HOLISM AND THE GEOMETRIZATION AND UNIFICATION OF FUNDAMENTAL PHYSICAL INTERACTIONS

(an essay on philosophy of physics)

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To the memory of my teacher
Professor Zdzisław Augustynek

Abstract

We are looking for a paradigm of modern physics and this essay is devoted to this problem. Thus in the essay we consider very important problems of philosophy of physics: a unification and a geometrization of fundamental physical interactions and holism. We are looking for holistic approaches in many different domains of physics. Simultaneously we look for some holistic approaches in the history of philosophy and compare them to approaches in physics. We consider a unification and a geometrization of physical interactions in contemporary physics in order to find some connections with many philosophical approaches known in the history of philosophy. In this way we want to connect humanity to natural sciences, i.e. physics. In the history of philosophy there are several basic principles, so-called "*arche*". Our conclusion is that a contemporary "*arche*" is geometry as in the Einstein programme connected to a holistic approach. In our meaning geometry as "*arche*" of physics will be a leading idea in a fundamental physics. Physics itself will be a leading force in philosophy of science and philosophy itself. Cultural quasi-reality (in R. Ingarden's meaning) and also biology, medicine and social sciences will be influenced by physics. In this way a cultural quasi-reality will be closer to physical reality.

Motto

The main concern in all scientific work
must be the human being himself.
This one should never forget among
all those diagrams and equations.
Albert Einstein

Preface

This essay is devoted to an important approach in philosophy of physics, according to which “the whole” is more important and takes precedence over “the part”. In the treatment here offered, holism links the geometry and the unification of fundamental interactions. In this way one tries to give a certain version of philosophy of physics. Let us define briefly what is to be understood as philosophy of physics. Under this term we are going to mean a general-theoretic superstructure spanned over the physical laws and methods. One includes into the philosophy of physics all such physical problems, which at present could not be solved by the exact methods of physics. The domain in question is therefore a kind of meta-science in its relation to physics, and one would quite naturally employ the name of metaphysics here, should not this term already been rooted inside definite and a completely different semantic tradition.

The impact of problems dealt with by the philosophy of physics is quite considerable for physics itself, as well as for the philosophy, since it offers a unique conceptual framework, within which one might embark onto the effort of reflecting upon the physical laws, methods, development, trends etc. Further, also within this conceptual picture one could tackle a number of general world-view topics, which have a bearing onto determinism, indeterminism, unity of nature, geometrization of physics and onto the unification of the fundamental interactions. Philosophy of physics thus assumes, within a perspective adopted — the status of a domain of inquiry occupying the borderline between the humanities and the mathematized natural sciences. Naturally for such a domain of inquiry frequent references to methods like analogy and value judgements shall find its place, quite typical of any philosophy. Such references (I hope) shall contribute to the humanization of the physics itself. Another important aspect of the approach adopted here consists in drawing attention to the culture and civilization advancement component of the role played by physics; so long as the human being is going to remain at the heart of all our undertakings — the humanization of physics deserves and shall assume very remarkable status. The thrust of that humanistic message is being emphasized by an excerpt from Prothagoras (in Albert Einstein spelling), serving as motto to this work.

Prothagoras known as a creator of humanity simply tells us that a measure of everything is a human being himself. According to our ideas developed in this essay we have to do with *Scientia humanitatis* which contains all sciences and arts and especially *Studia humanitatis*. The last one contains philologies, history, history of art, musicology, linguistics, archeology, theory of literature (studies of literature). In this type of scientific activity we use methods known in sciences, i.e. we are describing facts, and interpret them. This is very sound in linguistics and in archeology and history. In some sense we are using methodology of physics. Thus there is a place here for philosophical considerations. Philosophy of physics seems to be very important from this point of view because it connects physics (in general sciences) with remaining human scientific activity.

Physics can be considered as a mother science of chemistry and biology. On the molecular level there is no difference between physics, chemistry, biology or medicine. There is only physics. Physics can also be employed in evolution processes and even in psychology. To be honest, we should mention on a different approach to understand the reality. It is an *anthropic principle*. In this approach we explain everything as a postulate that the world, the universe exists, because we (the mankind) can explore it. In this way the mankind exists.

Thus all the parameters of physical world must allow a human being to exist. Moreover, it seems that the principle is very simplimistic and all history of science is going to avoid this principle. (This principle has in some sense a religious origin.) Some interesting problems in geometrization of physics and cosmology raised a concept of the Multiverse. It means, a concept with many universes considered at once. These universes are very different with different dimensions, different physics, different geometries. Many of them cannot be habitated by human beings. Moreover, no one proves that our Universe can be distinguished among remaining universes except this fact that we are living here. This is in favour of the anthropic principle. Moreover, this fact demands more investigations. The geometry of some of those universes can be very complex. The topology of them can also be complex.

One of the interesting problems is a problem of limits of science. This is a problem of philosophy of science and of course of philosophy of physics. L. Chwistek in his *Limits of Science* (Chwistek 1935) deals with this problem. The first problem is of course a problem with insolvability. Moreover, we have more problems. This is a problem of complexity. Some of calculational problems in theoretical physics (I mean here not only numerical calculations, but also some symbolic manipulations) can be non-polynomial or even hard. We have some academic examples in foundations of mathematics (some parts of Peano axiomatics). Thus some of the problems (if they exist) can be outside our calculational capabilities. This could happen also for our memory capacity. There is also a problem with our capabilities in constructing mathematical models of the Nature. Moreover, we have also possibilities (in future) to construct quantum computers to solve some hard problems. Maybe it is enough to find a model (a theory) to encompass the Nature. The second possibility is a pessimistic one. In this case there is a gap between complexity of the Nature and our (as the mankind) capability to construct models (theories) of the Nature. This will be an end (a limit) of science. Moreover, this is beyond the scope of the essay.

We can find such possibilities in the science-fiction literature. For example, in *Master's Voice* and *Solaris* by S. Lem (Lem 1961; Lem 1968). The conclusion is such that *One day we can find in our environment something which does not fit to our understanding of the world.* This is similar to the *Ignorabimus* by E. du Bois Reymond and really pessimistic. Let us sketch what it means.

E. du Bois Reymond put in his *Ignorabimus* (*Ignoramus et ignorabimus*) all we do not know and we will not know. It happens some of his predictions will not fulfil (e.g. a chemical composition of distant stars). This can give us a hope that there are no limits of science. The human mind is unbounded, the only boundaries are boundaries of the complexity of the Universe which is cognizable. (D. Hilbert in opposition to *Ignorabimus* wrote: *Wir müssen wissen — wir werden wissen.*) The assumption of cognizability of the Universe can be included to principles of the philosophy of science (moreover, not necessarily).

The very interesting point in any theoretical investigations on foundation of nature (I can say a physical reality) is an incredible successful application of mathematics. There is no physics without mathematics. It seems that a physical reality has a mathematical structure. In our meaning this mathematical structure is a geometry and because of this we are looking for a geometrization of physics. This geometrization can simultaneously unify physics of fundamental interactions.

Due to the status of physics as a standard, most developed among the natural sciences, the conclusions arrived at in this essay, shall bear its validity also in the remaining natural sciences — a fact it seems to us of equal importance.

This work does not claim to have completely solved the question of relationship between a holistic approach to physics and the one of having the fundamental physical interactions geometrized and unified. Nonetheless it does put forward a claim to the effect that the tasks of geometrization and unification of the fundamental physical interactions are reciprocally very strongly associated. Simultaneously, the geometrization and unification of fundamental physical interactions are implying the holistic approach, which in its turn gives rise for using of certain definite mathematical theories as tools for theoretical (mathematical) physics. It is just in the above sense a kind of attempt aimed at approximating the meaning of the philosophy of physics, emphasized in the subtitle of this essay.

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Marek Wojciech Kalinowski

Contents

Introduction and General Remarks

This work is devoted to philosophy of physics (Bunge 1980, 1973; Schultz 2013; Van Melsen 2012; Kemeny 1959; Kuhn 1970). A definition of its scope would be in order. We have got two possibilities. One of them places philosophy of physics inside philosophy of science. By this way philosophy of physics becomes a kind of metascience. One could call it metaphysics if this term have not already been earlier reserved for a completely different area. Philosophy of physics according to such an approach would deal with metaproblems, that is the methodology of physics, or to put it in another way to an empirical science called physics. We are all aware that a methodology of deductive sciences under the name of metamathematics scored a number of successes. Let me recall only such facts like: theory of truth, Gödel theorem, degrees of unsolvability, etc. Looking with envy onto metamathematics we might have desired to obtain certain similar results in the philosophy of physics. To prove the "Gödel theorem" as a thesis in the methodology of physics. Certainly the importance of such an achievement could have been great, I presume that even far greater than the value of Gödel theorem for mathematics. However, I would like to clearly stress that such a result is impossible at present and never shall become possible. Let us not be deceived by the mathematical character of theoretical physics, by its extensive apparatus, its abstractiveness, especially in the field theory or theory of elementary particles. The issue here has nothing to do with neither mathematical apparatus, nor quantitative results, or even abstract notions. Physics is simply an empirical science and this is its *differentia specifica* in comparison to mathematics. Of course this is truism, but one has to emphasize it, to avoid any doubts.

Mathematical theories are handsome when undertaking metamathematical type of investigations. Examining their logical structure within the framework of model theory and extracting general conclusions is very useful and interesting. Here we could say that sufficiently well axiomatized and mathematized physical theories shall also be amenable for similar treatment. Clearly, that is the case. But nothing would follow out from such an approach. Theoretical physics would have become a strange variety of mathematics, towards which hardly any mathematician would show any predilection. Such an approach would miss certain feature of physics, a very basic one, mainly the fact that physical theories describe reality somehow that there is a degree of conformity of their predictions with empirical evidence, and therefore physical theories might be either confirmed or rejected on the basis of experiment. (D. Hilbert tried, quite successfully, to axiomatize theory of elasticity, classical mechanics and even general relativity.)

It is precisely this problem, the most important for the whole issue which is unlikely to be formalized along the terms one is accustomed to within the realm of mathematical problem. It is hardly possible expressing all the variability exhibited by physical theories, which still better and more adequately describe reality. After all each physical theory contributes to yet another picture of physical phenomena. Let us consider for example Newtonian mechanics and non-relativistic classical mechanics. Both these theories present us with completely different world-views, but they are linked via the principle of correspondence. We also know that Newtonian mechanics is not false at all. The predictions offered by it agree with experiments to a definite extent. The relationship between classical mechanics and the quantum one in known *post factum*. We are occupying ourselves with this relationship only because quantum mechanics better describes reality. We see therefore, how unfounded is the claim that methodology of physics will gradually become indiscernible from the methodology of mathematics. Presumably we shall never grasp the suitable variability of physical theories which are leading towards still a better description of the world.

Should it be otherwise, we would become omniscient and would one day know the most fundamental physical theory, just the one most precise and the best possible. Let us just call it superunified field theory or something of that kind. For such a theory we could have used the apparatus of metamathematical type and arrive by this way to results quite analogous with the ones obtained for methodology of deductive sciences. Let us notice that in this case we should have assumed (God alone knows on what grounds) that our supertheory is the one most precise and best available description of reality. Thus we would invalidate the issue of whether it has any relationship to reality or not. Our theory would be the best possible and could not be improved. Every other physical theory would be either a paraphrase or a certain approximation of this one. Therefore by axiomatizing it and examining its structure we shall reach all possible philosophical implications of its contents, of the type like degrees of unsolvability and Gödel theorem. But this would be clearly absurd and impossible. That is why all those who would like to see the philosophy of physics in the role of metamathematics for physics must take into account this sort of consequences. Philosophy of physics in this sense will never play the role of metamathematics for physics. That is why a question arises as to whether the philosophy of physics should be methodology for physics, or it has to be a part of philosophy of science?

There is though another idea — the second possibility, which I had mentioned at the very beginning. According to this idea the philosophy of physics constitutes a part of physics itself. How could this be possible? — physicists as well as philosophers would immediately ask. The puzzle is to be explained by determining what is to be understood as problems in philosophy of physics. They are not methodological problems, but the problems of physics, which as yet are unsolvable by methods of physics alone. Here I mean of course the theoretical physics. By this token the problems in the philosophy of physics will assume their place inside the cone of growth of contemporary physics (and it would be the most fundamental one here). The basic question which we have to ask now is: What is the role of philosophy in this problem area? If physics by using its contemporary methods is not able to solve problems of this kind, then how should they be solved by philosophy? We will reply this question right now — it is not philosophy which would try to solve them. Philosophy after all, as we know, in principle does not solve any problems. Its responsibility lies in posing them and at the most showing some directions, presumably leading towards solutions. Such a role for philosophy of physics is interesting from the cognitive point of view and at the same time is of certain practical importance for physics. It shall play the germinal role. In order to enable the better shape of this role, let us give some problems, which are to be classified as ones belonging to the realm of philosophy of physics and have them presented briefly. Here belong:

1. problem of the unification of the fundamental interactions in physics
2. problem of the existence of elementary particles
3. problem of geometrization of physics
4. problem of relationship between physics and cosmology.

Let us first deal with problem 1). As we know there are four fundamental interactions: gravitational, weak, electromagnetic and strong. In order to acquaint ourselves with the issue of unification, let us return to the time period preceding C. Maxwell, that is the time when electricity and magnetism were conceived as separate types of phenomena related in a certain no fully understandable manner.

C. Maxwell by inventing his famous equations managed to unify electricity with magnetism. In doing so he was guided exclusively by his perception of mathematical elegance, rather than new, experimental evidence. Strictly speaking, he had adjoined to the equations a new term, displacement current. He took advantage in this case of a certain mechanical model for electric and magnetic phenomena and also had striven that a charge conservation principle would follow from his equations. At the same time he managed, somewhat unconsciously to unify not only electricity and magnetism but also optics, since in his theory light has turned out to be an electromagnetic wave.

In this manner electricity and magnetism revealed themselves as phenomena of the same type, examples of the same interaction. We call this interaction an electromagnetic one. One might separate electricity and magnetism only under special circumstances, they reveal themselves to us as the two sides of the same medal. At the same time phenomena become admissible, where electricity and magnetism occur simultaneously in a symmetric fashion. We mean here electromagnetic waves, whose existence was soon experimentally confirmed. Since the times of Maxwell a lot of things have naturally changed. New interactions were found, that is weak and strong ones (what concerns the gravitational ones, we shall consider later). The former is responsible among other things for the so called β-decay, observed in nuclear physics whereas the latter for the keeping together of protons and neutrons (components of the nucleus). There are numerous attempts of relating these interactions somehow, and also to unify them with electromagnetic ones, or at least to find a unified description of them similar to one valid for the case of electromagnetic interactions. All these efforts remained futile for quite long. A lot of physicists started to even doubt, whether it could ever be achieved. It was even claimed that relating these interactions among themselves could amount to something like joining kilogram with meter (since the electric charge — a coupling constant of electromagnetic interactions and Fermi constant — characterising the weak ones — have different physical dimensions). Nonetheless it has turned out that the relating of them is possible, and it occurred due to the introduction of a new theoretical concept (i.e. Yang–Mills fields).

These fields, known also under the name of gauge fields, are very much like the electromagnetic fields, but are endowed with a richer structure, due to being associated with the so called non-Abelian (non-commutative) gauge groups. Roughly speaking they differ from the electromagnetic field in that they possess several types of “photons” (photon — a quantum of an electromagnetic field). These “photons” called intermediate bosons in the case of weak interactions (strictly speaking weak-electromagnetic ones) and gluons in the case of strong interactions (in the so called quantum chromodynamics — Q.C.D.), carry interactions in a manner similar to usual photons carrying electromagnetic interactions. A special mechanism called after Higgs endows masses to these intermediate bosons. Glashow–Salam–Weinberg (GSW) model, since it is the one which unified the electromagnetic and weak interactions, employs gauge fields with the group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, whereas quantum chromodynamics is using the $SU(3)_C$ group.

One has to note that the electromagnetic and weak interactions are far less tightly related then is the case of electricity and magnetism in electromagnetic forces (Maxwell theory). Strictly speaking there is no full unification within Glashow–Salam–Weinberg model, but only the mixing of both the interactions in a certain sense. Strong interactions are not here linked with the weak and electromagnetic ones, and a special theory was put forward for them — Quantum Chromodynamics. Instead we have unified description of interactions in terms of gauge fields — Yang–Mills ones (excluding gravitation). Presently attempts are being made in unifying strong, electromagnetic and weak interactions by postulating a wider gauge group, which would include as its subgroups: a group $\mathrm{SU}(3)_c$ and $SU(2)_L \times \mathrm{U}(1)_Y$. This would lead to the creation of theory capable of unifying all interactions (except gravitational one). Only employs at present groups like SU(5), SO(10), E6, E7, etc.

Concerning gravity, there are some attempts at joining it with electromagnetic interactions in the so called Kaluza–Klein theory. Of course the General Relativity Theory is used as theory of gravitation. It is also possible to join any gauge field theory with General Relativity in the form of a generalized Kaluza–Klein theory. Let us deal with the second problem, that is one of the existence of elementary particles.

In order to understand what is involved here, we shall employ several examples from solid state physics. As we know, any solid consists of a periodical lattice formed by atomic nuclei together with their outer electron shells. To the entire lattice belong also the valency electrons, more or less localized with respect to the nodes of it. Such a system is very well described by non-relativistic many body quantum mechanics. This is only a theoretical possibility of course, since the solution of such a many-body problem far exceeds our calculation capabilities. To that end we use some approximate methods. For example, we may factor lattice vibrations into its elementary components and quantize them. We obtain in this manner a system of bosons called phonons (phonon — a quant of acoustic energy, of lattice vibrations). This gives us in accordance with the second quantization the operators of phonon creation and annihilation. This allows us to view the interactions of electrons with lattice as the interaction of electrons with the phonon field.

Due to the fact that there are certain anharmonic couplings, phonons are capable of interacting between themselves. These new particles (phonons) are not real ones. Their existence follows as a result of the approximation adopted. They are quasi-particles in principle the electrons, when we consider them as interacting with the phonon field, are also quasi-particles. They have a mass different from usual one and a different wave function (it is the so called Bloch's function). In the solid state physics we often have to deal with quasi-particles. The so called holes in semiconductors are of this kind, Cooper's pairs (in superconductivity) and also the so called excitons, which represent bounded states of holes and electrons. The latter ones are much like hydrogen atoms. The so called Bogolyubov transformation is closely related to quasi-particles. In it two types of objects are related — particles and quasi-particles.

Quasi-particles are endowed with a certain physical realm. Namely, experiments confirm their "existence". We could experimentally observe excited states of excitons, exciton liquids, hole current, scatterings between electrons and phonons, etc. On the other hand we are well aware that a solid body consists of real particles-atomic nuclei and electrons, but not quasi-particles. The same situation might be related also with regard to elementary particles and a question asked: maybe they are also quasi-particles of same kind more basic structure similar to crystalline lattice. Perhaps it is a very intriguing question and in spite of its strangeness, it ceases to be perceived as such, once we recall that in the theory of black holes the Bogolyubov transformation relates particles e.g. photons in $-\infty$ time-like with particles in $+\infty$ time like (we have an event horizon). Due to this we obtain there the evaporation of "black holes" (Hawking effect).

The third problem of the philosophy of physics is the issue of its geometrization. This topic is to be discussed in the sequel. Let us notice some additional meaning of a geometrization of physics. First of all we have an application of symplectic geometry in a canonical formalism. There is also a geometrical approach by Carathéodory to phenomenological thermodynamics, e.g. to the second law of thermodynamics. Similarly it is possible to extend to irreversible thermodynamics by Onsager and Prigogine. The geometrical quantization approach in mechanics with some extension to field theory is also an example of geometrical methods in physics which we do not consider as geometrization of physics. Our meaning of geometrization of physics is as follows. It is a geometrization of fundamental physical interactions. This will be described later in this work and we give many examples of such a procedure.

We shall only state at this point that we mean here Einstein's programme of reducing all the physical interactions to geometry, in much the similar manner as it occurred with gravity in the General Relativity Theory. Let us note here that Yang–Mills fields, which had found their place in the unification of the fundamental interaction occur there as connections on fiber bundles, therefore behave like geometrical objects, and also note that Kaluza–Klein theory is a geometrical theory like General Relativity. As we might see, the task of geometrization of physics and that of interactions seem very much related.

Let us give the following remark. Geometry works in Physics via invariants in a way independent of a coordinate system. Moreover, in order to apply mathematical results in physical sciences we should choose a coordinate system and replace “invariants” by “variants”. Afterwards we can proceed calculations and compare our results with an experiment or an observation. This is a clear methodological remark on any application of Geometry in Physics, not only in our case of geometrization of fundamental physical interactions.

The fourth problem, that of the relation between cosmology and physics, could be stated as follows: are the properties of the Universe implied by our local physics or rather the other way round — does the global properties of the Universe determine how the local physics looks like? The first point of view claims that one ought to go from physics to cosmology, the second that from cosmology to physics. The above dilemma is also very intriguing due to the fact that the Universe, by definition, is unique.

Having completed the above tour of the viewpoints, let us inquire, how the philosopher of physics is able to inspire anybody towards the solution of these problems. Such a thought provoking postulate would undoubtedly arise out of identifying the first and the third problem. Another such postulate shall be constituted by a holistic viewpoint, which for the fourth problem would bring a reply: from cosmology to physics, whereas with respect to second problem — it should push us towards seeking fundamental structure, as quasi-particles within that structure. Certainly the scientists like A. Einstein, W. Heisenberg, C.-F. von Weizsäcker, etc. had been developing the kind of philosophy of physics described here. In such cases it would be hardly possible to discern the physicist from philosopher, since these scientists were both at the same time. They had combined the two roles in one person. In a certain sense therefore — they were self-motivating themselves.

