# How many postulates are needed for deriving the Lorentz transformation?

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## Abstract

It is generally believed that Einstein derived special relativity from two postulates, the principle of relativity and the constancy of the speed of light, without paying much attention to the results of the Michelson-Morley experiment and Lorentz's interpretation. In this study, the minimum conditions for deriving the Lorentz transformation are investigated, and one of Einstein's derivations and one textbook derivation of the Lorentz transformation analyzed. It was found that Einstein's two postulates are not sufficient for deriving the Lorentz transformation and at least 4 postulates are necessary. In order to obtain the Lorentz transformation, Einstein made several logical mistakes by using same symbols for different quantities and variables. When these symbols are made unequivocal, Einstein's derivation cannot lead to the Lorentz transformation. Therefore, it is a false assertion that Einstein derived special relativity from only two postulates. This study demonstrates that there are an infinite number of linear and non-linear transformations that are consistent with the principle of relativity and the constancy of the speed of light. Having four postulates for deriving the Lorentz transformation is effectively equivalent to postulating the Lorentz transformation directly.

**Key words: Lorentz transformation; speed of light; principle of relativity;**

## 1 Introduction

It is a belief held by most physicists as well as the general public that Einstein derived special relativity from two postulates, the principle of relativity and the constancy of the speed of light. This feat of Einstein has been told for generations and admired by generations of scientists and people in other walks of life. According to

Einstein's own account, the idea of special relativity occurred to him when he reflected on the propagation of light and Maxwell's electromagnetic theory; the result of the Michelson-Morley experiment had little influence on this achievement. The account of historians on the development of special relativity tends to trace its origin to the Michelson-Morley experiment, and place Einstein's contribution as the completion of an evolving process.

The history of special relativity is a bit more complicated than what the general public perceives. Since the wave theory of light became prevalent, the relationship between objects with mass and the medium of light around them was an issue of debates. The dominant view was that an object could not fully drag the medium of light around it. This was supported by the experiments conducted by Arago (1818), Fizeau (1851), and Airy (1871) among others, so that it would be possible to measure the velocity of the object relative to the medium of light. The Michelson-Morley experiment was carried out to measure the velocity of the earth relative to the medium of light, but they obtained a null result (Michelson 1881; Michelson and Morley 1887). If the earth cannot drag the ether fully to move with it, the null result has to be explained. Fitzgerald (1889), Lorentz (1892, 1899), Larmor (1897, 1900), and Voigt (1887) proposed length change hypotheses, which culminated in the Lorentz transformation with Lorentz ether theory (Lorentz 1904).

Lorentz ether theory hypothesizes that the length of an object moving relative to the ether will contract in the direction of its velocity and the time unit of clocks moving relative to the ether will dilate (i.e., the clocks become slower) , such that the Galilean transformation between reference frames is no longer applicable. The new spatial and temporal transformations, the Lorentz transformation can ensure that the speed of light in vacuum measured in all inertial frames of reference is the same value, *c*. Poincaré (1905,1906) also made essential contributions to the Lorentz ether theory and modified the Lorentz transformation to its present forms.

The Lorentz ether theory could explain almost all the new macroscopic phenomena that had challenged classical physics. Then Einstein (1905) published his first paper on relativity, deriving the Lorentz transformation from the two postulates and obtained results from the Lorentz transformation on time dilation, length contraction, velocity addition, Doppler's effect, aberration, longitudinal mass,

transverse mass, and so on. In his paper, Einstein did not cite any references, so it is not clear whether he had prior knowledge of the works by Lorentz and Poincaré. Einstein's own account denies that he had prior knowledge of the works by Lorentz and Poincaré. Historians could not agree on this issue, but later researchers emphasize less on Einstein's derivation of the Lorentz transformation than on Einstein's new space-time concept.

Now the physics community has two competing theories which cannot be distinguished by any feasible experiments so far, because all testable predictions from the two theories are the same. Lorentz ether theory is based on the length contraction and time dilation hypotheses which ensure the relative speed of light to be constant, and it also needs the medium of light for the theory to be logically consistent. Einstein's special relativity is based on two postulates for deriving the Lorentz transformation from which length contraction and time dilation can be derived, and it needs no ether. The mainstream of the physics community has adopted Einstein's special relativity because it was perceived to be superior to Lorentz ether theory on philosophical grounds. There is no experimental evidence so far to favor either of the two theories.

Although most physicists and the general public believe that special relativity is derived from the two postulates by Einstein, it seems that the sufficiency of the two postulates for deriving the Lorentz transformation which lies in the core of special relativity has not been investigated. One reason for lack of studies on the completeness of the two postulates is that physicists and philosophers with some physics background may be more interested in the experimental validation of the predictions from special relativity than metaphysical issues such as the sufficiency or completeness of the postulates for special relativity. The other reason is that the theory of relativity is perceived to be too difficult to comprehend for most people, so many philosophers with or without some physics background are not confident enough to examine the sufficiency of the two postulates. The aim of the present study is to investigate whether the two postulates, the principle of relativity and the constancy of the speed of light, are sufficient for deriving the Lorentz transformation. This study does not question the correctness or validity of special relativity, nor does it question the logical consistency of special relativity per se. It only tries to establish

whether additional postulates or assumptions are needed and implicitly used in Einstein's derivation of the Lorentz transformation.

The rest of the paper is organized as follows: section 2 presents logical inference rules and looks into the sufficiency conditions from the logical inference rules; section 3 analyzes what the two postulates mean for deriving the Lorentz transformation; section 4 provides a derivation of the Lorentz transformation based on these general requirements; section 5 examines Einstein's simple derivation in "Relativity: the special and the general theory"; section 6 examines the derivation in *Berkeley Physics Course*; section 7 concludes.

## 2 Logical inference rules

A rule of inference is a logical form consisting of a function which takes premises, analyzes their syntax, and returns a conclusion or conclusions. In a Hilbert-style deduction system, the syntax of the logical operators is mostly defined by means of axioms and only one inference rule, *modus ponens*, is used. This inference rule has the form

$$((P \to Q) \wedge P) \to Q \qquad (1)$$

Equation (1) takes two premises, $P \to Q$, i.e. "If $P$ then $Q$", and "$P$", and returns the conclusion "$Q$". The rule is valid in the sense that if the premises are true, so is the conclusion. $P \to Q$ means that $P$ must be a sufficient condition of $Q$. If P is sufficient condition of $Q$, then $Q$ must be a necessary condition of $P$.

