QUANTUM MODIFICATION OF GENERAL RELATIVITY

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Abstract

This work is based on modification of the general relativity with two additional terms in the Einstein equations. The additional terms give macroscopic description of the quantum effects of production /absorption of matter by the vacuum. The theory (without fitting parameters and without hypothesis of inflation) is in good quantitative agreement with cosmological observations (SnIa, SDSS-BAO and reduction of acceleration of the expanding universe). In this theory, there is no Big Bang at the beginning, but some local bangs during the evolution are probable. Also, there is no critical density of the universe and, therefore, no dark energy (no need in the cosmological constant). Based on exact Gaussian solution for the scale factor, it is shown that an effective age of the universe is about 327 billion years. Production of primary dark matter particles (possibly, gravitons) have started 43 billion years later. It is shown that characteristic distance between particles is much smaller than the thermal de Brogle wavelength, so that quantum effects, including formation of the Bose-Einstein condensate, can dominate, even for high temperature. "Ordinary" matter was synthesized from dark matter (with estimated small electric dipole moment (EDM)) in galaxies. Supplementary exact solutions are obtained for various ranges of parameters. From the theory we get an interface between dark and ordinary matter (IDOM), which very likely exist not only in cosmos, but everywhere, including our body and our brain.

Key words: modification of general relativity; cosmology; age of the universe; dark matter (gravitons); interface between dark and ordinary matter.

1. Introduction

The standard theory (ST) in conventional cosmology is based on three major assumptions: Big Bang (BB), Cosmological Constant (CC) and Inflation (INF). Huge and useful work have been done in frames of ST. But, doubts about the basic assumptions are remaining.

BB corresponds to a particular Friedmann solution [1] of the classical equations of general relativity (GR). Yes, it is interesting solution, but, is it natural and physical? I do not think so [2-4] and I am far from been alone. There is growing evidence that age of many stars are inconsistent with assumed in the BB theory 13.8 billion years age of the universe. Data for other cosmic objects are hardly compatible with ST(see, for example, recent collection of such data in [5]). These observations are not at the level to proof that BB did not took place, but, at least, they give some warning. Note, that Friedmann solution created controversial critical density of the universe, which in turn created controversial dark energy.

As concern to CC, it is known, that inventor of CC Einstein was unhappy about it, especially after his friend Gödel gave him, as a birthday present, unphysical GR solution with CC [6]. It was a long before it turns out, that in order to be consistent with global scale observation, CC should be unphysically small. Numerical solutions of GR with such CC [7] contradict observations on the scale of galaxies (see also evidence for dark matter creation [8]).

INF is an interesting idea, which appeared as a rescue mission, when it was found that BB+CC contradict observations. Recent hopes to support INF by observations [9] turn sour [10]. Again, the data did not proof that there was no INF, but creates some doubts. There are also theoretical difficulties in matching BB with INF (see recent review of difficulties with INF [11]).

My primary motivation for this work was unphysically small CC with unclear physical sense.

Below we consider a different theory, supported by observations, which dismisses all three major assumption of ST. In order to better explain such fundamental change, let us briefly describe how this theory came about. I became interested in gravitation in late 1960-th. Because of my experience in fluid dynamics, two things surprised me at the time: the Lagrangian description of gravity (LDG) were not used and situations with spatial dimension less than 3 were not considered (a taboo?). So, I decided to do both and obtained Lagrangian invariant (relative acceleration of particles) and exact general analytical solution for (1+1)-dimensional Newtonian gravitation [12]. This is an example of trivialisation, which I always enjoy (see below). Before publication, this paper was discussed with Ya. B. Zel'dovich, who express great enthusiasm and a few months later told me that he and his collaborators have a continuation of ideas presented in my paper. The editor E. M. Lifshitz was surprised, but did not object publication, even did not object the remark in the paper: <<We present here one fantastic conjecture. Perhaps the universe was not always (3+1)-dimensional. The dimensionality might change during a transition through the singular state with zero space-dimensionality. Only starting with space-dimensionality equal to three did the universe gain the possibility "to survive">>[12]. Than came Zel'dovich approximation [13], "pancakes" and further development in this direction [14].

