z_2, \cdots, z_m)$ and $p(z_1, z_2, \cdots, z_m) = \omega(t_1, t_2, \cdots, t_n)$. Then calculation shows that

$$\begin{aligned} pgp^{-}(\omega(x_1, x_2, \cdots, x_n)) &= pg(y_1, y_2, \cdots, y_m) \\ &= p(z_1, z_2, \cdots, z_m) = \omega(t_1, t_2, \cdots, t_n) \in (\mathbf{R}^n, \omega). \end{aligned}$$

Whence, pgp^{-1} is a continuous mapping and $pgp^{-1} : (\mathbf{R}^n, \omega) \to (\mathbf{R}^n, \omega)$. $\natural$

Corollary 6.1.2 *Let $C(\mathbf{R}^m)$ and $C(\mathbf{R}^n, \omega)$ be sets of continuous mapping on an Euclid space $\mathbf{R}^m$ and an pseudo-metric space $(\mathbf{R}^n, \omega)$. If there is a pseudo-space for $\mathbf{R}^m$ in $(\mathbf{R}^n, \omega)$. Then there is a bijection between $C(\mathbf{R}^m)$ and $C(\mathbf{R}^n, \omega)$.*

For a body $\mathcal{B}$ in an Euclid space $\mathbf{R}^m$, its shape in a pseudo-face $(\mathbf{R}^n, \omega(p(\mathbf{R}^m)))$ of $\mathbf{R}^m$ in $(\mathbf{R}^n, \omega)$ is called a *pseudo-shape* of $\mathcal{B}$. We get results for pseudo-shapes of a ball in the following.

Theorem 6.1.3 *Let $\mathcal{B}$ be an $(n+1)$-ball of radius R in a space $\mathbf{R}^{n+1}$, i.e.,*

$$x_1^2 + x_2^2 + \cdots + x_n^2 + t^2 \leq R^2.$$

Define a continuous mapping $\omega : \mathbf{R}^n \to \mathbf{R}^n$ *by*

$$\omega(x_1, x_2, \cdots, x_n) = \varsigma t(x_1, x_2, \cdots, x_n)$$

for a real number ς *and a continuous mapping* $p : \mathbf{R}^{n+1} \to \mathbf{R}^n$ *by*

$$p(x_1, x_2, \cdots, x_n, t) = \omega(x_1, x_2, \cdots, x_n).$$

Then the pseudo-shape of $\mathcal{B}$ *in* $(\mathbf{R}^n, \omega)$ *is a ball of radius* $\frac{\sqrt{R^2-t^2}}{\varsigma t}$ *for any parameter* $t, -R \leq t \leq R$. *Particularly, for the case of* $n = 2$ *and* $\varsigma = \frac{1}{2}$, *it is a circle of radius* $\sqrt{R^2 - t^2}$ *for parameter* t *and an elliptic ball in* $\mathbf{R}^3$ *as shown in Fig.*6.1.

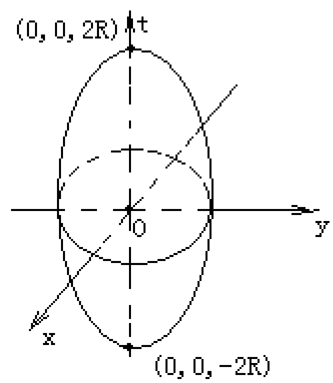

Fig.6.1

Proof For any parameter t, an $(n+1)$-ball

$$x_1^2 + x_2^2 + \cdots + x_n^2 + t^2 \leq R^2$$

can be transferred to an n-ball

$$x_1^2 + x_2^2 + \cdots + x_n^2 \leq R^2 - t^2$$

of radius $\sqrt{R^2 - t^2}$. Whence, if we define a continuous mapping on $\mathbf{R}^n$ by

$$\omega(x_1, x_2, \cdots, x_n) = \varsigma t(x_1, x_2, \cdots, x_n)$$

and

$$p(x_1, x_2, \cdots, x_n, t) = \omega(x_1, x_2, \cdots, x_n),$$

then we get an n-ball

$$x_1^2 + x_2^2 + \cdots + x_n^2 \leq \frac{R^2 - t^2}{\varsigma^2 t^2},$$

of $\mathcal{B}$ under p for any parameter t, which is the pseudo-face of $\mathcal{B}$ for a parameter t by definition.

For the case of $n = 2$ and $\varsigma = \frac{1}{2}$, since its pseudo-face is a circle in an Euclid plane and $-R \leq t \leq R$, we get an elliptic ball as shown in Fig.6.1. $\natural$

Similarly, if we define $\omega(x_1, x_2, \cdots, x_n) = 2\angle(\overrightarrow{OP}, Ot)$ for a point $P = (x_1, x_2, \cdots, x_n, t)$, i.e., an angle function, then we can also get a result like Theorem 6.1.2 for these pseudo-shapes of an $(n+1)$-ball.

