A critical review of MOND from the perspective of the 'Dark Matter Halo for Every Elementary Particle' Model

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Abstract

In this paper I look at MOND from the perspective of my elementary particle Dark matter halo hypothesis. First I repeat the core elements of my model, in order for the paper to be self contained. Then I show how the energy equation for the rotation curve with an extra constant term can give the natural but deceptive impression that Newton's Laws have to be corrected for the ultra low regime, if this energy perspective is missing. Special attention is given to the virtual aspect of the acceleration, virtual as in not caused by Newton's force of gravity, due to a constant kinetic energy caused by the Dark Matter halo at large distances. The rotation curve equations are discussed and the one from my model is given. Conclusions are drawn from the ΛCDM core-cusp problem in relation to this new perspective on MOND as hiding a Dark Matter model perspective. My model might well be the bridge between MOND at the galactic scale and ΛCDM at the cosmic level.

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I. THE ELEMENTARY PARTICLE DARK MATTER HALO MODEL

It is common knowledge that a Dark Matter mass function linear in r can explain the flatness of galaxy rotation curves at large r. This is the empirical starting point of our Dark Matter model, first presented in [1] and in [2]. Given a rest mass m_0 at $r = 0$, it will have an additional spherical Dark Matter halo containing an extra mass, with Dark Matter properties only, in the sphere with radius r as

$$m_{\text{DM}} = \frac{r}{r_{\text{DM}}} m_0. \tag{1}$$

The Dark Matter radius r_{DM} should have a galaxy specific value somewhere in between 10 kpc and 20 kpc, so approximately once or twice the radius of an average luminous galaxy. As everything indicates that Dark Matter only interacts as being a source mass of gravity, this extra mass m_{DM} only comes into play when the rest mass m_0 acts as a source of gravity. So m_{DM} doesn't contribute to the inertial mass of m_0, nor does it contribute to its gravitational charge when acted upon by a force of gravity due to some other mass distribution acting as the source of gravity.

We immediately have two differences with the Cold Dark Matter models or CDM, our Dark Matter Halo originates in baryonic particles and CDM particles are supposed to act as gravitational charges in the same way all baryonic particles do. Our choise of Dark Matter halo's as being only sources of a field of gravity and not charges in a field of gravity based on what astrophysicist do not see, as for example in a science journalists impression of a galaxy cluster collision research:

> Surprisingly, the study discovered that dark matters in galaxy cluster collisions simply pass through each other. This implies that dark matter particles do not interact with themselves, which would have caused dark matter to slow down. Instead, it appears that while dark matter could interact "non-gravitationally" with visible matter, this is not the case when it interacts with itself. More importantly, the study challenges the view that dark matter consists of proton-like particles - or perhaps any particles whatsoever. "We have now pushed the probability of two 'dark matter particles' interacting below the probability of two actual protons interacting, which means that dark matter is unlikely to consist of just 'dark-protons'," says David Harvey. "If it did, we would expect to see

them 'bounce' off each other". [3]

In our model the total gravitational source mass m_g of an elementary particle m_0 contained within a sphere with radius r will then be given by

$$m_g = m_0 + m_{\text{DM}} = m_0 + \frac{r}{r_{\text{DM}}} m_0 = m_0 \left(1 + \frac{r}{r_{\text{DM}}}\right). \tag{2}$$

So the total gravitating source mass m_g of an elementary particle at rest inside a galactic sphere of radius r_{DM} wil be twice the original rest mass at $r = 0$.

In our model we identify the Dark Matter halo with the de Broglie matter wave medium. This assures our proposal to be in full accordance with Special Relativity and it gives our theory a direct link with the micro-physics of pre-spin Wave Quantum Mechanics. In our view we connect the de Broglie's later thoughts on the matter wave field as a subquantum thermodynamic medium to Verlinde's ideas of gravity as emergent from quantum information [4], [5]. Disturbances in the elementary Dark Matter halo as changes in probability densities of position and momentum of m_0 at $r = 0$ will travel with matter wave velocity through the halo, with the usual matter wave - particle relation

$$v_{wave} = \frac{c^2}{v_{particle}} \tag{3}$$

If the elementary particle has velocity zero, the disturbances at the source can travel instantaneously through the entire halo, producing a Newtonian instantaneous force field of gravity. Particles that will be kicked out of an original $r = 0, p = 0$ position and momentum and acquire a velocity approaching c from below will have a matter wave velocity approaching c from above and there will be a considerable delay regarding the Dark Matter halo adjustment to the new situation of the source particle. In such circumstances, large retardation effects should be expected, diluting the proposed direct 'conspiracy' between dark-matter and baryons on cosmic scales.