I returned to fluid dynamics for a long time until acceleration of the universe was observed [15, 16]. The acceleration was explained by using CC, which is hundred orders smaller than can be predicted in the frames of classical GR. That was to much for me to accept. GR has to be modified. But how? Major player in GR is the spacetime curvature, which is balanced by the energy. But global curvature is close to zero in our universe. So, what else can play a role compatible to curvature? I had no desire to deal with new unknown fields and , in accord with trivialisation (Occam's razor), was looking for something very simple. And here, again, came help from fluid dynamics: divergency (stretching) of velocity field, which is related to creation/absorption of particles by the vacuum. So, to begin with, I have invented a new type of fluid, namely, dynamics of distributed sources/sinks (DODSS), in which divergency is Lagrangian invariant [17]. With constant initial distribution of divergency, it gives effect similar to CC [17].

The next step was relativistic DODSS [18] with the covariant divergency (2). Finally, came quantum modification of general relativity (QMOGER) [2], which is described below. I think, Einstein will be happy with such modification. This alternative to CC did not occurred to him, probably, because he came to GR from electricity, so to speak.

2. Quantum modification of general relativity (QMOGER)

Now, from words we are coming to equations of QMOGER [2]:

$$R_i^k - \frac{1}{2}\delta_i^k R = 8\pi G_* T_i^k + \lambda_N \delta_i^k, \; T_i^k = w u_i u^k - \delta_i^k p, \; w = \varepsilon + p, \tag{1}$$

$$\lambda_N = \lambda_0 + \beta \frac{d\sigma}{ds} + \gamma\sigma^2, \; \sigma = \frac{\partial u^k}{\partial x^k} + \frac{1}{2g}\frac{dg}{ds}, \; \frac{d}{ds} = u^k \frac{\partial}{\partial x^k} \tag{2}$$

Here R_i^k is the curvature tensor, p, ε and w are pressure, energy density and heat function, respectively, $G_* = Gc^{-4}$(G- gravitational constant, c- speed of light), u^k - components of velocity (summation over repeated indexes is assumed from 0 to 3, $x^0 = \tau = ct$), λ_0 is CC (which we will put zero), σ is the covariant divergency, β and γ are nondimensional constants (which we will put $\beta = 2\gamma = 2/3$) and g is the determinant of the metric tensor. With $\beta = \gamma = 0$ we recover the classical equation of GR. Let us note that curvature terms in lhs of (1) and additional terms $d\sigma/ds$ and σ^2 all contain second order (or square of first order) derivatives of metric tensor, which make these terms compatible. The importance of σ also follows from the fact that it is the only dynamic characteristic of media, which enters into the balance of the proper number density of particles $n : dn/ds + \sigma n = q$, where q is the rate of particle production (or absorption) by the vacuum. So, if n is constant (see the exact analytical solution (5) below) or changing slowly, than the σ-effect is, certainly, very important in quantum cosmology.

Some exact analytical solutions of equations (1,2) where obtained in Ref. 2. On the basis of these solutions, it was concluded that the effect of spacetime stretching (σ) explains the accelerated expansion of the universe and for negative σ (collapse) the same effect can prevent formation of singularity. Equations (1,2) reproduce Newtonian gravitation in the nonrelativistic asymptotic, but gravitational waves can propagate with speed, which is not necessary equal to speed of light [3]. This give us a hint that gravitons may have finite mass (see below).

In the case $\beta = 2\gamma$ equations (1,2) can be derived from the variational principle by simply replacing the cosmological constant λ_0 (in the Lagrangian) by $\lambda = \lambda_0 - \gamma\sigma^2$[3].