If $P$ is only a necessary condition of $Q$, i.e. $P \leftarrow Q$, then $(P \longleftarrow Q) \wedge P$ does not lead to $Q$. $P \leftarrow Q$ indicates that some conditions which are not $Q$ lead to $P$. If $P$ is a necessary condition of $Q$ and $Q$ is a necessary of $P$, then $Q$ must be a necessary and sufficient condition of $P$ and $P$ must be a necessary and sufficient condition of $Q$, $\big((P \leftarrow Q) \wedge (Q \leftarrow P)\big) \longrightarrow (P \leftrightarrow Q)$ .

To prove that $P$ is not a sufficient condition or a necessary and sufficient condition of $Q$, we only need to show that there is at least one instance where $P$ is true, which is not $Q$. If there is a $R$, $R \to P$ and $R \notin Q$, even though $Q \to P$, we still have $P \nrightarrow Q$.

To derive $Q$ from $P$, $P$ must be a sufficient or a necessary and sufficient condition of $Q$. If $P \rightarrow Q$, $Q$ must be a necessary or a necessary and sufficient condition of $P$, which means that other conditions cannot be sufficient for $P$ without $Q$. Therefore, as long as an R that does not belong to $Q$ is sufficient for $P$, a derivation of $Q$ from $P$ is invalid. When there is such an $R$, $P$ is only a necessary condition of $Q$ rather than a sufficient or a necessary and sufficient condition of $Q$.

When $P$ can be derived from $Q$, $Q$ is a sufficient condition of $P$. Then it is not possible to derive $Q$ from $P$, unless $Q$ is also a necessary condition of $P$. If there is also an $R$ that is a sufficient condition for $P$, $Q$ is not a necessary and sufficient condition of $P$. These rules of inference can help us assess the validity of derivations.

**3 What do the principle of relativity and constancy of the speed of light mean?**

From the viewpoint of logic, whether the Lorentz transformation can be derived from the two postulates, the principle of relativity and the constancy of the speed of light, depends on whether the Lorentz transformation is a necessary (and sufficient) condition of the two postulates. Whether the Lorentz transformation is a necessary (and sufficient) condition of the two postulates depends on whether there is another transformation that satisfies the principle of relativity and leads to the constancy of the speed of light. If there are other transformations that satisfy the principle of relativity and lead to the constancy of the speed of light, then Lorentz transformation is not a necessary condition for the principle of relativity and the constancy of the speed of light. If Lorentz transformation is not a necessary condition for the principle of relativity and the constancy of the speed of light, then it cannot be uniquely derived from the two postulates.

If Lorentz transformation is not a necessary condition for the principle of relativity and the constancy of the speed of light, then a derivation from the two postulates must be one of the two situations: 1) there is a logical mistake in the derivation, and 2) other conditions are used implicitly.

To investigate whether Lorentz transformation is a necessary condition for the principle of relativity and the constancy of the speed of light, we need to know what the two postulates mean. The constancy of the speed of light is relatively easy to interpret: the speed of light measured in all reference frames is $c$. The principle of

relativity may have different representations from different perspectives. To interpret the principle of relativity, we may first list some examples that satisfy the principle of relativity. Does Newtonian mechanics satisfy the principle of relativity? Does Lorentz ether theory satisfy it? Does special relativity satisfy the principle of relativity?

For many (if not all) physicists, Newtonian mechanics, Lorentz ether theory and special relativity all satisfy the principle of relativity. The usual expression of the relativity principle is that physical laws are the same in all reference frames. A simpler expression should be that absolute velocity cannot be measured by physical methods. Some physicists would object to such a representation, because the Michelson-Morley experiment was intended to measure the absolute velocity of the earth in space. For them, the principle of relativity implies that absolute velocity cannot be measured by mechanic methods instead.

As a matter of fact, even if the Michelson-Morley experiment had been positive, there would have been no method to ascertain that ether is absolutely stationary in space other than by definition or postulation. The velocity of the earth relative to ether is just its velocity relative to ether. Therefore, a simplistic representation of the principle of relativity is that no reference frame can measure its own absolute velocity. When the absolute stationarity of ether cannot be proved by any physical means, the best physicists can achieve through Michelson-Morley type experiments is the relative velocity between the earth and the medium of light surrounding the earth. The differences between Newtonian mechanics, Lorentz ether theory and special relativity can be summarized as follows:

1) In Newtonian mechanics (Galilean principle of relativity), objects' absolute velocity (their velocity relative to the absolutely stationary space) cannot be measured. For those who insist on ether being absolute stationary in space, electromagnetic methods such as the Michelson-Morley experiment can test absolute velocity in space. However, according to Galilean principle of relativity, there is no way to verify ether being absolutely stationary.
2) In Lorentz ether theory, objects' velocity relative to the surrounding medium of light cannot be measured (due to length contraction and time

dilation). For those who believe that ether is absolutely stationary in space, Lorentz ether theory provides a mechanism for absolute velocity being immeasurable.

3) In special relativity, there is no medium of light and the absolute velocity of objects cannot be measured.

In my view, a positive Michelson-Morley experiment result still could only show earth's velocity relative to its surrounding medium of light, not its absolute velocity, because there is no way to prove the surrounding ether being absolutely stationary. However, in order to avoid unnecessary disputes over whether a positive result of the Michelson-Morley experiment can show absolute velocity of the earth, I would present the principle of relativity as that the absolute velocity of a reference frame cannot be measured by mechanic methods. Deriving the transformation rules of space and time coordinates between two reference frames from the two postulates is to find a set of rules which are necessary conditions of the two postulates. That is, when the set of rules do not exist, the two postulates cannot arise. Since the principle of relativity is about non-measurability of absolute velocity, that absolute velocity cannot be measured by mechanic method is a necessary condition for the principle of relativity. Since the principle of relativity implies non-measurability of absolute velocity and the non-measurability of absolute velocity implies the principle of relativity, the principle of relativity is a necessary and sufficient condition of non-measurability of absolute velocity.

The relationship between the two postulates and the Lorentz transformation need to be examined. If we look at the principle of relativity, the Lorentz transformation shows non-measurability of objects' velocity relative to the ether frame and implicitly requires non-measurability of objects' velocity relative to the absolute space. As non-measurability of absolute velocity is a necessary and sufficient condition for the principle of relativity and the Lorentz transformation contains more than non-measurability of absolute velocity, the principle of relativity is a necessary condition, but not a sufficient condition of the Lorentz transformation.