As for the density function of Dark Matter, we start with the observations by astronomers. From a recent paper by Koopmans et.al. we quote:

> In both spiral and elliptical galaxies with prominent baryonic components, there appears to be a conspiracy between dark-matter and baryons, leading to a nearly universal total mass distribution out to the largest measured radii that is very close to isothermal (i.e. $\rho \sim r^{-2}$), with only a small intrinsic scatter between systems. [6]

This is a key motivation for our proposed axiom. The observation in the quote indicates towards some kind of a source like connection between baryons and their Dark Matter halo. The elementary particle Dark Matter halo mass content has been derived from a mass density that is inversely proportional to $4\pi r^2$. So the mass density of the halo drops of or dilutes at the surface of an ever larger sphere in the same way that all classical central sources do. We therefore define a Dark Matter halo mass density as

$$\rho_{\text{DM}} = \frac{m_0}{4\pi r^2 r_{\text{DM}}} \tag{4}$$

and then the spherically symmetric gravitational source mass m_g inside a sphere of radius r is given by

$$m_g = \int_V \rho_{\text{DM}} dV = \int_r \rho_{\text{DM}} 4\pi r^2 dr = \int_r \frac{m_0}{r_{\text{DM}}} dr = \frac{m_0}{r_{\text{DM}}} r + m_0 \tag{5}$$

with the last factor as the obvious constant of integration, given the starting point of our model that we have $m_g = m_0$ at $r = 0$.

II. THE VIRIAL THEOREM AND THE GALAXY ROTATION CURVES

Given the definition of the gravitational potential as

$$\phi = -\frac{GM}{r} \tag{6}$$

with gravitatinal source mass M as

$$M = M_0 + \frac{r M_0}{r_{\text{DM}}} \tag{7}$$

we get a gravitational potential at r as

$$\phi = -\frac{GM_0}{r} - \frac{GM_0}{r_{\text{DM}}} = \phi_0 + \phi_{\text{DM}} \tag{8}$$

For the resulting force of gravity on a classical mass m we get the Newtonian result

$$\mathbf{F} = -m\nabla\phi = -m\nabla\phi_0 + -m\nabla\phi_{\text{DM}} = -m\nabla\phi_0 = \frac{GM_0 m}{r^2}\hat{r}. \tag{9}$$

This is due to the fact that the new mass factor varies linear over r and thus results in a additional potential term that is constant. Our Dark Matter halo acts as a gauge term in the source that produces a constant term ϕ_{DM} in the potential and thus has zero effect on the force. This means that our model is not a MOND theory, we do not modify Newtonian

Dynamics. Nor do we modify General Relativity. Our model is theory of gravity neutral because it is based upon a hypothesis regarding the source of the field only.

Although the extra term in m_g doesn't effect Newtons force law of gravity, it is effecting the gravitational energy of a satellite mass m in the field of a source mass M. This gravitational energy is given by

$$U_g = m\phi = m\phi_0 + m\phi_{\text{DM}} = -\frac{GM_0 m}{r} - \frac{GM_0 m}{r_{\text{DM}}}. \tag{10}$$

Now we assume that the virial theorem is still valid. In this case we might use Bohr's correspondence principle to justify this assumption. If the virial theorem would be valid for m_0 only, we would have an energy discontinuity while going from far from to close to the source. Using $2U_k = -U_g$ we get $v^2 = -\phi$ for orbiting satellites and

$$v^2 = -\phi = \frac{GM_0}{r} + \frac{GM_0}{r_{\text{DM}}}. \tag{11}$$

If we let $r \to \infty$ then

$$v_f^2 = \frac{GM_0}{r_{\text{DM}}}, \tag{12}$$

which is a constant, the galaxy rotational velocity curves' final constant value. In Fig.(1) the result is compared to the Newtonian virial expectation for v.