Theorem 6.1.4 *Let $\mathcal{B}$ be an $(n+1)$-ball of radius R in a space $\mathbf{R}^{n+1}$, i.e.,*

$$x_1^2 + x_2^2 + \cdots + x_n^2 + t^2 \leq R^2.$$

Define a continuous mapping $\omega : \mathbf{R}^n \to \mathbf{R}^n$ by

$$\omega(x_1, x_2, \cdots, x_n) = 2\angle(\overrightarrow{OP}, Ot)$$

for a point P on $\mathcal{B}$ and a continuous mapping $p : \mathbf{R}^{n+1} \to \mathbf{R}^n$ by

$$p(x_1, x_2, \cdots, x_n, t) = \omega(x_1, x_2, \cdots, x_n).$$

Then the pseudo-shape of $\mathcal{B}$ in $(\mathbf{R}^n, \omega)$ is a ball of radius $\sqrt{R^2 - t^2}$ for any parameter $t, -R \leq t \leq R$. Particularly, for the case of $n = 2$, it is a circle of radius $\sqrt{R^2 - t^2}$ for parameter t and a body in $\mathbf{R}^3$ with equations

$$\oint \arctan(\frac{t}{x}) = 2\pi \quad \textit{and} \quad \oint \arctan(\frac{t}{y}) = 2\pi$$

for curves of its intersection with planes XOT and YOT.

Proof The proof is similar to the proof of Theorem 6.1.3. For these equations

$$\oint \arctan(\frac{t}{x}) = 2\pi \quad \text{or} \quad \oint \arctan(\frac{t}{y}) = 2\pi$$

of curves on planes XOT or YOT in the case of $n = 2$, they are implied by the geometrical meaning of an angle function. $\natural$

For an Euclid space $\mathbf{R}^n$, we can get a subspace sequence

$$\mathbf{R}_0 \supset \mathbf{R}_1 \supset \cdots \supset \mathbf{R}_{n-1} \supset \mathbf{R}_n,$$

where the dimensional of $\mathbf{R}_i$ is $n - i$ for $1 \leq i \leq n$ and $\mathbf{R}_n$ is just a point. But we can not get a sequence reversing the order, i.e., a sequence

$$\mathbf{R}_0 \subset \mathbf{R}_1 \subset \cdots \subset \mathbf{R}_{n-1} \subset \mathbf{R}_n$$

in classical space theory. By applying Smarandache multi-spaces, we can really find this kind of sequence by the next result, which can be used to explain a well-known model for our cosmos in M-theory.

Theorem 6.1.5 *Let $P = (x_1, x_2, \cdots, x_n)$ be a point of $\mathbf{R}^n$. Then there are subspaces of dimensional s in P for any integer $s, 1 \leq s \leq n$.*

Proof Notice that in an Euclid space $\mathbf{R}^n$, there is a basis $e_1 = (1, 0, 0, \cdots, 0)$, $e_2 = (0, 1, 0, \cdots, 0)$, $\cdots$, $e_i = (0, \cdots, 0, 1, 0, \cdots, 0)$ (every entry is 0 unless the i-th entry is 1), $\cdots$, $e_n = (0, 0, \cdots, 0, 1)$ such that

$$(x_1, x_2, \cdots, x_n) = x_1 e_1 + x_2 e_2 + \cdots + x_n e_n$$

for any point $(x_1, x_2, \cdots, x_n)$ of $\mathbf{R}^n$. Now we consider a linear space $\mathbf{R}^- = (V, +_{new}, \circ_{new})$ on a field $F = \{a_i, b_i, c_i, \cdots, d_i; i \geq 1\}$, where

$$V = \{x_1, x_2, \cdots, x_n\}.$$

Not loss of generality, we assume that $x_1, x_2, \cdots, x_s$ are independent, i.e., if there exist scalars $a_1, a_2, \cdots, a_s$ such that

$$a_1 \circ_{new} x_1 +_{new} a_2 \circ_{new} x_2 +_{new} \cdots +_{new} a_s \circ_{new} x_s = 0,$$

then $a_1 = a_2 = \cdots = 0_{new}$ and there are scalars $b_i, c_i, \cdots, d_i$ with $1 \leq i \leq s$ in $\mathbf{R}^-$ such that

$$x_{s+1} = b_1 \circ_{new} x_1 +_{new} b_2 \circ_{new} x_2 +_{new} \cdots +_{new} b_s \circ_{new} x_s;$$

$$x_{s+2} = c_1 \circ_{new} x_1 +_{new} c_2 \circ_{new} x_2 +_{new} \cdots +_{new} c_s \circ_{new} x_s;$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots;$$

$$x_n = d_1 \circ_{new} x_1 +_{new} d_2 \circ_{new} x_2 +_{new} \cdots +_{new} d_s \circ_{new} x_s.$$

Therefore, we get a subspace of dimensional s in a point P of $\mathbf{R}^n$. $\natural$

Corollary 6.1.3 *Let P be a point of an Euclid space $\mathbf{R}^n$. Then there is a subspace sequence*

$$\mathbf{R}_0^- \subset \mathbf{R}_1^- \subset \cdots \subset \mathbf{R}_{n-1}^- \subset \mathbf{R}_n^-$$

such that $\mathbf{R}_n^- = \{P\}$ and the dimensional of the subspace $\mathbf{R}_i^-$ is $n - i$, where $1 \leq i \leq n$.