The postulate at the centre of Einstein's first derivation is the constancy of the speed of light. Since the constancy of the speed of light can be derived from the

Lorentz transformation, the Lorentz transformation is at least a sufficient condition for this postulate which is a necessary condition of the Lorentz transformation. Then whether the Lorentz transformation or even special relativity can be derived from the two postulates depends on whether the constancy of the speed of light (plus the principle of relativity) is a sufficient condition of the Lorentz transformation.

If the two necessary conditions of the Lorentz transformation, the principle of relativity and the constancy of the speed of light together, are also its sufficient condition, then there would be no other transformation which is sufficient condition for the principle of relativity and the constancy of the speed of light.

Here the Lorentz transformation is used in a broad sense, including all forms of the Poincaré transformation. When there is one transformation that does not belong to the Lorentz/Poincaré transformation, the principle of relativity and the constancy of the speed of light together are not sufficient conditions, hence the Lorentz transformation cannot be derived logically from the two postulates. As a matter of fact, the Voigt transformation is such a space-time transformation that is sufficient to make absolute velocity immeasurable (even for those who believe in absolutely stationary ether) and ensure the constancy of the speed of light.

The existence of the Voigt transformation shows that the principle of relativity and constancy of the speed of light are not sufficient conditions of the Lorentz transformation. Therefore, the Lorentz transformation cannot be derived logically from only these two postulates. Either logical mistakes exist or other premises must have been used implicitly when the Lorentz transformation is allegedly derived from only those two postulates.

## 4 How to derive the Lorentz transformation?

The Lorentz transformation is initially intended to ensure Maxwell's electromagnetic equations invariant in different inertial reference frames. In the context of special relativity, the Lorentz transformation ensures constancy of the speed of light. Therefore, to derive the Lorentz transformation between two reference frames, which view the other reference frame moving at speed $v$ and $v'$ respectively

($v' = -v$) along the x-axis, from the two postulates is to derive equations with following characteristics,

$$x' = f(x, y, z, t, v) \quad (2)$$

$$y' = g(x, y, z, t, v) \quad (3)$$

$$z' = h(x, y, z, t, v) \quad (4)$$

$$t' = i(x, y, z, t, v) \quad (5)$$

$$x = f(x', y', z', t', v') \quad (6)$$

$$y = g(x', y', z', t', v') \quad (7)$$

$$z = h(x', y', z', t', v') \quad (8)$$

$$t = i(x', y', z', t', v') \quad (9)$$

$$\frac{x}{t} = \frac{x'}{t'} = c \quad (10)$$

The symmetry of the transformations (2) - (9) between the primed and un-primed systems satisfies the principle of relativity. Equation (10) satisfies the constancy of the speed of light. Equations (2) - (9) do not give the specific function forms of these transformations, so additional conditions or postulates might be needed to derive the Lorentz transformation.

### 4.1. Derivation of space and time transformations with two or three postulates

As Einstein showed, the space-time transformation should have a linear relationship between reference frames, so we might assume some general linear function forms for equations (2) - (9).

$$x' = \alpha_x x + bvt \quad (11)$$

$$y' = \alpha_y y \quad (12)$$

$$z' = \alpha_z z \quad (13)$$

$$t' = nt + \delta x \tag{14}$$

$$x = \alpha_x x' + bv't' \tag{15}$$

$$y = \alpha_y y' \tag{16}$$

$$z = \alpha_z z' \tag{17}$$

$$t = nt' + \delta x' \tag{18}$$

The constancy of the speed of light in all reference frames might be defined by the propagation of spherical light wave to have the following relations:

$$x^2 + y^2 + z^2 = c^2 t^2 \tag{19}$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \tag{20}$$

When transformation equations (11)-(14) are substituted into (20), we obtain

$$(\alpha_x x + bvt)^2 + (\alpha_y y)^2 + (\alpha_z z)^2 = c^2 (nt + \delta x)^2 \tag{21}$$

In equation (21), since all coefficients are to be determined, it is unlikely that we are able to derive the Lorentz/Poincaré transformation where $y$ and $z$ are not involved in the transformation between $x$ and $x'$. Therefore, we need to impose some constraints on (21), and the first obvious constraints are

$$\alpha_y = 1, \alpha_z = 1 \tag{22}$$

Then, using $\alpha$ for $\alpha_x$, we have

$$(\alpha x + bvt)^2 + y^2 + z^2 = c^2 (\delta x + nt)^2 . \tag{23}$$

Expanding the above equation, we have

$$\alpha^2 x^2 - c^2 \delta^2 x^2 + 2\alpha bxvt - 2c^2 \delta nxt + y^2 + z^2 = c^2 n^2 t^2 - b^2 v^2 t^2 , \tag{24}$$

which can be simplified to

$$(\alpha^2 - c^2\delta^2)x^2 + 2(\alpha bv - c^2\delta n)xt + y^2 + z^2 = (c^2n^2 - b^2v^2)t^2. \qquad (25)$$

In the above equations, the initial values of *x*, *x′*, *t* and *t′* are $x_0 = 0$, $x'_0 = 0$, $t_0 = 0$ and $t'_0 = 0$. It can be converted readily to the scenarios where $x_0 \neq 0$, $x'_0 \neq 0$, $t_0 \neq 0$ and $t'_0 \neq 0$, simply by replacing *x*, *x′*, *t* and *t′* in the transformation with $x - x_0$, $x' - x'_0$, $t - t_0$ and $t' - t'_0$.