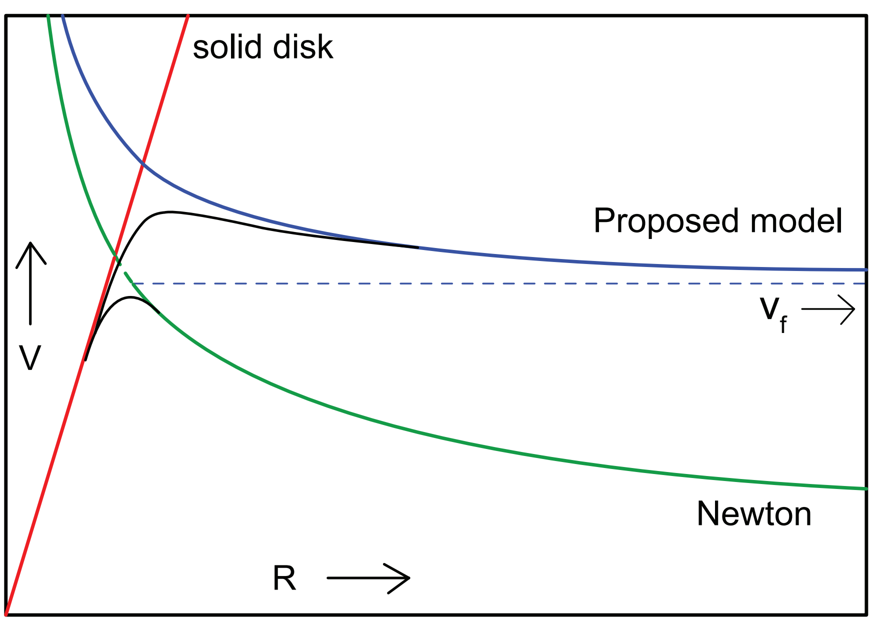

FIG. 1. Rotation velocity versus radius curves with the Newtonian and our new model expectation.

This result also allows us to give an estimate of $r_{\rm DM}$ by applying this to the Milky Way galaxy. We get

$$r_{\rm DM} = \frac{GM_0}{v_f^2} \approx \frac{6,67.10^{-11} \cdot 1,99 \cdot 10^{30} \cdot 1,4.10^{11}}{(230.10^3)^2} = 3,5.10^{20} m = 11,4 kpc. \qquad (13)$$

Actual galaxy velocity rotation curves vary considerably from our model with its point like mass distribution. Real galaxies have disk like or spherical like mass distributions which cause deviations from our single particle model. But given the luminous mass distribution, the associated galaxy Dark Matter halo as a summation over all the elementary particle Dark Matter halo's should be computable using Eqn.(4) with the value of $r_{\rm DM}$ as calculated in Eqn.(13).

III. THE DARK MATTER CONSTANT $r_{\rm dm}$ AND THE BARYONIC TULLY-FISHER RELATION

The question whether the Dark Matter radius $r_{\rm DM}$ is a galaxy specific constant or that it might even be a universal constant can be answered using the Baryonic Tully-Fisher relation, BTF. The fundamental relation underpinning the Tully-Fisher relation between galaxy luminosity and rotational velocity is one between final rotation velocity v_f and total baryonic disk mass M_d. In the 2005 paper of McGaugh the baryonic version of the LT relation has the form

$$M_d = 50 v_f^4, \qquad (14)$$

see [7] and Fig(2). In this form, M_d is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30} kg$ units and the final velocity of the galactic rotation velocity curve v_f is expressed in km/s. If we express the galactic mass in kg and the velocity in m/s we get the total baryonic mass, final velocity relations in SI units as

$$M_b = 1,0 \cdot 10^{20} v_f^4. \qquad (15)$$

In order to interpret this emperical result in the context of our model we start again with Eqn.(10)

$$U_g = U_b + U_{\rm DM} = m\phi_b + m\phi_{\rm DM} = -\frac{GM_b m}{r} - \frac{GM_b m}{r_{\rm DM}}. \qquad (16)$$

With $r \gg r_{\rm DM}$ this approaches

$$U_g \approx U_{\rm DM} = m\phi_{\rm DM} = -\frac{GM_b m}{r_{\rm DM}}. \qquad (17)$$

