Proof of Fermat's last theorem (Part III of III) $a^n + b^n = c^n$ (n > 1 and odd)

Objet:

- Another form of Fermat's last theorem : I prove that the Fermat's last theorem consist in finding 3 integers (x, y, and z) such as
  $(x+z)^n + (y+z)^n = (x+y+z)^n$
- From the Pythagorean triple we obtain a square equals the sum of three squares
  If $c^2 = a^2 + b^2$, and where d is the complement of c to (a + b) was $(c-d)^2 = (a-d)^2 + (b-d)^2 + d^2$.
- From each even integer we obtain at least a Pythagorean triple
- *The surface of the Pythagorean triangle*

Any number $s = \frac{w^3 - w}{4}$ is the surface of a Pythagorean triangle

$$w^2 + \left(\frac{w^2-1}{2}\right)^2 = \left(\frac{w^2+1}{2}\right)^2$$

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Another form of Fermat's last theorem:

$a^n + b^n = c^n$

$c = a + b - d$

$(a)^n + (b)^n = (c)^n$

$(a)^n + (b)^n = (a + b - d)^n$

$(a - d + d)^n + (b - d + d)^n = (a - d + b - d + d)^n$

$(a - d + d)^n + (b - d + d)^n = (a - d + b - d + d)^n$

If we take :

$x = a - d$

$y = b - d$

$z = d$

Fermat's last theorem consist in finding 3 integers (x, y, and z) such as

$$(\boldsymbol{x} + \boldsymbol{z})^{\boldsymbol{n}} + (\boldsymbol{y} + \boldsymbol{z})^{\boldsymbol{n}} = (\mathbf{x} + \mathbf{y} + \boldsymbol{z})^{\boldsymbol{n}}$$

Using the new form of Fermat's last theorem

From the Pythagorean triple we obtain a square equals the sum of three squares

$(x+z)^2 + (y+z)^2 = (\mathrm{x}+ \mathrm{y}+z)^2$

$(x+z)^2+(y+z)^2 = x^2+2xy+y^2+2yz++2xz$

$(x+z)^2+(y+z)^2 = x^2+2xy+y^2+2yz++2xz$

$(x+z)^2+(y+z)^2 = (x+y)^2-y^2+(y+z)^2-z^2+(x+z)^2-x^2$

$(x+z)^2+(y+z)^2 = (x+y)^2-y^2+(y+z)^2-z^2+(x+z)^2-x^2$

$0=(x+y)^2-y^2-z^2-x^2$

$(x+y)^2 = y^2+z^2+x^2$

This means that: $(c-d)^2 = (a-d)^2+d^2+(b-d)^2$

If $c^2 = a^2 + b^2$, and where d is the complement of c to $(a + b)$ was $(c\text{-}d)^2 = (a\text{-}d)^2 + (b\text{-}d)^2 + d^2$.

A square equals the sum of three squares

From each even integer we obtain at least a Pythagorean triple.
For every even integer the list of Pythagorean triple is limited.

$a^2 + b^2 = c^2$
$(x+z)^2 + (y+z)^2 = (\mathrm{x} + \mathrm{y} + z)^2$
$x^2 + 2xz + z^2 + y^2 + 2yz + z^2 = x^2 + z^2 + y^2 + 2xz + 2yz + 2xy$
After simplification we get
$z^2 = 2xy$ (z is even, since z = d)
$\frac{z^2}{2} = xy$

Take couples xy dividers such as xy $= \frac{z^2}{2}$, it is sufficient to calculate
$(x+z)^2 + (y+z)^2 = (\mathrm{x} + \mathrm{y} + z)^2$

Of each pair of dividers we obtain a Pythagorean triple
Each integer has at least a pair of dividers 1 and itself.

- ✓ **To find all Pythagorean triples**
- ✓ **Take an even integer z**
- ✓ **Find x and y as xy $= \frac{z^2}{2}$**

We have $(\boldsymbol{x}+\boldsymbol{z})^2 + (\boldsymbol{y}+\boldsymbol{z})^2 = (\mathbf{x} + \mathbf{y} + \boldsymbol{z})^2$

The surface of the Pythagorean triangle

$(x+z)^2 + (y+z)^2 = (x+ y+z)^2$

$2xy = z^2$

$xy = \frac{z^2}{2}$

xy is a number and every number a is the form a * 1

$xy = 1 * \mathrm{xy}$

$\mathrm{X} = 1$ et $\mathrm{y} = \frac{z^2}{2}$

$$S = \frac{(x+z)(y+z)}{2}$$

$$S = \frac{(x+z)(y+z)}{2} = \frac{(1+z)(\frac{z^2}{2}+z)}{2}$$

$$2s = (1+z)(\frac{z^2}{2}+z)$$

$$2s = \frac{z^2}{2} + \frac{z^3}{2} + z + z^2$$

$$2s = \frac{z^2}{2} + \frac{z^3}{2} + \frac{2z}{2} + \frac{2z^2}{2}$$

$$4s = z^2 + z^3 + 2z + 2z^2$$

$$4s = z^3 + 2z + 3z^2 + z - z + 1 - 1$$

$$4s = z^3 + 3z + 3z^2 + 1 - z - 1$$

$$4s = z^3 + 3z + 3z^2 + 1 - (z+1)$$

$$4s = (z+1)^3 - (z+1)$$

$$S = \frac{(z+1)^3 - (z+1)}{4}$$

$$S = \frac{w^3 - \mathrm{w}}{4}$$

Any number s $= \frac{w^3 - w}{4}$ is the surface of a Pythagorean triangle

$$w^2 + \left(\frac{w^2-1}{2}\right)^2 = \left(\frac{w^2+1}{2}\right)^2$$