Comparing (25) with (19), we can see that if

$$\alpha^2 - c^2\delta^2 = 1, \qquad (26)$$

$$\alpha bv - c^2\delta n = 0, \qquad (27)$$

$$c^2n^2 - b^2v^2 = c^2, \qquad (28)$$

the transformation equations will ensure that the speed of light will be the same *c* in both reference systems. From Eq. (27), we get

$$b = \frac{c^2\delta n}{\alpha v}, \qquad (29)$$

Substituting (29) into (28), we obtain

$$c^2n^2 - \frac{c^4\delta^2n^2}{\alpha^2} = c^2, \qquad (30)$$

which can be simplified to

$$\alpha^2n^2 - c^2\delta^2n^2 = \alpha^2. \qquad (31)$$

From (26), we have

$$\alpha^2 = 1 + c^2\delta^2. \qquad (32)$$

Substituting (32) in (31), we obtain

$$(1+c^2\delta^2)n^2 - c^2\delta^2 n^2 = 1+c^2\delta^2, \tag{33}$$

which can be simplified to

$$n^2 = 1+c^2\delta^2. \tag{34}$$

From (32) and (34), it is obvious that

$$n^2 = \alpha^2. \tag{35}$$

The original Eqs. (26), (27) and (28) are simplified to

$$\alpha^2 - c^2\delta^2 = 1, \tag{36}$$

$$bv - c^2\delta = 0, \tag{37}$$

$$c^2\alpha^2 - b^2v^2 = c^2. \tag{38}$$

It appears that we can solve for the parameters $\alpha$, $b$ and $\delta$ because we have three unknowns in three simultaneous equations, but this is an illusion. The three equations are not mutually independent, since given Eq. (37), Eqs. (36) and (38) are the same equation. We only have two mutually independent equations, and we can get infinite groups of solution for parameters $\alpha$, b and $\delta$. From (36) and (37), we obtain

$$\alpha = \sqrt{1+c^2\delta^2}, \tag{39}$$

$$b = \frac{c^2\delta}{v}. \tag{40}$$

For any real valued $\delta$, which can be $v$ or any constant or any combination of constants and $v$, as long as $\alpha$ and $b$ are chosen according to Eqs. (39) and (40), the transformation Eqs. (11) - (18) satisfy the principle of relativity and the constancy of the speed of light and there are an infinite number of such transformations.

### 4.2. Derivation of the Lorentz transformation

The Lorentz transformation is an instance of the general transformation equations where $\alpha = -b$, that is,

$$x' = \alpha x - \alpha v t . \tag{41}$$

This stipulation $\alpha = -b$ means

$$-\frac{c^2\delta}{v} = \sqrt{1 + c^2\delta^2}$$

which implies

$$\delta^2 = \frac{1}{\frac{c^4}{v^2} - c^2} = \frac{v^2/c^4}{1 - v^2/c^2}$$

$$\alpha = \sqrt{1 + c^2\delta^2} = \sqrt{1 + \frac{v^2/c^2}{1 - v^2/c^2}} = \sqrt{\frac{1}{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{42}$$

$$b = -\alpha = -\frac{1}{\sqrt{1 - v^2/c^2}} \tag{43}$$

$$\delta = \frac{bv}{c^2} = -\frac{v/c^2}{\sqrt{1 - v^2/c^2}} \tag{44}$$

Substituting these coefficients, $n = \alpha$ and equations (22) in equations (11)-(14), we obtain the Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \qquad y' = y, \qquad z' = z,$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} \tag{45}$$

The transformation from $x'$ to $x$ has the same form,

$$x = \alpha x' - \alpha v' t' = \alpha x' + \alpha v t \tag{46}$$

Equation (41) reveals that we need to impose another constraint $\alpha = -b$ to derive the Lorentz transformation. Therefore, we must have at least four premises to derive the Lorentz transformation:

1) The constancy of the speed of light, which is reflected by equations (19) and (20)

2) The principle of relativity, which is reflected by equations (11)-(18). The transformations from the un-primed frame have the same forms as those from the primed frame.

3) The length in the directions perpendicular to the direction of the velocity is not affected by the velocity between the two frames, which is reflected by equation (22), i.e. $y' = y, z' = z$ when the velocity is along the *x*-axis.

4) The space transformation in the velocity direction has the form $\Phi' = \alpha\Phi - \alpha vt$, in which $\Phi$ indicates the ordinate of the axis along which the frame is moving.

When we use all the four conditions to derive space-time transformation equations, we have

$$(\alpha x - \alpha vt)^2 + y^2 + z^2 = c^2(\delta x + nt)^2 \qquad (47)$$

Expanding the above equation, we have

$$\alpha^2x^2 - c^2\delta^2x^2 - 2\alpha^2vxt - 2c^2\delta nxt + y^2 + z^2 = c^2n^2t^2 - \alpha^2v^2t^2$$

which can be simplified to

$$(\alpha^2 - c^2\delta^2)x^2 - 2(\alpha^2v - c^2\delta n)xt + y^2 + z^2 = (c^2n^2 - \alpha^2v^2)t^2 \qquad (48)$$

Solving the following equations for α, δ and n will give us the coefficients for the Lorentz transformation,

$$\alpha^2 - c^2\delta^2 = 1, \qquad (49)$$

$$\alpha^2v - c^2\delta n = 0, \qquad (50)$$

$$c^2n^2 - \alpha^2v^2 = c^2. \qquad (51)$$

### 4.3. Derivation of the Poincaré transformation and the nature of the Lorentz/Poincaré transformation

In fact, the fourth premise in the preceding section should be that the space transformation has the form $\Delta\Phi' = \alpha(\Delta\Phi - v\Delta t)$, or more informatively

$$\Phi' - \Phi'_0 = \alpha(\Phi - \Phi_0) - \alpha v(t - t_0) \qquad (52)$$

Derivation using (52) leads to the Poincaré transformation. Derivation of the Lorentz transformation is achieved by imposing the condition $\Phi'_0 = 0$ and $\Phi_0 = 0$ when $t_0 = 0$ to equation (52). Equations (11) and (14) implicitly contain this condition. From (52) and the Poincaré transformation we may conclude that the nature of the Lorentz/Poincaré transformation is (coordinate) interval transformations rather than coordinate transformation.

If we want to derive true coordinate transformations, that is, equations (11) and (14) can be used when $\Phi'_0 \neq 0$ and $\Phi_0 \neq 0$, the space transformation would be the Galilean transformation. We can prove this by examining the transformation equation when initial values of $t$ and $t'$ are not zero, $t_0 \neq 0$ and $t'_0 \neq 0$.