The natural next step was quantitative comparison with cosmological data and choice of nondimensional constants β and γ. Let us consider equations for the scale factor $a(\tau)$ in homogeneous isotropic universe (Eq. (8,9) in Ref. 2):

$$(2 - 3\beta)\frac{\ddot{a}}{a} + (1 + 3\beta - 9\gamma)(\frac{\dot{a}}{a})^2 + \frac{k}{a^2} - \lambda_0 = -8\pi G_* p, \tag{3}$$

$$-\beta\frac{\ddot{a}}{a}+(1+\beta-3\gamma)(\frac{\dot{a}}{a})^2+\frac{k}{a^2}-\frac{\lambda_0}{3}=\frac{8\pi}{3}G_*\varepsilon. \qquad (4)$$

Here points indicate differentiation over τ, the discrete curvature parameter $k=0,+1,-1$ corresponds to flat, closed and open universe, respectively.

With indicated in [2] unique choice $\beta=2\gamma=2/3$, these equations take simple form:

$$\frac{k}{a^2}=\lambda_0-8\pi G_* p, \qquad (3^*)$$

$$\dot{H}=\frac{3k}{2a^2}-\frac{\lambda_0}{2}-4\pi G_*\varepsilon,\ H\equiv\frac{\dot{a}}{a} \qquad (4^*)$$

From (3*) with $\lambda_0=0$, we see that sign of curvature is opposite to sign of pressure. From observations we know that global curvature is close to zero. So, the dust approximation ($p=0$) is natural for this theory with $\lambda_0=0$ and $\beta=2\gamma=2/3$.

In the dust approximation with $\lambda_0=0, k=0$, two special cases for system (3-4) have been indicated [2]: 1) for $\beta=2/3$ and $\gamma\neq 1/3$ stationary solution exist; 2) for $\beta=2\gamma$ the global energy is conserved, except for $\beta=2\gamma=2/3$. The choice $\beta=2\gamma=2/3$ is exceptional and in the dust approximation with $\lambda_0=0, k=0$, equation (3*) is identity and from (4*) we have exact analytical Gaussian solution:

$$a(\tau)=a_0\exp[H_0\tau-2\pi(\tau/L_*)^2],\ L_*=(G_*\varepsilon_0)^{-1/2} \qquad (5)$$

Here subscript 0 indicate present epoch ($\tau=0$) and H_0 is the Hubble constant. In the analogous solution, obtained in [3], instead of ε_0 was $w_0=\varepsilon_0+\lambda_0/8\pi G_*$, for generality. In the Appendix we present other supplementary solutions of system (3)-(4) for various ranges of parameters.

Solution (5) corresponds to continuous and metric-affecting production of dark matter (DM) particles out of vacuum, with its density $\rho_0=\varepsilon_0 c^{-2}$ being retain constant during the expansion of spatially flat universe. In this solution there is no critical density of the universe, which is a kind of relief.

The solution (5) is shown [3] to be stable in the regime of cosmological expansion until $t_{\max}$ about 34 billion years from now. After that time, the solution becomes unstable and characterizes the inverse process of dark matter particle absorption by the vacuum in the regime of contraction of the universe. This can imply the need for considering the change of regime (5) at $t>t_{\max}$ to a different evolutionary regime, possibly, with a different value of the parameter γ or with the more general model (2) from [2].

In this context, it is tempting to consider equations (1,2) without curvature terms in the left side of (1). In the dust approximation (with $\lambda_0=0, k=0$), equations (3,4) give not only stationary regime with $\varepsilon=0$, but also dynamical solution:

$$a(\tau)=a_0(1+\theta_1 H_0\tau)^{1/\theta_1}, \theta_1=3\gamma/\beta \qquad (6)$$

With $H_0 > 0, \theta_1 > 0$, from (6) we get: $a = 0$ at $\tau = -1/H_0\theta_1$, $a \approx a_0(\theta_1 H_0 \tau)^{1/\theta_1}$ for $\tau \gg 1/\theta_1 H_0$ - power-law expansion. With $H_0 > 0, \theta_1 < 0$, (6) gives: $a \to 0$ at $\tau \to -\infty$, $a \to \infty$ at $\tau \to 1/|\theta_1| H_0$ - blowup at finite time. With $\theta_1 = 0$: $H = H_0$, $a(\tau) = a_0 \exp\{H_0\tau\}$.