Proof Applying Theorem 6.1.5 repeatedly, we get the desired sequence. $\natural$

§6.2. Relativity Theory

In theoretical physics, these spacetimes are used to describe various states of particles dependent on the time parameter in an Euclid space $\mathbf{R}^3$. There are two kinds of spacetimes. An *absolute spacetime* is an Euclid space $\mathbf{R}^3$ with an independent time, denoted by $(x_1, x_2, x_3|t)$ and a *relative spacetime* is an Euclid space $\mathbf{R}^4$, where time is the t-axis, seeing also in [30] – [31] for details.

A point in a spacetime is called an *event*, i.e., represented by

$$(x_1, x_2, x_3) \in \mathbf{R}^3 \quad \text{and} \quad t \in \mathbf{R}^+$$

in an absolute spacetime in the Newton's mechanics and

$$(x_1, x_2, x_3, t) \in \mathbf{R}^4$$

with time parameter t in a relative space-time used in the Einstein's relativity theory.

For two events $A_1 = (x_1, x_2, x_3|t_1)$ and $A_2 = (y_1, y_2, y_3|t_2)$, the *time interval* $\triangle t$ is defined by $\triangle t = t_1 - t_2$ and the *space interval* $\triangle(A_1, A_2)$ by

$$\triangle(A_1, A_2) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}.$$

Similarly, for two events $B_1 = (x_1, x_2, x_3, t_1)$ and $B_2 = (y_1, y_2, y_3, t_2)$, the *space-time interval* $\triangle s$ is defined by

$$\triangle^2 s = -c^2 \triangle t^2 + \triangle^2(B_1, B_2),$$

where c is the speed of the light in vacuum. In Fig.6.2, a spacetime only with two parameters x, y and the time parameter t is shown.

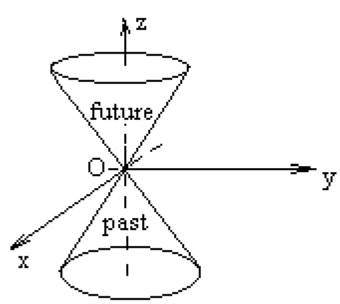

Fig.6.2

The Einstein's spacetime is an uniform linear space. By the assumption of linearity of a spacetime and invariance of the light speed, it can be shown that the invariance of the space-time intervals, i.e.,

For two reference systems S_1 and S_2 with a homogenous relative velocity, there must be

$$\triangle s^2 = \triangle s'^2.$$

We can also get the Lorentz transformation of spacetimes or velocities by this assumption. For two parallel reference systems S_1 and S_2, if the velocity of S_2 relative to S_1 is v along x-axis such as shown in Fig.6.3,

Fig.6.3

then we know the *Lorentz transformation of spacetimes*

$$\begin{cases} x_2 = \frac{x_1 - vt_1}{\sqrt{1-(\frac{v}{c})^2}} \\ y_2 = y_1 \\ z_2 = z_1 \\ t_2 = \frac{t_1 - \frac{v}{c}x_1}{\sqrt{1-(\frac{v}{c})^2}} \end{cases}$$

and the *transformation of velocities*

$$\begin{cases} v_{x_2} = \frac{v_{x_1} - v}{1-\frac{vv_{x_1}}{c^2}} \\ v_{y_2} = \frac{v_{y_1}\sqrt{1-(\frac{v}{c})^2}}{1-\frac{vv_{x_1}}{c^2}} \\ v_{z_2} = \frac{v_{z_1}\sqrt{1-(\frac{v}{c})^2}}{1-\frac{vv_{x_1}}{c^2}}. \end{cases}$$

In the relative spacetime, the *general interval* is defined by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu,$$

where $g_{\mu\nu} = g_{\mu\nu}(x^\sigma, t)$ is a metric dependent on the space and time. We can also introduce the invariance of general intervals, i.e.,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g'_{\mu\nu}dx'^\mu dx'^\nu.$$

Then the *Einstein's equivalence principle* says that

There are no difference for physical effects of the inertial force and the gravitation in a field small enough.

An immediately consequence of the Einstein's equivalence principle is the idea of the *geometrization of gravitation*, i.e., considering the curvature at each point in a spacetime to be all effect of gravitation([18]), which is called a *gravitational factor* at this point.

Combining these discussions in Section 6.1 with the Einstein's idea of the geometrization of gravitation, we get a result for spacetimes in the theoretical physics.