When initial values are $x = x_0$, $x' = x'_0$, $t = t_0$ and $t' = t'_0$, for the propagation of light waves we have

$$x = x_0 + c(t - t_0), x' = x'_0 + c(t' - t'_0). \tag{53}$$

Substituting Eqs. (53) in Eqs. (41) and (46), we obtain

$$x' = \alpha x - \alpha v t = \alpha[x_0 + c(t - t_0)] - \alpha v(t - t_0), \tag{54}$$

$$x = \alpha x' + \alpha v t' = \alpha[x'_0 + c(t' - t'_0)] + \alpha v(t' - t'_0). \tag{55}$$

Using initial values $x = x_0$, $x' = x'_0$, $t = t_0$ and $t' = t'_0$, we get

$$x_0' = \alpha x_0, x_0 = \alpha x'_0, \tag{56}$$

which suggests

$$\alpha^2 = 1, \alpha = \pm 1. \tag{57}$$

That implies

$$x' = x - vt \tag{58}$$

The outcome will be the same, if $t_0 = 0$ , $t'_0 = 0$, while $x_0 \neq 0$and $x'_0 \neq 0$. This demonstrates again that the Lorentz spatial transformation only works for $x_0 = 0$ and $x'_0 = 0$; when $x_0 \neq 0$ and $x'_0 \neq 0$, the spatial coordinate transformation has to be the Galilean transformation which also works for $x_0 = 0$ and $x'_0 = 0$. One solution to this difficulty (i.e. spatial transformation has to be Galilean when $x_0 \neq 0$ and $x'_0 \neq 0$) is to transform the coordinate intervals instead of coordinates per se. Transformation of the coordinate intervals is the Poincaré transformation. The Lorentz transformation can therefore be viewed as transformation between the difference of $(x, y, z, t)$ and $(x_0, y_0, z_0, t_0)$ and the difference of $(x', y', z', t')$ and $(x'_0, y'_0, z'_0, t'_0)$ when $x_0 = 0$, $x'_0 = 0$, $t_0 = 0$ and $t'_0 = 0$.

When the spatial coordinate transformation is Galilean $x' = x - vt$, to ensure the constancy of the speed of light the time transformation would be

$$t' = \frac{x' - x'_0}{c} = \frac{x - x_0 - vt}{c} = \frac{ct - vt}{c} = t - \frac{vt}{c} = t - \frac{vct}{c^2} = t - \frac{v(x - x_0)}{c^2} \qquad (59)$$

**4.4. Derivation of the Voigt transformation**

The Voigt transformation can be derived from equations (11) – (18) with different specifications on $\alpha_x$, $\alpha_y$, $\alpha_z$ and $b$ than those for deriving the Lorentz/ Poincaré transformation. If instead of imposing equation (22) to (21), we impose

$$\alpha_x = -b = 1, \alpha_y = \alpha_z = \alpha \qquad (60)$$

Then, equation (21) becomes

$$(x - vt)^2 + \alpha^2 y^2 + \alpha^2 z^2 = c^2(nt + \delta x)^2 \qquad (61)$$

Expanding and simplifying (61), we obtain

$$(1 - \delta^2 c^2)x^2 - 2(v + \delta n c^2)xt + \alpha^2 y^2 + \alpha^2 z^2 = (c^2 n^2 - v^2)t^2 \qquad (62)$$

The constancy of the speed of light will be ensured if we have

$$1 - \delta^2 c^2 = \alpha^2, \qquad (63)$$

$$v + \delta n c^2 = 0, \tag{64}$$

$$c^2 n^2 - v^2 = \alpha^2 c^2. \tag{65}$$

Solving equations (63)-(65) for α, δ and *n* will arrive at the Voigt transformation,

$$x' = x - vt,$$

$$y' = y\sqrt{1 - v^2/c^2}\,,$$

$$z' = z\sqrt{1 - v^2/c^2}\,,$$

$$t' = t - vx/c^2. \tag{66}$$

With other constraints than those for the Lorentz transformation and the Voigt transformation, there are an infinite number of transformations that satisfy both the principle of relativity and the constancy of the speed of light. Therefore, it is impossible to derive the Lorentz transformation from only these two postulates.

The Lorentz transformation and the more general Poincaré transformation are necessary conditions for the above four premises, and the four premises are also necessary conditions for the Lorentz/Poincaré transformation. Therefore, they are necessary and sufficient conditions of each other. In such a situation, postulating the Lorentz/Poincaré transformation directly is equivalent to postulating the four premises and then deriving the transformation from the four premises. Lorentz was fully justified to postulate the Lorentz transformation as the foundation of his ether theory. Einstein or anyone else cannot logically derive the Lorentz transformation from the principle of relativity and the constancy of the speed of light, whereas postulating four premises seems more burdensome than postulating the Lorentz transformation directly. Special relativity could start directly from using the Lorentz transformation as postulates/laws. Then the constancy of the speed of light would be a theorem derived from the Lorentz transformation.

## 5 Einstein's simple derivation and its logical flaws

Einstein gave a simple derivation of the Lorentz transformation in his book *Relativity: the special and the general theory* (Einstein 1952), which has been

imitated and modified in various textbooks and popular science books. Einstein's derivation is based on the constancy of the speed of light in all inertial frames of reference no matter in which frame the light source is stationary. The basic setup for his derivation is two coordinate systems *K* and *K′*. As I have shown in the preceding section, the principle of relativity and the constancy of the speed of light are insufficient for deriving the Lorentz transformation. In Einstein's simple derivation and its likes, that $y' = y, z' = z$ when the velocity is along the *x*-axis are taken for granted, although it is an essential assumption for obtaining the Lorentz transformation. What is missing in those derivations is an explicit assumption of the function form $\Phi' - \Phi'_0 = \alpha(\Phi - \Phi_0) - \alpha v(t - t_0)$ for spatial transformation.

### 5.1. "Deriving" $x' = ax - bct$ and $t' = act - bx$ from $x = ct$ and $x' = ct'$

Instead of making an explicit assumption, Einstein tried to derive it from the constancy of the speed of light. Since the constancy of the speed of light does not contain information on the specific function form of spatial transformation, Einstein's effort can only be a logical mistake. Einstein's derivation is quoted in the following paragraphs.

"For the relative orientation of the co-ordinate systems …, the *x*-axes of both systems permanently coincide. In the present case we can divide the problem into parts by considering first only events which are localised on the *x*-axis. Any such event is represented with respect to the coordinate system *K* by the abscissa *x* and the time *t*, and with respect to the system *K′* by the abscissa *x′* and the time *t′*. We require to find *x′* and *t′* when *x* and *t* are given. A light-signal, which is proceeding along the positive axis of *x*, is transmitted according to the equation

$$x = ct$$

or $$x - ct = 0. \qquad [(67)]$$

Since the same light-signal has to be transmitted relative to *K′* with the velocity *c*, the propagation relative to the system *K′* will be represented by the analogous formula

$$x' - ct' = 0. \qquad [(68)]$$

Those space-time points (events) which satisfy [(67)] must also satisfy [(68)]. Obviously this will be the case when the relation

$$(x - ct) = \lambda(x' - ct') \qquad [(69)]$$

is fulfilled in general, where $\lambda$ indicates a constant; for, according to [(69)], the disappearance of $(x - ct)$ involves the disappearance of $(x' - ct')$."

