Consequence of a comparison of Numerical E folds for a non singular initial cosmology bounce versus E folds in the classical GR case, combined with Friedman Equation evolution of the Inflaton, in the case of a 'quantum bounce'. Tie in With DM suggested

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Abstract. This document goes to the Friedman equation and initially gives an argument as to a quantum bounce , for a modified consideration of the inflaton. Once an inflaton for a non zero initial starting point is established we look at the ratio of the Numerical E folds for when there is a quantum bounce, to the Numerical E folds for when classical GR is assumed. This rests upon looking at M. Bojowald's reformation of the Friedman equation, with some profound physics coming from a defined length, as given by Banerjee and Date, as of 2005. A tie in with initial production of DM is suggested as a compliment to work L. Randall and others have initiated with the synthesis of DM.

i. Introduction

The numerical e fold factor of inflation, with a quantum bounce is calculated, using a modified Friedman equation as given by Freeze [1], and then followed up by Bojowald, [2] for a non singular universe. This is given by the result from Freeze [1] on page 153, with $\sigma = A_1 \cdot M_P$, and A_1 a constant

$$H^2 = \frac{8\pi}{3M_P^2} \cdot \left[\rho - \frac{\rho^2}{2\sigma} \right] \tag{1}$$

Eq.(1) will be tied into both , from [2]

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k_{curvature}}{a^2} = \frac{4\pi G}{3} \cdot \frac{p_\phi^2}{a^6} + \Lambda \tag{2}$$

And

$$p_\phi = a^3 \cdot \dot{\phi} \tag{3}$$

Leading to

$$\dot{\phi}^2 \approx a^{-6} \cdot (12\pi G) \cdot V^{(4)} \cdot \left(H^2 + |\Lambda| \right) \tag{4}$$

We will treat, then the Hubble parameter, as leading to , after applying [3] , and

$$N = \int_{t_{INITIAL}}^{t_{END}} H_{initial} \cdot dt \tag{5}$$

Then

$$\begin{aligned} N(bounce) &= \sqrt{\frac{8\pi}{3M_P^2}} \int_{t_{INITIAL}}^{t_{END}} e^{-2H_{INITIAL}\cdot t} \cdot \sqrt{1 - \frac{e^{-4H_{INITIAL}\cdot t} \cdot a_{INITIAL}^{-4}}{2A_1 M_P}} \cdot dt \\ N(no-bounce) &= \sqrt{\frac{8\pi}{3M_P^2}} \int_{t_{INITIAL}}^{t_{END}} e^{-2H_{INITIAL}\cdot t} \cdot dt \end{aligned} \tag{6}$$

Then

$$\frac{N(bounce)}{N(no-bounce)} = \frac{\int_{t_{INITIAL}}^{t_{END}} e^{-2H_{INITIAL}\cdot t} \cdot \sqrt{1 - \frac{e^{-4H_{INITIAL}\cdot t} \cdot a_{INITIAL}^{-4}}{2A_1 M_P}} \cdot dt}{\int_{t_{INITIAL}}^{t_{END}} e^{-2H_{INITIAL}\cdot t} \cdot dt} \leq 1 \tag{7}$$

We will from here, for this article, evaluate the consequences of Eq. (7) and Eq.(4), with then the following approximation made $H_{initial} \leq m_{3/2}$, with the initial value of the Hubble parameter bounded above by gravitino mass due to $H_{initial} \leq m_{3/2}$ [3,4]. Also, we should keep in mind that

$$\begin{aligned} &\dot{\phi}^2 \approx a^{-6} \cdot (12\pi G) \cdot V^{(4)} \cdot \left(H^2 + |\Lambda|\right) \\ &\xrightarrow[H \to m_{3/2}]{} a^{-6} \cdot (12\pi G) \cdot V^{(4)} \cdot \left({m_{3/2}}^2 + |\Lambda|\right) \\ &\Leftrightarrow \phi^2 \approx t_{time}^2 \times a^{-6} \cdot (12\pi G) \cdot V^{(4)} \cdot \left({m_{3/2}}^2 + |\Lambda|\right) \end{aligned} \tag{8}$$

ϕ is meant here to be the initial inflaton, whereas $V^{(4)} \sim l_P^3 \times t_P$, and t_P is Planck time. l_P^3 is the cube power of Planck length, whereas $a \sim a_{initial} \sim 10^{-55}$ is a common guess .