In the work we will deal with all the questions touched upon thus far. We shall discuss briefly the problem of geometrization of physics. Geometrization of physics is a certain methodological doctrine which one might define as using of the geometrical methods in every such place within physics where possible. Such a definition from the outset quite vague and undetermined calls for a number of comments and precization. At the same time we will examine the examples of using geometrization of physics and which definitely not. To that goal let us return to Albert Einstein original contributions here, namely the General Relativity Theory and the so called Unitary Field Theory. Recalling briefly what was the most important on the road from Special Relativity Theory and the GRT. Namely, Special Relativity is the theory of a "space-time background", in which the Poincaré group plays the role of space-time symmetry group, Minkowski space endowed with Minkowski geometry is just the space-time of Special Relativity. Minkowski tensor is a principal geometrical invariant in this theory. Now, in accordance with Erlangen programme of F. Klein, this geometry constitutes a set of invariants of the Poincaré group and Minkowski tensor is to become a principal quantity occurring in this theory. Development of this theory consists of relating some geometrical quantities occurring in geometry of the Poincaré group with physical ones. In fact we shall be able to construct a four-tensor of electromagnetic field strength out of the strengths of electrical and magnetic fields. Energy and momentum shall serve for the construction of a four-momentum. The density of a charge and the density of a current shall constitute the current four-density. In a similar manner the notions of a four-force, four-velocity etc. should arise. According to the symmetry group selection postulate, we shall require that all these quantities should transform along a suitable representation of the Lorentz group. Therefore, we see that space-time geometry of Minkowski space-time constitutes a certain fixed, invariant background for the motions of bodies and fields. This is a setup characterizing also the mechanics of Newton and Aristotle. In these two cases also the geometry of space-time is a durable non-dynamical background. The only difference among these three systems of mechanics lies in the choice of space-time symmetry group, and consequently ends up in arriving at different geometrical invariants, which means various geometries — in accordance with F. Klein's programme. Undoubtedly it is a considerable difference. When seen in historical perspective it amounts to the abandoning of space-time concept, the notion of Universe and the overall world view. Let us recall that the transition from the Aristotelian to Newtonian mechanics has led to complete transformation of world-view by a Middle Age men. It has ruined the world order described e.g. in Dante's *Divine Comedy*, and therefore the foundations which were at the origin of that world-view. It was a really tremendous change, one of the greatest one in the history of human thought. One should not underestimate its impact by any means. Nonetheless from the standpoint of F. Klein's programme it is to be considered as "only" a change of space-time symmetry group, change of the background for mechanical movement. In short, it consisted of passing from the Aristotelian space-time with the group of rotations and absolute rest to a Galilean space-time with a Galileo group acting upon it. As a matter of fact, it amounted to the extending of a former group to a Galileo group. In this way we have admitted the possibility of mixing of space and time coordinates in a definite way. At the same time in this new theory (Galilean mechanics) there appeared the idea of an absolute space and of the absolute time with accompanying relativization of the notion of rest. Both Aristotle and Newton had attached great importance to absolute notions. Aristotle considered the absolute rest as a natural state of physical bodies and was linking that with his idea of four elements, four basic elements serving as building blocks of all earthly substances. In turn Newton has considered absolute time and absolute space to be sensorium of God. We therefore see that both notions of space-time were very closely connected with the metaphysics of both thinkers. The relationship between the mechanics of Aristotle and a general world-view of the ancient and Middle Age people is without any doubt natural. One could not have existed without the other, with the fall of a first one, the second has fall down as well. Clearly Aristotle has never expressed his theory in the form of geometrical axioms and equations. What we have been undertaking here is only a certain reconstructions of his theory in contemporary terms. Nonetheless it is very useful and interesting. It offers us the example of translating certain metaphysical postulates to mathematical language, namely the geometrical one. In the case of Newton we need not to do that. Newton has done it for us. He managed to translate his metaphysical views concerning time and space into the mathematic language. He expressed them in quantitative terms in the principles of dynamics. Afterwards it has turned out that in this manner he obtained a space-time with the Galileo group acting on it. Although Aristotle and Newton had dealt with different qualities involved in their theories, it turns out that they are of the same type. There are namely geometries serving as backgrounds for mechanics which were linked between themselves in a certain intriguing way. Once we introduce for comparison the Special Relativity Theory the situation will not change since we are going to obtain an absolute component again. It will be Minkowski space-time — an arena of all events. The geometry of this space-time will be of course different, because we have another symmetry group. This new symmetry group, an inhomogeneous Lorentz one, is a Poincaré group. It turns out that there is an interesting relationship among Lorentz group and the Galileo one. Namely, the Lie algebra of a Galilean group is a contraction of Lorentz group Lie algebra. This relation could be considered as a correspondence principal linking relativistic mechanics with the Newtonian one. This is an interesting fact since it enables us to avoid the not so well determined limit $c \to \infty$ (where c is the velocity of light in a vacuum) when performing this kind of transition between the corresponding theories. Let us note also that relativistic mechanics (Special Relativity Theory) decides a dispute between Leibniz and Newton — on behalf of the latter. Recall that this dispute dealt with the relativeness of space. Leibniz took the relativistic side, whereas

Newton advocated the concept of an absolute space. In relativistic mechanics there occurs an absolute notion of Minkowski space-time, which expresses the absoluteness of the geometry adopted here as a background. In spite of the parallelism between the names, Minkowski's idea of space contradicts that of Leibniz. Once this discussion of the geometrical meaning of different space-time from Aristotle to Einstein is over, let us return to our main topic — geometrization of physics. We are emphasizing at this point that all these notions are not to be taken as geometrization of physics in our sense. They constitute only mere application of certain geometrical methods. This is so because geometry as dealt with within these approaches is at most a background, not a dynamic variable. We might say here that in our approach to geometrization of physics, geometry cannot assume the role of a fixed, invariable background — the "invariant" of a theory. It has to be a dynamical quantity. It follows from such a postulate that we have to link it with some physical magnitudes, which would be subject to the variation. To put it in more exact terms, we shall require that some physical field corresponds to it. More specifically these should be field of the fundamental physical interactions.

Let us now start considering basic examples. There are a gravitational field in General Relativity Theory and the electromagnetic field. Let us notice that GRT is now a quite old, i.e. 100 years old, theory. In this way a relation between matter and geometry is also quite old. It is an established idea. General Relativity is a theory of a space-time and gravitational field. In this way according to Albert Einstein we can summarize the theory of relativity in one sentence: *Time and space and gravitation have no separate existence from matter.*

Albert Einstein geometrized the gravitational field by transferring from Special to General Relativity Theory. He has performed this in an usually simple and natural way. He postulates the geometry to be variable, making it into a dynamical quantity. Space-time metric tensor assumed the role of a dynamical quantity, ceased to be a "constant" of a theory. At the same time this tensor was linked to the gravitational field. The fact that the curvature of a Riemannian connection generated by this tensor was related to the existence of a non-vanishing gravitational field. Einstein's equations managed to link space-time geometry with masses, with non-gravitational fields. Such a relationship might be taken as the first case of equations, where on the left side occur geometrical quantities (Einstein tensor) whereas on the right hand side "material" ones (the energy-momentum tensor of matter). This is not to say that gravitation is "non-material". At this point our idea needs recapitulating.

The geometry taken as a dynamical quantity corresponds to a certain physical field-gravity. Sources of gravity are massive fields — the right hand sides of the Einstein equations. These have not been geometrized yet. The physics geometrization postulate requires obtaining new such equations, i.e. equations of the form: matter on the right hand side and “geometry” on the left. This idea of a unified geometrical field theory seems to be in the light of present state of knowledge correct. One has only to abandon this too rigorous a postulate, to the effect that all geometrical quantities be defined on a space-time. One has to define them on multi-dimensional (of more than four dimensions) manifolds. This type of a theory was known even to Einstein, it was the Kaluza–Klein theory. He was even able to contribute something of his own. Strictly speaking, the ultimate classical solution is due to him. Let us recall here that the Kaluza–Klein theory is a geometrical theory of the electromagnetic and gravitational fields. This theory satisfies the postulate of moving the right hand side of field equations as the geometrical quantities onto the left. This postulate is fulfilled for the electromagnetic field in a vacuum. Formally in classical formulation of this theory we obtain a five-dimensional counterpart of General Relativity Theory without sources. We shall obtain out of the equations contained in it Einstein’s equations for electromagnetic field with energy-momentum tensor in the vacuum and also Maxwell’s equations for the vacuum simultaneously. One has to strive for placing the maximum possible number of right-hand quantities to the left. This corresponds to the geometrization of quantities thus displaced. The maximal programme for the geometrization of physics would consist of finally arriving at a single equation (or system of equations). The right-hand side in this equation shall be zero, while on the left a certain geometric quantity. Such a left-hand side quantity shall depend on the geometry capable of describing all fundamental interactions. This would be a dream programme, Einstein’s dream. When it was put forward by Einstein, it had failed as we recall in its classical form. Namely, Einstein had required that all geometrical quantities be the ones determined over a space-time. Einstein’s Non-symmetric Field Theory, while it constituted the culmination of this idea, was doomed to fail. Currently it seems that this was an inevitable outcome. Space-time geometry (dealt with as a dynamical quantity) is too poor for explaining the electromagnetic fields of strong interactions and weak ones. Einstein had attempted to unify solely the electromagnetic and gravitational interactions. He failed even to envisage the existence of the weak and strong interactions. He was unable to foresee their occurrence. That is why his theory from the very beginning was incomplete.

Bianchi identities for a Riemannian connection defined on 5-dimensional manifold give us energy-momentum and charge conservation principles. This theory differs formally from General Relativity not only in the dimension of the manifold. Another difference being in that metric tensor on that manifold has also the Killing vector. The occurrence of this vector is connected with the fact that the fifth dimension could not be directly observable. To put it softly, no physical quantity could depend on the fifth coordinate. Nonetheless the fact of the occurrence of that dimension has serious consequences, since it is linked with gauge transformations of the four-potentials. The four-potentials themselves are related with certain original manifold's metric tensor's components. Generally speaking, the geometry of this manifold describes the electromagnetic field and the gravitational one while taking into account their gauge symmetries. This fact is quite interesting. However this theory has some drawbacks. They consist mainly in the fact that from its equations we obtain both Einstein and Maxwell equations in their classical form. We do not observe any interference effects between the gravitational and electromagnetic fields which were hitherto unknown. That is why Einstein was very skeptical to Kaluza–Klein theory. He objected against considering it as a unified theory of electromagnetic and gravitational fields. On the other hand there is Utiyama's theory of electromagnetic field, generally of a gauge field with any gauge group G. U(1) is a gauge group of an electromagnetic field. One might represent a theory of electromagnetic field in accordance with Utiyama's theory as a theory of principal bundle connection over the space-time with structural group U(1). In such a situation the gauge transformations turns out to be changes of sections of a bundle, one could link a four-potential with the connection, whereas the strength of the electromagnetic field with a connection's curvature. The minimal coupling scheme shall consist in substituting of all partial derivatives in the equations by covariant derivatives in the bundle's connection. The principal bundle, mentioned above, we shall be calling electromagnetic one. It turns out that there is a procedure linking Kaluza–Klein and Utiyama theories in a bundle formulation. Namely, Trautman and Tulczyjew had noticed that once the electromagnetic bundle is metrized in a natural manner, we obtain the metric tensor known from the Kaluza–Klein theory. This observation is very interesting and allows generalization to Yang–Mills' fields. Let us recall at this point that Yang–Mills fields are connections on principle bundles over the space-time endowed with a structural group G. Most often we assume that G is semisimple. In the case of a classical Yang–Mills field $G = \mathrm{SU}(2)$. At present it seems that Yang–Mills with non-Abelian gauge groups describe the fundamental physical interactions, that is weak and strong ones. It turns out that in this theory strong interactions have the so called colour gauge group, whereas the weakly electromagnetic ones within Glashow–Salam–Weinberg model have the group $\mathrm{SU}(2)_L \times U(1)_Y$. At present it is assumed that $SU(3)_C$ is the colour gauge group. Investigations are underway aimed in joining the strong electromagnetic and weak interactions into a unified theory of Glashow–Salam–Weinberg type of model for the weak and the electromagnetic interactions. On the other hand it is known that there is a natural link between gauge field theory with General Relativity Theory. In order to obtain such a relationship, one has to metrize the principal bundle and further to proceed as in Kaluza–Klein theory. Glashow–Salam–Weinberg model uses spontaneous symmetry breaking and the Higgs mechanism, the geometrization of which in a proper way was as yet unsuccessful, in spite of repeated attempts. Promising attempts include ideas requiring the application of nonsymmetric metric connections. In the case of GRT (dimension 4) we get the theory of Einstein-Cartan. In such a theory the spin of material fields serves as a source of torsion. The introducing of torsion into Kaluza–Klein theory supplies the geometrical interpretation of the electromagnetic polarization. Electromagnetic polarization becomes a source for a certain part of torsion. This theory might be generalized to the case of any gauge field. Once this is done the polarization of Yang–Mills becomes the source of torsion for higher dimensions. However in this type of theory, not all torsion components are involved. Probably they might be linked with the Higgs field and geometrize in this manner spontaneous symmetry breaking and Higgs mechanism.

Summing up we see that the problem of geometrization of fundamental interactions is open and it seems that Albert Einstein was right but had put forward this idea too early. At present, the geometrization of physics in that sense seems to be approaching completion due to the progress in the theory of elementary particles. At the same time there is a strong link between the geometrization of fundamental interactions and the idea of Unitary Field Theory which seems not to be accidental.

The possibility of geometrization of all the fundamental interactions together with their unification, emerging in recent years is presented in this work. These are precisely the gauge fields which have given rise to the establishment of a basic language, offering the description *modo geometrico* for the theories of fundamental interactions. Relationships between gravity — GRT, Einstein–Cartan theory and gauge theories has also been shown here. There is also a scheme presented for unifying two theories T1, T2 with supporting examples. These examples come from recent investigations dealing with the unification of gravity and Yang–Mills fields. Kaluza–Klein theory, which unifies gravitational interactions with the electromagnetic ones, plays here the role of a principal example.

The origins of supersymmetry and supergravity was presented in the work. Examples of applications were shown and analogies found with Yang–Mills type of theories. It was also demonstrated that these theories could be geometrized once extending the notion of geometrical structures to include anti-commuting coordinates (i.e. supermanifolds) is performed. The relationship between the supersymmetric and supergravitational formalism for fermion fields has been given in our proceedings. The fact that geometrization and unification of fundamental interaction's fields are strongly linked, as is the case for ordinary gauge theories was presented too.

The present work is also devoted to holism in contemporary physics and aims at proving that this research direction in spite of its origin stemming from biology remains very useful in physics because of its anti-reductionistic standpoint. According to holism, the "whole" is not reducible into "parts" that in a sense it precedes "part" and is more fundamental. In this manner, often the division of a "whole" into "parts" is a conventional one, is a question of convenience, constitutes a procedure without any counterpart in reality. Very often such a division is possible only in specific circumstances. For example, the division of interactions into weak and electromagnetic ones as it follows from recent investigations, turn out to be a question of mere convention.

The holistic programme in physics, which claims that elementary particles are solutions of the field equations, once abandoned for computational reasons, comes back today in a new form. It has taken the shape of a new version of a Nonlinear Unitary Field Theory linking together the programme of simultaneous geometrization and unifying of physics by employing the notion of soliton as a model for particle — the individuum.

We shall make an attempt at abolishing some myths rooted in the philosophy of physics, and show the real one, holistic picture of contemporary physics.

We shall be going also to link the physics geometrization programme with the holistic approach to physics. It is interesting here to note that Einstein's programme of the so called Nonsymmetric Field Theory is exactly the case for a holistic attitude towards physics. Contemporary programmes aimed at unifying the theory of elementary particles and of the fundamental interactions constitute exactly the continuation of the same general programme.

Finally this work has to prove that the holistic approach is most productive one in contemporary physics, and also that the holistic attitude itself implies the anticipation of such particular research directions, which for the time being could no be solved by strictly physical means.

An attitude of this type substantially influences the researcher's approach towards conducting investigations, the horizon of his research interest, causes even the usage of definite mathematical means. Here I mean employing methods of differential geometry (Kobayashi, Nomizu 1963), algebraic geometry and topology.

It seems that there are precisely these mathematical theories which are best capable of grasping the hiatus between "whole" and "part" in contemporary physics. This shall be particularly discernible in the subsequent chapters, where the possibility emerging in recent years, of geometrizing all fundamental interactions is being demonstrated. Let us give some details on ideas of geometrization of physics. The postulate of geometrization of physics is a methodological doctrine, which may be defined as the application of geometrical methods in physics whenever it is possible. This postulate, somehow vague and undefined in principle, requires certain remarks and definitions. Simultaneously we have to consider instances of using geometrical methods in physics and answer the question: *Which of them shall we take as geometrization of physics and which not?* In order to do this we turn back to the conception of Albert Einstein, i.e., to the general theory of relativity and to the so called unified field theory (Bergmann 1947; Thirring 1972; Tonnelat 1965). We recall, in short, the most fundamental items in transition from the Special Relativity Theory to the General Relativity Theory. The Special Relativity Theory is such a theory of "space-time background" in which the Poincaré group is a group of space-time symmetries. Minkowski's space with Minkowski's geometry is a space-time in the Special Relativity Theory. The fundamental geometrical invariant of this theory is Minkowski's tensor. According to F. Klein's programme this geometry is the set of invariants of the Poincaré group whereas Minkowski's tensor is a basic quantity appearing in the theory. The developing of the theory (Special Relativity Theory) consists in combining some appropriate geometrical quantities of the geometry of the Poincaré group with physical quantities. Namely we construct a four-dimensional tensor of strength of electromagnetic field from three-dimensional vectors of the strength of electric and magnetic fields. Energy and the three-dimensional vector of momentum serve to construct a four-dimensional momentum. Three-dimensional current density $\vec{j}$ and charge density ρ form a four-dimensional density of current j_μ. In the similar way we get the concepts of four-dimensional force, four-dimensional velocity, etc. According to the postulate of choice of an appropriate symmetry group (this postulate really means that laws are invariant with respect to the action of the group, i.e., they are symmetric) we require these quantities to transform according to an appropriate representation of this group, in this case the Poincaré or Lorentz group. Thus we see that geometry of space-time

(Minkowski's space-time) is a certain constant, unchanging background of bodies and fields. This situation is also characteristic for the mechanics of both Newton and Aristotle (Trautman 1970). In these two cases also space-time geometry is a constant non-dynamic background. The only difference among these three mechanics consists of a choice of space-time symmetry group. Consequently it consists of achievement of other geometrical invariants and thus another geometry according to F. Klein's programme. (We mention it earlier.) Undoubtedly the difference is significant. From a historical point of view it leads to a change in the conception of space-time, Universe, and general philosophy of nature. Let us recall that the shift from the Aristotelian mechanics to Newtonian caused a complete change in the philosophy of medieval man. It destroyed the order of the Universe described for instance in Dante's *Divine Comedy* (Alighieri 1317) and consequently the basis on which this philosophy was built. Indeed, this change was enormous, one of the greatest ones in the history of human thought. Its importance cannot be overestimated. However, from the point of view of F. Klein's programme it was "only" a change of group of space-time symmetry, a change of background on which mechanical movement took place. In short, this was a transition from Aristotelian space-time with absolute rest and rotation group O(3) to Galileo's space-time with the Galileo group (Trautman 1970). In fact, it was an extension: the previous group to Galileo's (O(3) is a subgroup of Galileo's group). Thus, we have admitted a possibility of mixing coordinates of space and time in a certain way. In Aristotle's mechanics it was impossible. We recall that Aristotle's space is finite and because of this, non-translationally invariant. Aristotle's time is not invariant with respect to time translations as well. At the same time, in this new theory (Galileo–Newton mechanics) a conception of absolute space and absolute time with the concept of relativized rest appeared. Both Aristotle and Newton paid much attention to absolute concepts. In absolute rest Aristotle saw a natural state of physical bodies and related this to his theory of the four elements, the four basic elements of all substances. Newton, on the other hand, regarded absolute time and absolute space as God's *sensorium.* So we see that both conceptions of space-time were very strongly connected with the metaphysics of the two thinkers. A connection between Aristotelian mechanics and the general philosophy of medieval and ancient people is obvious. One cannot exist without the other, and when one is destroyed the other must be destroyed as well. Of course, Aristotle never described his theory in geometrical language. What we do now is only a certain reconstruction of his theory in terms of geometry. It is interesting, however, in that it gives us an example of the translation of certain metaphysical postulates into a language of mathematics, more precisely, geometry. In Newton's case we do not have to do this, since Newton himself did it for us. He transformed his metaphysical concepts concerning space and time into a language of mathematics. He expressed them in quantitative form on the basis of principles of dynamics. Later it was revealed that he had achieved a space-time with the Galileo group as a symmetry group. Although Aristotle and Newton wrote about different absolute concepts, it is clear that both theories of space-time are of the same type. Namely, they are the geometries which are backgrounds for mechanical movements. The moment we introduce, for comparison also, the special theory of relativity (see Augustynek 1972, 1975, 1997 for a definition of a time by an abstraction), the situation does not change because again we get an absolute element. It is Minkowski space-time — i.e., the background of all events. The geometry of this space-time is of course different because a symmetry group changes. The new symmetry is the non-homogeneous Lorentz group — Poincaré group. It is interesting that there exists a connection between the Lorentz group and the Galileo group. Specifically, the Lie algebra of the Galileo group is a contraction of the Lie algebra of the Lorentz group. This connection may be regarded as the correspondence law connecting relativistic mechanics with Newtonian mechanics. We mention about it earlier. This is interesting since in such a way we do not have to do with a not quite well-defined transition $c \to \infty$ (c velocity of light in vacuum). Let us also notice that relativistic mechanics (the Special Relativity Theory) settles the dispute between Leibniz and Newton in favour of the latter. Let us recall, too, that the dispute concerned relativity of space. In this dispute Leibniz represented a relativistic standpoint. He believed that space is relativized with respect to bodies. Newton, on the other hand, maintained that space is absolute. In relativistic mechanics there appeared an absolute concept, Minkowski space-time being an expression of the absoluteness of geometry which is taken for background here. Thus there exists a parallelism, the conception of Minkowski space-time, which contradicts Leibniz's conception. We consider this problem below. Moreover, it is so important that we repeat some notions in a different context.

Following this discussion of the geometrical conception of various space-times from Aristotle (Aristotle 1954, 1984) to Einstein let us turn back to the subject i.e., geometrization of physics. Here we stress that all these conceptions are not geometrization of physics in our meaning. They are only applications of certain geometrical methods. This results from the fact that geometry in the discussed conceptions is only a background, and not a dynamic quantity. Now we can say that in our meaning of geometrization of physics, geometry cannot be a constant, unchanging background, a "constant" of the theory. It must be a "dynamical quantity". This postulate requires that we must, connect geometry with certain physical quantities.

More precisely we demand it to be connected with physical fields, the fields of fundamental physical interactions. Now, let us turn to some basic examples. They are: gravitational field in General Relativity Theory and electromagnetic field. Albert Einstein has performed a geometrization of a gravitational field passing from the special theory of relativity to the general theory of relativity. He did it in an unusually simple and natural way. He postulated inconstancy of geometry and made it a dynamic quantity. The metric tensor of space-time is no longer a "constant" of the theory. At the same time it is connected with the gravitational field. The non-zero curvature of the Riemannian connection generated by this tensor is connected with the existence of a non-vanishing gravitational field. Einstein's equations connect the geometry of space-time with masses and non-gravitational fields. On the left-hand side of the equations there are geometrical quantities (Einstein's tensor) and on the right-hand side material ones (energy-momentum tensor of matter). Obviously this does not mean that gravitation is "non-material".

Geometry as a dynamic quantity corresponds to a certain physical field — i.e., gravitation. The source of gravitation is other fields (the right-hand side of Einstein's equations). They have not been geometrized. The postulate of geometrization of physics now requires such equations, i.e., the equations of the type: "matter" on the right-hand side, "geometry" on the left. Simultaneously one should aim at shifting as many quantities as possible from the right to the left side. It corresponds to geometrization of the shifted quantities. A maximal programme of geometrization of physics consists in achieving one equation, or system of equations. In this equation we shall have zero on the right and geometrical quantities on the left. This quantity will depend on geometry describing all fundamental interactions. This is a dream programme, Einstein's dream. This programme raised by Einstein ended with fiasco in its classic form. Einstein required all geometrical quantities to be quantities defined on a space-time. Einstein's Nonsymmetric Field Theory, being the final form of this idea, ended up with the implication that weak and strong interactions would not occur. In light of the above arguments such a failure seems inevitable. Geometry of a space-time (regarded as a dynamic quantity) is too poor in order to be able to explain gravitational, electromagnetic, strong and weak interactions. Einstein tried to combine only electromagnetic and gravitational interactions, and did not even foresee the weak and strong interactions. He simply could not foresee them. That is why, from the very beginning, his theory was incomplete. However, it does not mean that his idea was wrong. This idea, i.e., the idea of unified, geometrical field theory, seems right in the light of contemporary knowledge. What one should do is to abandon the too rigorous postulate that all geometrical quantities are defined on a space-time. They should be defined on multi-dimensional (more than 4) manifold. This type of theory was known by Einstein himself. This was the theory of Kaluza–Klein (Bergmann 1947; Lichnerowicz 1965; Tonnelat 1965; Kaluza 1921; Rayski 1965). He even introduced his own contributions to it. In fact, the final classical formulation belongs to him (Bergmann 1947). Let us remember here that the Kaluza–Klein theory is the geometrical theory of gravitational and electromagnetic interactions. The theory satisfied the postulate of shifting a part of the right-hand side of field equations, as geometrical quantities, to the left. This postulate is satisfied for an electromagnetic field in vacuum. Formally, in the classical expression of this theory we got a five-dimensional analogue of the general theory of relativity without sources. From the equations for such an analogue we achieve Einstein's equations with a tensor of energy-momentum of electromagnetic field in a vacuum and Maxwell's equations in a vacuum. This theory differs formally from the general theory of relativity not only in the dimension of manifold, but also in the metric tensor of this manifold possessing Killing's vector. The existence of this vector is connected with the fact that the fifth dimension cannot be directly observed. In short, any physical observation cannot depend on the fifth coordinate. Nevertheless, the existence of this dimension has considerable consequences. It is connected with gauge transformation of electromagnetic potentials. The electromagnetic potentials are components of metric tensor of five-dimensional manifold. Generally speaking, the geometry of this manifold describes electromagnetic and gravitational fields. We mention it earlier. This is quite interesting. This theory, however, has some drawbacks, which are mainly that from the equations of this theory we achieve Einstein's equations and Maxwell's equations in the classic form. We do not get any "interference" effects between gravitational and electromagnetic fields which were not known before. Because of that, Einstein's attitude was skeptical about the Kaluza–Klein theory. He did not want to treat it as a unified theory of electromagnetic and gravitational fields. On the other hand it was known that the theory of Utiyama of electromagnetic field (Utiyama 1956), was more general than the theory of gauge group U(1). The theory of electromagnetic field, according to Utiyama's theory, may be described as a theory of a connection of principal fibre bundle over a space-time with structural group U(1) (Utiyama 1956). In such a case the gauge transformations are sections of the principal fibre bundle. The electromagnetic potentials are related to this connection and the strength of the electromagnetic field with a curvature of a connection (a curvature in the fifth dimension). The rule of minimal coupling of other fields with an electromagnetic field consists (in this language) in substituting of all partial derivatives in the equations by covariant derivatives with respect to a connection on principal fibre bundle.