In order to solve equations (3,4) in more general case, we choose the simplest equation of state, which does not introduce a dimensional constant: $p = \varkappa\varepsilon$, where $\varkappa$ is nondimensional constant. Particularly, with $\varkappa = 0$ we return to the dust approximation, $\varkappa = 1/3$ corresponds to ultrarelativistic matter. From (3,4) with $\lambda_0 = 0$, we obtain invariant:

$$I = (\dot{a}^2 + \mu)(\frac{a_0}{a})^{2(1-\theta)} = (\dot{a_0}^2 + \mu),\ \mu = \frac{k(1+\varkappa)}{1+3\varkappa+3(1+\varkappa)(\beta-3\gamma)},\ \theta = \frac{3(1+\varkappa)(1-3\gamma)}{2-3\beta(1+\varkappa)} \qquad (7)$$

This is a generalization of invariants, introduced first in [2] and than used in [3] for more special cases. Without stretching effects ($\beta = \gamma = 0$), we have $\mu = k(1+\varkappa)/(1+3\varkappa)$, $\theta = 3(1+\varkappa)/2$.For $k > 0$ and $\varkappa > -1/3$, we get $\mu > 0$, $\theta > 1$. With such parameters, gravitational collapse ($\dot{a_0} < 0$), according to (7), will lead to singularity ($a \to 0, \dot{a} \to -\infty$). Stretching effects can prevent singularity. With $\theta < 1, \mu > 0$, from (7) it follows that gravitational collapse will lead to a finite core:

$$a \to a_* = a_0 \left[\frac{\mu}{(\dot{a_0})^2 + \mu}\right]^{\frac{1}{2(1-\theta)}} \qquad (8)$$

From (7) we can get general solution in quadrature, which has special cases, detectable from expressions for coefficients in (7). One of these cases corresponds to system (3*-4*) with $k \neq 0$ and $\varkappa \neq 0$ and considered in Appendix. Expression for θ indicates another special case with $\gamma = 1/3$ (as before) and $\beta(1+\varkappa) = 2/3$. Assuming, as above, $\lambda_0 = 0$, from (3) -(4) we have $ka^{-2}(1+3\varkappa) = 0$. For $k = 0$, from (4) we get solution similar to (5) with substitution ε_0 by $\varepsilon_0(1+\varkappa)$. Solution for $\varkappa = -1/3$ (Appendix) deserves detailed consideration in future.

Mass m_0 of dark matter particles, which we identify with graviton, have been estimated [4] by comparing characteristic scale L_* from (5) with the relativistic uncertainty of particle position [19] (or Compton wavelength) $\hbar/m_0 c$, where $\hbar$ is the Plank constant. We have:

$$m_0 \sim \hbar(G_*\rho_0)^{1/2} \sim 0.5 \cdot 10^{-66} gram. \qquad (9)$$

Here, according to WMAP data, we use $\rho_0 \approx 0.26 \cdot 10^{-29} gcm^{-3}$, which includes dark and ordinary matter, but not dark energy. In this theory, flatness of the universe is supported by the divergency (σ) terms in (1-2).

Estimate, similar to (9), we got before [3] from more complicated consideration, which involves solution of a model equation for a quantum field, so, this is also an example of trivialisation. According to (9), DM particles are ultralight and their uncertainty of position $L_* \approx 76$ billion light years (bly) is of the same order as size of the visible universe $a_0 \approx 46,5\ bly$. So, we can say, that mass of primary dark matter particles (PDMP) m_0 predetermined the size of universe (see also next section). It also means that universe has a halo of DM particles. This halo potentially can influence the visible part of universe, producing effects similar to influence of hypothetical multiverse. The same effect (large uncertainty of DM particle position) can explain halo of a galaxy, which is more easy to observe (see, for example, paper [20] and references therein). Formula (5) does not have any fitting parameters and shows good quantitative agreement with cosmological observations (SnIa, SDSS-BAO and reduction of acceleration of the expanding Universe [21]) [3,4].