Theorem 6.2.1 *Every spacetime is a pseudo-face in an Euclid pseudo-space, especially, the Einstein's space-time is* $\mathbf{R}^n$ *in* $(\mathbf{R}^4, \omega)$ *for an integer* $n, n \geq 4$.

By the uniformity of a spacetime, we get an equation by equilibrium of vectors in a cosmos.

Theorem 6.2.2 *By the assumption of uniformity for a spacetime in* $(\mathbf{R}^4, \omega)$, *there exists an anti-vector* ω_O^- *of* ω_O *along any orientation* $\overrightarrow{O}$ *in* $\mathbf{R}^4$ *such that*

$$\omega_O + \omega_O^- = 0.$$

Proof Since $\mathbf{R}^4$ is uniformity, By the principle of equilibrium in a uniform space, along any orientation $\overrightarrow{O}$ in $\mathbf{R}^4$, there must exists an anti-vector ω_O^- of ω_O such that

$$\omega_O + \omega_O^- = 0. \quad \natural$$

Theorem 6.2.2 has many useful applications. For example, let

$$\omega_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu},$$

then we know that

$$\omega_{\mu\nu}^- = -8\pi G T_{\mu\nu}.$$

in a gravitational field. Whence, we get the *Einstein's equation of gravitational field*

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

by the equation in Theorem 6.2.2 which is widely used for our cosmos by physicists. In fact, there are two assumptions for our cosmos in the following. One is partially adopted from the Einstein's, another is just suggested by ours.

Postulate 6.2.1 *At the beginning our cosmos is homogenous.*

Postulate 6.2.2 *Human beings can only survey pseudo-faces of our cosmos by observations and experiments.*

Applying these postulates, the Einstein's equation of gravitational field and the *cosmological principle*, i.e., *there are no difference at different points and different orientations at a point of a cosmos on the metric* $10^4 l.y.$, we can get a standard model of cosmos, also called the *Friedmann cosmos*, seeing [18],[26], [28],[30] – [31],[79] and [95] for details. In this model, its line element ds is

$$ds^2 = -c^2dt^2 + a^2(t)[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]$$

and cosmoses are classified into three types:

Static Cosmos: $da/dt\ = 0$;

Contracting Cosmos: $da/dt\ < 0$;

Expanding Cosmos: $da/dt\ > 0$.

By the Einstein's view, our living cosmos is the static cosmos. That is why he added a cosmological constant λ in his equation of gravitational field. But unfortunately, our cosmos is an expanding cosmos found by Hubble in 1929. As a by-product, the shape of our cosmos described by S.Hawking in [30] $-$ [32] is coincide with these results gotten in Section 6.1.

§6.3 A Multi-Space Model for Cosmoses

6.3.1. What is M-theory

Today, we have know that all matter are made of atoms and sub-atomic particles, held together by four fundamental forces: *gravity, electro-magnetism, strong nuclear force* and *weak force*. Their features are partially explained by the *quantum theory* and the *relativity theory*. The former is a theory for the microcosm but the later is for the macrocosm. However, these two theories do not resemble each other in any way. The quantum theory reduces forces to the exchange of discrete packet of quanta, while the relativity theory explains the cosmic forces by postulating the smooth deformation of the fabric spacetime.

As we known, there are two string theories : the $E_8 \times E_8$ heterotic string, the *SO(32) heterotic string* and three superstring theories: the *SO(32) Type I string*, the *Type IIA* and *Type IIB* in superstring theories. Two physical theories are *dual* to each other if they have identical physics after a certain mathematical transformation. There are *T-duality* and *S-duality* in superstring theories defined in the following table 6.1([15]).

	fundamental string	dual string
$T-duality$	$Radius \leftrightarrow 1/(radius)$ $Kaluza-Klein \leftrightarrow Winding$	$charge \leftrightarrow 1/(charge)$ $Electric \leftrightarrow Magnet$
$S-duality$	$charge \leftrightarrow 1/(charge)$ $Electric \leftrightarrow Magnetic$	$Radius \leftrightarrow 1/(Radius)$ $Kaluza-Klein \leftrightarrow Winding$

table 6.1

We already know some profound properties for these spring or superspring theories, such as:

(i) *Type IIA and IIB are related by T-duality, as are the two heterotic theories.*

(ii) Type I and heterotic $SO(32)$ are related by S-duality and Type IIB is also S-dual with itself.

(iii) Type II theories have two supersymmetries in the 10*-dimensional sense, but the rest just one.*

(iv) Type I theory is special in that it is based on unoriented open and closed strings, but the other four are based on oriented closed strings.

(v) The IIA theory is special because it is non-chiral(parity conserving), but the other four are chiral(parity violating).

(vi) In each of these cases there is an 11th dimension that becomes large at strong coupling. For substance, in the IIA case the 11th dimension is a circle and in IIB case it is a line interval, which makes 11-dimensional spacetime display two 10-dimensional boundaries.