The above-mentioned principal bundle is called the electromagnetic bundle. It is interesting that there exists a link between the Kaluza–Klein theory and Utiyama's theory (Utiyama 1956) in a fibre bundle formalism. In fact, Trautman and Tulczyjew have observed that if we metrize the electromagnetic bundle in a natural way (Trautman 1970), we achieve a metrical tensor known from the Kaluza–Klein theory. This is a very interesting observation. In fact, by generalization we can achieve a unified theory of Yang–Mills' field theory and gravitation (Kerner 1968; Cho 1975). Let us remember here that Yang–Mills' fields are connections on principal fibre bundles over a space-time with a structural group G. Most often we assume that G is semisimple. These fields are certain generalizations of electromagnetic field. At present it seems that Yang–Mills' fields with non-Abelian gauge groups describe fundamental physical interactions, i.e., weak and strong interactions (Kalinowski 1983). It appears that in this theory strong interactions are Yang–Mills' fields with colour gauge group $\mathrm{SU}(3)_C$. The weak electromagnetic interactions in the Glashow–Salam–Weinberg model are Yang–Mills' field with $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ gauge group. There are researches to unify strong, electromagnetic and weak interactions. Some attempts known from the literature use groups SU(5), SO(10), E6, E8, etc. On the other hand it is known that there exists a natural link between the theory of gauge field and the general theory of relativity. In order to get this we should metrize the principal bundle and then follow the Kaluza–Klein theory.

The classical Kaluza–Klein theory unifies two major concepts in physics: (1) local coordinate invariance; and (2) local gauge invariance. The first is a basic for General Relativity Theory and the second is fundamental for electrodynamics. The Kaluza–Klein theory reduces these two concepts to the first, but in more than a four-dimensional world. In the electromagnetic case we deal with a five-dimensional manifold.

Now we know the principle of a local gauge invariance is fundamental also for weak and strong interactions (Glashow–Salam–Weinberg model, Q.C.D.), but the gauge groups are non-Abelian. Thus it seems natural and important to generalize the Kaluza–Klein procedure from Abelian U(1) group to non-Abelian groups. It was done by Kerner and Cho and Freund. The authors work with the Riemann connection on a $(n+4)$-dimensional manifold. Einstein equations with the energy momentum tensor of Yang–Mills' fields and Yang–Mills' equations were obtained as a general results. Unfortunately these Einstein equations have a cosmological term and the cosmological constant is enormous, about $1/l_{pl}^2$ where $l_{pl} = \sqrt{G_N/c^3}$ is a Planck's length. This cosmological constant is 10^{127} times greater than the upper limit from observational data. It is a pity and one may suspect that the Kaluza–Klein approach failed. But there are other obstacles. In the classical Kaluza–Klein theory (five-dimensional) there are no "interference effects" between gravitational and electromagnetic fields, as I have written before. W. Pauli in 1933 said that electricity and gravity were separated like oil and water in this theory. This theory reproduces (in the five-dimensional case) the well-known Einstein and Maxwell equations. But one may obtain some gravitational-electromagnetical effects if one introduces spinor fields on a five-dimensional manifold and generalizes minimal coupling scheme. In this way we may obtain a new effect, i.e., dipole electric moment of fermion of value about 10^{-31} [cm]q, where q is an elementary charge. It is very well known that if a fermion has a dipole electric moment then PC symmetry breaking must take place. Thus we see that "interference effect" between gravitational and electromagnetic fields in the Kaluza–Klein theory framework violates time-reversal symmetry. PC symmetry breaking involves both shifting coordinates $[x, y, z]$ to $[-x, -y, -z]$ and replacing all particles with their anti-particles. The breaking of PC symmetry is equivalent to the breaking of time-reversal symmetry. This has fundamental philosophical consequences.

In the theory which unifies gravitational and electromagnetic interactions is a difference between future and past. It is due to this dipole electric moment of fermion. This is a very small but significant difference. It was done first by W. Thirring (Thirring 1972). Unfortunately Thirring's results were obtained at some price, namely, the existence of an unwanted minimal mass of fermion (of order $1\,\mu$g) — Planck's mass term. Summing up, one may say that this approach failed. But the general idea is beautiful and elegant and it would be very important to avoid all these troubles. The general way is as follows: to change geometry of $(n+4)$-dimensional manifold to cancel cosmological constant (Kopczyński 1978; Kalinowski 1981a, 1981b; Orzalesi & Pauri 1981; Kalinowski 1983, 1988). In order to cancel Planck's mass term in Dirac equation in the Kaluza–Klein theory one is forced to introduce a new kind of gauge derivative for spinor field (five-dimensional case) (Kalinowski 1981a, 1981b). This new gauge derivative induces a new connection on a five-dimensional manifold.

Recently there has been a significant progress in obtaining an upper limit on the EDM (Electric Dipole Moment) of an electron by using a polar molecule thorium monoxide (ThO) and ^{199}Hg (Baron et al. 2013; Heckel 2011; Raidal et al. 2008). The upper limit obtained is $|d_e| < 8.7 \cdot 10^{-29}$[cm]$q$ which is bigger of three orders of magnitude than the result from Kaluza–Klein theory ($d_{\mathrm{KK}} = -\frac{4\ell_{\mathrm{pl}}}{\sqrt{\alpha}} q \simeq -7.57 \times 10^{-32}$[cm]$q$). From the other side there is also a progress in calculation of SM prediction of EDM for an electron coming from phase δ_{CP} of CKM matrix. This calculation gives $d_e \sim 10^{-38}[\mathrm{cm}]q$ which is still smaller of six orders of magnitude than the result from Kaluza–Klein Theory (Booth 1993; Pospela & Ritz 2013).

Let us consider the following problem. What would it mean for Physics if someone measured an EDM for an electron of the value $d_{\mathrm{KK}} = -\frac{4\ell_{\mathrm{pl}}}{\sqrt{\alpha}}q$ as predicted by Kaluza–Klein Theory? It would mean the fifth dimension is a reality in the sense of 5-dimensional Minkowski space.

An experiment which measures such a quantity strongly supports the idea of rotations around the fifth axis in this space (the fifth dimension is a space-like). This EDM exists only due to these rotations. Otherwise spinor fields couple to ordinary connection and there is not a new effect. Even $\underline{P}$ (a bundle manifold) is a 5-dimensional manifold, the additional fifth dimension is not necessarily of the same nature as the remaining four dimensions, in particular three space dimensions. This dimension is a gauge dimension connected to an electromagnetic field. Moreover, we can develop this theory using Yang–Mills' fields and also Higgs' fields using dimensional reduction procedure, expecting some additional effects. It means we can expect something as "travelling" along additional dimensions. This perspective would have a tremendous importance for Physics and Technology. Simultaneously an existence of an EDM of an electron has also very great impact on our understanding of PC and T symmetries breaking. This is also very important. Thus a mentioned measurement, with an answer: Yes, would have very important physical, technological and philosophical consequences.

It is very easy to generalize this connection to the $(n + 4)$-dimensional case. One may ask what about cosmological constant for such a Kaluza–Klein theory. The answer is, it vanishes. Thus we avoid two basic troubles: enormous cosmological constant in Einstein equations and Planck's mass term in Dirac equation. Simultaneously we get some "interference effects" between gravitational and gauge fields (electromagnetic or Yang–Mills).

The Glashow–Salam–Weinberg model uses a spontaneous symmetry breaking and Higgs mechanism (Albers & Lee 1973). It is very interesting that Higgs' fields and Higgs' mechanism may be geometrized by introducing a large gauge group (Forgács & Manton 1980; Mayer 1981; Manton 1979). In the case of the Glashow–Salam–Weinberg model we must extend $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ to G2 and extend the four-dimensional manifold (space-time) to six-dimensional space. In this case, the base space is a six-dimensional (Manton 1979) manifold and we have fibre bundle with structural group G2 over this manifold. In this way we obtain Higgs' fields from the Glashow–Salam–Weinberg model as a part of gauge fields. Thus we have the bosonic sector of the Glashow–Salam–Weinberg model with correct Weinberg angle value. There are some troubles with fermionic sector of this model in geometrical language, but maybe supersymmetry and geometrization of the sector. It seems that supergravity will help us in a geometrical theories of the Kaluza–Klein type are able to describe a unification of fundamental interactions in a geometrical manner. This scheme of unifying fundamental interactions is very exciting and worth development. From a philosophical point of view geometrization and unification are very important. Perhaps it is the greatest problem of human thought. It seems that the geometrization and unification in the Kaluza–Klein theory framework is similar (not only in spirit) to philosophical systems of greater philosophers from the medieval East: Ibn Sina, Ibn Rushd and Moses Maimoun.

Summing up, we see that the problem of geometrization of fundamental interactions is open and it seems that Albert Einstein was on a right track but he formulated his theory too soon. At present the idea of geometrization of physics in this sense seems to be close to realization due to the progress in the theory of elementary particles. The simultaneous geometrization of fundamental interaction and the idea of a unified field theory is therefore not accidental. The geometrization of physics may represent a necessary psychological precondition for the development of a non-trivial unification. Let us repeat some important points. The Einstein programme consists in a geometrization of physics. In such an approach geometry is a dynamical quantity. It is connected to physical fields, the fields of fundamental physical interactions. In the case of General Relativity or other (alternative) they of gravitation we have on the left-hand side or the equations geometrical quantities (it is an Einstein tensor), on the right hand side we have some non-geometrical quantities. Thus we get "matter" on the right hand side, "geometry" on the left. Simultaneously one should aim at shifting as many quantities as possible from the right to the left side. It corresponds to the geometrization of the shifted quantities. A maximal programme of geometrization of physics consists in achieving an equation (a system of equations). In such an equation we should have zero on the right and geometrical quantities on the left. These quantities will depend on geometry describing all fundamental interactions. This is a dream programme, Einstein's dream. This programme raised by Einstein ended with fiasco in its classic form. Namely, Einstein required all geometrical quantities defined on a space-time (Hlavatý 1957; Einstein 1955; Tonnelat 1966). Einstein's Nonsymmetric Field Theory, being the final from of this idea, ended with being forced to conclude that weak and strong interactions would not occur. In light of the above arguments, such as a failure seems inevitable. Geometry of the space-time (regarded as a dynamical quantity) is too poor in order to be able to explain gravitational, electromagnetic, weak, strong and the (possible) fifth force. The reinterpretation of this theory as a theory of gravity seems to be right. What one should do is to abandon the too rigorous postulate that all geometrical quantities are defined on a space-time. They should be defined on multi-dimensional (more than 4) manifolds. Such a theory is a generalized Kaluza–Klein Theory. The theory satisfies the postulate of shifting a part of the right hand-side of field equations, as geometrical quantities to the left. This postulate is satisfied for an electromagnetic field, non-Abelian Yang-Mill's field, Higgs' field. Thus it can geometrize a bosonic sector of electromagnetic, weak and strong interactions. In order to geometrize the fermionic sector it is necessary to employ supergravity or supersymmetry — like theories. Such a theory due to possible miraculous cancellations of ultraviolet divergences would be finite in perturbation calculus in Quantum Field Theory. The programme of this investigation seems to be quite unambigous. The only one ambiguity is connected to groups in the theory. They could be fixed by a consistency (anomaly cancellations). It seems that our model (Nonsymmetric Kaluza–Klein Theory) would be more unambigous than superstrings models. We mean here ambiguities connected to a spontaneous compactification of a superstring. The existing super Kaluza–Klein theory could help us in this programme similarly as superspace generalization of NGT. In this place we should give a remark on geometrical view of physics of fundamental interactions. *Nature likes theories that are simple when stated in coordinate-free, geometrical language.* According to this principle, Nature must love Kaluza–Klein Theory and hate phenomenological theories.

In philosophy of physics we should consider a problem of an existence. Let us notice the following fact concerning an existence. What does it mean "to exist" and is it possible to prove that "something exists". This problem has been raised by A. Husserl and R. Ingarden (Ingarden 1947–1974, 1992). This is the famous *dispute* (*controversy*) *on the existence of the world* in the philosophical (realistic) phenomenology (this is not the phenomenology from physical theories). The famous dispute (controversy) is not finished. Moreover, R. Ingarden was forced to consider not so ambitious problem. He wanted to prove only that "something exists" (not the whole world). In this way he turns his interests to philosophical aesthetics. Roughly speaking, in order to find some important differences between something which "really exists" and something which "does not really exist" he considers a hero of a novel (a literary work). R. Ingarden developed in very great details an analysis of a literary work, which is interesting for itself (Ingarden 1977). Moreover, he developed afterwards a philosophical phenomenology in order to describe the reality. This description happens to be purely classical (as it was pointed out by A. Szczepański). Quantum mechanics is beyond this description. In this way the philosophical phenomenology by Husserl and Ingarden cannot be a philosophical foundation of the physical world. This is really a pity because such a philosophy is very promising in any different domains of human activity. Maybe some descenders of R. Ingarden would be able to extend philosophical phenomenology to quantum world. Moreover, in our investigations we will consider (without any ambitions to prove an existence) three types of beings:

1. beings with a space-time existence,
2. beings with only a time existence,
3. beings without a space-time existence.

According to W. Krajewski beings of the first type are physical objects, beings of the second type are psychological impressions, and beings of the third type are mathematical or logical notions. In this way to be a physical object is to be extended in a space and to exist for some time. In further considerations we are interested in geometrization and unification of physical interactions with holistic aspects and space-time beings can be considered. Psychological impressions can be considered as physical processes in a brain. Mathematical and logical notions (especially geometrical objects) are behind physical interactions (it means also behind physical objects). According to our philosophy there is only a geometry behind a matter (a space-time existence). It would be a formidable task to connect R. Ingarden's phenomenology to our geometrization and unification of fundamental physical interactions with holistic aspects. In this way we could get a connection of a philosophical system with an ancient Greek idea of an *arche* (here a geometry) to very modern ideas of philosophical (realistic) phenomenology as in R. Ingarden's works: *Controversy on an existence of the world* (in Polish, vol. I, II (parts I, II) and vol. III). (We should remember A. Szczepański's criticism.)

Let us sketch some important elements of R. Ingarden's phenomenology. According to him, philosophy is divided into ontology and metaphysics. In our case philosophy of nature is also divided into its own ontology and metaphysics. Every being is a triple unity of matter (content), form (of the matter) and existence (in a certain mode). In this way ontology (a theory of beings) is divided into material, formal and existential ontology. R. Ingarden considers four spheres of beings: absolute (supratemporal), ideal (timeless), real (temporal). He considers the following modes of existence: real, ideal, intentional and absolute. The physical world is of course real. Moreover, it is possible that behind the real world there are some different worlds. Philosophical phenomenology consists in an analysis without any assumptions. It means, we should reject any assumptions on the world and start an analysis from obvious sentences. Further discussion on philosophical realistic phenomenology is beyond the scope of this essay.

The work has been divided into fifteen chapters. The field theoretic approach in physics and its relation to geometry is shown in the first one. The second chapter deals with the problem of classical electrodynamics when seen as the task of geometrizing and unifying of electrical and magnetic interactions. The problem of geometrization of physics is being considered in the third chapter on the example of General Relativity Theory (GRT). The fourth chapter touches on the interactions in the light of quantum chromodynamics (QCD), gauge fields, Glashow–Salam–Weinberg model and the unification of all physical interactions. Extensive elaboration was devoted here to topics such as: theory of Kaluza–Klein type, nonsymmetric field theory, spontaneous symmetry breaking etc. In the fifth and sixth chapters, we write about the relationships between geometrization of physical interactions and the dialectical notion of matter and ideas proclaimed by Latin Averroists. In the seventh chapter we consider a geometrizability criterion as a practical one; also some thoughts about the very notion of praxis in physics are being offered. P. Feyerabend's ideas and the topic of geometrizing physics are taken up in the eight chapter. The ninth chapter reflects upon problems concerning symmetry in theory of elementary particles and also the attempts aims at linking of the internal and space-time symmetries. The tenth chapter deals with hidden symmetries and the supersymmetric algebras. Also the theory of strings is touched upon. Anti-commuting coordinates and Lie supergroups enter the scene in the eleventh chapter, also the supermanifolds are being considered, while in the twelfth our attention is focused on the problem of supersymmetric gauge transformations and supersymmetric gauge fields, The topics of supergravity and supersymmetric (supergravitational) extension of the Kaluza–Klein theory conclude the twelfth chapter. Chapter thirteen is devoted to the relationships between holism and reductionism in contemporary physics. Holism in physics of nonlinear phenomena and in cosmology constitutes the subject of the fourteenth chapter, followed by a summary of the whole work sketched against the broader background in the history of physics as well as the history of philosophy in the last chapter.

We give in the work four notes concerning Laplace's cosmic mind, Nonsymmetric Kaluza–Klein theory, first philosophers and reduction of all natural sciences to SM (Standard Model) physics.

We have abstained from citing of the original bibliographic references since due to their sheer number doing so would take more space than the whole work offered here. Except some sporadic original papers only references to available collections of original papers and to monographs were retained in order to keep a form of an essay.

In this essay we use the notion of a Higgs' mechanism or a Higgs–Kibble mechanism. Moreover, there are more researchers who had important ingredients in the discovery of this mechanism. The full name of the mechanism should be as follows: Anderson–Brout–Guralnik–Hagen–Higgs–Kibble–t'Hooft (ABEGHH'tH)–mechanism (the notion proposed by P. Higgs). Moreover, we add also Y. Nambu, L. D. Landau, N. N. Bogolyubov and maybe more.

1 Field Theoretic Worldview and Geometry

The question of the relationship between field theory and geometry is being considered in this chapter.

In a physical world there occur four types of interactions: gravitational, electromagnetic, weak and strong ones. Recently there is also the possibility for the existence of the fifth type of interaction being mentioned; this one would be very weak and capable of modifying the gravitational one. We shall also comment here on some problems connected with the fifth force. Gravitational interaction is the most wide-spread in nature and the most universal. It is responsible for the fall of a stone onto the earth as well as for the cosmological properties of the Universe. Celestial mechanics, black holes, neutron stars — these are but a few of the forms of gravity.

Electromagnetic interactions are also very wide-spread; they govern a lot of physical phenomena. They are responsible for chemical bonds, plasma properties, light properties etc. All applications of electric current, that is electronics, electrical motors etc. constitute the forms of appearance of electromagnetic interactions. The weak and strong interactions reveal themselves in the domains of nuclear physics and elementary particles, they govern e.g. beta decay, are responsible for the fact that nucleons inside the nucleus are being kept together. All these interactions are known in the so called low energy area, that is within the energy area accessible without the recourse of special machines, which serve the purpose of accelerating elementary particles (accelerators). Energy ranges accessible with the help of accelerators are being called middle and high energy ranges. Within these ranges and especially in the range of high energies the physicists envisage lot of new interactions, which cause effects observable within that domain of energies. Sometimes some of these predictions might have great importance in the description of the early phases of the Universe and could serve for the creation of new cosmological models. These model in turn might be tested on the basis of observations. In general, however, the predictions of new interactions within the high energy ranges do not have any influence onto the low energy physics and also onto our everyday's life. It would be utmost interest to find some traces of these high-energetic interactions in our everyday's life, and it is quite certain that such an interaction would be called the fifth one or the fifth force.

There is only one question — what phenomena would be revealing this new interaction. Now, it is the fact that for some time already, reports about this types of interactions appear in the low-energy physics and even in the everyday physics. One type of such a report which had focused the attention of physicists all over the world was one by Fischbach and collaborators about the reexamining of data from a famous experiment by Eötvös, Pekar and Fékéte. Let us recall that in this experiment the equivalence of inertial and gravitational mass has been checked with very great precision, as well as the independence of the relationships between the inertial and gravitational mass from the chemical composition of the body. Already Newton has adopted the assumption about the equality of the two masses, and later it became the foundation of General Relativity Theory (GRT) (Wald 1984; Papapetrou 1974; Kopczyński & Trautman 1992; Roseveare 1982; Will 1981). To tell the truth, Galileo was already aware of the fact, when he mentioned that all bodies fall in a vacuum in the same way. In his times this claim constituted the abandoning of an out-dated view of Aristotle, which held that the heavier bodies fall quicker. E. Fischbach et al. maintain that the bodies do not fall equally in a vacuum, and that this depends on their chemical composition. It is supposedly following out from a reinterpretation of experimental data published by Eötvös already in 1922. The author named above has explained the original Eötvös experiment, while many others had tried to explain the same phenomenon.

Alternative theories of gravity (other than GRT) were called for the assistance and also traces of high-energy interactions were being spotted as likely explanations of this type of falling bodies behaviour. Simultaneously attempts are being made finding other appearances of this phenomenon by examining geophysical, oceanographical or astronomical data, or in other experiments. Others convinced as to the equivalence of inertial and gravitational mass keep trying to find other consistent interpretations of this historic experiment. The question of the fifth force as we see is still being scrutinized and discussed. Since the emergence of General Relativity Theory and Maxwellian electrodynamics, theories of the electromagnetic and gravitational interactions are available. The interactions are being carried across the physical field according to a scheme particle-field-particle. Therefore a particle interacts via the fields with another particle and becomes a source of the field. The mediating agent located between interacting particles, that is to say — field carries the interaction over with a finite speed, equal to the velocity of light. Until the creation of Maxwell's electrodynamics there were no physical theories which would describe the interactions among the bodies with the help of some mediating agent — the field.

Newton's theory of gravitation introduced the mysterious action at-a-distance. Coulomb interactions between the electric charges and magnetic masses are constructed similarly as in Newton's gravitation theory. There were Maxwell's equations which managed to make a breakthrough in the hitherto understanding of the physical interactions. They introduced a new material object, equally important as the charges — the electromagnetic field. This object acquired autonomy with respect to charges-sources of this field. A situation even become possible where the field alone would remain without sources. The equations admitted ondulatory solutions — electromagnetic waves and it turned out that light is just an electromagnetic wave. Already at Maxwell's times, repeated attempts had been undertaken towards the creation of a similar field theory for gravitation. Maxwell himself undertook some unsuccessful attempts. It turned out that the gravitational field is not easily amenable for this type of procedures. A certain intermediate step was necessary which failed to occur; namely the creation of Special Relativity Theory. Maxwell's equations possess certain internal symmetries which while transforming among themselves the fields, left the form of the equation invariant. In addition to that Maxwell's equations are not invariant with respect to Galileo transformation. Attempts at supplementing the Maxwell's equations with additional terms in such manner that they become invariant with respect to Galileo transformation had inevitably led to contradiction with experimental evidence.