In retrospect, some early theoretical papers are relevant to our work, particularly, [23-25]. These and others relevant papers are discussed in [3]. The physical nature of the ultralight dark matter particles is also discussed in [3] and arguments in favor of scalar massive photon pairs are presented there. Now, taking into account indicated above unusual gravitational waves, I am inclined to suggest that dark matter consists of (or connected to) gravitons with tiny EDM (see below). Irrespective of this particular interpretation, the quantity m_0 defined in (9) can also serve as a basis for subsequent reconsideration of the problem of divergence in quantum field theory [26, 27].

3. Age of the universe

According to (5), our universe was born in infinite past from small fluctuation. But, physically speaking, we can choose some initial scale for an effective beginning of the universe. From (5) we get:

$$T = h_0 \pm (h_0^2 + s)^{1/2}, \quad T = \tau/L_*, \quad h_0 = H_0 L_*/4\pi, \quad s = \frac{1}{2\pi} \ln\left(\frac{a_0}{a(\tau)}\right) \quad (10)$$

For $\tau < 0$ we have $s > 0$ and in formula for T the sign is minus. It seems natural to choose Planck length $l_P = (G_* c\hbar)^{1/2}$ as an initial scale, at which we can expect beginning of a smooth metric. With $a(\tau) = l_P$ and $h_0 \approx 0.45$ ($H_0 c \approx 2.4 \cdot 10^{-18} s^{-1}$), from (10) we get corresponding time $t_1 \approx -327$ billion years. So, at the effective beginning of the universe there was a spec of matter, which we will call Premote, with size l_P and mass $M_1 = \rho_0 l_P^3 \approx 10^{-128} gram$. The uncertainty of position for Premote is $L_1 = \hbar/M_1 c \approx 10^{90} cm \approx 10^{63} bly$. So, the probability of finding Premote can be estimated by $(l_P/L_1)^3 \sim 4 \cdot 10^{-369}$.

The next step is when universe is ready to accommodate production of PDMP with mass (9). Solution (5) corresponds to constant mass density ρ_0 with concentration of particles n and characteristic scale l :

$$n = \rho_0/m_0 \sim 0.5 \cdot 10^{37} cm^{-3}, \ l = n^{-1/3} \sim 0,27 \cdot 10^{-12} cm. \quad (11)$$

With that scale from (10) we get $t_2 \approx -284$ billion years. So, it took about 43 billion years to accommodate universe for production of PDMP. The mass of the universe at time t_2 was $M_2 = \rho_0 l^3 \sim m_0$. As was said above, the uncertainty of position L_* predetermined the size of the visible universe and, from (5) we get $a_{\max}/a_0 \approx 3.56$.