(vii) The strong coupling limit of either theory produces an 11-dimensional spacetime.

(viii) $\cdots$, etc..

The M-theory was established by Witten in 1995 for the unity of those two string theories and three superstring theories, which postulates that all matter and energy can be reduced to *branes* of energy vibrating in an 11 dimensional space. This theory gives us a compelling explanation of the origin of our cosmos and combines all of existed string theories by showing those are just special cases of M-theory such as shown in table 6.2.

$$M - theory \begin{cases} E_8 \times E_8 \; heterotic \; string \\ SO(32) \; heterotic \; string \\ SO(32) \; Type \; I \; string \\ Type \; IIA \\ Type \; IIB. \end{cases}$$

Table 6.2

See Fig.6.4 for the M-theory planet in which we can find a relation of M-theory with these two strings or three superstring theories.

Fig.6.4

As it is widely accepted that our cosmos is in accelerating expansion, i.e., our cosmos is most possible an accelerating cosmos of expansion, it should satisfies the following condition

$$\frac{d^2a}{dt^2} > 0.$$

The *Kasner* type metric

$$ds^2 = -dt^2 + a(t)^2 d^2_{\mathbf{R}^3} + b(t)^2 ds^2(T^m)$$

solves the $4+m$ dimensional vacuum Einstein equations if

$$a(t) = t^{\mu} \quad \text{and} \quad b(t) = t\nu$$

with

$$\mu = \frac{3 \pm \sqrt{3m(m+2)}}{3(m+3)}, \nu = \frac{3 \mp \sqrt{3m(m+2)}}{3(m+3)}.$$

These solutions in general do not give an accelerating expansion of spacetime of dimension 4. However, by using the time-shift symmetry

$$t \to t_{+\infty} - t, \quad a(t) = (t_{+\infty} - t)^{\mu},$$

we see that yields a really accelerating expansion since

$$\frac{da(t)}{dt} > 0 \text{ and } \frac{d^2a(t)}{dt^2} > 0.$$

According to M-theory, our cosmos started as a perfect 11 dimensional space with nothing in it. However, this 11 dimensional space was unstable. The original 11 dimensional spacetime finally cracked into two pieces, a 4 and a 7 dimensional cosmos. The cosmos made the 7 of the 11 dimensions curled into a tiny ball, allowing the remaining 4 dimensional cosmos to inflate at enormous rates. This origin of our cosmos implies a multi-space result for our cosmos verified by Theorem 6.1.5.

Theorem 6.3.1 *The spacetime of M-theory is a multi-space with a warping* $\mathbf{R}^7$ *at each point of* $\mathbf{R}^4$.

Applying Theorem 6.3.1, an example for an accelerating expansion cosmos of 4-dimensional cosmos from supergravity compactification on hyperbolic spaces is the *Townsend-Wohlfarth type* in which the solution is

$$ds^2 = e^{-m\phi(t)}(-S^6dt^2 + S^2dx_3^2) + r_C^2 e^{2\phi(t)} ds_{H_m}^2,$$

where

$$\phi(t) = \frac{1}{m-1}(\ln K(t) - 3\lambda_0 t), \quad S^2 = K^{\frac{m}{m-1}} e^{-\frac{m+2}{m-1}\lambda_0 t}$$

and

$$K(t) = \frac{\lambda_0 \zeta r_c}{(m-1)\sin[\lambda_0 \zeta |t + t_1|]}$$

with $\zeta = \sqrt{3 + 6/m}$. This solution is obtainable from space-like brane solution and if the proper time ς is defined by $d\varsigma = S^3(t)dt$, then the conditions for expansion and acceleration are $\frac{dS}{d\varsigma} > 0$ and $\frac{d^2S}{d\varsigma^2} > 0$. For example, the expansion factor is 3.04 if $m = 7$, i.e., a really expanding cosmos.

6.3.2. **A pseudo-face model for p-branes**

In fact, M-theory contains much more than just strings, which is also implied in Fig.6.4. It contains both higher and lower dimensional objects, called *branes*. A *brane* is an object or subspace which can have various spatial dimensions. For any integer $p \geq 0$, a p-*brane* has length in p dimensions, for example, a 0-*brane* is just a point; a 1-*brane* is a string and a 2-*brane* is a surface or membrane $\cdots$.

Two branes and their motion have been shown in Fig.6.5 where (a) is a 1-brane and (b) is a 2-brane.

Fig.6.5

Combining these ideas in the pseudo-spaces theory and in M-theory, a model for $\mathbf{R}^m$ is constructed in the below.

Model 6.3.1 *For each m-brane $\mathbf{B}$ of a space $\mathbf{R}^m$, let $(n_1(\mathbf{B}), n_2(\mathbf{B}), \cdots, n_p(\mathbf{B}))$ be its unit vibrating normal vector along these p directions and $q : \mathbf{R}^m \to \mathbf{R}^4$ a continuous mapping. Now for $\forall P \in \mathbf{B}$, define*

$$\omega(q(P)) = (n_1(P), n_2(P), \cdots, n_p(P)).$$

Then $(\mathbf{R}^4, \omega)$ is a pseudo-face of $\mathbf{R}^m$, particularly, if $m = 11$, it is a pseudo-face for the M-theory.