Here Einstein failed to understand that given (67) and (68), Eq. (69) is true for λ of any arbitrary value, that is, λ in (69) is undefined. When $x - ct \equiv 0$ and $x' - ct' \equiv 0$, $(x - ct) = \lambda(x' - ct')$ means only $0 = \lambda \cdot 0$.

"If we apply quite similar considerations to light rays which are being transmitted along the negative *x*-axis, we obtain the condition

$$(x + ct) = \mu(x' + ct') \qquad [(70)]$$

By adding (or subtracting) equations [(69)] and [(70)], and introducing for convenience the constants a and b in place of the constants $\lambda$ and $\mu$ where

$$a = \frac{\lambda + \mu}{2}$$

and $$b = \frac{\lambda - \mu}{2}$$ ,

we obtain the equations

$$\left.\begin{aligned} x' &= ax - bct \\ t' &= act - bx \end{aligned}\right\} \qquad [(71)]"$$

Again Eq. (70) is true for μ of any value because for wave front transmitted along the negative x-axis $x + ct = 0$ and $x' + ct' = 0$. It is impossible to obtain equations (71) from (69) and (70) without making mathematical and logical mistakes. In his next operation, Einstein failed to understand that although *x* in (70) looks the same as *x* in (69), they are different. Since $x = ct$ in the positive end of the *x*-axis and $x = -ct$ in the negative end, the sum of *x* in (69) and *x* in (70) is zero,

$$ct + (-ct) = 0,$$

and *x* in (69) minus *x* in (70) is $|2x|$,

$$ct - (-ct) = 2ct,$$

while *x* in (70) minus *x* in (69) is $-|2x|$,

$$-ct - ct = -2ct.$$

The same is true for $x'$ in (69) and (70), and it is impossible to derive (71) correctly from (69) and (70), given (67) and (68). The correct result of adding (69) and (70) is

$$0 = (\lambda - \mu)\left(x'_{positive\ direction} - ct'\right)$$

Since $x_{positive} - ct = 0$, we have

$$2x_{positive\ direction} - 2ct = (\lambda - \mu)\left(x'_{positive\ direction} - ct'\right)$$

$$x_{positive\ direction} - ct = b\left(x'_{positive\ direction} - ct'\right)$$

Two sides of (69) minus two sides of (70) respectively should be

$$2x_{positive\ direction} - 2ct = (\lambda + \mu)\left(x'_{positive\ direction} - ct'\right)$$

The result of (70) minus (69) is

$$2x_{negative\ direction} + 2ct = (\mu + \lambda)(x'_{negative\ direction} + ct')$$

$$x_{negative\ direction} + ct = a(x'_{negative\ direction} + ct')$$

In the above equations, the subscripts "positive direction" and "negative direction" indicate that $x$ or $x'$ takes values on the positive and negative directions of the $x$- or $x'$ -axis respectively. For example, $x_{positive\ direction}$ takes the value of $x$ in equation (67). Therefore, the correct equations (71) should be the same equations as (69) and (70), with which Einstein tries to obtain the incorrect equations (71).

As we have commented, λ and μ are undefined, it is impossible to derive logically any defined value from them. From Einstein's above mistakes, we can see how important to use unequivocal notations in derivations. In order to avoid Einstein's mistake, equations (67) and (68) should be written as

$$x_{positive\ direction} = ct,$$

$$x_{negative\ direction} = -ct.$$

And we have the following relations,

$$x_{positive\ direction} + x_{negative\ direction} = 0$$

$$x_{positive\ direction} - x_{negative\ direction} = 2x_{positive\ direction} = -2x_{negative\ direction}$$

$$x_{negative\ direction} - x_{positive\ direction} = -2x_{positive\ direction} = 2x_{negative\ direction}$$

Although Einstein's derivation is so obviously wrong, generations of physicists have been trained to believe that it is possible to obtain equations (71) from $x = ct$ and $x' = ct'$.

## 5.2. Coordinates of the wave front substituted with coordinates of a fixed point in a reference frame

As I showed in the preceding section that (71) will not lead to a unique transformation, derivation of the Lorentz transformation needs the coefficient of *t* in the spatial transformation to be $av$. Therefore, even with equation (71), Einstein still had to make logical mistakes to derive the Lorentz transformation.

"We should thus have the solution of our problem, if the constants *a* and *b* were known. These result from the following discussion. For the origin of *K′* we have permanently *x′ =0*, and hence according to the first of the equations [(71)]

$$x = \frac{bc}{a}t\ ." \qquad (72)$$

In Eqs. (61)-(65), *x* and *x′* are wave fronts of the light beam in the coordinate systems *K* and *K′* respectively. Einstein should not use $x'_{origin\ of\ K'}$ to substitute $x'_{wave\ front}$. It is wrong that Einstein used the following inference to derive an expression of $x_{origin\ of\ K'}$,

Premise 1: $x'_{wave\ front} = ax_{wave\ front} - bct$

Premise 2: $x'_{origin\ of\ K'} = 0$

Conclusion: $x_{origin\ of\ K'} = \frac{bc}{a}t$

Because $x'_{origin\ of\ K'} = x'_{wave\ front}$ only when $x'_{wave\ front} = 0$, Einstein's conclusion $x_{origin\ of\ K'} = \frac{bc}{a}t$ only works when $t = 0$ and $x_{wave\ front} = 0$, which are starting assumptions for deriving the Lorentz transformation, so we only have $0 = 0$.

"If we call $v$ the velocity with which the origin of $K'$ is moving relative to $K$, we then have

$$v = \frac{bc}{a}. \qquad [(73)]$$

The same value $v$ can be obtained from equations [(71)], if we calculate the velocity of another point of $K'$ relative to $K$, or the velocity (directed towards the negative $x$-axis) of a point of $K$ with respect to $K'$. In short, we can designate $v$ as the relative velocity of the two systems."

According to this paragraph, $x$ in (72) is the $x$-ordinate of the origin of $K'$, $x_{origin\ of\ K'}$. Since $x_{origin\ of\ K'} = \frac{bc}{a}t$ is true only when $t = 0$, equation (73) obtained by dividing both sides of (72) with 0 is not valid.