On the other hand, it was known that Newton's equations are invariant with respect to this group. The study of symmetries of Maxwell's equations led to discovery of special Lorentz transformation (instead of special Galilean transformation) and Lorentz group (instead of Galileo group). Later it was found that the relationship between Galileo group and Lorentz one is very interesting. Namely Lie algebra of Galileo group is a contraction of Lie algebra of Lorentz group. In a sense one might consider this as an example of correspondence between the relativistic mechanics and Newtonian one expressed in a very precise language dealing with symmetries of Newtonian space-time (Galilean) and relativistic one. We shall refer in the sequel to the relationship between different theories of space-time and of gravity. The discovery of special Lorentz transform and Lorentz group as a space-time symmetry group provided direct impulse for the revision of views about the space-time nature, and later on has led to the creation of relativistic mechanics. I would like to stress at this point that it is not without any support that I am using here the notion of Newton or Galileo space-time and of its geometry because is in itself interesting and throws light onto the correspondence between the subsequent mechanics from Aristotle to Einstein. Perhaps it shall allow us to understand the complicated paths which lead towards still better — more adequate models of Nature. In fact it is possible to establish interesting links between the space-times of Aristotle, Newton, Einstein or Weyl.

We shall write more about these and many others interesting relationships, connected with this problem, but for a while we shall devote to it the following comments. Namely a widely spread conviction that with the advent of Special Relativity Theory *the absolute time and absolute space had gone* and that *new notion, that of relativistic space-time continuum was born* — is false. I would like to emphasize the fact that although undoubtedly the abandonment of Newtonian views about absolute space and absolute time, which would exist independently and originate out of itself has really taken place (Newton even called them *God's sensorium*). It does not signify the birth of some brand new notion. We could speak about the Newtonian space-time, having in mind the space-time endowed with a geometry different from that characterizing the Minkowski space-time. Within that geometry, some privileged geometrical objects shall exists — absolute time and a space — like hypersurface — the absolute space. Of course in such a space-time, Galileo group should act. Nonetheless Newton's space-time and Minkowski's one shall have the same absolute character, in spite of their different geometrical structure.

They shall play the role of a background, fixed and invariant for the events taking place. In both the above named cases it shall be different background, but anyhow — a background. Here it is worth stressing that the Newton's standpoint in his dispute with Leibniz by offering new evidence on his behalf — that of an absolute space-time. Let us only recall that in this dispute Newton defended (in Clark's spelling) the absolute character of time and space as a background for events — whereas Leibniz held the view about he relative nature of space. Namely is was to be a "material" space associated with a given body and moving together with it, and as such existing only in connection to the said body. There were repeated attempts aimed at advancing a paradoxical claim to the effect that supposedly Special Relativity Theory would confirm Leibniz position overturning the idea of an absolute nature of space. Of course it is true that we abandon the notion of an absolute space, but it is not at all to say that we thus accept Leibniz view, since we are introducing an equally absolute concept — that of a relativistic space-time. Now, recalling that there is something of a kind as Newton's space-time, we are able to see that there occurs a passage from one absolute space-time to another, with a due change of the geometrical structure from a Galilean to that of Minkowski.

The point of this fact seems to me very important since in the theories of the type like Aristotelian, Galileo, Minkowski or conformal mechanics, the space-time geometry is absolute and invariable. It is not a dynamical variable, one cannot associate with it interactions of any kind. Therefore there arises the contradiction of a sort between the background space-time and the matter which fills it. There arises a need to formalize the space-time, associating its structure with physical interactions. This is only possible within the General Relativity Theory which turns the geometry of space-time into a dynamical quantity and thus associated it with gravity. Let us note that the Newton's space-time with "non-flat" geometry might be taken for an analogy of General Relativity Theory with regard to Galileo's space-time and is interesting enough — a Newtonian gravitational theory. Let us return to Maxwell's electrodynamics. As we have said above, Maxwell's equations turned out to be invariant with respect to Lorentz group acting in Minkowski space-time. It turned out also that one might connect certain physical quantities to tensors or vectors defined on the Minkowski space-time. Namely, electromagnetic fields by making first the so called "four-quantity" defined on the space-time and capable of covariant transformation upon changing of the system of coordinates. It was possible to construct in a similar manner out of current and the charge a four-current and the other relativistic quantities. It was also possible to build the invariants, scalars — quantities which remained invariant upon changes to the system of four-coordinates, the so called electromagnetic field invariants. These invariants of course are invariants of Lorentz group and would loose that property once this group would be extended.

The Maxwell equations in a vacuum have a wider symmetry group than Lorentz group — it is the so called the conformal group. It is a very intriguing property and it enables associating with this group of a new mechanics, new space-time — the conformal one. It was already done and the resulting one is analogous in its appearance to Galileo and Minkowski space-time. Returning now upon completion of this digression to the main thread of our reasoning, let us note that the development of Einstein's mechanics leads to the construction of four-quantities on Minkowski space-time, which is endowed with a constant — invariable geometry generated by Minkowski tensor and Lorentz symmetry group. In accordance with Klein's programme of studying geometry with the aid of groups, one might say that the geometry of Minkowski space constitutes a set of invariants of the Lorentz group and more precisely — Poincaré group. By analogy we may refer also to the geometry of Galileo, Aristotle and the conformal one. Let us emphasize at this point one very important moment on the passage between Maxwell's electrodynamics in a traditional vector rendering (with the aid of field strength and current vectors) and the very same electrodynamics expressed in terms of the Minkowski space formalism. Namely we had constructed the electromagnetic field strength tensor defined on the space-time — with the help of electric and magnetic field vectors (usual ones) in a three-dimensional space in a certain coordinate system. The transformations of electric and magnetic fields upon changing of the coordinate system turned out to be equivalent to a covariant transformation of an antisymmetric electromagnetic field strength's four-tensor.

We have constructed a four-dimensional quantity out of the three-dimensional ones, which had turned out to be a geometrical one (in Schouten's sense) on a four-dimensional space-time. We shall employ this kind of a procedure several times more in the sequel, therefore it needs emphasizing and term it is a crucial one. To further narration we could say that this kind of procedure shall appear in the context of geometrization of physical interactions during the associating of them with Yang–Mills field theories, with connections on multidimensional differential manifolds and also during the different attempts aimed at the creation of a theory unifying physical interactions. In the sequel we shall try to describe a general scheme of this procedure which turns out to be at the same time the unifying and geometrizing one.

2 Classical Electrodynamics as an Example of the Unification of the Electric and Magnetic Interactions

This chapter explains the geometrization of electric and magnetic interactions on the example of classical electrodynamics.

Let us return to Maxwell's theory casted in the language of Minkowski space geometry, that is with the aid of two electromagnetic field tensors $F_{\alpha\beta}$, $H_{\alpha\beta}$ and the current density four-vector j^μ. According to what was said until now, the tensor $F_{\alpha\beta}$ shall decompose upon the adoption of a certain system of coordinates into the three-dimensional vectors $\vec{E}$, $\vec{B}$, the tensor $H_{\alpha\beta}$ into $\vec{D}$, $\vec{H}$, whereas the four-vector j^μ could be expressed by the current density $\vec{j}$ and charge density ρ. From the equations

$$\begin{aligned} \partial_{[\lambda} F_{\mu\nu]} &= 0 \\ \partial_\mu H^\mu{}_\nu &= 4\pi j_\nu \end{aligned} \tag{2.1}$$

we derive as a corollary the continuity equation of j^ν, $\partial_\nu j^\nu = 0$, that is electrical charge conservation law. This is interesting since as we shall notice in the sequel, it is no accident that the conservation law is a corollary from the second pair of Maxwell equations. More generally — conservation law from the equations of motion for the fields. From the differential geometry point of view, it is simply Bianchi identity for a certain connection, the topic to be dealt in the sequel.

There are material relations between the quantities $F_{\alpha\beta}$, $H_{\alpha\beta}$ when we deal with relativistic electrodynamics of continuous media, or there is a deeper one, once we pass to the nonlinear non-Maxwellian electrodynamics. There, the relationships between $F_{\alpha\beta}$, $H_{\alpha\beta}$ are more complicated and involve the nonlinear electrodynamical Lagrangian $L(S, P)$ (Plebański 1970) where

$$\begin{aligned} S &= \frac{1}{4}\, F_{\mu\nu} F^{\mu\nu} \\ \overset{*}{F}_{\mu\nu} &= \frac{1}{2}\, \varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho} \\ P &= -\frac{1}{4}\, F_{\mu\nu} \overset{*}{F}{}^{\mu\nu} \end{aligned} \tag{2.2}$$

where $\varepsilon^{\mu\nu\lambda\rho}$ is a Levi-Civita antisymmetric symbol, $e^{1234} = 1$.

There are lot of interesting attempts at generalizing the electrodynamics onto the nonlinear cases. The most successful one being that of Born–Infeld;

nonetheless the experiment is not able to detect any significant nonlinear effects which would be associated with classical electrodynamics (not a quantum one). In addition to this, nonlinear electrodynamics has no conformal symmetry (even in a sourceless case). In Maxwellian case, to which we shall be limiting for a while, we have:

$$F_{\alpha\beta} = H_{\alpha\beta} \tag{2.3}$$

The equations of Maxwell's electrodynamics written above admit some interesting and very deep symmetries called electromagnetic field gauge symmetries. Namely it turns out that one might introduce the quantity A_μ — the four-potential in such a manner that

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.4}$$

and to perform the transformations

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \chi. \tag{2.5}$$

These transformations constitute just the electromagnetic gauge transformations. As a result of this transformation, Maxwell equations do not change. The quantity A_μ could only be determined up to the choice of gauge. It seems therefore that this quantity does not have any physical meaning. Its existence follows from the first pair of Maxwell equations, which might be written by using the dual tensor:

$$\partial^\mu \overset{*}{F}_{\mu\nu} = 0, \quad \partial^\mu F_{\mu\nu} = 4\pi j_\nu. \tag{2.6}$$

Let us note here that there is a certain asymmetry between the two "pairs" of Maxwell equations, the first pair is sourceless. Its sourcelessness — lack of magnetic charges — constitutes the immediate reason for the existence of the four-potential. There is also possible the theory of magnetic monopoles within the Maxwellian theory, that is — theory of Dirac monopoles with a discontinuous four-potential.

Let us return to Maxwell equations in the form just written. There are sources j^μ for them, which might be expressible via complex scalar fields, spinor ones, etc., carrying charges. These fields couple themselves with the electromagnetic field on the basis of the least coupling. We could also consider these fields, their Lagrangians, in separation from the electromagnetic field. It would turn out then that their Lagrangians are invariant with respect to gauge transformations of the first kind, that is

$$\begin{aligned} \Phi &\to \Phi' = \Phi e^{-i\chi} \\ \Phi^* &\to \Phi'^* = \Phi^* e^{-i\chi}, \end{aligned} \tag{2.7}$$

the so called phase transformation, $x = \text{const}$. In accordance to E. Noether theorem we could construct the quantity j^μ fulfilling the continuity equation. It is just the principle of charge conservation. Should we however require $\chi(x)$ to be a function of a point in the space-time (the so called rotations with point-dependent phase) we shall introduce the compensating field A_μ, which would covariantly transform in such a manner, as the electromagnetic field potential would do. This field should be coupled to field Φ in conformance with the minimal coupling scheme. Maxwell equations — with sources stemming from the field Φ — would follow from the variational principle. Simultaneously these equations shall be invariant with respect to the transformation

$$\begin{aligned} \Phi \to \Phi' &= \Phi e^{-i\chi} \\ \Phi^* \to \Phi'^* &= \Phi^* e^{i\chi} \\ A_\mu \to A'_\mu &= A_\mu + \partial_\mu \chi, \end{aligned} \tag{2.8}$$

called gauge transformations of the second kind, or transformations with point-dependent phase. Let us notice here the characteristic fact: starting with the theory of a charged field which was invariant with respect to phase transformation (gauge of the first kind) and assuming the dependence of phase of the point in space-time, we end up with the notion of a compensating field associated with the gauge transformations of the second kind. This field is responsible for the interactions of the charges, whose conservation law is a corollary via E. Noether theorem from the invariance with respect to the gauge transformations of the first kind. The passage from the first kind gauge transformation to second kind one, could be described physically as the expression of the fact that the (conserved) charges become the sources of the field strength lines. This field is a compensating one, in this case the electromagnetic field. Let us notice that it is not always possible to extend transformations of the first kind to those of the second kind as was the case for electric charge. It turns out that for the case of baryon charge (we have here the phase of baryon transformation) one cannot introduce the gauge field similar to that of an electromagnetic one, for this simple reason that these fields do not interact among themselves. There is no compensating field which might be associated with the baryon charge. This claim is being recently questioned and one introduces the gauges of the second kind for baryon or hyperion charges and hence gets a new compensating field of the electromagnetic type. This is related to the so called fifth force (mentioned above). Similar considerations apply to isospin, strangeness, lepton charge etc. With such quantities of the charge type there are associated gauge transformations of the first kind leading to symmetries which are termed as global ones. To put it from a historical point of view, there were attempts at associating the gauge field (compensating one) with isospin. By the way, it was the first case of introducing into physics the gauge fields with non-Abelian gauge groups. After the names of their creators we call them Yang–Mills fields. This was an unsuccessful attempt because at that time the role of isospin within the theory of elementary particles was not yet understood.

We are aware today that the group SU(2) is not to be linked with any single compensational interaction. This is a group of the first kind (phase) gauge transformation appearing only due to the fact that there are two quarks "u" and "d" where difference of mass is very small. The relation between occurring in the physics of elementary particles groups of global and local symmetries is not completely clear. Let us return to the gauge transform of the second kind of the charged field and of the electromagnetic one, decoupled to it and compensating the phase change. We shall try to find the geometrical sense of these transformations and also the geometrical meaning of the electromagnetic field's four-potentials.

It turns out the theory of electromagnetic field interacting with the charged fields has a clear and simple geometrical form, given the theory of fibre bundles as our mathematical tool. Within this approach the electromagnetic field turns out to be a connection of the principle fibre bundle over the space-time with a structural group U(1). Under this picture the gauge transformations of the second kind turn out to be the changes of bundle cross-section. Taking the bundles associated with the electromagnetic bundle (e.g. a spinor one), we could (by a covariant derivative in the bundle connection) obtain the scheme of minimal coupling for the electromagnetic field.

All hitherto known properties of electromagnetic field shall be obtained. Namely from the connection we can get the potential form in any gauge. The bundle curvature form will turn out to be an electromagnetic field strength form. Introducing the covariant derivatives of the fields carrying the electric charge we will obtain the coupling between them and electromagnetism. The compensation field — the electromagnetic one will appear to us as non-integrable bundle connection. This is an extremely interesting and intriguing picture. In fact, it turns out that certain quantities defined on the space-time such as $F_{\mu\nu}$, A_μ, Φ, Φ^* etc. become the geometrical ones — the curvature connection, the field on the bundle, provided we take into account not the space-time itself, but a bundle defined over the space-time. It calls for the introduction of an additional (the fifth one) gauge dimension connected with the U(1) group. The four-potential is a basic quantity as one associated with the connection, while the strength of an electromagnetic field is a derived quantity — the bundle curvature.

This theory of electromagnetism presented here cursorily bears the name of Utiyama since he was the first who had introduced it, emphasizing the role of gauging and the gauge derivative. The geometrical casting of Utiyama's idea is due to A. Trautman and W. Tulczyjew. At its core lies the fact that a certain physical quantity, the electromagnetic field has been substituted by a connection on a certain manifold — on the principal bundle. In this manner a geometrization of electromagnetic interactions has been achieved. Clearly, should our aim be obtaining Maxwell's theory, we have to attach here also the second pair of Maxwell's equations. Summing up we might conclude that the electromagnetic field — its model, became identical with the geometrical structure of a five-dimensional manifold. All the quantities had obtained clear interpretations. In this sense the strength of an electromagnetic field turns out to be a curvature in the fifth dimension, while the bundle connection — an electromagnetic field in an arbitrary gauge. This is to say that with the aid of an operation of cross-section we could obtain an arbitrary electromagnetic gauge. This kind of representing of the electromagnetic theory clearly establishes the meaning of four-potential and singles it out from among the remaining quantities. It becomes a basic one. In classical electrodynamics it was without any significance, but turned out to be very important in quantum mechanics.

Namely, in the famous Bohm–Aharonov experiment the interaction of electromagnetic field with electrons was obtained, which caused the displacement of the diffraction pattern. The resulting displacement was depending upon magnetic flux grasped within the electron trajectories. In the place where the electrons were moving, the strength of electromagnetic field was equal to zero. In contrast the four-potential was different from zero. R. Feynman considers this experiment as an evidence supporting the view about the primacy of four-potentials in electrodynamics. One should agree with this opinion since otherwise it would be necessary to abandon the field theoretical picture and return to the action at-a-distance one. The theory of electromagnetism in its geometrical form using the bundle (named also as Utiyama theory) solves this controversy.

This picture is quite standard and in the case of gravity and other interactions, will retain this structure. Namely, we should try to represent the physical interactions as connections on the respective manifolds or certain other geometrical elements. In our attempt at achieving the unification of physical interactions, we are going to build from the existing geometrical elements describing e.g. the gravitational and electromagnetical interactions — the new geometrical quantities comprising the interactions being unified. Due to some requirements of the mathematical-geometrical nature, one will expect some interference effects between various interactions which would not follow from the previous theories. This kind of heuristic approach might lead towards the unified theory of physical interactions, with the simultaneous geometrization of them. This approach is an interesting one and is undergoing wide-spread development, as evidenced by theories appearing recently, aimed at unifying electromagnetic, weak and even strong interactions. Maxwell equations in vacuum have an additional symmetry. It is a conformal symmetry in Minkowski space-time. The conformal group is locally isomorphic to $\mathrm{SO}(2,4)$ of $\mathrm{SU}(2,2)$. The space of Maxwell field equations solutions has additional symmetries, e.g. $\mathrm{SU}(2,2) \otimes \mathrm{O}(2)$ and so on. After 150 years of discovery of Maxwell equations they still have very unknown interesting properties to be discovered.

3 General Relativity Theory and the Programme of Geometrization of Physics

In this chapter we will discuss General Relativity Theory within the framework of geometrization of physics.

When seen from the historical view-point General Relativity Theory was the first geometrical theory of space-time and gravity. It originated from Special Relativity Theory in such a way that the fixed invariable geometrical background of Minkowski space-time had been made a variable one. It was only assumed that this geometry is a Riemannian one, locally Minkowskian (a choice of the metric tensor signature). Einstein adopted a view that space-time geometry represents the gravitation itself. Components of the space-time metric tensor became gravitational potentials, while the non-vanishing space-time curvature indicated the occurrence of gravitational forces. At the same time it was Einstein who introduced the so called generally covariant invariance, which postulated the equivalence of all coordinate systems. This principle is often being rightfully seen as an analogue of gauge invariance for gravitation. Simultaneously Einstein assumed that free falls of bodies that more solely under the influence of gravitation and inertial forces reveal themselves in space-time as the geodesics. At this point the issues like equations for the space-time geometry (gravitation), their relationship with a weighting mass and the remaining fields was left behind.

In this way according to Albert Einstein we can summarize the theory of relativity in one sentence: *time and space and gravitation have no separate existence from matter*.

In this year we have 100th anniversary of General Relativity and we are close to 100th anniversary of Kaluza's idea.

After examining several possibilities, Einstein selected the one which we call today Einstein equations. They are second order hyperbolic nonlinear differential equations. At the same time they constitute generally covariant relationships between geometrical quantities and the "material" ones. This last notion will occupy us in the sequel, but now let us review some important aspects of General Relativity Theory, which are distinguishing it from other ones. Namely in all the theories mentioned thus far, that is in Galileo's mechanics, Special Relativity Theory, Maxwell's electrodynamics (in its classical, non-geometrical casting) we were dealing with an absolute space-time with its absolute geometry suitably selected. This geometry served as an invariable background, non-dynamical one. In General Relativity Theory the situation has been changed, since the gravitational forces were substituted by a suitable change of geometry, its deviation from Minkowski geometry.

Mass had "curved" the space-time. At the same time, what is of great importance, the bodies should be moving along the geodesics. In the case of Minkowski space, these lines were straight ones and were describing the motion in accordance with the Galileo principle (the first principle of Newton).

Their generalization in a curved space become just the geodesics as the simplest lines on the Riemannian manifold, that is such lines passing alongside it, whose tangent vector is being paralelly transported with respect to the connection. The important innovation consisted in preserving the simplicity of the motions taking place under the in influence of gravitation via the significant change of the background — the space-time within which the motion was occurring. All the couplings between the gravity and the remaining fields were introduced via the substitution of ordinary derivatives with covariant ones in Riemann's connection. Let us notice here the deep analogy between General Relativity Theory and the Utiyama's theory of electromagnetic field. There, also the geometry of an electromagnetic bundle was a dynamical quantity, and the connection's curvature was also associated with the strength of the field of interactions. In Utiyama's theory it was associated with the strength of the electromagnetic field. In both cases, in order to introduce the interaction with the gravitational or electromagnetic field one had to take covariant derivatives. The only fundamental difference would appear to be the occurrence in Utiyama's theory of an extra dimension associated with the electromagnetic gauge. This difference is, however, only superficial. Namely as we had described it above, the general Einsteinian invariance serves in General Relativity Theory as an analogue of gauge invariance. Due to that, one might view the General Relativity Theory as the theory of connection of the bundle of orthogonal repers in the sense of Minkowski metric over the space-time. This bundle is a principal one with a structural group SO(1,3), i.e. the Lorentz group. Its dimension is 10, because the number of parameters in Lorentz group is 6.

In both cases a physical world — the space-time — is a four-dimensional one. There is no fear of having more than four-dimensional space. The bundles are constructed over the space-time.

Let us now turn our attention onto general characteristics of Einstein's equations. A. Einstein has postulated them in the form

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = \kappa T_{\alpha\beta}$$
$$\kappa = 8\pi\, \frac{G_N}{c^4}\,.$$

On the left side of this equation there are geometrical quantities whereas on the right-hand side, the "material" ones. In this manner the massive matter and the remaining non-gravitational fields become sources for the space-time geometry. Let us note that these equations have a structure very much like the second pair of Maxwell's equations in Utiyama's theory. There, the non-electromagnetic fields also become the sources of geometry. Let us observe as well that Bianchi identity leads to the covariant conservation laws. In the case of General Relativity Theory — to the energy momentum conservation principle. This is very interesting.

To continue further along the lines included here, let us consider a certain theory, namely that of Einstein-Cartan. It represents the generalization of classical Einsteinian gravitational theory in a sense. We introduce here into the space-time the Cartanian connection, metrical one, but not necessarily Riemannian. As a source of torsion there appears the spin of material fields. We obtain additional equations, that of Cartan. They relate spin with the space-time torsion and are of the similar form as was the case of General Relativity Theory and Utiyama's theory. On the right side we have the "material" quantities, on the left the geometrical ones. The matter becomes a source for geometry. For the sake of completeness of this picture let us add that from Bianchi identity for Cartanian connection we get covariant conservation laws: that of energy-momentum and angular momentum. From the theories enumerated above, a certain very brave idea follows, interesting from both heuristic as ontological points of view. Perhaps all the fundamental interactions have model as connections on certain fibre bundles? Could it be so that conservation laws in all these cases are corollaries from Bianchi identity for the said connections? Maybe all field equations should have the form: geometrical quantities on the left side, whereas the "material" ones on the right? There is yet another question associated with this type of questions. Should all these cases be OK, therefore one might conclude that gauge fields are the same as the fields of physical interactions. It is known however that the theory of principal fibre bundle connection is a gauge theory. Therefore it appears here something of the emerging unified language for a field theory — the language of theory of physical interactions. Differential geometry constitutes this language. We are going to identify the programme leading to the creation of this type of theory as the fundamental physical interactions geometrization programme. We therefore see that such a programme has led to a complete success in the case of gravitational and electromagnetic interactions. In the case of weak and strong interactions in the physics of elementary particles, it is just in the process of being implemented, and leads towards the unification of all physical interactions not only what concerns the language being employed, but in the theory describing physical interactions as well. Such a theory is already implemented for the case of weak and electromagnetic interactions in the so called Glashow–Salam–Weinberg model. It has in turn taken the form of Kaluza–Klein theory for the case of electromagnetical and gravitational interactions. Before we embark on reviewing these theories and their role, we are going to deal for a while with Yang–Mills field theory. These fields are very similar in their structure to the electromagnetic field in Utiyama's form. The difference consists mainly in a gauge group different than U(1). In the case of classical Yang–Mills it was a group SU(2). These authors had introduced this field in order to explain the strong interactions. In their theory, the gauge field had to be identified with meson fields, which in Yukawa theory are hypothetical carriers of the nuclear interactions between the nucleons. Yang–Mills fields, very much like the electromagnetical ones are massless. The mesons have the rest mass. Because of that Yang and Mills had to introduce mass terms breaking the gauge symmetry associated with SU(2). In spite that the general idea remained correct it has however turned out that one cannot associate with isospin the compensating — gauge field, which was already mentioned. Yang–Mills equations are identical in their structure to Maxwell equations. Let us notice that their form agrees with the postulate: on the left side the geometry, on the right side the "matter". By a suitable choice of gauge group we might try to describe other physical interactions — the weak and strong ones.