We will after making a more phenomenological guess as to answering an optimal

2. Making sense out of Eq.(7) so it is consistent.

$$\begin{aligned} &\frac{e^{-4H_{INITIAL}\cdot t} \cdot a_{INITIAL}^{-4}}{2A_1 M_P} < 1 \\ &\Leftrightarrow e^{-4H_{INITIAL}\cdot t} < 2A_1 M_P \times a_{INITIAL}^4 \end{aligned} \tag{9}$$

Then for all time from the initial to the final time of inflation, we will have

$$
\begin{aligned}
& e^{-4H_{INITIAL}\cdot t} < 2A_1 M_P \times a_{INITIAL}^4 \\
& \Leftrightarrow -4H_{INITIAL}\cdot t \sim \ln(2A_1 M_P \times a_{INITIAL}^4) \\
& \Leftrightarrow t \sim -\frac{\ln(2A_1 M_P \times a_{INITIAL}^4)}{4H_{INITIAL}}
\end{aligned}
\tag{10}
$$

If time is of the order of, say $t \sim t_{final} \propto 10^{-32}\,\mathrm{sec}$, $A_1 \sim O(1)$

$$
a_{INITIAL} \xrightarrow[t \sim t_{final} \propto 10^{-32}\,\mathrm{sec}]{} \tilde{a} \approx 2^{-1/4} M_P^{1/4} \exp(-m_{3/2} t_{final}) \propto 10^{-50} - 10^{-20}
\tag{11}
$$

The expected radius of the universe, in the final time is of the order of 1 meter or more.

3. Tie in with Dark Matter, via a publication by L. Randall and others.

In [5] Randall, Scholtz, and Unwin write that "DM is not part of the Standard Model sector and a population of DM too is generated at the end of inflation", i.e. our choice of inputs as to Eq.(11) and also look at the following with this generation of DM at the end of inflation via the following equation. Namely if Φ is a scalar field which collapse to a field of the standard model of physics, with g_Φ the degrees of freedom of the scalar field Φ , and we take advantage of the situation defined in the following quote from [5], i.e. quote" *We therefore assume the late decay of a heavy field Φ produced at early times adds entropy to the Standard Model at late times, thereby associating the dark matter relic density with the lifetime of a long-lived particle*".

$$
H^2(a) \approx \frac{g_\Phi \pi^2}{90} \cdot \frac{m_\Phi^2}{M_{Pl}^2} \cdot \left[\left(\frac{a}{a_0}\right)^3 + \left(1 + \frac{\rho_{SM} + \rho_{DS}}{\rho_\Phi}\right) \right] \cdot \left(\frac{a_0}{a}\right)^4
\tag{12}
$$

Here, in this situation, a_0 as when the field Φ becomes non relativistic. Our supposition here is that the scale factor in Eq. (12) $a \to \tilde{a}$, with $\tilde{a}$ the scale factor as given in Eq. (11). If so, then, $\tilde{a}$ would be at the end of inflation, whereas the end of inflation is where $\tilde{a}$ is, as given by Eq.(11). Then the above

$$
H^2(\tilde{a}) \approx \frac{g_\Phi \pi^2}{90} \cdot \frac{m_\Phi^2}{M_{Pl}^2} \cdot \left[\left(\frac{\tilde{a}}{a_0}\right)^3 + \left(1 + \frac{\rho_{SM} + \rho_{DS}}{\rho_\Phi}\right) \right] \cdot \left(\frac{a_0}{\tilde{a}}\right)^4
\tag{13}
$$