"Furthermore, the principle of relativity teaches us that, as judged from $K$, the length of a unit measuring-rod which is at rest with reference to $K'$ must be exactly the same as the length, as judged from $K'$, of a unit measuring-rod which is at rest relative to $K$. In order to see how the points of the $x'$-axis appear as viewed from $K$, we only require to take a "snapshot" of $K'$ from $K$; this means that we have to insert a particular value of $t$ (time of $K$), e.g. $t = 0$. For this value of t we then obtain from the first of the equations [(71)]

$$x' = ax."$$

The unequivocal expression of equation (71) is

$$x'_{wave\ front} = ax_{wave\ front} - bct$$

When $t = 0$, since $x_{wave\ front} = ct$ and $x'_{wave\ front} = ct'$, we have $x = 0, t' = 0$, and $x' = 0$. Therefore the expression $x' = ax$ from (71) just indicates $0 = 0$. A snapshot of a fixed distance on the $x$-axis at $t = 0$ is not an instance of equation (71) which describes relationships between coordinates of wave fronts.

"Two points of the $x'$-axis which are separated by the distance $x' = 1$ when measured in the $K'$ system are thus separated in our instantaneous photograph by the distance

$$\Delta x = \frac{1}{a}. \qquad [(74)]"$$

As the expression $x' = ax$ inferred from equation (71) at $t = 0$ just indicates $0 = 0$ , Einstein's equation (74) is meaningless in that context.

"But if the snapshot be taken from $K'$ ($t' = 0$), and if we eliminate t from the equations [(71)], taking into account the expression [(73)], we obtain

$$x' = a\left(1 - \frac{v^2}{c^2}\right)x.$$"

We may guess how Einstein obtained $x' = a\left(1 - \frac{v^2}{c^2}\right)x$ from the Eqs. (71) taking into account the expression (73). Because $t' = act - bx$ of (71) and Einstein takes a snapshot at $t' = 0$,

$$t = \frac{bx}{ac}.$$

Remember the starting assumptions: when $t' = 0, t = 0, x' = 0, x = 0$ , this equation is still $0 = 0$. Substituting $\frac{bx}{ac}$ for $t$ in $x' = ax - bct$ of (71), Einstein obtains

$$x' = ax - \frac{b^2 cx}{ac} = ax - \frac{b^2 x}{a}.$$

Using (73), Einstein obtains $b^2 = \frac{a^2 v^2}{c^2}$ and substituting it into the above equation leads to

$$x' = ax - \frac{a^2 v^2 x}{ac^2} = ax\left(1 - \frac{v^2}{c^2}\right).$$

"From this we conclude that two points on the $x$-axis and separated by the distance 1 (relative to $K$) will be represented on our snapshot by the distance

$$\Delta x' = a\left(1 - \frac{v^2}{c^2}\right). \qquad [(74a)]$$

But from what has been said, the two snapshots must be identical; hence $\Delta x$ in [(74)] must be equal to $\Delta x'$in [(74a)], so that we obtain

$$a^2 = \frac{1}{1 - \frac{v^2}{c^2}}. \qquad [(74b)]"$$

Einstein started with equations for wave fronts of light, and then "derived" equations for the position of the origin of frame K′ in frame K from the equations for wave fronts. Following that, he took a "snapshot" of a length of 1 unit in K′ from K at $t = 0$, which means $x = 0, t' = 0$, and $x' = 0$ for wave front equations described earlier. Finally Einstein obtained the expression of *a* needed for the Lorentz transformation.

"The equations [(73)] and [(74b)] determine the constants *a* and *b*. By inserting the values of these constants in [(71)], we obtain the first and the fourth of the equations given in Section XI.

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t' = \frac{t - vx/c^2}{\sqrt{1 - \frac{v^2}{c^2}}}. \qquad [(75)]"$$

It is difficult to find a proof or derivation in the history of modern science with more logical and mathematical mistakes than this simple derivation of the Lorentz transformation by Einstein. Interestingly, even though millions of copies of *Relativity: the special and general theory* have been printed and sold, there seem to be only generous praises and admirations of Einstein's genius demonstrated in this small book. Few people raised doubts about the derivations and expositions in this book.

Einstein firstly failed to understand that from $x - ct = 0$ and $x' - ct' = 0$, nobody can derive the complicated expression of the Lorentz transformation. He secondly failed to understand that some general or specific function forms must be postulated for the transformation, while the constancy of the speed of light can only be a constraint for deriving the transformation. Without specifying $x' = ax - avt$ and using spherical light wave propagation equation, he cannot derive the Lorentz transformation. The only way to compensate for not specifying all necessary premises is to make logical mistakes.

## 6 Derivation in the *Mechanics* of *Berkeley Physics Course*

Although there might still be millions of scholars who would defend Einstein's simple derivation of the Lorentz transformation as a correct one and a demonstration of his unmatched genius, Einstein's simple derivation would not be able to appear in reputable physics textbooks. Physics textbooks use modified versions of Einstein's simple derivation. To derive the Lorentz transformation, plane wave light beam model

is not adequate, so spherical light waves model has to be used. In the *Mechanics* of *Berkeley Physics Course* (Kittel et al 1973), the Lorentz transformation is derived as follows.

"We shall use the same ideas here with two different frames of reference *S* and *S′*, moving with uniform velocity *V* with respect to each other.… If we assume that in the frame *S* a light source is at the origin, the equation of a spherical wave front emitted at *t=0* is

$$x^2 + y^2 + z^2 = c^2 t^2 . \qquad [(76)]$$

In the frame of reference *S′* in which the coordinates are *x′*, *y′*, *z′*, and *t′*, the equation of the spherical wave front must be

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 . \qquad [(77)]$$

The speed of light c is the same in both Eqs. [(76)] and [(77)].

We can try the galilean transformation to see whether it gives results in agreement with Eqs. [(76)] and [(77)].

$$x' = x - Vt,\ y' = y,\ z' = z,\ t' = t . \qquad [(78)]$$

When we substitute Eq.[(78)] in Eq.[(77)] we obtain directly

$$x^2 - 2xVt + V^2 t^2 y^2 + z^2 = c^2 t^2 .$$

This result is certainly not in agreement with Eq.[(76)]. Thus the galilean transformation fails, and we must attempt to find some other transformation. It must reduce to the galilean transformation when the velocity *V* becomes very small compared with the velocity of light *c*.