For the case of $p = 4$, interesting results are obtained by applying results in Chapters 5.

Theorem 6.3.2 *For a sphere-like cosmos $\mathbf{B}^2$, there is a continuous mapping $q : \mathbf{B}^2 \to \mathbf{R}^2$ such that its spacetime is a pseudo-plane.*

Proof According to the classical geometry, we know that there is a projection $q : \mathbf{B}^2 \to \mathbf{R}^2$ from a 2-ball $\mathbf{B}^2$ to an Euclid plane $\mathbf{R}^2$, as shown in Fig.6.6.

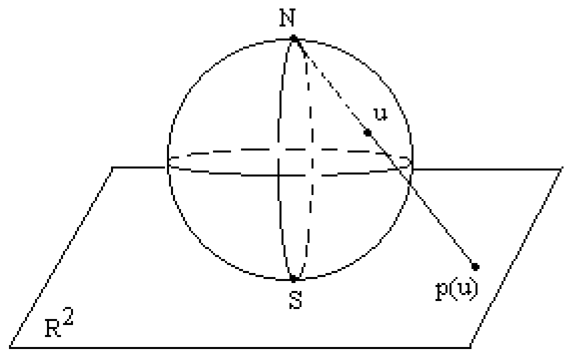

Fig.6.6

Now for any point $u \in \mathbf{B}^2$ with an unit vibrating normal vector $(x(u), y(u), z(u))$, define

$$\omega(q(u)) = (z(u), t),$$

where t is the time parameter. Then $(\mathbf{R}^2, \omega)$ is a pseudo-face of $(\mathbf{B}^2, t)$. $\natural$

Generally, we can also find pseudo-surfaces as a pseudo-face of sphere-like cosmoses.

Theorem 6.3.3 *For a sphere-like cosmos $\mathbf{B}^2$ and a surface S, there is a continuous mapping $q : \mathbf{B}^2 \to S$ such that its spacetime is a pseudo-surface on S.*

Proof According to the classification theorem of surfaces, an surface S can be combinatorially represented by a $2n$-polygon for an integer $n, n \geq 1$. If we assume that each edge of this polygon is at an infinite place, then the projection in Fig.6.6 also enables us to get a continuous mapping $q : \mathbf{B}^2 \to S$. Thereby we get a pseudo-face on S for the cosmos $\mathbf{B}^2$. $\natural$

Furthermore, we can construct a combinatorial model for our cosmos by applying materials in Section 2.5.

Model 6.3.2 *For each m-brane $\mathbf{B}$ of a space $\mathbf{R}^m$, let $(n_1(\mathbf{B}), n_2(\mathbf{B}), \cdots, n_p(\mathbf{B}))$ be its unit vibrating normal vector along these p directions and $q : \mathbf{R}^m \to \mathbf{R}^4$ a continuous mapping. Now construct a graph phase $(\mathcal{G}, \omega, \Lambda)$ by*

$$V(\mathcal{G}) = \{p - branes\ q(\mathbf{B})\},$$

$$E(\mathcal{G}) = \{(q(\mathbf{B}_1), q(\mathbf{B}_2)) | there\ is\ an\ action\ between\ \mathbf{B}_1\ and\ \mathbf{B}_2\},$$

$$\omega(q(\mathbf{B})) = (n_1(\mathbf{B}), n_2(\mathbf{B}), \cdots, n_p(\mathbf{B})),$$

and

$$\Lambda(q(\mathbf{B}_1), q(\mathbf{B}_2)) = \textit{ forces between } \mathbf{B}_1 \textit{ and } \mathbf{B}_2.$$

Then we get a graph phase $(\mathcal{G}, \omega, \Lambda)$ *in* $\mathbf{R}^4$. *Similarly, if* $m = 11$, *it is a graph phase for the M-theory.*

If there are only finite p-branes in our cosmos, then Theorems 6.3.2 and 6.3.3 can be restated as follows.

Theorem 6.3.4 *For a sphere-like cosmos* $\mathbf{B}^2$ *with finite p-branes and a surface* S, *its spacetime is a map geometry on* S.

Now we consider the transport of a graph phase $(\mathcal{G}, \omega, \Lambda)$ in $\mathbf{R}^m$ by applying results in Sections 2.3 and 2.5.

Theorem 6.3.5 *A graph phase* $(\mathcal{G}_1, \omega_1, \Lambda_1)$ *of space* $\mathbf{R}^m$ *is transformable to a graph phase* $(\mathcal{G}_2, \omega_2, \Lambda_2)$ *of space* $\mathbf{R}^n$ *if and only if* $\mathcal{G}_1$ *is embeddable in* $\mathbf{R}^n$ *and there is a continuous mapping* τ *such that* $\omega_2 = \tau(\omega_1)$ *and* $\Lambda_2 = \tau(\Lambda_1)$.