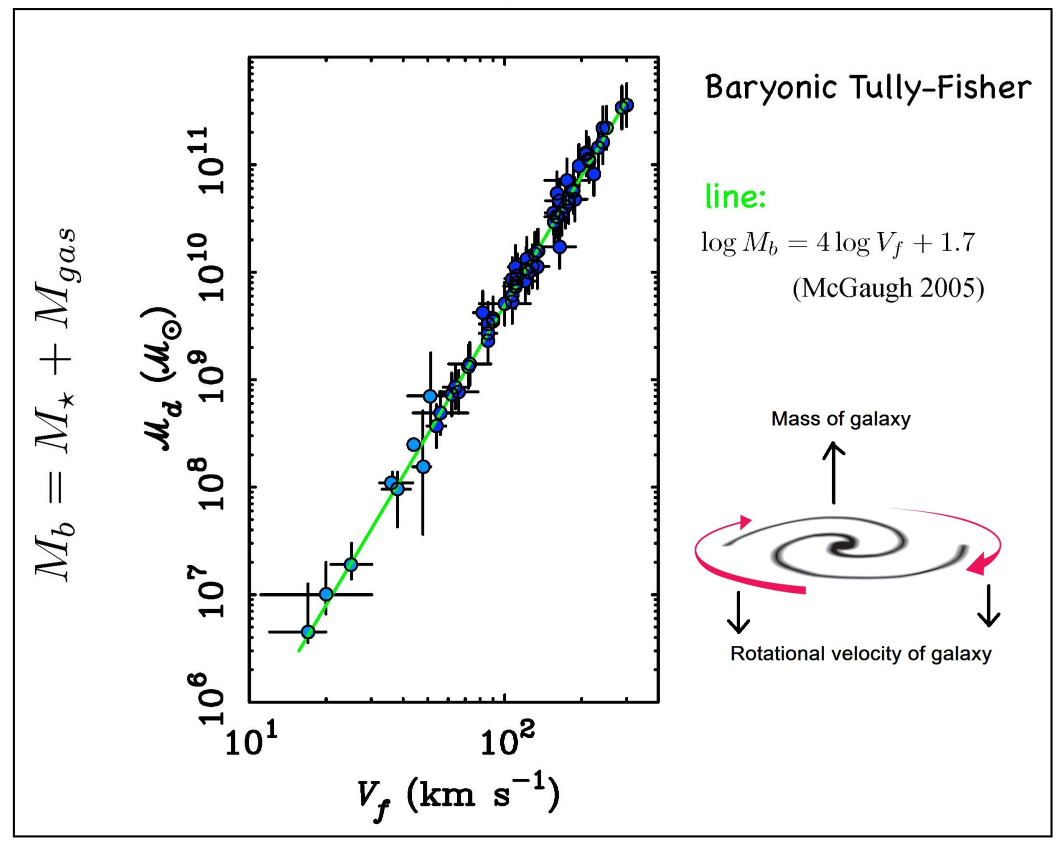

FIG. 2. The graph for the Baryonic Tully-Fisher relation from McGaug.

and with the virial theorem we get for $r \gg r_{\text{DM}}$ the Dark Matter virial theorem

$$v_f^2 = \frac{GM_b}{r_{\text{DM}}} \tag{18}$$

which combines with Eqn.(15) to

$$v_f^2 = 6,64 \cdot 10^9 \frac{v_f^4}{r_{\text{DM}}} \tag{19}$$

and this leads to

$$\frac{v_f^2}{r_{\text{DM}}} = 1,5 \cdot 10^{-10} \frac{m}{s^2} = a_{\text{DM}} \tag{20}$$

This relation with $a_{\text{DM}} \approx 2 \cdot 10^{-10} \frac{m}{s^2}$ was first obtained by Milgrom in 1983 in his second MOND paper [8]. In our model this acceleration is not caused by a modified Newtonian force equation but derived from the energy expression of the virial theorem, in a situation where the derivative of the potential, the real force, should be near zero.

The result allows us to determine the galaxy specific r_{DM} for every galaxy with known v_f

without knowledge of the galactic baryonic mass as

$$r_{\text{DM}} = \frac{v_f^2}{a_{\text{DM}}} = \frac{v_f^2}{1,5 \cdot 10^{-10} \frac{m}{s^2}} \tag{21}$$

leading to

$$m_g = m_0 + m_{\text{DM}} = m_0 \left(1 + \frac{a_{\text{DM}}}{v_f^2} r\right). \tag{22}$$

and

$$\rho_{\text{DM}} = \frac{m_0 a_{\text{DM}}}{4\pi r^2 v_f^2} \tag{23}$$

with m_0 as the individual baryonic particles in the specific galaxies.