For the case of electromagnetic field we are able to obtain the gauge of the second kind by changing the phase transformation in the global gauges into the function dependent on point of space-time. In that manner we extend the gauge transformation of the first kind. In accordance with the Noether theorem we have a charge conservation principle which is a corollary from invariance with regard to the gauges of the first kind. The introduction of the second kind gauges will cause a significant change. Namely an additional field will arise, to be called a compensating one. The interactions between the charges would be described by this field; the charge conservation principle here has been obtained via the Noether theorem from the invariance with regard to gauges of the first kind. In the case of U(1) gauge group this is the electromagnetic field, and the photons should function as its carriers (quanta). There are just photons, and more precisely their exchanges which are responsible for the repelling of charges of the same type, and the attraction of the charges of the opposite type. The appearance of the gauge field will cause various complications. In fact the field undergoing the gauge of the first kind should be interacting with this field. This will reveal itself in the change of field derivative appearing in Lagrangian or in the equations of motion. Here, the gauge derivative should appear, describing in accordance with the minimal coupling scheme the interaction between this gauge field and the original one. This original field could be a spinorial field, complex vector, or complex scalar one. Real fields (real-valued ones) are out of the question here, since the charges (carried by them) and currents are equal to zero. In the case of U(1) based gauge, there will be usual currents and electrical charges. Here, the gauge derivatives will be the ones well known for the electromagnetic field.

In such a case, the Lagrangian of the original field would change as a result of introducing new derivatives. The total Lagrangian here should comprise the gauge field Lagrangian. In the case of electromagnetic field we have Maxwell's Lagrangian. By changing gauge group we have another gauge field. Taking $G = \mathrm{SU}(2)$ we get Yang–Mills field. The cases of most interest represent here the fields with non-Abelian gauge groups such as SU(2), SU(3) etc. Groups of this kind lead to the other charge and current conservation principles. These charges create the gauge group (of the first kind) Lie algebra. The compensation field (Yang–Mills) describes the interactions between the charges, their repulsion or attraction. The so called intermediate bosons will be the carriers of that field's interactions. These particles are massless like photons and the number of their various kinds is equal the number of independent parameters in gauge group G. For the case $G = \mathrm{SU}(2)$ we have three such bosons, whereas for $G = \mathrm{SU}(3)$ eight. Of course for U(1) it is clearly seen that there would be one. The interaction between charges consists of exchanging this type of particles which has been given the name of intermediate bosons: There is however a certain very significant difference between the electromagnetic case and the general one (with an arbitrary gauge group). In the case where the gauge group G is non-Abelian the gauge field equations are nonlinear. Thus a self-interaction of the field would appear. It is easy to understand why this is so, since for the electromagnetic case the photon has not carried any electric charge, whereas in the gauge field case the intermediate bosons could be charged. In other words the field strength lines of Yang–Mills field might attract or repeal themselves.

The Yang–Mills fields due to the masslessness of their carriers have an infinite range. The fundamental static solution for them has also to be Coulomb solution. These equations in the vacuum will possess a conformal symmetry, similarly like the ones of Maxwell. There is a very natural and elegant gauge field description in the language of differential geometry. This is a description based on the fibre bundle theory. We consider here the principal fibre bundle with a structural group G and with the base being a space-time. The connection on such a bundle will be simply Yang–Mills field, while the connection's curvature is equal to the strength of Yang–Mills field. The bundle cross-sections will become the choice of the gauge (of the second kind). The covariant derivative of the bundle in a given connection shall be the gauge derivative. Bianchi identities for the connection's curvature would constitute one pair of equations (Maxwell, Yang–Mills). These will follow from the existence of the four-potential. As we see, all the physical quantities obtained have the natural geometrical interpretations. In this way, Maxwell's theory of electromagnetism has been geometrized and analogously to it the Yang–Mills field theories with an arbitrary gauge group G.

It is a fact that the greatest achievement thus far on this path was the Glashow–Salam–Weinberg model. That model unifies the electromagnetic and weak interactions. From this moment onwards we should be speaking about the weak-electromagnetic interaction. It turned out that these two interactions became the two sides of the same medal, quite like electricity and magnetism in Maxwell's theory. They could be separated only under special circumstances, which might be compared to magnetostatics or electrostatics in Maxwell's theory. Curiously enough we could observe that the difference masses between the proton and the neutron is of the weak origin and not of the electromagnetic one, due to the links between both types of interactions. We know from experiments however that the weak interactions (e.g. those associated with the beta decay) are of the finite range. This indicates that the intermediate bosons responsible for carrying through this interaction have non-vanishing rest masses. On the other hand however, we know that the introduction of massive terms will break the gauge symmetry. The only way of introducing (generating) intermediate boson masses without breaking gauge symmetry is the spontaneous symmetry breaking and the Higgs–Kibble mechanism. In this place we are not going to enter into the details of these interesting phenomena. We have only to say that this is associated with the introduction of the additional fields — the so called Higgs fields and also with a degeneration of the ground state. Glashow–Salam–Weinberg model uses $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge group, due to which there appear the neutral currents detected experimentally. We obtain two charged intermediate bosons W^+, W^-, one neutral Z^0 and a photon γ in this theory. Bosons $W^\pm$, Z^0 have masses endowed upon them via Higgs mechanism (Mohapatra & Lai 1984; Yndur̈ain 1983; Lai 1981; Commins & Bucksbaum 1983; Cheng & Li 1984; Lee 1984; Aitchson & Hey 1983; Zee 1984; Herman 1978; Konuma & Maskawa 1981; de Sabatta & Schmutzer (eds.) 1983).

We have also leptons e, μ, ν_e, ν_μ in this model. One could also add a new sequence τ, ν_τ. The leptons are also massless and they also assume masses by Higgs mechanism. The geometrization of Higgs mechanism appears to be an extremely interesting problem. There are attempts under way to do this. Some successes have been reported. It seems that Higgs fields are of the similar nature as Yang–Mills ones, hence they are geometrizable in the above described sense. Observe therefore that the unified weak-electromagnetic interactions turned out to be a connection on a principal bundle with a structural group $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ over the space-time. In spite that there is no complete geometrization of this model in the lepton sector, one is entitled to speak about a success on the road to the geometrization of all interactions. One might speak at this moment about a full experimental confirmation of this unification. The neutral current predicted in this theory has been discovered. The due price of that success was the awarding of the Nobel prize in physics for 1979 to the creators of the unifying models: S. Weinberg, A. Salam and S. L. Glashow. Several years later the intermediate bosons $W^{\pm}$, Z^0 had been discovered with the masses predicted by the theory. This was also honoured with the Nobel prize in physics for the year 1984.

The Higgs boson also has been discovered in 2013 and also honoured with the Nobel prize in physics. Peter Higgs and François Englert had been awarded the 2013 Nobel Prize in physics.

There are some extensions of the GSW model with more than one Higgs' doublet. In general we can have n-doublet model. The most popular are 2-doublet models (sometimes 3-doublet models). Very popular is an inert doublet model (2-doublet). Additional doublet of selfinteracting scalar fields are not really Higgs' fields, because we have not a spontaneous symmetry breaking and Higgs' mechanism connecting to these fields. Moreover, in general additional doublets of scalar fields can complicate a structure of a vacuum going to some experimental predictions. The additional Higgs' fields can serve as a source of "dark matter" particles. Those models are far away from our considerations of geometrization of physics.

In this way we can define a Standard Model (SM) as GSW model and QCD with fermions in 3-generations including of course also quarks: (e, ν_e, u, d), (μ, ν_μ, s, c), (τ, ν_τ, b, t). In the model all neutrinos are massless. Masses for massive fermions are obtained due to Yukawa mechanism. Moreover, they are not really massless (except maybe ν_e). They are oscillating. In this way ν_μ and ν_τ should be massive. The oscillations among ν_e, ν_μ and ν_τ have been discovered in solar, atmospheric and reactor and accelerator neutrino (or antineutrino) sources. This has been honoured with the Nobel Prize in physics. T. Kajita and A. B. McDonald have been awarded the 2015 Nobel Prize in physics.

Mixing between neutrinos is described by PMNS (Pontecorvo–Maki–Nakagawa–Sakata) matrix. There is still a controversy: are neutrinos Dirac particles or Majorana particles? Neutrinoless double β decay ($\beta\beta_{0\nu}$) is still not confirmed.

In neutrino physics we have flavour states ν_e, ν_μ, ν_τ and massive states (states with defined masses) ν_1, ν_2, ν_3. In this way ν_e, ν_μ, ν_τ are mixtures of ν_1, ν_2, ν_3. During a motion a definite flavour state oscillates due to different velocities of massive states which compose it. Thus we get a mixture of flavour states which can be distinguished by detectors. In some sense to speak of masses of ν_e, ν_μ, ν_τ is an abuse of nomination for they have not definite masses. The real puzzle in neutrino physics is also a mass hierarchy. Is it normal or reversed? We are still looking for PC violation in neutrino mixing, the so-called δ_{CP} phase.

The important investigation in physics of fundamental physical interactions is to look for parity and CP non-conservation in strong interactions. There are theoretical predictions to get some traces of P and PC non-conservation in heavy ions collisions. These traces are some correlations in scatterings which can be tested in experiments. Up to now we do not see such phenomena. The only one place to see PC breaking is a Cabbibo–Kobayashi–Maskawa matrix with nonzero δ-phase coming to K^0 and $\overline{K}^0$ and also to B^0 and $\overline{B}^0$ mixing. An additional prediction is a nonzero dipole electric moment of a neutron (extremely small). In the case of quark-gluon plasma (after a deconfinement) we can get very strong CP violation effects due to an additional term in QCD lagrangian which is a full divergence. This term is important only on quantum level due to tunnel effects between degenerate states of a vacuum. The mentioned term is purely hypothetical.

It is worth to mention of G. Zoupanos higher-dimensional unification with continuous and fuzzy coset spaces as extra dimensions. The best model in this approach is based on $N = 1$, 10-dimensional E8 gauge theory reduced to nearly Kähler manifold. The corresponding programme is considered with fuzzy coset spaces as extra dimensions. The interesting programme is to include here a nonsymmetric gravity.

4 Quantum Chromodynamics, Gauge Fields and the Unification of the Fundamental Interactions

In this chapter we will consider the applications of the gauge fields in the domain of strong interactions, and attempts in unifying all interactions. Quantum chromodynamics provides another interesting example of gauge fields application in the theory of elementary particles. This theory (QCD) is a likely candidate for becoming a recognized one in the domain of strong interactions. It is a field theory with $\mathrm{SU}(3)_c$ as a gauge group. Its intermediate bosons, the so called gluons are being interchanged between the strongly interacting particles but they do not occur in the asymptotic states. They glue quarks inside the hadrons, which explains their name.

The gauge group $\mathrm{SU}(3)_c$ is the so called colour one and has nothing in common with the group SU(3) known from Gell-Mann classification.

At this point we would like to emphasize that all the groups classifying the hadrons via irreducible representations, that is SU(2), SU(3), SU(4) etc. according to predominant views have nothing in common with the symmetry of the strong interactions. They express rather the fact that at the base of hadron mass spectra one finds 2, 3, 4 etc. quarks of different kinds, with different “flavour”. Each of these quarks might appear in one of the three different colour states i.e. red, blue, green. However in accordance with the hypothesis of quantum chromodynamics the hadrons are white and thus they could only appear in singleton states with respect to colour. The additional quantum number — the colour has been introduced in order to preserve Fermi statistics for the quarks without the need of substituting it with a parastatistics of order 3. Quark Lagrangian is symmetric (i.e. invariant) with respect to $\mathrm{SU}(3)_c$ group, that is with respect to gauges of the first kind. The introduction of the second kind gauges and the compensating fields associated with them, constitutes just the chromodynamics. The fundamental theorem of this theory not yet adequately proven is the quark confinement hypothesis; more generally the colour quark confinement. According to this hypothesis, the forces which bind quarks together increase proportionally with the distance and that is why we cannot observe free quarks. Quantum chromodynamics is a renormalized theory like that of Glashow–Salam–Weinberg. It is also asymptotically free. That is to say, at small distances quarks behave in such a way as they were free. Hence we have the infrared confinement and ultraviolet freedom. The predictions of Quantum Chromodynamics in the domain of weak coupling near the ultraviolet freedom, could be considered as gluonic corrections to the parton model.

According to recent views, we have inside this region good agreement with experiment. What concerns the infrared confinement (big distances small energies) no decisive results have been obtained thus far, in spite of great efforts, using of instantons and methods of algebraic geometry. The majority of experts favours the opinion that Quantum Chromodynamics constitutes a proper foundation for the theory of strong interactions. It might turn necessary the complementing of this theory with some additional elements. Let me remark also that the efforts are under way aimed at the so called Grand Unification of the weak electromagnetic and strong interactions by introducing of gauge theories with different gauge groups: E6, SU(5). This work is however not been accomplished as yet. The evidence supporting the idea of a linkage between the strong interactions and weak electromagnetic ones follows from the relationship which constitutes a corollary from Glashow–Salam–Weinberg model. Namely, the number of leptons has to be equal to the number of quark flavours. It is exactly due to this fact that the prediction of 'charm" and "beauty" originated within this model. At present there is also the so called "truth" being included. The models of Grand Unification try to get this relationship as a corollary from the theory. Theory of this type with certain gauge group G includes the corresponding multiplet of Higgs field in such a way that the spontaneous symmetry breaking might leave massless only the photon and the gluons. Therefore the symmetry breaking occurs from G toward its subgroup $\mathrm{SU}(3)_c \otimes U(1)_{el}$. One of the more successful models within this category (called GUT — Grand Unified Theories) is the one based on SU(5) group. In a certain way it is a minimal model. It predicts proton decay. Such a decay has not as yet been experimentally observed. This does not mean that the very idea of GUT is an improper one. Probably we have to pass to other groups e.g. SU(10) and find the new predictions amenable for experimental verification. Let us notice that in a lot of instances we have obtained the physical interactions as connections on the respective fibre bundles.

The equations of motion for these fields obtained the form: the geometrical quantities on the left, whereas the "material" ones on the right. It seems quite natural to pose a question, which had been posed by Einstein himself, whether it would be possible to move e.g. in the equations for gravity the most of the terms from its right side to the left. The ideal situation would be having all terms on the left side. This would amount to a complete geometrization of the interactions. A. Einstein always maintained that only the left side of this equation is completely trustworthy for him and what regards the right one, he would put identically zero there. It would follow from all this that the ideal situation should be the vacuum equation for the geometry describing all interactions. This idea provided the basis for the concept of Unitary Field Theory. This concept in its extreme form called for the creation of such equations for gravity and electro-magnetism. The so called Nonsymmetric Field Theory was the last version of his theory, published by Einstein in 1950. He did not manage to show, whether the equations for gravity and electromagnetism would follow from the equations of his theory. The Unitary Field Theory also required the solving of the field equations in the interior of the elementary particles themselves. This would mean that the particles ceased to be considered as singular points of the field and had to belong to the solutions of these equations. In this sense, the Unitary Field Theory proposed the geometrical field theoretical description of all physical interactions and elementary particles. A. Einstein hoped that due to certain interference effects between the electromagnetic field and a gravitational one he should obtain a new phenomenon amplified due to the non-linearity of the equations. That new phenomenon could be responsible for stability of the elementary particles and in this way the "sufficiently dense" field might describe the "matter" on the right side of equations for gravity and electromagnetism in their classical form. This type of an extreme geometro-unification programme has failed. Simply — other interactions were missing here — the weak and strong ones; without them, one is hardly in a position to imagine the theory of elementary particles. However such an approach was further to be continued in Wheeler's geometrodynamics. It has turned out that the geometrical structure of space-time is too poor in order to describe all the interactions. In recent times we observe a kind of return to the mathematical structure of a Nonsymmetric Field Theory, the so called Einstein–Strauss and Einstein–Kaufman theory. Some investigators use it as a generalized (the so called nonsymmetric) gravity theory, the others as a macroscopic theory of gravity and electromagnetism. Happily, there was still another unifying, geometrical approach, which postulated the description of the electromagnetic and gravitational fields with the aid of geometry on manifolds of more than four dimensions. Here belong the five-dimensional Kaluza–Klein and Jordan–Thiry theories.

Weyl's theory, which used projective geometry in space-time was also of a similar kind. From historical standpoint, these theories considered five-dimensional manifolds, where no physical effects depend on the fifth dimension. They were assuming the appearance of Killing vector and have linked the electromagnetic four-potential with $g_{\alpha 5}$. On the basis of vacuum analogues to Einstein equations for the five-dimensional theory, Einstein equation with energy-momentum tensor for electromagnetic field and also Maxwell's equations in the vacuum were obtained. In this way, a unitary theory of the gravitational and electromagnetic fields was arrived at, in which both fields were generating the Riemannian connection on a five-dimensional manifold. This theory had suffered from the inconvenience that the space-time (four-dimensional) was the hyper-surface in a five-dimensional "cylinder". Possibilities exist of introducing sources into this theory in such a manner that the equations for electromagnetism and gravitation would be written as one five-dimensional equation with sources. Bianchi identity for Kaluza–Klein connection provided here the covariant energy-momentum and charge conservation laws. Such an approach, computationally quite difficult, turned out to be equivalent with Utiyama's one as shown by A. Trautman and W. Tulczyjew. It turns out that there is a natural metrization of the electromagnetic bundle. The connection generated by this metric tensor is identical with Kaluza–Klein connection. In this way, the geometrical theory of gravity and of electromagnetism was obtained as a theory of the connection for the bundle of bases over the metrized electromagnetic bundle. The equations of course remained the same as in the classical version of this theory. One could extend the principal fibre bundle metrization procedure without the slightest obstacles to the case of arbitrary gauge group G (semisimple). The geometrical theories of gravitation and of Yang–Mills fields were obtained in this manner. From this theory resulted the equations of Yang–Mills fields and Einstein equations with a source in the form: gauge field energy-momentum tensor plus a cosmological term. The cosmological term used always to appear whenever the theory has a non-Abelian gauge group. There was a possibility of introducing the external sources and obtaining of the covariant conservation laws from Bianchi identity for Riemannian connection generated from metrics obtained in a canonical way. Gravity and Yang–Mills theory turned out to be the connection theories of a bundle of bases over the metrized principal bundle with a structural group G on a space-time. The interference effect appeared also — the cosmological term in the non-Abelian case. Kaluza–Klein theory described above admitted the extension to a Kaluza–Klein with torsion. Resulting extended theory constitutes geometrical unification of Kaluza–Klein and Einstein-Cartan theories. We obtain it introducing metric connection onto the metrized electromagnetic bundle. This connection needs not to be Riemannian; it might possess the non-vanishing torsion. Assuming certain natural properties of the connection — invariance with respect to U(1) and the horizontality of curvature, plus by introduction of sources into it, we will obtain the interpretation of the torsion linked with the fifth dimension — as an electromagnetical polarization of the sources. There is going to appear an additional equation of the type described above. Namely, on the left side we shall obtain a geometrical quantity — the torsion in the fifth dimension and on the right side, a physical quantity — the electromagnetic polarization. This type of theory (with torsion) could be extended to an arbitrary gauge group G.

Then the gauge (Yang–Mills) field polarization will become the source of torsion in higher dimensions. From Bianchi identity for the connection in question we will obtain the covariant conservation laws for: energy-momentum, angular momentum and electrical charge in the five-dimensional case, or colour charges for arbitrary gauge group G. The interpretation of the equations for the geodesics in Kaluza–Klein types of theories is an interesting topic, both in the case with and without torsion. As a matter of fact, it turns out that these equations reduce themselves to the equations of motion of a material point in the gravitational and electromagnetic field. There appears in these equations a term associated with Lorentz force. In the formula for Lorentz force (the case with torsion) there appears a full electromagnetic field (polarization included). The extending of this theory onto the case of an arbitrary gauge group G, introduces a term analogous to Lorentz force for Yang–Mills field. The case with torsion generalizes the formulas via the occurrence of a complete gauge field — together with polarization. The equations for the geodesics in all the cases enumerated above have the first integrals. They have the interpretation of electric charges or the colour ones for test particles. Let us note that the usage of the equations for the geodesics on the multi-dimensional manifolds in the suitably constructed connections, has provided us with the description of the test particles motion under the influence of gravity and of the gauge field. These equations include the terms with Lorentz force plus the feature that test particles have constant charges during the motion (the first integrals). This is a remarkable fact, since the geodesics provide the simplest paths on a manifold. They are analogous to straight lines for the flat spaces. Similarly as it was in the case in General Relativity Theory, "the geometry became a physical interaction". Now this concerns not only the gravitational but also the electromagnetic or gauge ones, with preservation of a postulate about the maximal simplicity of the test particles trajectories.

This is in sharp contrast to the situation we had in Newton's theory, where the force is considered to be a measure of motion's deviation from the uniform one — thus the motion occurring along straight lines with constant velocity and the simplest possible. Here we change the space-time geometry and also that of the manifold, but still have the test particles trajectories identical with the simplest possible paths. This represent a viewpoint accepting the assumption to the effect that the interaction is "geometry". The condition requiring that the test particle paths be identical with geodesics, makes the search for a geometry suitable for describing a given interaction

— a unique enterprize in a certain sense. Due to this it seems that the programme aimed to geometrize all the interactions appears to be relatively uniquely defined and maybe it is going to decide the dispute between various phenomenologies in the area of elementary particles. These phenomenologies often bring similar experimental predictions within the given range of measurements. The choice performed among these phenomenologies or perhaps certain modifications of the existing ones, when carried along the geometrical guidelines, might contribute to the discovery of a suitable theory. The choice or modifications of this type could be compared with Maxwell's decision about the introducing; of a displacement current into the theory of electromagnetism. As we know, Maxwell was helped by the charge conservation idea. Precisely this idea has made unique the choice between great number of theories (phenomenologies) predominant in the times of Maxwell. Later on it has turned out that this was a theory of geometrical character, and Maxwell's choice represented in fact geometrization of the electromagnetic interactions. Let us notice that there is not too many phenomenologies, which would be equivalent to a certain geometrical theory of physical interactions (in a sense given above). This explains why the postulate of geometrizability could be a very convenient heuristic hint during the construction of theories capable of unifying the physical interactions. At the same time it could become an ontological hypothesis. According to it, the physical world becomes a geometrical one. The "matter" vanishes and there remains only geometry. The "vanishing" of the matter need not necessarily be interpreted in an idealistic sense. The matter need not to vanish, simply its range gets extending. Geometry unifying physical interactions constitutes this sort of extending the notion of matter.