Proof By the definition of transformations, if $(\mathcal{G}_1, \omega_1, \Lambda_1)$ is transformable to $(\mathcal{G}_2, \omega_2, \Lambda_2)$, then there must be $\mathcal{G}_1$ is embeddable in $\mathbf{R}^n$ and there is a continuous mapping τ such that $\omega_2 = \tau(\omega_1)$ and $\Lambda_2 = \tau(\Lambda_1)$.

Now if $\mathcal{G}_1$ is embeddable in $\mathbf{R}^n$ and there is a continuous mapping τ such that $\omega_2 = \tau(\omega_1)$, $\Lambda_2 = \tau(\Lambda_1)$, let $\varsigma : \mathcal{G}_1 \to \mathcal{G}_2$ be a continuous mapping from $\mathcal{G}_1$ to $\mathcal{G}_2$, then (ς, τ) is continuous and

$$(\varsigma, \tau) : (\mathcal{G}_1, \omega_1, \Lambda_1) \to (\mathcal{G}_2, \omega_2, \Lambda_2).$$

Therefore $(\mathcal{G}_1, \omega_1, \Lambda_1)$ is transformable to $(\mathcal{G}_2, \omega_2, \Lambda_2)$. $\natural$

Theorem 6.3.5 has many interesting consequences as by-products.

Corollary 6.3.1 *A graph phase* $(\mathcal{G}_1, \omega_1, \Lambda_1)$ *in* $\mathbf{R}^m$ *is transformable to a planar graph phase* $(\mathcal{G}_2, \omega_2, \Lambda_2)$ *if and only if* $\mathcal{G}_2$ *is a planar embedding of* $\mathcal{G}_1$ *and there is a continuous mapping* τ *such that* $\omega_2 = \tau(\omega_1)$, $\Lambda_2 = \tau(\Lambda_1)$ *and vice via, a planar graph phase* $(\mathcal{G}_2, \omega_2, \Lambda_2)$ *is transformable to a graph phase* $(\mathcal{G}_1, \omega_1, \Lambda_1)$ *in* $\mathbf{R}^m$ *if and only if* $\mathcal{G}_1$ *is an embedding of* $\mathcal{G}_2$ *in* $\mathbf{R}^m$ *and there is a continuous mapping* τ^{-1} *such that* $\omega_1 = \tau^{-1}(\omega_2)$, $\Lambda_1 = \tau^{-1}(\Lambda_2)$.

Corollary 6.3.2 *For a continuous mapping* τ, *a graph phase* $(\mathcal{G}_1, \omega_1, \Lambda_1)$ *in* $\mathbf{R}^m$ *is transformable to a graph phase* $(\mathcal{G}_2, \tau(\omega_1), \tau(\Lambda_1))$ *in* $\mathbf{R}^n$ *with* $m, n \geq 3$.

Proof This result follows immediately from Theorems 2.3.2 and 6.3.5. $\natural$

This theorem can be also used to explain the problems of *travelling between cosmoses* or *getting into the heaven or hell* for a person. For example, water will go from a liquid phase to a steam phase by heating and then will go to a liquid phase by cooling because its phase is transformable between the steam phase and the liquid phase. For a person on the earth, he can only get into the heaven or hell after death because the dimension of the heaven is more than 4 and that of the hell is less than 4 and there does not exist a transformation for an alive person from our cosmos to the heaven or hell by the biological structure of his body. Whence, if black holes are really these tunnels between different cosmoses, the destiny for a cosmonaut unfortunately fell into a black hole is only the death ([30][32]). Perhaps, there are really other kind of beings in cosmoses or mankind in the further who can freely change from one phase in a space $\mathbf{R}^m$ to another in $\mathbf{R}^n$ with $m \neq n$, then the travelling between cosmoses is possible for those beings or mankind in that time.

6.3.3. **A multi-space model of cosmos**

Until today, many problems in cosmology are puzzling one's eyes. Comparing with these vast cosmoses, human beings are very tiny. In spite of this depressed fact, we can still investigate cosmoses by our deeply thinking. Motivated by this belief, a multi-space model for cosmoses is constructed in the following.

Model 6.3.3 *A mathematical cosmos is constructed by a triple* (Ω, Δ, T)*, where*

$$\Omega = \bigcup_{i\geq 0} \Omega_i, \quad \Delta = \bigcup_{i\geq 0} O_i$$

and $T = \{t_i; i \geq 0\}$ *are respectively called the cosmos, the operation or the time set with the following conditions hold.*

(1) (Ω, Δ) *is a Smarandache multi-space dependent on* T*, i.e., the cosmos* (Ω_i, O_i) *is dependent on the time parameter* t_i *for any integer* $i, i \geq 0$.