IV. THE MOND HYPOTHESIS EXPLAINED FROM OUR POINT OF VIEW

We can write the above results for the mass distribution in MOND form using

$$a_c = \frac{v_f^2}{r} \tag{24}$$

as

$$m_g = m_0 + m_{\text{DM}} = m_0 \left(1 + \frac{a_{\text{DM}}}{a_c}\right). \tag{25}$$

From Eqn.(20) and Eqn.(25), Milgrom concluded that Newton's force laws needed to be revised.

We can use our energy equations prior to the application of the virial theorem to derive his correction regarding Newtonian dynamics. Divide our Eqn.(10) by r gives

$$\frac{U_g}{r} = -\frac{GM_b m}{r^2} - \frac{GM_b m}{r_{\text{DM}} r} = -\frac{GM_b m}{r^2} \left(1 + \frac{r}{r_{\text{DM}}}\right). \tag{26}$$

In this energy equation divided by r we can see a modification of Newton's law of gravity, where the second part on the right represents the correction term as α/r. This is of course not what Milgrom did, he empirically guessed the correction needed. And he was remarkably close, his model performed beyond expectations in the galaxy domain, especially if the circumstances he worked in are taken into account. With $U_g/r = ma$ we can write the last result also as

$$\frac{ma}{\left(1 + \frac{a_{\text{DM}}}{a_c}\right)} = \frac{GM_b m}{r^2}. \tag{27}$$

and then a modification of Newton's second law in the extremely weak regime is the logical interpretation [9].

The remarkable thing is that it took so long to explain (to exactly derive them is of course not possible) his results without changing Newton's laws. Two reasons can be considered why it took so long. First, General Relativity has the Newtonian dynamics as its weak field limit and the galaxy rotation curve problem is in the extreme weak field regime. It is simply not possible to modify GR consistently in such a way that in the solar systems moderate weak limit Newtonian dynamics results as its weak field limit and then in even much weaker realm of outer galaxy fields deviate again from the limit. That is why the answer had to come from the input side, so from elementary particle physics.

V. THE ROTATION CURVE EQUATION AND DARK MATTER

The nice thing about all this is that all of the evidence gathered for MOND can be looked at as evidence for our model, whereas most of MOND's troubles do not apply to our model. Our model is in accordance with SR, has no conflict with GR and is Newtonian for the small velocity, weak regime limit. We do have a Dark Matter halo as this halo is 'seen' by the astrophysicists.

Most of MOND's results regarding rotation curve fits came from the MOND equivalent of the combination of the BTF relationship and our energy equation

$$v(r)^2 = \frac{GM_b(r)}{r} + \frac{GM_b(r)}{r_{\rm DM}} = GM_b(r)\left(\frac{1}{r} + \frac{1}{r_{\rm DM}}\right) = GM_b(r)\left(\frac{1}{r} + \frac{a_{\rm DM}}{v_f^2}\right). \tag{28}$$

With galaxy mass distribution dependent Newtonian rotation velocity $v_{\rm N}$ this leads to a velocity function as

$$v(r) = \sqrt{\frac{GM_b(r)}{r}} \cdot \sqrt{\left(1 + \frac{a_{\rm DM} r}{v_f^2}\right)} = v_{\rm N}(r, M_b(r)) \cdot \sqrt{\left(1 + \frac{a_{\rm DM} r}{v_f^2}\right)} \tag{29}$$

also given as

$$v(r) = v_{\rm N}(r, M_b(r)) \cdot \sqrt{\left(1 + \frac{r}{r_{\rm DM}}\right)} \tag{30}$$

A more exact function isn't possible because $M_b(r)$ is an empirical imput. And if one doesn't connect $a_{\rm DM}$ and v_f to Dark Matter but to a universal acceleration almost close to cH_0, as is usual in MOND, then Dark Matter isn't needed at all. This new velocity curve function of Eqn.(30) should be tested, or it already matches on of the succesfull MOND functions. This is a falsification/verification prediction of my model.

It might however be that the best way to match the velocity curve will need the density formula and then the calculus will become way more complicated. The formula Eqn.(30) assumes a spherical symmetry. The Milky Way is a disk with a bulge, a bar and spirals, so it isn't spherically symmetric and the density formula might have to be used for a real thorough test. Every good theory usually has a smart way out, towards more complicated mathematics resulting from the confrontation of the principle with reality. And where my theory uses the crucial $\frac{r}{r_{\rm DM}}$, MOND uses the accelerations in the expressions. But in both theories, due to BTF, it is the (virtual) acceleration $a_{\rm DM} = \frac{v_f^2}{r_{\rm DM}}$ that is the universal constant over a hugh galactic range.