Let us try to present a general scheme of unifying two physical theories T_1 and T_2 in accordance with the ideas put forward thus far. Let T_1 comprise the theoretical notions $U_1, U_2, \ldots, U_n$ while T_2 include $V_1, V_2, \ldots, V_m$. The first step would consist of finding such a description of the theories T_1, T_2 that on the theoretical notions occurring here one could find their geometrical counterparts such as: connections, metric tensors, curvatures on certain manifolds etc. Let us assume that in $G(T_1)$ such quantities are $G(U_1), G(U_2), \ldots, G(U_n)$, and $G(V_1), G(V_2), \ldots, G(V_m)$ — in $G(T_2)$. The second step would consist of constructing a new theory T out of the geometrical elements $K_1, K_2, \ldots, K_p$ in turn composed from $G(V_1), G(V_2), \ldots, G(V_m)$ and $G(U_1), G(U_2), \ldots, G(U_n)$. While the first step is relatively simple to a certain degree, the second one in contrast includes lot of the unclarified issues. Thus, we could only offer some examples of implementing the second step. This step contains in itself a substantial progress when compared to $G(T_1)$ or $G(T_2)$; the theory T is not equivalent to the theories $G(T_1) \cup G(T_2)$.

One is entitled to expect some interference effects, between the interactions described by the theories $G(T_1), G(T_2)$. We are going to follow through the scheme sketched above on the exemplary construction of Kaluza–Klein theory and Kaluza–Klein with torsion. In the first case General Relativity Theory plays the role of T_1, whereas Maxwell's electrodynamics in vacuum is to play the role of T_2. In the second case Einstein–Cartan theory will become T_1 whereas Maxwell electrodynamics (taking into account the electromagnetic polarization of matter) is going to become T_2.

Let us begin with usual Kaluza–Klein theory. General Relativity Theory is already a geometrical one and hence $G(\mathrm{GRT}) = \mathrm{GRT}$. In the case of $T_2 =$ Maxwell's electrodynamics in the vacuum, the geometrization consists in the introducing of the electromagnetic bundle. We have described this earlier.

Hence $G(T_1)$ contains the theoretical notions being simultaneously geometrical quantities; $g_{\mu\tau}$ — metric tensor of a space-time E, ω^{α}_{β} — linear Riemannian connection compatible with $g_{\mu\tau}$, Ω^{α}_{β} — curvature of the connection, θ^{α} — fundamental forms on E. These quantities describe the fields in GRT and have the interpretation which is known. We might say that T_1 is a certain theory of the connection ω^{α}_{β} on a bundle of orthogonal (in the sense of Minkowski metric) bases over E. In the case of existence of gravitational field, the curvature of this bundle is different from zero, but the torsion vanishes, which is an assumption adopted in GRT. Thus GRT might be presented in a fold manner. As $(E, \omega^{\alpha}_{\beta}, g_{\alpha\beta}, \theta^{\alpha})$ or $(M, \omega^{\alpha}_{\beta})$. M denotes a 14-dimensional principal bundle of bases over E with Poincaré structural group. In either of these descriptions (in accordance with general Einstein's invariance) there is no privileged coordinate system.

Of course the first and the second description, represents only the "kinetics" of gravitation. Its "dynamics" is to be determined only by the equations linking the geometry with the external sources — heavy matter. Einstein equations play this role for the case of GRT. In the variational setting, the choice of a suitable dynamics will be associated with the choice of the Lagrangian for gravity. Then Euler–Lagrange equations will become the dynamical ones for gravity. In the case of GRT, scalar curvature constitutes Lagrangian density, and Einstein equations follow from the Hilbert principle.

In a case when the electromagnetic field is a source for gravity, we will put the energy-momentum tensor for the electromagnetic field on the right side of Einstein equations.

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = \kappa T^{em}_{\alpha\beta} \tag{4.1}$$

where

$$T^{em}_{\alpha\beta} = -\frac{1}{4\pi}\left(F_{\alpha\mu} F^{\mu}{}_{\beta} - \frac{1}{4}\, g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}\right) \tag{4.2}$$

and $F_{\alpha\beta}$ satisfies Maxwell equations for vacuum. The electromagnetic field and its interaction with gravity are given in a phenomenological, non-geometrical way. Clearly, in Maxwell equations one has to substitute the partial derivatives with the covariant derivatives in the connection ω^{α}_{β}. Since the connection w^{α}_{β} is Riemannian, the relationship between the four-potential A_μ and the field strength $F_{\mu\nu}$ remains intact. Gauge invariance of the four-potential is also going to be preserved. Therefore the scheme of least coupling plus Einstein equations with an electromagnetic source describe the interaction of electromagnetism with gravity. Let us notice that electromagnetism has become here a source for gravity, but the gravity has not become a source for electromagnetism. A certain quantity is missing, which would become a gravitational source in the second pair of Maxwell equations, in a manner reminiscent of $T^{em}_{\alpha\beta}$ being an electromagnetic source in Einstein equations. The minimal coupling scheme has not introduced such a source — the gravitational current. The full symmetry between the interactions would call for the existence of such a quantity. Hence, summing up we have a geometrical theory T_1 of gravity and its application to electromagnetic sources. Only the quantities associated with gravity are geometrized here. The electromagnetic field is treated phenomenologically. On the other hand, we have the theory T_2, Maxwell's electrodynamics and its geometrization $G(T_2)$ in the sense of Utiyama–Trautman–Tulczyjew. A pair (P, α), where $P = P(P, F, \mathrm{U}(1), \mathrm{U}(1), \pi)$ constitutes a principal bundle with a structural group $\mathrm{U}(1)$ and the connection α on P describes a kinetic part of the theory.

The dynamical part: the choice of Lagrangian or the second pair of Maxwell equations. In this place there is a possibility of using a nonlinear dynamics. The choice of dynamics means exactly the choice of a Lagrangian L $(S\ P)$. The choice $L = S$ introduces the Maxwellian (the second pair of Maxwell's equation in the vacuum), whereas the decision that $L = R$ introduces the Einsteinian dynamics, the General Relativity Theory. Hence we have got two geometrical theories of interactions T_1, T_2.

We shall perform a unification procedure by constructing a theory T — in this case Kaluza–Klein theory. Therefore we have the structure $(E, \omega^{\alpha}_{\beta}, g_{\alpha\beta}, \theta^{\alpha})$ and (P, α). This procedure could be uniquely defined here. We metrize P in a natural way given by Trautman and introduce on P the Riemannian connection, generated by the metric adopted. We also introduce a frame on P. In this fashion we obtain a kinetic part of the theory T.

$$(P, \omega^{A}_{B}, \gamma_{AB}, \theta^{A}) \tag{4.3a}$$

where

$$\gamma_{AB} = \begin{bmatrix} g_{\alpha\beta} & 0 \\ 0 & -1 \end{bmatrix}, \quad \theta^A = (\theta^\alpha, \lambda\alpha), \quad \lambda > 0, \tag{4.3b}$$

$$\widetilde{D}\gamma_{AB} = 0. \tag{4.3c}$$

$\widetilde{D}$ is the covariant differentiation in the connection $\omega^A_B \gamma_{AB}$ is represented in a frame θ^A. In this way we have obtained a theory, where a kinetic part is described with the aid of a certain connection ω^A_B. Similarly as in the case of General Relativity Theory there is a description using the bundle of bases:

(M, ω^A_B) where M is the principal fibre bundle of orthogonal bases in the sense of de Sitter metric (the signature $-{+}{+}{+}{+}$) over P.

De Sitter SO$(1,4)$ group plays here the role of Lorentz group in General Relativity Theory. The frame θ^A for the non-vanishing electromagnetic field (the non-integrability of a connection α) is a non-holonomic frame.

In the holonomic frame

$$dx^A = (dx^\alpha, dx^5)$$

we have

$$\gamma_{AB} = \begin{bmatrix} g_{\alpha\beta} + \lambda^2 A_\alpha A_\beta & -\lambda A_\alpha \\ -\lambda A_\beta & -1 \end{bmatrix} \tag{4.4}$$

where A_μ is a four-potential.

In the classical Kaluza–Klein approach, the geometrization had taken place via the introduction of a five-dimensional metric tensor defined in the manner given above.

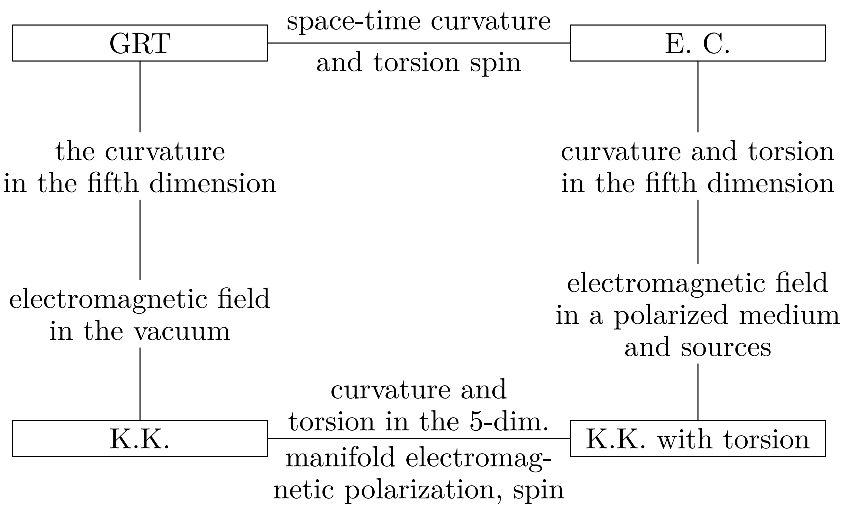
Let us occupy ourselves with the dynamics of the theory T–Kaluza–Klein. In accordance with a traditional approach we introduce a five-dimensional Lagrangian. It is a curvature scalar in the connection ω^A_B. From the Hilbert variational principle extended onto the five-dimensional case we obtain the equations for gravity and electromagnetism. It turns out that they are Einstein equations with an electromagnetic source $T^{em}_{\alpha\beta}$ and the second part of Maxwell's equations. The interesting fact is that the second part of Maxwell's equations appears here with taking into account the minimal coupling scheme. For the case of absence of any external sources we do not obtain any new interference effects linking gravitational field to the electromagnetic one, which were not known in General Relativity Theory. Once we introduce the external sources described by other fields e.g. Dirac spinors we could obtain interesting predictions. To that end one has to define the spinorial fields ψ and $\overline{\psi}$ over the metrized P. Then it turns out that the fields ψ are going to carry a dipole electric moment, the breaking of PC will occur. The dipole electric moment will have a value expressed by the elementary constants and its magnitude is of order $10^{-32}\,q[\mathrm{cm}]$. The PC symmetry breaking brings very interesting consequences. It is equivalent (on the assumption of the PCT symmetry existence) breaking of the symmetry T. This in turn means that we could discern the past and the future at microscopic level. Hence we get the "arrow of time" in a unification theory. This effect is due to the taking into account of $\mathrm{SO}(2,3)$ as a symmetry of a local five-dimensional manifold. There appears a certain structure of fermionic charge, which does not result as a corollary from General Relativity Theory, Maxwell's electrodynamic or Dirac's equations. Thus Kaluza–Klein theory with Dirac sources predicts the effects which are new in comparison to the previous ones. Here belong the interference effects previously described. Summing up: it turned out that the theory T described by a "kinetics" (P, ω^A_B) upon transition to the theories $G(T_1)$ and $G(T_2)$ will break down into two kinetics:

$$(P, \alpha), \quad (E, \omega^\alpha_\beta) \tag{4.5a}$$

following the scheme

$$\begin{aligned} \theta^A &= (\theta^\alpha, \lambda\alpha) \\ \omega^A_B &= (\omega^\alpha_b, F_{\alpha\beta}) \\ \gamma_{AB} &= (g_{\alpha\beta}, A_\alpha) \\ P &= P(P, E, \mathrm{U}(1), \mathrm{U}(1), \pi) \end{aligned} \tag{4.5b}$$

Therefore starting from the quantities defined in the last instance on the space-time E one managed to construct a connection on a multi-dimensional bundle P describing simultaneously the gravitational and electromagnetic interactions. The spinors ψ and $\overline{\psi}$ and Dirac's Lagrangian on P had been similarly constructed. We obtained thus an illustration of a general geometrico-unifying scheme put forward previously. The unification might be similarly carried through in the case with torsion, too. Here T_1 is Einstein–Cartan theory whereas T_2 — Maxwell's electrodynamics with the polarization of matter. The scheme presented below will illustrate us its place among the theories involved.



GRT — General Relativity Theory, E.C. — Einstein–Cartan theory
K.K. — Kaluza–Klein theory

Fig. 1

It is interesting to notice that $W^{\pm}$ and Z^0 have been discovered in an experiment. Also a Higgs' boson has been discovered in an experiment. For a Higgs' field can be geometrized as a part of Yang–Mills fields, this strongly suggests that geometrization and unification of fundamental physical interactions is a right direction in looking for an *arche* of the world. It is a geometry.

The idea of a unification through geometrization could also be applied to gauge fields models (non-Abelian ones) by unifying them with Higgs fields. In this manner a model with spontaneous symmetry breaking is described by a connection on a certain multi-dimensional fibre bundle and Higgs fields assume the natural geometrical interpretation. Spontaneous symmetry breaking and the generation of masses for the intermediate bosons also occur within this framework. This scheme has fulfilled its role for the case of a bosonic part of Glashow–Salam–Weinberg model, the theory for the weak-electromagnetic interactions. By compiling the Kaluza–Klein idea and the Higgs mechanism geometrization we are in a position, by using the scheme described above to obtain a geometrical unification of gravity (GRT) and gauge field models (Abelian and non-Abelian ones) with a spontaneous symmetry breaking and the Higgs mechanism. Still further this leads toward the geometrical unification of gravity with other interactions that is with the weak, the electromagnetic and the strong ones.

This supergravitational (supersymmetric) extension of the presented scheme allows one to hope for a natural inclusion of fermions and of their mass generation mechanism. The interference effects and the new predictions of the unified theory (appearing here) could be verified experimentally in certain extremal conditions. Let us observe some characteristic properties of this type of unification. Firstly, they leave relatively few possibilities, due to the rigidity of the geometrical theories' construction rules. Secondly, every single possibility from among those selected offers very precise answers with regard to the particle masses, coupling constants, mixing angles etc. These two features taken together prove that theories of this type nicely fit the picture of a good theory in Popper sense. In fact such a theory is easy to falsify, and consequently to abolish. It does not represent (in spite of a very abstract, geometrical appearance) something completely contemplative, fully separated from the realm of experiment. Let us consider for example that in a geometrical (6-dimensional) Glashow–Salam–Weinberg model we predict a correct (experimentally confirmed) value of Weinberg angle (it is a phenomenological parameter in this model) and Higgs particles masses (for the time being beyond the reach of experimental checking). Let us stress also that the using of other geometries (non-Riemannian ones) within Kaluza–Klein scheme could lead to the effective alternative theory of gravity (different than GRT), e.g. to Nonsymmetric Gravitation Theory put forward by J. W. Moffat (Moffat 1982). A scheme of this type has been implemented and interesting effects were obtained of the interference type, that is: removing the singularity from a Coulomb-type solutions (Coulomb potential) and a natural dielectric confinement model. This approach combines in itself two (completely independent and considered hitherto as orthogonal) unification schemes: Einstein's Nonsymmetric Field Theory and that of Kaluza–Klein (Jordan–Thiry). The geometry defined on multi-dimensional manifold constitutes the multi-dimensional counterpart of a geometry from Einstein's Nonsymmetric Field Theory. On the space-time we have the geometry (known from literature) from the Nonsymmetric Field Theory. The unification is similar as was the case of Riemannian, classical approach. The case of Jordan–Thiry, where a scalar field appears is also considered. This field as usually is a component of the effective gravitational constant.

Of interest here should be the presentation of motivations for using non-Riemannian connections on the bundle manifold and on a space-time. This is dictated by troubles of a Riemannian Kaluza–Klein theory, prominently occurring in the non-Abelian case. The using of the non-Riemannian geometry on a space-time is associated with the passing from the General Relativity Theory to an alternative theory for gravity, the so called Nonsymmetric Gravitation Theory. In principle the topic under consideration presents a natural extension of a corollary from Kaluza–Klein theory with torsion, where on the space-time we would have defined the theory of Einstein–Cartan whereas on a multi-dimensional gauge manifold — the multi-dimensional counterpart to the geometry of this theory. In the present case we deal with another theory for gravity and another geometry. We substitute General Relativity Theory with another gravitational theory (the selection criteria are to be given later), generalize the geometry of this theory to a multi-dimensional case and apply it to a multi-dimensional gauge manifold. Further we continue with the whole procedure *à la* Kaluza–Klein (Jordan–Thiry). We compute the curvature, the torsion and investigate the equations for the geodesics, in search for their interpretation. We write down the Lagrangian of a theory (scalar curvature) and derive the field equations from Palatini principle. We look for the interpretations of new elements of the theory. One important property is the fact that obtaining the same physical interpretations of the higher torsions in the theory as before. They equal the electromagnetic polarization. The only difference being, that whereas in the previous case there was the electromagnetic (Yang–Mills) polarization of the sources, now it is caused by an antisymmetric part of a tensor $g_{\mu\nu}$, from the Nonsymmetric Gravity Theory. In this fashion the torsion propagates in this theory.

The Nonsymmetric Gravity Theory is an alternative theory for gravity basing on the non-Riemannian space-time geometry. Two types of gravitational field sources occur in this theory, that is two kinds of "gravitational charges". They are-mass and fermionic charge. Because of that, this theory predicts gravitational acceleration of a test body in a gravitational field other than that from Newton's theory. It is given by the formula:

$$G_N = c = 1$$

$$\vec{a} = \frac{m\vec{r}}{r^3} + \frac{2m^2\vec{r}}{r^4} + \frac{2l^2\vec{r}}{r^6} - \frac{l_p^2 l^2 m\vec{r}}{m_p r^6} \tag{4.6}$$

where

m — mass,

l^2 — fermion charge,

m_p — test body mass,

l_p^2 — test body fermion charge.

The case is also being considered containing the spontaneous symmetry breaking and Higgs mechanism, by using the dimensional reduction technique for a principle fibre bundle constructed over the extended space-time. The usage of geometry from the Nonsymmetric Field Theory of A. Einstein is a new idea, and is associated with the new interpretation of the formalism contained in this theory. Nonsymmetric Gravitation Theory constitutes this new interpretation. Without that interpretation the whole procedure would be meaningless. For the case of the five-dimensional Kaluza–Klein theory using the same geometry with the old interpretation, we would get a redundancy. The electromagnetic field would be described two times, once as a part of Nonsymmetric Field Theory and the second time as the connection on a fibre bundle. But using the new interpretation of a Nonsymmetric Field Theory as an extended gravitation theory we avoid that type of inconveniences. Due to on that, we have obtained the unification of the Nonsymmetric Gravitation Theory, Yang–Mills theory, spontaneous symmetry breaking, Higgs mechanism and of the scalar field responsible for the effective "gravitational constant". Even if we should adopt the vanishing gauge field, the pure theory of gravitation thus obtained will be different from the nonsymmetric theory of gravity, due to the possibility of considering the effective gravitational constant, as for instance in Brans–Dicke theory.

In order to explain the merits and advantages of the approach adopted here, let us note that there are two basic schemes for the geometrical unification of gravity (described by GRT) and electromagnetism. The first of them is based on five-dimensional extension of GRT and is universally known as Kaluza–Klein theory. The second scheme comes from A. Einstein, uses non-Riemannian geometry defined on a space-time with the aid of a nonsymmetric tensor. It is universally called a Nonsymmetric Field Theory. The first scheme, in its modern version uses a fibre bundle as a mathematical model of a gauge field and by performing the metrization of this bundle, brings classical Kaluza–Klein results. Kaluza–Klein principle as a matter of fact is based on Riemannian geometry defined on a 5-dimensional manifold space and a structure generated by the electromagnetic bundle plus Riemann geometry on a space-time. At the heart of the Kaluza–Klein theory lies the reduction (unification) of the two basic physical invariance laws, that is gauge invariance and a generally covariant Einsteinian invariance. The first of them is fundamental in electrodynamics, the second in General Relativity Theory. Such a reduction is possible in a five-dimensional world, what has been used in an original Kaluza–Klein approach. At present we are aware that gauge invariance plays a fundamental role in weak interactions (Glashow–Salam–Weinberg model) and the strong ones (QCD). In both cases mentioned it is a principle based on non-Abelian gauge groups. In GUT (Grand Unified Theories) we also have to deal with a non-Abelian gauge invariance. Therefore the creation for a Kaluza–Klein theory seems to be something very interesting. Under the term "realistic" we mean one containing the physical interactions occurring in nature and giving the interference effects between the gravitational and gauge fields. Non-Abelian Kaluza–Klein theory, basing on the natural metrization of the principal fibre bundle (serving as a model for a gauge field) was created relatively long ago. This is in a matter of fact a multi-dimensional ($(n+4)$-dimensional, n — dimension of the gauge group) General Relativity Theory with the equations for vacuum. It has two fundamental inconveniences. For one, it predicts too big cosmological constant (10^{127} times bigger than the upper limit of the experimental data) and in addition to that fails to introduce some new effects in comparison with General Relativity Theory and the theory of gauge field (Yang–Mills). Both inconveniences are undoubtedly strictly associated with the using of Riemannian geometry on a multi-dimensional manifold (bundle of bases over the metrized fibre bundle). Taking into account here of the naturally metrized fibre bundle *à la* A. Trautman and W. Tulczyjew seems fundamental, nonetheless Riemannian geometry assumption should be discarded. There are a lot of approaches which resign from Riemann geometry and are thus able to avoid too big a cosmological constant, reducing it down to zero. In other approaches one had resigned from Riemann geometry even for the case of electromagnetism, substituting it with a geometry from Einstein–Cartan theory. This resulted in composition of the two known extensions of the classical General Relativity Theory: Einstein–Cartan and Kaluza–Klein theories. In this fashion, additional predictions not envisaged neither in Einstein-Cartan nor by Kaluza–Klein mentioned earlier had been obtained. Here belong new Cartan equations linking the torsion in the fifth dimension and the electromagnetic polarization of the sources, additional contact terms in the energy-momentum tensor of the (electromagnetic moment) $\times$ (electromagnetic moment) type, electric field's energy-momentum tensor in the form given by W. Israel and the additional electric current associated with spin. This approach could be without obstacles extended onto a case of arbitrary gauge group. It has been called Kaluza–Klein theory with torsion. We were discussing more widely this topic during our remarks concerning the geometrical unification of the two theories T_1 and T_2. One gets the feeling therefore that taking into account non-Riemannian geometries in the generalized Kaluza–Klein theories is reasonable and interesting. Kaluza–Klein theory has very simple and natural generalization based on Riemannian geometry too. At its heart lies the abandoning of the so called Kaluza Ansatz, that is the condition $g_{55} = -1$, assuming instead that $g_{55} = \phi(x)$, where $\phi(x)$ is a scalar field defined on a space-time. This generalization is called the Jordan–Thiry theory and results in coupling between the electromagnetic field, scalar field and General Relativity Theory. The scalar field enters in a non-trivial manner to the equations and reveals itself as an effective “gravitational” constant independent on the space-time point. This theory is very much like the Brans–Dicke scalar-tensor theory with the electromagnetic sources. Its extension for the case of non-Abelian gauge group is easy to come by. Then the scalar field couples itself with cosmological constant. Hence, there the same problem arises as was the case with Riemannian Kaluza–Klein theories. Jordan–Thiry theory has yet another drawback. Namely, the scalar field ϕ in some approaches is the so called “ghost” — that is, it has a negative kinetic energy in the Lagrangian of this theory. It seems also that this inconvenience might be avoided by passage to the non-Riemannian geometry on multi-dimensional manifold. Thus, the search for a realistic Kaluza–Klein theory should be based mainly on the choice of a suitable non-Riemannian geometry defined on multi-dimensional bundle manifold. This geometry is to be of a type similar to that on a space-time. One might see therefore that Kaluza–Klein theory with torsion, which was mentioned above, fits this criterion.