(2) *For any integer* $i, i \geq 0$*, there is a sub-cosmos sequence*

$$(S): \ \Omega_i \supset \cdots \supset \Omega_{i1} \supset \Omega_{i0}$$

in the cosmos (Ω_i, O_i) *and for two sub-cosmoses* (Ω_{ij}, O_i) *and* (Ω_{il}, O_i)*, if* $\Omega_{ij} \supset \Omega_{il}$*, then there is a homomorphism* $\rho_{\Omega_{ij},\Omega_{il}} : (\Omega_{ij}, O_i) \to (\Omega_{il}, O_i)$ *such that*

(*i*) *for* $\forall(\Omega_{i1}, O_i), (\Omega_{i2}, O_i)(\Omega_{i3}, O_i) \in (S)$*, if* $\Omega_{i1} \supset \Omega_{i2} \supset \Omega_{i3}$*, then*

$$\rho_{\Omega_{i1},\Omega_{i3}} = \rho_{\Omega_{i1},\Omega_{i2}} \circ \rho_{\Omega_{i2},\Omega_{i3}},$$

where $\circ$ *denotes the composition operation on homomorphisms.*

(*ii*) *for* $\forall g, h \in \Omega_i$*, if for any integer* i*,* $\rho_{\Omega,\Omega_i}(g) = \rho_{\Omega,\Omega_i}(h)$*, then* $g = h$.

(*iii*) *for* $\forall i$*, if there is an* $f_i \in \Omega_i$ *with*

$$\rho_{\Omega_i,\Omega_i \bigcap \Omega_j}(f_i) = \rho_{\Omega_j,\Omega_i \bigcap \Omega_j}(f_j)$$

for integers $i, j, \Omega_i \bigcap \Omega_j \neq \emptyset$*, then there exists an* $f \in \Omega$ *such that* $\rho_{\Omega,\Omega_i}(f) = f_i$ *for any integer* i.

Notice that this model is a multi-cosmos model. In the Newton's mechanics, the Einstein's relativity theory or the M-theory, there is just one cosmos Ω and these sub-cosmos sequences are

$$\mathbf{R}^3 \supset \mathbf{R}^2 \supset \mathbf{R}^1 \supset \mathbf{R}^0 = \{P\},$$

$$\mathbf{R}^4 \supset \mathbf{R}^3 \supset \mathbf{R}^2 \supset \mathbf{R}^1 \supset \mathbf{R}^0 = \{P\}$$

and

$$\mathbf{R}^4 \supset \mathbf{R}^3 \supset \mathbf{R}^2 \supset \mathbf{R}^1 \supset \mathbf{R}^0 = \{P\} \supset \mathbf{R}_7^- \supset \cdots \supset \mathbf{R}_1^- \supset \mathbf{R}_0^- = \{Q\}.$$

These conditions in (2) are used to ensure that a mathematical cosmos posses a *general structure sheaf* of a topological space, for instance if we equip each multi-space (Ω_i, O_i) with an abelian group G_i for an integer $i, i \geq 0$, then we get a structure sheaf on a mathematical cosmos. For *general sheaf theory*, one can see in the reference [29] for details. This structure enables that a being in a cosmos of higher dimension can supervises those in lower dimension.

Motivated by this multi-space model of cosmos, we present a number of conjectures on cosmoses in the following. The first is on the number of cosmoses and their dimension.

Conjecture 6.3.1 *There are infinite many cosmoses and all dimensions of cosmoses make up an integer interval* $[1, +\infty]$.

A famous proverbs in Chinese says that *seeing is believing but hearing is unbelieving*, which is also a dogma in the pragmatism. Today, this view should be abandoned by a mathematician if he wish to investigate the 21st mathematics. On the first, we present a conjecture on the problem of travelling between cosmoses.

Conjecture 6.3.2 *There must exists a kind of beings who can get from one cosmos into another. There must exists a kind of being who can goes from a space of higher dimension into its subspace of lower dimension, especially, on the earth.*

Although nearly every physicist acknowledges the existence of black holes, those holes are really found by mathematical calculation. On the opposite, we present the next conjecture.

Conjecture 6.3.3 *Contrary to black holes, there are also white holes at where no matters can arrive including the light in our cosmos.*

Conjecture 6.3.4 *Every black hole is also a white hole in a cosmos.*

Our cosmonauts is good luck if Conjecture 6.3.4 is true since they do not need to worry about attracted by these black holes in our cosmos. Today, a very important task in theoretical and experimental physics is looking for dark matters. However, we do not think this would be success by the multi-model of cosmoses. This is included in the following conjecture.

Conjecture 6.3.5 *One can not find dark matters by experiments since they are in spatial can not be found by human beings.*

Few consideration is on the relation of the dark energy with dark matters. But we believe there exists a relation between the dark energy and dark matters such as stated in the next conjecture.

Conjecture 6.3.6 *Dark energy is just the effect of dark matters.*

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