Studies on the Accuracy of the Results Predicted by the Simple Beam Theory in the case of Short Cantilever Beams

Kirana Kumara P
Department of Automobile Engineering
Dayananda Sagar College of Engineering
Bengaluru, India
kiranakumarap@gmail.com

***Abstract*—The simple beam theory can predict the deflection of the loaded end of cantilever beams with a high level of accuracy if the length-to-diameter ratios for the beams are high. Hence the readily available formula (which is based on the simple beam theory) for cantilever beams may be used with confidence when the length-to-diameter ratios for the beams are high. However, studies on the accuracy of the results predicted by the simple beam theory are not common when the length-to-diameter ratios are small, especially when the Young's modulus is low. Hence the present work compares the results obtained through the use of the well-known formula (based on the simple bending theory) with the results obtained by making use of the well-known commercial finite element software ANSYS, for cantilever beams of different length-to-diameter ratios, in order to get an idea of the accuracy of the results that are obtained by making use of the well-known formula. The effect of Young's modulus is also looked into, by solving the same problems but for different Young's moduli. All these results are tabulated. The present study would be of use to designers who are concerned over the accuracy of the results that are obtained through the use of the well-known analytical formula.**

Keywords—cantilever; beam; short; accuracy; deflection; simple bending; pure bending; formula

I. INTRODUCTION

Predicting the deflection of the cantilever beam subjected to end load has wide applications in the design of mechanical components. The formula that is based on the simple (pure) bending theory is widely used for the prediction. Literature (e.g., [1] and [2]) tells that the analytical formula gives reasonably accurate results when the length-to-diameter ratios for beams are high (>10, say). However, extensive and exhaustive studies on the deflection of short beams are rarely available, although such studies could have practical applications too. Hence the present work is an attempt to study the deflection of short cantilever beams. Of course, a few sources in the literature (e.g., [1]) do contain material on this topic. But many a times the coverage of the topic is not comprehensive and exhaustive; for example, error in the prediction of stresses might have been addressed well but error in the prediction of deflection might have been neglected. So the present work aims to make a fresh attempt towards the quantification of the error when the analytical formula that is based on the pure bending theory is used for the prediction of the deflection of the loaded end of a cantilever beam when the length-to-diameter ratio for the beam is low (i.e., beam is short).

Of course, all the problems considered in the present study are liner elastostatic problems, and body forces are not considered.

II. METHODOLOGY

The geometry considered is a solid cylinder of 1 mm diameter. The length of the cylinder could be 1 mm, 3 mm, or 10 mm (which correspond to length-to-diameter ratios 1, 3, or 10 respectively). The Poisson's ratio is assumed to be equal to 0.33, for each of the simulations. For each of the length-to-diameter ratios, three values for the Young's modulus (E) are tried out. These three values are: 200 N/mm^2, 2000 N/mm^2, and 200000 N/mm^2.

The model is constructed in the commercial finite element software ANSYS. A fine mesh is used always, so that the solutions given by ANSYS are accurate. The element type used is Tet 10node 187, and the geometry is divided into tetrahedral elements using the "free" meshing option.

One end (the entire circular surface) of the beam is completely fixed (no displacement is allowed along any of the directions). The other end is subjected to a point load of 1 N, at a node which is very close (if not the closest) to the centerline of the cylinder (of course, the node is located on the loaded end). The problem is to find the deflection at the loaded (by 1 N) node, along the direction of the load.

Results are tabulated for each of the (three dimensional) finite element simulations (ANSYS is used for the simulations). These results are presented in Table I. Results obtained by using the (one dimensional) analytical formula for cantilever beams (the formula neglects the effect of shear forces) are also tabulated and these results are presented in Table II.

III. RESULTS

As mentioned previously, the results from ANSYS are presented in Table I whereas the results from the analytical formula are presented in Table II, for different length-to-diameter ratios (L/d) and Young's moduli (E).

The percentage error is also calculated for each of the cases, and this is presented in Table III. Error is calculated here by taking the results from ANSYS as the reference values.

TABLE I. RESULTS OBTAINED BY USING ANSYS

	E = 200 N/mm^2	**E = 2000 N/mm^2**	**E = 200000 N/mm^2**
L/d = 1	0.203 mm	0.020 mm	0.000 mm
L/d = 3	1.048 mm	0.105 mm	0.001 mm
L/d = 10	33.913 mm	3.391 mm	0.034 mm

TABLE II. RESULTS OBTAINED BY USING THE ANALYTICAL FORMULA

	E = 200 N/mm^2	**E = 2000 N/mm^2**	**E = 200000 N/mm^2**
L/d = 1	0.034 mm	0.003 mm	0.000 mm
L/d = 3	0.917 mm	0.092 mm	0.001 mm
L/d = 10	33.953 mm	3.395 mm	0.034 mm

TABLE III. PERCENTAGE ERROR

	E = 200 N/mm^2	**E = 2000 N/mm^2**	**E = 200000 N/mm^2**
L/d = 1	83.251	85.000	0.000
L/d = 3	12.500	12.381	0.000
L/d = 10	-0.118	-0.118	0.000

IV. CONCLUDING REMARKS

Although literature does contain some results that are similar to the ones presented in this work, those results are not exhaustive. Further, extensive and exhaustive studies on the effect of Young's modulus on the accuracy (and error) of the results obtained using the simple analytical formula are not readily available in the literature. The present work is a small step towards bridging these gaps.

Future work would be to generate and tabulate more results, by considering more number of length-to-diameter ratios, different cross sections (e.g., square cross section), and different material properties (different Young's moduli and Poisson's ratios). This would result in a study that is really extensive and exhaustive.

REFERENCES

[1] Warren C. Young, Richard G. Budynas, Roark's Formulas for Stress and Strain, Seventh Edition, McGraw-Hill: New York, 2002.

[2] K. Mahadevan, K. Balaveera Reddy, Design Data Handbook, Fourth Edition, CBS Publishers & Distributors: New Delhi, 2013.