Other approaches modify geometry on a multi-dimensional bundle keeping Riemannian geometry on a space-time. The change of geometry on a space-time means the abandoning of General Relativity Theory and substituting it with one of the alternative theories for gravity. There are however not so many alternative gravitation theories which could be “valiable” according to a classification put forward by C. Will. In addition to that, it has to be a theory of gravity with a Lagrangian linear in curvature, since otherwise we would have been forced to take into account e.g. quadratic Lagrangians in Kaluza–Klein, which would eliminate the so called “Kaluza–Klein miracle” — that is the appearance of a Yang–Mills field Lagrangian. The majority of the alternative theories for gravity (those with quadratic Lagrangian included) are only partially geometrical theories. They comprise certain non-geometric elements, which spoil the General Relativity Theory’s elegance and most often contradict the observations or experimental data. Alternative theory of graviton will include in itself General Relativity Theory as a limit or special case and be identical with it in these instances where General Relativity Theory fits the observational data. The examples of Einstein–Cartan and Kaluza–Klein with torsion prove that it is possible. An additional criterion to be fulfilled by such a theory is this: it should possess additional sources for geometry, capable of being interpreted in terms of interactions other than gravitational ones, that is to say — there should be conserved currents, associated with internal (non-space-time) symmetries. The gravitational theory with a tele-parallelism which might have been taken into account upon carrying through of such a procedure mentioned above, does not satisfy this criterion.

Einstein–Cartan theory by introducing spin as an additional material source also fails to comply with the said conditions, in spite of the fact that spin is very closely associated with the properties of the elementary particles. Nonetheless spin constitutes a purely space-time — like quantity. The supergravitational extension of GRT is of the same type as Einstein–Cartan theory. It is even equivalent to this theory when we limit ourselves to Rarita–Schwinger sources (a spin 3/2 spinor field). One should rather look for supergravitational extension of the alternative gravity theory just mentioned, or look for its Einstein–Cartan type extension. The only theory suitable for that purpose is Nonsymmetric Gravitational Theory put forward by J. W. Moffat, or to be precise, its geometrical foundation, being a reinterpretation of a geometry from Einstein's Nonsymmetric Field Theory. Because of this, it has well defined geometry, displaying certain common features with Riemannian geometry. This theory has a Lagrangian linear in curvature and introduces an additional material source for the space-time geometry — the current associated with the fermion charge. The adoption of a Lagrangian linear in curvature for a gravitational field is dictated by a kind of simplicity rule. Once adopted in this form, the Lagrangian is linear (and hence the simplest) interaction of gravitational potentials gap with space-time geometry and also covariantly invariant. In a search for a realistic Kaluza–Klein theory (Jordan–Thiry) one has to geometrize the spontaneous symmetry breaking and Higgs mechanism in its language. This is possible once the extension of the space-time with the additional manifold of degenerated vacuum states is performed and after applying the dimensional reduction on a gauge group principal bundle with the extended space-time being its basis. Further one has to endow the bundle manifold with Riemannian geometry, in our case it will be a geometry from Einstein's Nonsymmetric Field Theory in its real form. In order to understand why we could use this geometry in our approach, let us go on to characterize the second scheme for the unifying of gravity and electromagnetism. This scheme, put forward by A. Einstein uses non-Riemannian geometry defined on the space-time. As a basic quantity, this geometry comprises a nonsymmetric tensor defined on the space-time. This tensor induces in a unique fashion a linear connection on the space-time. This is a nonsymmetric connection (torsion is not zero). Also the Ricci tensor is nonsymmetric. One gets the equations of this theory from Palatini principle for the curvature scalar obtained as a result of contracting Ricci tensor with a nonsymmetric tensor $g_{\mu\nu}$.

There is a problem of quantization of such theories. This can be achieved by an extension of the Ashtekar–Lewandowski method or using nonlocal quantization method by Efimov or Yukawa.

The relationship between the connection and the tensor $g_{\mu\nu}$ could be obtained as an equation of motion on the basis of the variational principle mentioned. Traditionally, this definition is taken as a definition of connection in the theory under consideration. There is a large bibliography, a purely mathematical one devoted to the investigation of properties of geometry defined in the above fashion. Two possibilities were analyzed. Under the first, the tensor g was complex Hermitean (as a matrix) while under the second it was purely real. In both these cases, the quantity refereed to above has two independent components: the symmetric $g_{(\mu\nu)}$ and the antisymmetric one $g_{(\mu\nu)}$. According to the established tradition, the first of them is being called Einstein–Strauss theory, while the second Einstein–Kaufman. The hope which inspired the creators of both the theories was to obtain the equations for electromagnetism and gravity from the vacuum equations of the theory. The unification programme has also assumed the obtaining of the equations of motion via a method similar to Einstein–Infeld–Hoffmann for the charged particles and the derivation of Lorentz force. The basic problem which appeared from the very outset here, was the question how to construct the space-time metric (the metric tensor familiar to us in GRT) and the electromagnetic field tensor $F_{\mu\nu}$ starting from an original quantity, that is the nonsymmetric tensor $g_{\mu\nu}$. A lot of possibilities were considered and the obvious choice of $g_{(\mu\nu)}$, as the space-time metric and of $g_{\mu\nu}$ as $F_{[\mu\nu]}$ was not an only one. Summing up, all of these approaches had had some drawbacks. It looked like this theory could not describe electromagnetism. Currently one has managed to overcome these difficulties by adopting additional assumptions concerned with the nonsymmetric connection. They enable us to interpret the above theory as a macroscopic theory of gravity and electromagnetism. The relationship between the space-time metric and the nonsymmetric tensor $g_{\mu\nu}$ reveals itself through a complicated first order differential equation, which only makes sense for a certain subclass of tensors $g_{\mu\nu}$. The electromagnetic field tensor $F_{\mu\nu}$ is being identified here with $R_{[\mu\nu]}$ — the antisymmetric part of Ricci tensor.

There is however yet another attempt to applying the scheme of Einstein–Strauss or Einstein–Kaufman theories. This approach relies on a certain new achievement in physics, unknown in A. Einstein's times. This is a Nonsymmetric Gravitational Theory put forward by J. W. Moffat. In his interpretation the gravitational field is more complicated and described by two kinds of potentials: $g_{(\mu\nu)}$ — the metric and $g_{[\mu\nu]}$ — the antisymmetric tensor. This antisymmetric tensor has nothing to do with the electromagnetism. It was proved that in a linear approximation of the equation, the spinor composition of the theory is $(2, 0)$, that is a graviton plus a particle with a spin zero called "skewon", associated with $g_{[\mu\nu]}$. There is no room here for the electromagnetism since a photon caries a spin of 1. The notion of sources for the gravitational field obtains an extended sense in this theory. Two types of gravitational charges — mass and fermion charge, usually determined as $F = B - L$, where B — the baryon number, L — the lepton number. In classical gravitational theory mass as a measure for the quantity of matter, is a source of the gravitational field. However, we know from investigations in the area of Grand Unified Theories (GUT) for the elementary particles, that there is yet another "measure of the quantity of matter". It is just the fermion charge. It is a conserved quantity, and as scalar, that is additive quantity assumes non-zero and significant magnitudes for the macroscopic bodies consisting of matter, that is from fermions (without anti-fermions). Roughly speaking, the fermionic charge of a macroscopic body will be proportional to the quantity of neutrons in this body. The universal nature of gravitational interactions is being conserved here, because both gravitational charges are measures of the quantity of matter. In this way there appear two sources in the theory mentioned above: energy-momentum tensor (in general nonsymmetric one) and the fermion current. The introducing of the second type of gravitational charge — the fermion one is associated with the appearance of an additional coupling constant similar to Newton's constant. The theory satisfies the Birkhoff theorem. Spherically-symmetric solution of the field equations (reinterpreted spherically-symmetric solution of the field equations in Einstein's Nonsymmetric Theory, found by A. Papapetrou and J. R. Vanstone) provides corrections to the motion of a test particle in a gravitational field. These corrections could be associated with the relativistic corrections to GRT in Schwarzschild field. The difference comes from the new post-Newtonian correction, associated with the fermionic charge. It is a higher order correction. In Einstein's Unitary Field Theory it was one of the fundamental obstacles. For by interpreting the solution as a geometrized field — the gravitational field and the electrical one of a charged body, one had hoped to get a term with Lorentz force stemming from Coulomb field acting onto the test particle. Here it is rather a merit, since otherwise we would be led to contradiction with the experiment. As a matter of fact we do not observe long-range Coulomb forces proportional to the quantity of neutrons inside a body. The correction mentioned above has a noticeable influence onto the perihelion advance of Mercury, provided that we employ the Nonsymmetric Theory of Gravitation for Solar System description. This enables us to evaluate the new coupling constant occurring in the theory. By associating this with the new data concerning the quadrupole moment of mass of the sun, we could get an approximation of that constant. Additional terms could play a significant role only in a very strong gravitational fields e.g. in closed binary systems, gravitational collapse and the early cosmological phases. It had in fact been used in such cases and has managed to clarify certain discrepancies between the observation and theory for DI Hercules case (closed binary), X — ray bursts, etc.

In the case of cosmological models it gives interesting outcomes for the problems of the inflationary Universe and cosmological horizon. This theory has a well posed Cauchy problem. J. W. Moffat and collaborators have managed to elaborate the PPN formalism for this theory. They examined also the models of stars described by it. A major issue here is the problem of predicting the gravitational radiation in this theory. It turns out that in its linear version there is only the quadrupole radiation, given by the same formula as in GRT. The dipole radiation occurring here, is being only obtained for higher order of the coupling constant. For that reason the theory remains in a complete agreement with the observational data originating from the pulsar (closed binary) PSR1913 + 16. As we know, these data had falsified many from among the alternative theories for gravity e.g. Rosen's bimetric theory. One significant thing which makes the Nonsymmetric Gravitational Theory similar to GRT is the fact that the test particles and light move according to it along the Riemannian geodesics generated by a symmetric part of the tensor $g_{(\mu\nu)}$. One could obtain this result provided that suitable assumptions about the energy-momentum tensor are made. Using the non-Riemannian geodesics also leads to the successes. It seems that these geodesics are nearer the generalized Galileo principle and that one should modify this theory in this spirit. There is also an attempt to extend this theory onto the cases of sources with spin similar to the Einstein–Cartan theory. The black hole in this theory also has been the subject of consideration by methods similar to these of Hawking and Wald in GRT. It is interesting to note that this theory in one of its formulations uses the hypercomplex metric. This version is equivalent to one where $g_{\mu\nu}$ is nonsymmetric, but real. In the original versions of it also the complex case was considered. It was abandoned because of the following reasons. In a linear approximation there appeared in this case the so called "ghosts" that is the particles with negative energy or tachyons. Only the purely real case or hypercomplex one allows to avoid these contradictions. The hypercomplex version could not be extended in a consistent way to Kaluza–Klein case.

The behaviour of an antisymmetric part of the tensor $g_{[\mu\nu]}$ presents an interesting phenomenon in this theory. In linear approximation this field behaves like a generalized Maxwell field, that is Abelian gauge field with two subscripts. It is described by a massless two-form $h = h_{[\mu\nu]}\, dx^\mu\, dx^\nu$ and its strength $F = dh = h_{[\mu\nu,\lambda]}\, dx^\mu \wedge dx^\nu \wedge dx^\lambda$. The Lagrangian of this field

$F \wedge F$ is similar to Kalb–Ramond Lagrangian in string theory. This justifies the hope that this theory has something in common with strings or even superstrings. In nonsymmetric Kaluza–Klein theory one obtains the action of a generalized Maxwell field with non-Abelian gauge fields (Yang–Mills).

We see therefore that the alternative gravitation theory based on the formalism of Einstein's Nonsymmetric Field Theory has a lot of interesting properties. It is undoubtedly a "valiable theory" to use C. Will's terminology concerning the alternative theories of gravitation. It seems however that there are questions in the theory, which one cannot satisfactorily answer exclusively within the theory itself. These questions are associated with the very nature of the second gravitational charge, i.e. the fermion charge. This charge comes from a theory of elementary particles within GUT framework and e.g. in a theory based on SO(10) group is one of its generators. The local gauge symmetry which corresponds to this charge is being spontaneously broken by Higgs mechanism. This symmetry is: $\mathrm{U}(1)_F = \mathrm{U}(1)_{B-L}$. To this symmetry there corresponds a massive intermediate boson A^F_μ, the part of Yang–Mills field with gauge group SO(10). Of course it is possible to consider groups of higher rank. The relationships between a non-Abelian gauge field could not be explained by a formalism based only on the nonsymmetric connection defined on the space-time. Therefore the need arises for extending it to a kind of geometric scheme which would naturally include nonsymmetric geometry and gauge fields. Nonsymmetric Kaluza–Klein theory (Jordan–Thiry) constitutes the extension of this sort.

During the last several years the nonsymmetric Kaluza–Klein theory has been elaborated together with its extension to a nonsymmetric Jordan–Thiry theory. Also were found the non-Abelian extensions of nonsymmetric Kaluza–Klein and Jordan–Thiry theories. The nonsymmetric Kaluza–Klein theory has been extended for the case of non-vanishing external sources (including also spin). Spontaneous symmetry breaking has been introduced as well as the Higgs mechanism (the method of dimensional reduction). Linear versions of both theories, that is nonsymmetric Kaluza–Klein and nonsymmetric Jordan–Thiry were found. Also the first exact solutions of the equations of Kaluza–Klein theory were given. The results of the preceding works were also successfully extended onto the case of a nonsymmetric Kaluza–Klein theory, that is the obtaining of a dipole momentum of electric fermion and the breaking of CP. Nonsymmetric Kaluza–Klein theory (Jordan–Thiry) implements the real unification of gravitation and gauge fields in the following sense. In this theory we obtain interference effects between the gravitational field and gauge fields (electromagnetic one, in a 5-dimensional case) which are missing in a traditional approach. We have the interference effects as follows:

1) Additional term in a Lagrangian of an electromagnetic field equal $2(g^{[\mu\nu]}F_{\mu\nu})^2$ (for the gauge field $2(h_{ab}H^a H^b)$, $H^a = g^{[\mu\nu]}H^a_{\mu\nu}$, where $H^a_{\mu\nu}$ is the Yang–Mills field strength, the bundle curvature).
2) New energy momentum tensor for electromagnetic field (gauge field). There are several equivalent forms of that tensor.
3) Two electromagnetic field strength tensors (gauge field), that is $F_{\mu\nu'}H_{\mu\nu}(= H^a_{\mu\nu'}L^a_{\mu\nu})$. One of them represents a usual strength (bundle curvature) constructed from $\vec{E}$, $\vec{B}$ $(\vec{E}^a, \vec{B}^a)$. The second in turn is a tensor with taking into account of polarization $\vec{D}$, $\vec{H}$ $(\vec{D}^a, \vec{H}^a)$. The relationship between the two tensors could be considered as generalized material relationships and given a formula (for the first case).
$$g_{\delta\beta}g^{\gamma\delta}H_{\gamma\alpha} + g_{\alpha\delta}g^{\delta\gamma}H_{\beta\gamma} = 2g_{\alpha\delta}g^{\delta\gamma}F_{\beta\gamma}. \tag{4.7}$$
It is clear that for $g_{\alpha\beta} = g_{\beta\alpha}$ we have $H_{\mu\nu} = F_{\mu\nu}$. This is is going to prove that a gravitational field described by a nonsymmetric theory of gravity serves as a polarization source. The gravitational field therefore behaves like an electromagnetic polarization conserving medium (Yang–Mills in general case).
4) The source (current) for the second pair of Maxwell's (Yang–Mills) equations that is the current $i_\mu(i^\alpha_\mu)$. This current for the electromagnetic case is being identically conserved which proves that it might have been associated with certain topological properties (topological current).
5) Vacuum polarization (due to the gravitational field)
$$M_{\mu\nu} = -\frac{1}{4\pi}(H_{\mu\nu} - F_{\mu\nu})$$
$(M^\alpha_{\mu\nu} = -\frac{1}{4\pi}(L^a_{\mu\nu} - H^a_{\mu\nu})$, in a general Yang–Mills case). This polarization has the interpretation of torsion in higher dimensions.
$$\begin{aligned} Q^5_{\alpha\beta} &= 8\pi M_{\alpha\beta} \\ Q^a_{\alpha\beta} &= 8\pi M^a_{\alpha\beta} \end{aligned} \tag{4.8}$$
Equation (4.7) can be solved with respect to $H_{\nu\mu}$
$$H_{\nu\mu} = F_{\nu\mu} - \widetilde{g}^{(\tau\alpha)}F_{\alpha\nu}g_{[\mu\tau]} + \widetilde{g}^{(\tau\alpha)}F_{\alpha\mu}g_{[\nu\tau]}.$$

However the form of Eq. (4.7) is easier to handle from theoretical point of view. We get

$$Q^5_{\mu\nu} = 8\pi M_{\mu\nu} = 2\widetilde{g}^{(\tau\alpha)}\big(F_{\alpha\mu}g_{[\nu\tau]} - F_{\alpha\nu}g_{[\mu\tau]}\big), \quad \text{and}$$

$$\begin{aligned}
L^n{}_{\omega\mu} &= H^n{}_{\omega\mu} + \mu h^{na}k_{ad}H^d{}_{\omega\mu} + \big(H^n{}_{\alpha\omega}\widetilde{g}^{(\alpha\delta)}g_{[\delta\mu]} - H^n{}_{\alpha\mu}\widetilde{g}^{(\alpha\delta)}g_{[\delta\omega]}\big)\\
&- 2\mu h^{na}k_{ad}\widetilde{g}^{(\delta\tau)}\widetilde{g}^{(\alpha\beta)}H^d{}_{\delta\alpha}g_{[\tau\omega]}g_{[\beta\mu]} - 2\mu h^{na}k_{ad}\widetilde{g}^{(\delta\beta)}\widetilde{g}^{(\alpha\tau)}H^d{}_{\beta[\omega}g_{\mu]\tau}g_{[\delta\alpha]}\\
&+ 2\mu^2 h^{na}h^{bc}k_{ac}k_{bd}\widetilde{g}^{(\alpha\beta)}H^d{}_{\alpha\left[\omega\right.}g_{\left.[\mu]\beta\right]}
\end{aligned}$$

$$\begin{aligned}
Q^n{}_{\omega\mu} &= -2\Big(\mu h^{na}k_{ad}H^d{}_{\omega\mu} + \big(H^n{}_{\alpha\omega}\widetilde{g}^{(\alpha\delta)}g_{[\delta\mu]} - H^n{}_{\alpha\mu}\widetilde{g}^{(\alpha\delta)}g_{[\delta\omega]}\big)\\
&- 2\mu h^{na}k_{ad}\widetilde{g}^{(\delta\tau)}\widetilde{g}^{(\alpha\beta)}H^d{}_{\delta\alpha}g_{[\tau\omega]}g_{[\beta\mu]} - 2\mu h^{na}k_{ad}\widetilde{g}^{(\delta\beta)}\widetilde{g}^{(\alpha\tau)}H^d{}_{\beta[\omega}g_{\mu]\tau}g_{[\delta\alpha]}\\
&+ 2\mu^2 h^{na}h^{bc}k_{ac}k_{bd}\widetilde{g}^{(\alpha\beta)}H^d{}_{\alpha\left[\omega\right.}g_{\left.[\mu]\beta\right]}\Big)
\end{aligned}$$

in a general Yang–Mills case.

6) Additional term in the equation of motion of the test particle, that is, one of the form of Lorentz force in the electromagnetic case.

(Kalinowski 1990, 1991, 1992, 2014, 2015a, 2015b, 2016)

This term has the interpretation of the reaction force for the non-holonomic constraints. In the non-Abelian case, we get a comparable term modifying the Kerner–Wong equation. In the Jordan–Thiry case we obtain also: 1. Lagrangian for the scalar field ψ. 2. Energy momentum for that field (ψ). 3. Additional scalar forces, modifying the motion of a test particle (generalized Kerner–Wong equation). The scalar field ψ is associated with the effective gravitational constant via the formula

$$G_{\text{eff}} = G_N \exp(-(n+2)\psi) \tag{4.9}$$

where G_N is Newton's constant, and n is the dimension of the gauge group. Scalar field energy momentum sensor has a non-vanishing trace, which testifies that this is probably a massive field, acquiring the mass in time consequence of its interactions with other fields.

These additional effects, labelled as interference ones, do not lead to contradictions with either experimental or observational data for the weak fields case. They bring instead interesting effects at the level of exact solutions. It was due to these effects that one managed to find nonsingular solutions (in the electromagnetic and gravitational fields), which asymptotically approached towards the solutions known from the nonsymmetric gravitational theory. The search is under way to find the stationary axially-symmetric cosmological solutions.

Let us return to the dynamics of the geometrized theories, or to the means of obtaining Einstein type equations of motion or the second pair of Maxwell's equations. In the case of Maxwell's equations; it consists in choosing the suitable Lagrangian $L(S, P)$. We consider here the nonlinear electrodynamics. The relationship between $F_{\alpha\beta}$ and $H_{\alpha\beta}$ is given by

$$H_{\alpha\beta} = -\frac{\partial L}{\partial F_{\mu\nu}} = \frac{\partial L}{\partial S}\, F_{\mu\nu} + \frac{\partial L}{\partial P}\, F^{*}_{\mu\nu}\,. \tag{4.10}$$

This relationship is very interesting since $F_{\alpha\beta}$ has the meaning of curvature and in the kinetic picture of the theory is related to a four-potential A_μ (a connection). This relationship might be extended for the case of gravity by choosing instead of $F_{\alpha\beta}$ the curvature of the bundle associated with gravity and taking in place of L — the corresponding purely gravitational Lagrangian, e.g. the quadratic one.

Let us return now to the nonlinear electrodynamics and note that the arbitrariness in choice is in principle unlimited.

Born–Infeld choice

$$L(S, P) = b^2\left(1 - \sqrt{1 - \frac{2S}{b^2} - \frac{P^2}{b^4}}\right), \quad b = \mathrm{const} \tag{4.10a}$$

and Maxwell's

$$L = S \tag{4.10b}$$

are in a sense privileged. Namely, they are the only Lagrangians which do not result in bireffraction of the vacuum. What does the bireffraction of the vacuum mean? In general for the nonlinear electrodynamics there are two types of electromagnetic radiation with two different light cones. One of this cones is inside the Minkowski cone, the other is outside it. Therefore one type of radiation gets propagated with the superluminal velocity. This phenomenon is similar to a bireffraction of the cristals (e.g. icelandic spat), whence the name. It is an interesting thing to investigate the solutions of GRT with electromagnetic sources described by Born–Infeld electrodynamics. It turns out that we arrive at solutions without singularities in a gravitational field, asymptotically of Schwarzschild type. These solutions also do not have singularities in the electric field. At this point let us observe that there is an interesting relation between Born–Infeld electrodynamics and the Nonsymmetric Field Theory. Namely $L(S, P)$ from (4.10a) could be rewritten in the form

$$\sqrt{\det(\eta_{\mu\nu} + bF_{\mu\nu})} - \sqrt{\det(\eta_{\mu\nu})}$$

where $\eta_{\mu\nu}$ is a Minkowski tensor.