Let us try

$$x' = \alpha x + \varepsilon t,\ y' = y,\ z' = z,\ t' = \delta x + \eta t .$$

We know that for $x'=0, dx/dt=V$; and for $x=0$, $dx'/dt'=-V$. The algebra leads to

$$V=-\frac{\varepsilon}{\alpha}, \qquad -V=\frac{\varepsilon}{\eta}$$

or $\alpha=\eta$."

In the above derivation, the authors of Mechanics postulate that $y'=y$ and $z'=z$, which is an additional condition as I discussed earlier. They also postulated function forms $x'=\alpha x+\varepsilon t$and $t'=\delta x+\eta t$. These are two additional postulates. As I showed earlier, these function forms will not lead uniquely to the Lorentz transformation. Like Einstein, the authors did not make it unequivocal what $x'$ and $x$ stand for in $x'=\alpha x+\varepsilon t$and $t'=\delta x+\eta t$. In my view, the two equations should be

$$x'_{wave\ front}=\alpha x_{wave\ front}+\varepsilon t$$

$$t'=\delta x_{wave\ front}+\eta t$$

To replace the coordinates of wave fronts in the spherical light wave propagation equation, $x'$ and $x$ in the transformation equations need to represent coordinates of wave fronts. The variable $x'$ cannot be the origin of $x'$-axis in $K'$, because $x'_{origin\ of\ K'}=0$ which is not a function of $x$ and $t$. The expressions $V=-\frac{\varepsilon}{\alpha}$ and $-V=\frac{\varepsilon}{\eta}$ cannot be obtained by differentiating the above equations. However, the authors of *Mechanics* set $x'=0$ and $x=0$ in turn to obtain these two expressions of $V$, and their objective is to get $x'=\alpha x-\alpha vt$. Their differentiation should be more unequivocal,

$$\left.\frac{dx}{dt}\right|_{x'=0}=V_{x'=0}=-\frac{\varepsilon}{\alpha}, \qquad \left.\frac{dx'}{dt'}\right|_{x=0}=-V_{x=0}=\frac{\varepsilon}{\eta}$$

Or more generally,

$$\left.\frac{dx}{dt}\right|_{x'=x'_0}=V_{x'=x'_0}=-\frac{\varepsilon}{\alpha}, \quad \left.\frac{dx'}{dt'}\right|_{x=x_0}=-V_{x=x_0}=\frac{\varepsilon}{\eta}.$$

Therefore, the coefficients they obtained will only be valid for those particular fixed points, $x' = x'_0$ and $x = x_0$, which cannot be used to describe the position of wave fronts. By illogically using coefficients of equations of fixed points, $x' = x'_0$ and $x = x_0$, for equations of wave fronts of light, the authors of *Mechanics* imposed their fourth condition, $\alpha = -\varepsilon/V$, i.e. $x' = \alpha x - \alpha V t$. This is an additional assumption or postulate, because they have not proved and in fact they cannot prove that positions of wave fronts of light, $x = ct$ and $x' = ct'$, have the same coefficients in their transformation equations as fixed points, $x' = x'_0$ and $x = x_0$.

The derivation in *Mechanics* is superior to Einstein's simple derivation in terms of mathematical and logical correctness. The authors failed to acknowledge that they have used four premises instead of two, the principle of relativity and the constancy of the speed of light. They took the third premise $y' = y$ and $z' = z$ for granted, while in fact it plays a key role in obtaining the Lorentz transformation. By obtaining $x' = \alpha x - \alpha V t$ and $t' = \alpha t - \delta x$ illogically from $x' = \alpha x + \varepsilon t$ and $t' = \eta t + \delta x$, they effectively specify another postulate for deriving the Lorentz transformation. Therefore, implicitly and with logical mistakes, the authors of *Mechanics* used all the four postulates we listed earlier. The rest of their derivation is the same as the one I presented in section 4.

Einstein gave his first derivation of the Lorentz transformation in his first paper on relativity (Einstein 1905). Because it is a bit long, I will not give a detailed analysis of its logical shortcomings here other than pointing out the follows (Ma 2015).

1. Einstein's first derivation has the same logical mistake of mixing the origin of the x-axis and the x-ordinate of the wave front of a light beam as his simple derivation we examined earlier.

2. Einstein used the velocity of light along y-axis and z-axis being $\sqrt{c^2 - v^2}$ as viewed by the stationary system, which is an additional postulate that cannot be derived from the principle of relativity and the constancy of the speed of light.

3. In Einstein's model of a ray of plane light waves, the velocity of light along y-axis and z-axis does not exist, but the velocity of light along y-axis and z-axis plays a key role in Einstein's derivation.

4. Einstein assumed that after applying the Lorentz transformation to ($x$, $t$) with velocity being $v$ to obtain ($x'$, $t'$), applying it to ($x'$, $t'$) again with velocity being $-v$ will arrive at ($x$, $t$) again. This incorrect assumption is essential for his derivation.

**7 Conclusions**

Einstein's reputation is partly built on the belief in the mainstream physicists, many historians and philosophers of science, and the general public that Einstein derived special relativity from only two postulates, the principle of relativity and the constancy of the speed of light. The present study has demonstrated that this belief does not have a solid foundation and it is a belief rather than a fact.

The Lorentz transformation and consequently special relativity cannot be derived from only two postulates, the principle of relativity and the constancy of the speed of light without additional postulates on the function form of the transformation and length/distance of vertical directions. To obtain the Lorentz transformation through deduction, at least four postulates are needed:

1. The principle of relativity ensures that the transformation from S to S′ has the same form as the transformation from S′ to S in vector form.
2. Constancy of the speed of light ensures that the speed of light is the same in all the inertial frames of reference.
3. The length/distance in the directions orthogonal to the direction of the velocity does not change so that Voigt transformation and similar ones are excluded.
4. Basic function forms of the transformation

   $x' = ax - avt$

   $t' = mt - nx$

If the function forms in 4) are not postulated, there are infinite transformation functions that satisfy the constancy of the speed of light and the principle of relativity.

Since the Lorentz transformation cannot be derived from the constancy of the speed of light and the principle of relativity, any statement of Einstein having achieved this should be viewed as false assertion. Serious history of science or physics should not carry on making such false assertions.

Einstein's true contribution is not the derivation of the Lorentz transformation from only two postulates, but the expulsion of the medium of light from Lorentz ether theory and reinterpretation of the velocity in the Lorentz transformation as the velocity relative to the observing reference frame.

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