SOME BASIC CONCEPTS OF FRACTIONAL CALCULUS

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Theory I

Abstract

In this research section, a definition for Fractional Derivative is presented.

Theory

Definition For Fractional Derivative

Given

$0 < \alpha < 1$ and $N \in \{1, 2, 3, 4, ...\}$ i.e., a set of positive integers from 1 onwards,

We know that a derivative for a function is given by

$$f^{'}(x) = \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

(1)

In the same spirit, we define the Fractional Derivative in the following fashion:

$$f^{1+\alpha}(x) = \frac{d^{1+\alpha} f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f^{'}(x+\alpha\Delta x) - f^{'}(x)}{\alpha\Delta x}$$

(2)

$$f^{2+\alpha}(x) = \frac{d^{2+\alpha} f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f^{''}(x+\alpha\Delta x) - f^{''}(x)}{\alpha\Delta x} \qquad (3)$$

And similarly, we write

$$f^{N+\alpha}(x) = \frac{d^{N+\alpha} f(x)}{dx} = \lim_{\Delta x \to 0} \frac{f^{\overbrace{''......... ....''}^{N\ times}}(x+\alpha\Delta x) - f^{\overbrace{''......... ....''}^{N\ times}}(x)}{\alpha\Delta x}$$

(4)

Here, the notation, $f^{'}(x) = \frac{df(x)}{dx}$ and (5)

$$f^{\overbrace{''......... ....''}^{N\ times}}(x) = \frac{d^N f(x)}{dx^N} \qquad (6)$$

Theory II

Abstract

In this research section, a method to calculate Fractional Integral is detailed.

1 Theory

For the Integral

$$\int_a^b f(x) d^{\beta} x \; , \quad \text{where } 0 < \beta < 1 \tag{7}$$

we consider the Riemann sum of the kind

Definition 1:

$$\boxed{\int_a^b f(x) d^{\beta} x = \lim_{n \to \infty} \sum_{i=1}^{n} \left\{ f\left[a + (i+\beta)\left(\frac{b-a}{n+\beta} \right) \right] \right\} \left(\frac{b-a}{n+\beta} \right)} \quad \text{where} \tag{8}$$

$$dx = \left(\frac{b-a}{n+\beta} \right) \tag{9}$$

For the Integral

$$\int_a^b f(x) d^{1+\beta} x \; , \quad \text{where } 0 < \beta < 1 \tag{10}$$

$$\boxed{\int_a^b f(x) d^{1+\beta} x = \int_a^b p(x) d^{\beta} x} \quad \text{where} \tag{11}$$

$$p(x) = \int f(x) dx \tag{12}$$

$$\int_a^b f(x)d^{1+\beta}x = \int_a^b \left(\int f(x)dx \right) d^{\beta}x$$

(13)

Say , for $\int_a^b f(x)d^{N+\beta}x$ (14)

$$\int_a^b f(x)d^{N+\beta}x = \int_a^b \underbrace{\left(\int \left(\cdots \int \left(\left(\int f(x)dx \right) \right) dx \right) \cdots dx \right)}_{N-1 \text{ Integrals}} d^{1+\beta}x \qquad (15)$$

Where N is a positive Integer

If

$$g(x) = \underbrace{\left(\int \left(\cdots \int \left(\left(\int f(x)dx \right) \right) dx \right) \cdots dx \right)}_{N-1 \text{ Integrals}} \qquad (16)$$

Then

$$\int_a^b f(x)d^{N+\beta}x = \int_a^b g(x)d^{1+\beta}x \qquad (17)$$

2 A Definition For Functional Integration

The Integral

$$\int_a^b f(x)d^{h(x)}x$$

(18)

Is a curve where we find the Integral for every co-ordinate of $h(x)$ for every co-ordinate of x if $h(x)$ is a function of x

References

1. 'Calculus' by Thomas & Finney.
2. 'Calculus' by Apostol.

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Note

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