4. Dark matter

According to cosmic observations, DM interacts with ordinary matter (OM) only gravitationally. So far, in frames of our theory, we know the mass (9) of PDMP, which is very small, and averaged concentration (11), which is not only enormous, but also constant. It means, that these particles somehow communicate with each other and polarize vacuum in order to maintain averaged distance l (11). Remember, that we are dealing with unusual fluid [17]. Note, that the thermal de Brogle wavelength for the temperature of the universe $T \approx 2.73K$ is substantially bigger than l: $\hbar c(lk_BT)^{-1} \approx 3 \cdot 10^{11}$($k_B$ - Boltzmann constant). This estimate is for massless particle. For nonrelativistic PDMP (graviton) with mass m_0 (9), the scale factor is $\hbar l^{-1}(m_0 k_B T)^{-1/2} \approx 7 \cdot 10^{13}$. So, the quantum effects, such as Bose-Einstein condensate (compare with [11]), can dominate, even for high temperature. In the areas of gravitational condensation (future galaxies) the density was even much higher. With certain critical density, we can expect local bangs of multiple collisions with formation of new particles in some sort of "natural selection". During the steady and stable expansion of the universe, the OM was synthesized in this way, probably, starting with light particles. Baryon asymmetry can be explained, for example, by nonzero electric dipole moment of PDMP [28]. These processes were accompanied by radiation, which is reflected in Cosmic Microwave Background (CMB). The eqilibrium character of CMB and the global condition $R \approx 0$ are naturally explained by the large amount of time available for the evolution. Some peculiarities of CMB can be associated with synthesis of various particles in expanding universe. Particularly, the observed anisotropy of CMB can be connected with nonsynchronous processes in galaxies. In context of the type of evolution, which is described by exact solution (5), what we call ordinary matter is, in fact, an exotic matter, which was synthesized from PDMP and, so far, constitute about 15% of the total mass of the universe (or 4%, if we include dark energy). The theory of elementary particles should be modified by considering DM as primary basis for all particles. Moreover, we can not be sure that DM obeys all the rules of the conventional quantum theory. It is possible, that DM produces some quantum effects for "ordinary" matter (see new interpretation of quantum theory [29]).

However, this is not work for one person. The short list of what we need to do is: 1) based on equations (1,2) without fitting parameters ($\lambda_0 = 0$, $\beta = 2\gamma = 2/3$), or in more general case ($\lambda_0 = 0, \beta(1 + \varkappa) = 2/3, \gamma = 1/3$), calculate formation of galaxies and compare results with Sloan Digital Sky Survey and Canada-France-Hawaii Telescope Legacy Survey; 2) using the same equations (1, 2), for simplicity in spherically-symmetric case, calculate gravitational collapse and look what modification of the classical singular Black Holes we got in this theory; 3) develop detailed model for PDMP interaction and synthesis of OM particles; 4) calculate temperature and polarization anisotropies of CMB and compare with measurements (WMAP and Plank missions); 5) investigate nonlinear quantum-gravitional waves, based on equations (1, 2); 6) consider problem of baryon asymmetry, using QMOGER, mass of graviton (9) and EDM [28].

I will be happy if cosmologists, with experience in corresponding work in frames of ST, can contribute in this development. Below we indicate another important aspect of the theory, which can help in the project 3) in the above list.

5. Interface between dark and ordinary matter (IDOM)

In the described theory we got that DM is omnipresent background in the universe. As a result of gravitational condensation, from that background emerged OM. We can expect existence of a particle or a group of particles - mediators between dark and ordinary matter (MeDOM), which may have a superluminal component, related to indicated above communication between PDMP. These mediators can be produced spontaneously, or, more likely, during collisions. The "plasma" of PDMP and MeDOM produces ordinary matter, including photons. So, we got interface between dark and ordinary matter (IDOM). Such interface very likely exists not only in cosmos, but everywhere, including our body and our brain [30]. A model of IDOM is described in [32]. From that model it follows that our subjective experiences are manifestations of IDOM and can be used as a natural detector of dark-ordinary matter interaction, which may be not easy to detect in cosmic data or in the supercollider.

This paper is an example of what two simple terms can do, when they added to the powerful Einstein equations.

APPENDIX

From equation (7) with $\beta = 2\gamma = 2/3, k \neq 0, \varkappa \neq 0$, we get:

$$\dot{a}^2 = H_0^2[(1+\nu)a^2 \ - \nu a_0^2],\ \nu = \frac{k(1+\varkappa)}{2\varkappa a_0^2 H_0^2}. \tag{12}$$

The right side of equation (12) initially is $H_0^2 a_0^2 > 0$ and for continuous $a(\tau)$ remains positive, at least, for finite time. Assuming that $\nu > 0$, from (12) we have condition: $a \geq a_{\min} = a_0/\chi$, $\chi = (\frac{1+\nu}{\nu})^{1/2} > 1$. Solution of (12) is:

$$a(\tau) = a_{\min}\cosh[(1+\nu)^{1/2}H_0\tau + \eta),\ \eta = \ln[\chi + (\chi^2-1)^{1/2}\ ] > 0, \tag{13}$$

where constant η is determined by the initial condition. According to (13), $a(\tau) \to (a_{\min}/2)\exp[(1+\nu)^{1/2} \mid H_0\tau \mid]$ for $\mid H_0\tau \mid \to \infty$. So, with $H_0 > 0$, we get unlimited expansion of the universe.