VI. THE CORE-CUSP PROBLEM AND MY MODEL RELATIVE TO MOND VERSUS ΛCDM

What is the core-cusp problem? Let's quote an unusual source, Wikipedia:

> The cuspy halo problem (also known as the cusp-core problem) arises from cosmological simulations that seem to indicate cold dark matter (CDM) would form cuspy distributions that is, increasing sharply to a high value at a central point in the most dense areas of the universe. This would imply that the center of the Milky Way, for example, should exhibit a higher dark-matter density than other areas. However, it seems rather that the centers of these galaxies likely have no cusp in the dark-matter distribution at all.

The problem goes a bit deeper than just the core that isn't cuspy. The ΛCDM paradigm doesn't have a Dark Matter mass distribution model based upon first principles.

> This ΛCDM paradigm provides a comprehensive description of the universe at large scales. However, despite these great successes, it should be kept in mind that the cusp and the central dark matter distribution are not predicted from first principles by ΛCDM. Rather, these properties are derived from analytical fits made to dark-matter-only numerical simulations. While the quality and

quantity of these simulations has improved by orders of magnitude over the years, there is as yet no cosmological theory that explains and correctly predicts the distribution of dark matter in galaxies from first principles [10].

In MOND there is no core-cusp problem because the theory predicts the correct velocity rotation curves based on baryonic mass only without Cold Dark Matter.

Now, my model has the same basis predictions as MOND as derived from Milgrom's galactic Kepler Law. Let's state that MOND has a hidden Dark Matter formulation because it has the same basic predictions regarding deviations from 'Newton' as my model does, although in my model this is energy based and in MOND it is acceleration based. That implies that MOND is compatible with a Dark Matter halo according to my model. In other words, MOND is a Dark Matter theory in disguise. In my model, Dark Matter only acts as a source of gravity, not as a charge and thus doesn't interact directly gravitationally. The basis parameter is the mass doubling distance r_{DM}, which for a average galaxy is about $5 - 15kpc$. At the core, where $r = 0$, the Dark Matter density is zero by definition, for baryonic point masses that is. This mass doubling distance is fixed by v_f and a_{DM}, a fix that in MOND is represented as the distance where the centripetal acceleration meets a_{DM}. There is no Dark Matter cusp by first principle in my model, a model that fits with MOND, and MOND fits with the data.

Only one conclusion is possible here: ΛCDM has a non-fixable problem with its Dark Matter model on the scale of galaxies. A possible fix lies in adopting the basic properties of my Dark Matter model for the individual galaxy domain all along the BTF scale. Then the discrepancy with MOND would also disappear. In my model, the operational density distribution is a combination of first principles and empirical input. Because my model is build-in Special Relativistic and compatible with General Relativity, adopting my model of Dark Matter in ΛCDM shouldn't be a problem. That is, in principle.

VII. CONCLUSION

According to McGauch and Famaey, the results of MOND as a Galactic Kepler Law

> can all be summarized by Milgroms empirical formula, meaning that the observed gravitational field in galaxies is mimicking a universal force law generated by the baryons alone [11]

This is one of the key elements of our model. In the end McGaugh and Famaey stress "the need to continue to search for a deeper theory" to explain the successes of MOND from first principles. Our model is an axiomatic one, but in order to arrive at MOND, especially Milgrom's 'Kepler Law', the empirical BTF relation input was required. And our axioms are chosen to match the astrophysical knowledge regarding galaxies. In that sense we developed a theory that is emergent from the experimental data, with a minimal extra fundamental choices.

We showed how in our model, the illusion of modified Newtonian Dynamics can arise quite naturally and how it can be a good first approximation if a deeper theory is lacking. We showed at the beginning that in our model, there is no modified force law or dynamics, there is only a changed energy in the form of a constant that disappears during space-like differentiation.

Our model for Dark Matter is a basic one, but it can explain the average galactic rotation curve and is in conformance with the density distribution of Dark Matter and gives an interpretation of the BTF relation of Milgrom's MOND. The presented hypothesis is neutral relative to the theories of gravity, being a proposition solely about the source of gravity, but our model also seems to connects naturally to the theory of gravity of Verlinde. Using the density formula, our models area of verification/falsification should be extendable to galactic Einstein lensing.

Our model might well provide inside towards the deeper reason for the successes of MOND and provide a bridge towards ΛCDM.

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