Here, m_Φ^2 is the square of the mass of a particle representing the scalar field which would decay to a standard model framework. In addition ρ_Φ is the density of the scalar field, whereas ρ_{SM} and ρ_{DS} refer to standard model density, and dark sector density. i.e. our supposition is that Eq. (13) would in addition be bound by gravitino mass , squared $m_{3/2}^2$ as could be written as

$$
\begin{aligned}
& H^2(\tilde{a}) \approx \frac{g_\Phi \pi^2}{90} \cdot \frac{m_\Phi^2}{M_{Pl}^2} \cdot \left[\left(\frac{\tilde{a}}{a_0}\right)^3 + \left(1 + \frac{\rho_{SM} + \rho_{DS}}{\rho_\Phi}\right) \right] \cdot \left(\frac{a_0}{\tilde{a}}\right)^4 \sim m_{3/2}^2 \\
& \Leftrightarrow \left(\frac{g_\Phi \pi^2}{90} \cdot \frac{m_\Phi^2}{M_{Pl}^2}\right) \cdot \left(1 + \frac{\rho_{SM} + \rho_{DS}}{\rho_\Phi}\right) \cdot \left(\frac{a_0}{\tilde{a}}\right)^4 + \left(\frac{g_\Phi \pi^2}{90} \cdot \frac{m_\Phi^2}{M_{Pl}^2}\right) \cdot \left(\frac{a_0}{\tilde{a}}\right)^3 - m_{3/2}^2 \approx 0
\end{aligned}
\tag{14}
$$

The result is a highly non linear equation as to the ratio of $\left(\frac{a_0}{\tilde{a}}\right)$ with the numerator the scale factor of where the square of the mass of a scalar field contribution, namely m_Φ^2, is non relativistic, and the denominator satisfied Eq.(11) above as well.

4. Fixing a tie in with Dark Matter and Dark Matter physics. And the admissible values of $\left(\frac{a_0}{\tilde{a}}\right)$

The quest as to when DM forms, if it is at the "edge" of the end of the inflationary period is one of the hottest problem in contemporary physics. The ration of $\left(\frac{a_0}{\tilde{a}}\right)$, with its emphasis, upon finding operational links between the ratios of $\frac{\rho_{SM}+\rho_{DS}}{\rho_\Phi}$ of Eq.(14), the values of the denominator of $\left(\frac{a_0}{\tilde{a}}\right)$ as given in Eq. (11) as well as the necessity of a value less than or equal to 1 of Eq.(7) leading to the simple restriction given in Eq.(9) will numerically begin to confine the scale factors with DM formation properties.

Finally we assert that nonsingular start points for the scale factor are the same as a quantum "bounce" as given in [6], and that further work will show, with work, some of the quantum nucleation pathways to the formation of Dark Matter. i.e. this will require an interplay between Eq.(10), Eq. (11) and Eq. (14) to get further results. I.e. to this end we will in future work look at also including in an extension of the Friedman equation [2] as

$$\frac{\mathrm{Sin}^2(\mu P)}{\mu}=\frac{1}{12\pi G}\cdot\frac{p_\phi^2}{V_{Volume}} \tag{15}$$

Here, μ is a length scale [7] with P, and the numerator of the RHS of this equation defined as in [2]. i.e. μ is defined via an eventually quantized system as obeying

$$\begin{aligned}
&p_\phi = 2\cdot\sqrt{3\pi G}\cdot\mathrm{Im}(J)\\
&\&V = a^3/4\pi G\\
&\&J = V\exp(i\mu P)\\
&\Leftrightarrow\left[\hat{J},\hat{J}^\dagger\right]=\mu\hbar\hat{V}
\end{aligned} \tag{16}$$

The commutation bracket, as given below, in the bottom of Eq. (16) will be part of a quantization procedure, as to make sense of quantum processes which may explain further the formation of $\left(\frac{a_0}{\tilde{a}}\right)$ allowed values, and of Dark Matter formation at the end of the inflationary era. This may allow for answering some of the constraint issues brought up in [7].

Acknowledgements

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