In the case $\nu < -1$, from (12) follows condition: $a \leq a_{\max} = a_0 S$, $S = (\frac{|\nu|}{|\nu|-1})^{1/2} > 1$. Solution of (12) with $a \geq 0$ can be written in the form:

$$a(\tau) = a_{\max} \mid \cos[(\mid \nu \mid -1)^{1/2}H_0\tau + \varphi] \mid,\ \varphi = Arc\cos(S^{-1}). \tag{14}$$

In this case we got periodic pulsation of the universe.

Now, we consider indicated above special case: $\gamma = 1/3, \beta = 1$ and $\varkappa = -1/3$. Equations (3) and (4) with $\lambda_0 = 0$ become identical in this case and give:

$$\dot{H} = \frac{k}{a^2} - \frac{8\pi}{3} G_* \varepsilon. \tag{15}$$

If $k = 0$, than solution of (15) with $\varepsilon = \varepsilon_0$, by analogy with (4*) with $\lambda_0 = 0$, we get from solution (5) by change $\varepsilon_0 \to \frac{2}{3}\varepsilon_0$. For $k \neq 0$, we consider H as function of a and, after integration over a, we have equation

$$\dot{a}^2 = f(a) = H_0^2 a^2 + [2ka - (16\pi/3) G_* \varepsilon_0 a_0 a^2](a a_0^{-1} - 1), \; f(a_0) = H_0^2 a_0^2 > 0, \tag{16}$$

condition $f(a) \geq 0$ and formal solution:

$$\int_{a_0}^{a} \frac{db}{[f(b)]^{1/2}} = \pm\tau. \tag{17}$$

From system (3-4) with $p = \varkappa\varepsilon$, we get equation:

$$[2 - 3\beta(1+\varkappa)]\dot{H} + 3(1+\varkappa)(1-3\gamma)H^2 + k(1+3\varkappa)a^{-2} = 0. \tag{18}$$

By considering range of parameters:

$$3\beta(1+\varkappa) = 2, (1+\varkappa)(1-3\gamma) \neq 0, u^2 \equiv \frac{k(1+3\varkappa)}{3(1+\varkappa)(3\gamma-1)} \geq 0, \tag{19}$$

we get solution of (18) with constant velocity:

$$a(\tau) = a_0 \pm u\tau. \tag{20}$$

From (4) and (20), we see that $\varepsilon \sim a^{-2}$ and $\varepsilon a^3 \sim a$, so, in this case the global energy depends linearly on the scale instead of cubic dependence for solution (5).

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[28] E. A. Novikov (submitted for publication). From mass m_0 (9), Planck scale $l_P = (\hbar G/c^3)^{1/2}$ and c we have unique expression for EDM: $d \sim m_0^{1/2} l_P^{3/2} c \sim 2 \times 10^{-72} gram^{1/2} cm^{5/2}\,\mathrm{sec}^{-1}$.

[29] E. A. Novikov, arXiv:0707.3299.

[30] It did not escape my attention, that this approach has important philosophical consequences. Particularly, nonmaterial entities can be considered as interfaces (or collections of interfaces) between different types of matter. Also, the approach can be imbedded in a mathematical structure, similar to the category theory [31], with morphisms and formalized interfaces, but that is another story.

[31] See an excellent review: J. C. Baez and M. Stay, arXiv:0903.0340

[32] E. A Novikov, Gravity of subjectivity (submitted for publication).