Let us note also that the Lagrangian of a Maxwell's electrodynamics is quadratic in the fields and confront this fact with the linearity of gravitational Lagrangian in GRT. From this point of view we shall compare the dynamics of the gravitational and electromagnetic fields. To that end we'll try first to inquire whether one could consider the theory of gravity as a gauge field theory. In accordance to what was said above, the changes of coordinate system-general Einsteinian invariance constitutes the counterpart of gauge transformations of the second kind. What concerns the gauge group — it should be Lorentz or Poincaré one. It turns out that this problem is not unique and depends on what we decide to take as a basic quantity, metric tensor $g_{\alpha\beta}$ or connection $\omega^{\alpha\beta}(\Gamma^{\alpha}_{\beta\gamma})$. In the case when we take $g_{\alpha\beta}$ as basic quantity and formulate the theory with the help of tetrad h^{α}_{b}.

$$g_{\alpha\beta} = h^{a}_{\alpha} h^{b}_{\beta} \eta_{ab} \tag{4.11}$$

($\eta_{ab} = \mathrm{diag}(-1, -1, -1, +1)$, η_{ab} — Minkowski tensor). Then it shall turn out that the gauge group is A^4-translation group in 4-dimensional space. Here we might associate with h^{a}_{β} the Yang–Mills field in accordance with Kibble's Ansatz.

$$h^{a}_{\beta} = \delta^{a}_{\beta} + A^{a}_{\beta} \tag{4.12}$$

Under certain assumptions about the form of a Lagrangian, one could obtain GRT out of this. Let us note that the choice of A^4 as a gauge group for the gravitational field confirms the observation in linear theory of gravity. Namely the transformation

$$h_{\mu\nu} \to \overline{h}_{\mu\nu} = h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)} \tag{4.13}$$

constitute a natural generalization of the electromagnetic gauge for the tensor $h_{\mu\nu}$. This transform could be obtained in a linear approximation of GRT from Einsteinian invariance: Here from Einstein equations we obtain d'Alembert equation for

$$\begin{aligned} \overline{h}_{\mu\nu} &= h_{\mu\nu} - \frac{1}{2} h\eta_{\mu\nu}, \quad h = h^{\alpha}_{\alpha} \\ -\Box\overline{h}_{\mu\nu} &= 2\kappa T_m, \quad \frac{\partial\overline{h}_{\mu\nu}}{\partial x_{\mu}} = 0 \end{aligned} \tag{4.14}$$

Thus, linear theory of graviton is analogous to Maxwell's electrodynamics. Another ground for considering the gravity as gauge field would be adopting of ω^{α}_{b} as a connection on a bundle of orthogonal bases over the space-time. Then Lorentz group in a natural way will become a gauge group for the gravitational field as Yang–Mills field. Of course, the simultaneous adoption of both view-points together would be possible. Namely, let the tetrad h^a_α and connection ω^α_β describe gravitation at the same time. The relation between the metric tensor and connection given by the condition of its metricity could be considered as a condition for constraints. In the latter case, we need not to require the vanishing of the connection's torsion. Under such circumstances Poincaré group becomes the gravity gauge group. In this case let us notice that there are fundamental differences between the gravitational gauge theory and any other gauge group theory e.g. the electromagnetic one. Poincaré gauge group is a symmetry of space-time and everything is subordinated to it, whereas the phase transformation generated by U(1) group applies only to the charged fields. Of course this follows from the universality of gravitational interactions. On the other hand the bundle of frames over the space-time is a natural bundle, in contrast to the electromagnetic bundle, where the fibres are arbitrarily glued to the space-time (it does not follow from the bundle structure). Let us note that from the gauge theory point of view vanishing of torsion in GRT appears as something very arbitrary. The generalization of GRT to Einstein–Cartan theory seems to be from this point of view very natural. This generalization might also be dictated by the fact that we need two parameters, mass and spin to mark Poincaré group irreducible representations. In GRT mass constitutes a source for geometry, while in Einstein–Cartan theory a linear Lagrangian was chosen, leading to Einstein's equations and Cartan ones through Hilbert's variational principle. Of course other Lagrangians nonlinear in curvature are admissible, too. Here belongs Yang theory of quadratic gravity with a Lagrangian

$$L_{\mathrm{grav}} = R_{\alpha\beta} R^{\alpha\beta}. \tag{4.15}$$

Due to the quadraticity of L_{grav} in $R_{\alpha\beta}$ this is a theory corresponding to that of Yang–Mills, where the Lagrangian is quadratic. In accordance with on what we have said while discussing the nonlinear electrodynamics, we might associate with gravity a "second strength tensor" analogous to $H_{\alpha\beta}$ as a derivative of the Lagrangian with respect to the curvature associated with a gauge group-Poincaré one and write down the equation of motion for this tensor as well. It turns out that in a natural manner there are two curvatures, namely one associated with the tetrad h^a_α and the other with the connection ω^α_β. We get in this way two tensors $H_{\alpha\beta}$ and two equations of motion, the details of which depend on the form of the gravitational Lagrangian. One of these equations also leads to energy-momentum conservation law while the other — to the moment of momentum conservation law. Both conservation laws follow from the Bianchi identity. In the case where the Lagrangian of Einstein–Cartan theory is adopted as our Lagrangian — we get the equations of Einstein and Cartan. Under quite special choices of Lagrangians, we obtain the torsion propagation instead the algebraic relationship which we have in Einstein–Cartan theory. In spite of the great arbitrarity in choosing the gravitational Lagrangians, it looks like GRT and E.C. with their linear Lagrangians are similarly distinguished as are Maxwell's electrodynamics and classical Yang–Mills theory among the other gauge theories. Nonsymmetric Gravitation Theory is also a remarkable one among such other theories. At this point, I would like to note that Brans–Dicke scalar-tensor gravity belongs to the same class as GRT and E.C. in spite of the introducing of an additional element — the scalar field of the gravitational "constant". This quantity might obtain the geometrical interpretation in Jordan–Thiry type of unifying theories, as a part of a metric tensor on the multi-dimensional manifolds. There are some approaches which try to explain the 5-th force with the aid of a scalar field associated with a gravitational constant. Should this field assume mass, we could obtain a correction to Newton potential with Yukawa (exponential) behaviour; therefore it is finite-range one.

It is interesting to notice that A. Poincaré was in some sense a precursor of a geometrization of physics. However, even if he considered a geometrical language as appropriate for physics, he was very much tied to the Euclidean geometry of space and did not try to geometrize physical interactions. His philosophical attitude for geometry is possible to deduce from his conventionalism. H. Poincaré understood a difference between a mathematical 3-dimensional space and a physical one. A. Lubomirski (Lubomirski 1974) considers Poincaré's ideas in his book *Henri Poincaré's philosophy of geometry* (in Polish).

To conclude the above considerations, I would like to pay attention to a difference between the application of geometry in the theoretical physics and the unifying geometric schemes. The problem of geometrization of the fundamental physical interactions has arisen once General Relativity Theory was created, in spite of earlier attempts of using the geometrical methods in the theoretical physics. Namely, in the theoretical mechanics (the classical mechanics of Newton) certain the least action principles could have been expressed in the geometrical language. For that purpose certain abstract Riemannian manifolds were constructed in such a way that the principle of the least action would become a principle of the minimal (extremal) distance separating two points on this manifold. This could be achieved by introducing a suitable metric tensor associated with a given problem (e.g. a quadratic form of the kinetic energy). The physical system implemented the motion by choosing (from among on possible paths linking the starting and the terminal point) such a path, the motion along which would occur without any forcing. To put it simply, this motion presented a free-fall case on a given parametric manifold of independent coordinates. It was taking place along geodesic lines. Clearly the form of metric tensor depended on the choice of physical systems and varied quite widely. In addition to that, in a specific problem, the metric tensor was not a dynamic quantity, it played the role of a certain background. This metric tensor and the Riemannian connection associated with it had nothing to do with fundamental properties of the space-time. It remained Newtonian like the mechanics which was at the core of this whole theory. I would like to note that the idea of introducing a curved space (I discern it here from the space-time) originated due to Riemann himself, who was also interested with gravity, more specifically with the perihelion advance of Mercury. The attempt of fitting theory with the observational data, via the introduction of other gravitational potentials, the so called Riemann–Weber potentials is also due to him. This was, as we know, an unsuccessful attempt and we could refer to it as an anecdote at least due to the fact that this theory was in agreement with Mach principle. Therefore we see that A. Einstein was not the pioneer of geometrical methods in physics. One of his teachers H. Minkowski has introduced to the Special Relativity Theory the tensor and the space-time named after him. The contribution of D. Hilbert to General Relativity Theory is also worth mentioning here. He was the first who observed that the Lagrangian used by A. Einstein is equivalent to a scalar curvature R, and had formulated the variational principle in General Relativity Theory, bearing his name. The idea of associating the equivalence of the inertial and gravitational forces (plus of the general Einsteinian invariance) with the covariance of General Relativity Theory equations is also due to Hilbert. General Relativity Theory was the first attempt towards geometrization of the physical interactions. It was a conscientious geometrization, for the fact of an earlier geometrization of the electromagnetic field in Maxwell's electrodynamics remained for a long period unnoticed. This was after on caused by the nonexistence of suitable mathematical methods at the times of C. Maxwell's life. One has to admire the geniality of Maxwell's intuition, making such a choice among the different electromagnetic phenomenologies.

Recently some extensions of General Relativity to Aether existence appeared. It means, some researchers added to the theory a vector field (an Aether). In this way there is a preferred system of coordinates. This Aether is a dynamical field. These theories are called Einstein–Aether theories. These theories are geometrized from the very beginnings.

Let us give some remarks on Standard Model of fundamental interactions and elementary particles. The bosonic part of the model is based on Yang–

Mills fields with a gauge group $\mathrm{SU}(3)_c$ (QCD) and Glashow–Salam–Weinberg (GSW) model of electroweak interactions. The first part (QCD) describes strong interactions. GSW model contains $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ Yang–Mills fields and a doublet of Higgs' field. After a spontaneous symmetry breaking and Higgs' mechanism we get $\mathrm{U}(1)_{em}$ — gauge field, an electromagnetic field and massive $W^{\pm}, Z^0$ boson. We get also massive Higgs' boson. The bosonic part of SM model can be easily geometrized. Yang–Mills field with $\mathrm{SU}(3)_c$ describing strong interactions is mathematically a connection on a fibre bundle over a space-time. GSW model with Higgs' fields can be described as a connection with $G2$ group over $E \times S^2$ where S^2 is a two-dimensional sphere and E is a space-time. Supposing a SO(3) symmetry of the connection we get $\mathrm{SU}(2) \times \mathrm{U}(1)$-gauge field and a doublet of scalar fields with double quartic selfinteraction potential coupled to $\mathrm{SU}(2) \times \mathrm{U}(1)$ — Yang–Mills field. We get a spontaneous symmetry breaking and a Higgs' mechanism in the theory. This procedure, known as a dimensional reduction, geometrized GSW-model (a bosonic part of GSW). Thus geometrization and a SM model are strongly combined. One can get the further unification using Kaluza–Klein approach in the nonsymmetric version.

The Nonsymmetric Kaluza–Klein Theory is an example of the geometrization of fundamental interaction (described by Yang–Mills' fields and Higgs' fields) and gravitation according to the Einstein program for geometrization of gravitational and electromagnetic interactions. It means an example to create a Unified Field Theory. In the Einstein program now we have to do with more degrees of freedom, unknown in Einstein times, i.e. GSW model, QCD, Higgs' fields, GUT. Moreover, the program seems to be the same.

One of the directions in geometrization and unification is to use more general lagrangian than (4.15), i.e.

$$L_{\mathrm{grav}} = f(R, R_{\alpha\beta}). \tag{4.16}$$

In this case we should consider higher order field equations. Moreover, there is a different approach to such a theory. Due to Legendre transformation (advocated by J. Kijowski) we get ordinary General Relativity with additional scalar fields. This approach can be generalized to use a scalar field (as in Brans–Dicke theory) and also to higher dimensional theory. These scalar fields can be considered as a source of a dark matter (scalarons) and dark energy. J. Kijowski considers also more general lagrangians $f(R)$, $f(R_{\alpha\beta}R^{\alpha\beta})$, $f((R^{\alpha}{}_{\beta\mu\nu}\varepsilon^{\mu\nu\lambda\rho}R^{\beta}{}_{\alpha\lambda\rho})^2)$ where $R^{\alpha}{}_{\beta\mu\nu}$ means a curvature tensor (not necessarily Riemann–Christoffel tensor), $\varepsilon^{\mu\nu\lambda\rho}$ being a Levi-Civita pseudotensor. One can consider also some different lagrangians, i.e. $f(g_{\mu\nu}, R_{\alpha\beta})$. f means here an arbitrary function. It is possible of course using Legendre transformation techniques to define a new metric (symmetric) tensor and a Levi-Civita connection compatible with this tensor. The interpretation of this new connection and new tensor can be complex. They could not have clear physical interpretation. L. M. Sokołowski (Sokołowski 2007) is using $f(R)$, $f(g_{\mu\nu}, R_{\alpha\beta\lambda\delta})$ gravity theories in cosmology. He concludes that it is impossible to recover nonlinear Lagrangian gravity from pure cosmological observations because Friedmann Universe is so special. He concludes that Solar System observations are also blind for a difference between GRT and nonlinear gravities. Thus we cannot get correct theory of gravity from cosmology.

In GRT we have gravitational wave solutions (e.g. Robinson–Trautman solution). Such solutions describe very strong gravitational field. Moreover, we are looking for gravitational waves from linear theory of gravity (linearized version of GRT) using several very sophisticated detectors. These gravitational waves are very weak and we are up to now not successful.

In the Nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory we can find gravito-electromagnetic waves (e.g. generalized plane waves). We are looking for more sophisticated solutions, i.e. spherical waves and cylindrical waves. We are looking for axially symmetric stationary solutions in the electromagnetic case and in non-Abelian case. Also we are looking for such a solution in the case of Nonsymmetric Kaluza–Klein Theory with Higgs' mechanism and spontaneous symmetry breaking. They give us models of elementary particles (e.g. an electron). This is a holistic aspect of the Nonsymmetric Kaluza–Klein Theory. Let us give the following remark on classical Kaluza–Klein Theory. This theory unifies all known classical Physics, i.e. General Relativity and Maxwell–Lorentz electrodynamics. This unification is geometrical.

5 Dialectical Notion of Matter and the Geometrization of the Physical Interactions

This chapter is devoted to the geometrization of physics as interpreted from the dialectically-materialistic standpoint. There is a feeling that the idea of geometrization and unification of physics, developed in the preceding chapters, has an idealistic nature in the philosophical sense. However, a dialectically-materialistic interpretation of the very same notion seems also reasonable. Therefore we will bring the supporting evidence from the standpoint of dialectical materialism. In a matter of fact, the ideal geometrical forms of (Euclidean) geometry are found at the foundation of Plato's idealism. Plato and his followers were well aware that a triangle, a straight line, a point etc. in Euclid's geometry are not identical with the triangles, straight lines occurring in nature, or ones drawn on sand. They were equally aware that this has nothing to do with the appearance of the drawing. The same conviction is shared with them by contemporary draftsmen. That is why the ancient thinkers felt it necessary to create the notion of the ideal geometrical forms non-perfect counterparts of which are the forms appearing in nature. The furthering of the same notion lies at the very core of the famous story about the slaves in the cave. The linking of Plato's idea with the geometrization and unification programme presented thus far — would be unreasonable for at least two reasons.

For the first one, it is a fact that since the ancient times the geometry has come along evolutionary way and we no longer rely on drawing the elements of contemporary geometry on sand, as it might have been the case for Thales, Plato, or Archimedes. There is no approximate correspondence between the non-ideal geometry of everyday life and an ideal one within the realm of mathematical abstraction, where proven theorem would be exactly satisfied. One of Plato's arguments therefore becomes invalidated. Currently, the geometry is far more abstract and most often it has very little in common with the simple spatial relations of everyday life. Similar case as the information extraction about the properties of a right-angled triangle inscribed into the circle via the inspecting of the architectural forms will no longer be possible. It is highly unlikely that we shall get a theorem of algebraic geometry on the basis of this type of observations. This example with the triangle is associated with Thales; supposedly it was he who had come to the idea of proving of the said theorem just on the basis of the observation of this sort. At present, geometry is very much distant from the architectural forms. Nonetheless, there is in contemporary geometry still something left from this ancient geometry. This thing is constituted by a certain kind of geometric intuition, to such a degree different from the general mathematical one that we are still separating geometry from other branches of mathematics. We often adjoin to it such adjectives like analytical, protective, topological, differential or algebraic. Then it is being dictated by the fact that the problems being formulated in a given theory find this expression in a language very much like the geometrical one used in ancient times. It seems like the language in question has been derived from certain generalizations of everyday experience concerning the relationships between figures in space. The fact that contemporary geometry is able to describe successfully the physical interactions at the level of very deep structures of matter testifies only for the unity of matter, which admits the descriptions using similar means at the quite different level of its structure.

Another misconception is the one mentioned by V. I. Lenin in his work *Materialism and Empiriocriticism.* Namely, analyzing the problems accumulated as a result of "revolution in physics" and adopting a skeptical position against the exclamations envisaging "the crisis of materialism" V. I. Lenin chiefly emphasized the fact that it is completely inappropriate to mix this or other theory of matter with the concept of matter as a philosophical notion. Dialectical materialism is by no means to be reduced to a certain theory of matter. These theories could be variable and relative. Unchanging are however in dialectical materialism the claims to the effect that the matter constitutes the objective reality given to us in the perceptions. The matter is given to us in experiment and constitutes the objective source for human perceptions. It exists independently from human consciousness and finds its reflection in this consciousness. In this way, a fundamental difference between materialism and idealism, that is the issue: what is more basic — matter or thought, does not depend on the theory of physical matter. This issue could not be tackled solely on the basis of the structure of matter. According to this view, light, electromagnetic waves, fields and the elementary particles are all material. In this sense geometry is material, since it is being identified with the fields of physical interactions and probably could also be identified with the elementary particles, once they turn out to be exact solutions of the field equations. In this manner, geometry appears to be material, objective reality. It becomes material which is after on in consonance with the intuitive understanding of objective reality. Of course the interpretation of geometry along the lines of Plato's philosophy or neo-Platonism could still be possible and consistent. One should not follow a dialectic point of view if fells that the idealistic one fits better the reality.

The approach developed by dialectic materialism is now a part of history of philosophy as also Lenin's work *Materialism and Empiriocriticism* (Lenin 1967). Moreover, the problem and the work of V. I. Lenin were criticized by many prominent dialectic philosophers and politicians in Soviet Union, e.g. by Ï. V. Stalin. He called it "a storm in a teacup". Presently this work is in the history of philosophy (in particular, philosophy of natural sciences, i.e. physics) and in such a way it should be considered.

From the scientific point of view it is necessary to reject historiosophy (philosophy of history) by K. Marx, F. Engels, V. I. Lenin and Ï. V. Stalin and a brutal and criminal political practice deduced from it as nonscientific and false. Moreover, it is not necessary to reject all dialectical materialism which is connected to the critical rationalism and to a realistic trend in contemporary philosophy of science (philosophy of physics). In this way we agree with the last philosophical views by W. Krajewski (one of the greatest experts in marxist philosophy), see *Contemporary scientific philosophy: metaphilosophy and ontology* (in Polish) (Krajewski 2005). His last philosophical views are part of the so-called *scientism*.

The fundamental error committed in marxism historiosophy is as follows. They created a theory of development of human society. They supposed that after a revolution they would be able (being in a power, as leaders of dictatorship of the proletariat) to change the society using only administrative decisions (some of the leaders of the revolution were very skilled administrators) to communist society. This was a fundamental assumption (a dogma) of the so-called scientific communism. Simultaneously this was a fundamental error which goes to the brutal and criminal political acts (as in the October Revolution in Russia, after in Soviet Union). All these acts were justified by this *error in assumption* that they know laws of this development, as we know laws of thermodynamics. In this way marxism historiosophy is unscientific and false even if some of partial conclusions can be valid (of course without any political implications). The best way to call it is to say it was (*political*) *utopia in a power*. This could be considered as ridiculous if it were not tragic.

To be honest in our critique of marxism historiosophy we should mention two philosophers, i.e. Mao Zedong (also a leader of the revolution in China and a political leader—the Chairman Mao) (Mao Zedong 2009, 2014) and H. Marcuse. Mao Zedong added to marxism historiosophy a theory of a revolution as a fight of world's village against world's town (city) or the poor South against the rich North with a role of nuclear weapon (*Little Red Book* and Mao Zedong's Thought). H. Marcuse was a philosopher criticizing capitalism and so-called entertainment culture (*One-dimensional Man*) (Marcuse 1964). Simultaneously he wanted to add sexuality to marxism philosophy (Freudo-Marxism, Frankfurt school) in his work *Eros and Civilization* (Marcuse 1955). Both approaches by Mao Zedong and H. Marcuse in historiosophy seem to be unscientific and false. How they were important in a social life in the West (especially in USA) is explained by the slogan propagated by students: *MaMaMa* (three *Ma*—Marx, Mao, Marcuse).

6 Latin Averroists and the Geometrical Unification of the Physical Interactions

In this chapter we are going to compare the views of Latin Averroists to the geometrization of the physical interactions. I would like to stress certain similarities between the idea of multi-dimensional geometrical unification of physical interactions and some systems of medieval philosophy. Here we mean the systems of Arabic followers of Aristotle, like: Avicenna (Ibn Sina), Averroes (Ibn Rushd) and the Jewish ones, like M. Maimoun and the Latin Averroists: Siger de Brabant, Boetius from Dacia (Copleston 1964; Tatarkiewicz 2011; Kuksewicz 1968, 1971, 1973, 2005), etc. These systems originated as a result of mixing the system of Aristotle with the neo-Platonic ones (emanational) e.g. that of Plotynus tried to combine Aristotle with Platonism. Their creator's failed to realize that what they considered to be the original works of Aristotle or Plato — were in fact a mixture of both, obtained as a result of invasions and disorders. They were convinced that what they read and commented upon — were original ideas of Aristotle. They themselves held a view that following the essence of Aristotelian philosophy, according to these (Latin) authors, Averroes was the major commentator of Aristotle. Creating comments of their own with respect to the supposed works by Aristotle, they have managed to build original philosophical systems, which might be of interest until this day. It is worth noticing that the hierarchical structure of the Grand Unification and of Kaluza–Klein models is very much like the structure of an ontologic system put forward by Averroes, provided that we recognize the following analogies. To a space-time of today, one shall relate a sub-lunar region and to each successive supra-regions one shall relate the groups and homogenous spaces in the unification models. To the regress from a most ideal being down to a one most charged with matter inside a sub-lunar world, there is to correspond the passing from a higher to lower symmetries. With the emanation of being there will correspond a spontaneous symmetry breaking subject to the shrinking energy scale. To a matter (in the sense of Aristotle) — the geometry describing the physical interactions, to a form (in Aristotelian meaning) — there should correspond an exact solution of the field equation. To a composition of matter and form (that is — the being) — the elementary particle. To the Absolute — a state with the highest symmetry (gauge group) before the compactification and the spontaneous breaking of it. To the naturalness of the emanation of being, there corresponds a spontaneity of symmetry breaking. This is not to say that because of the above analogies we shall have to reinterpret idealistically the Grand Unification. It is not necessary; after on we are confronted here with something; that in his time T. Kotarbiński called the principal oppositions' contact point. Geometry describing physical interactions is a material one, there is no room here for the duality of the soul and matter coming from Averroes. The structural similarities occurring mere are probably not accidental, and might be associated with some archetypical cognitive categories of a human being. These parallels might bring us a lot of stuff for thought, particularly once we would embark onto their materialistic explanation on the basis of Kant's philosophy; I mean here the materialistic casting of Kant's philosophy according to the school from Baden. Naturally we have also the dialectically-materialistic explanation. According to on these view-points, all theoretic concepts take their root in nature itself. Due to this, we might conclude from the principle of uniqueness (Unity) of matter that a system similar to the geometrization of physical interactions might be discovered independently, and used in other applications for the description of the world. Let us note that in the models under consideration we use very involved mathematical apparatus, not only in the computational sense, but in the conceptual one, too. Contemporary mathematics provides us with a great variety of concepts; contemporary geometry is particularly productive in this regard. All these concepts get applied very, quickly. One of the latest achievements in the theory of finite groups might best explain this: the so called F group (called also friendly giant, monster, or Fischer–Griess group). This group and its representations have found an immediate application in the multi-dimensional string model. Another example of immediate applications to physics of new mathematical results is provided by the algebraic classification of knots, recently obtained. The knots are the usual ones, known e.g. from sailing. They found applications in the topological quantum field theory. Hence, we see that almost all mathematical theories, the old (those from XIX century) ones and new alike, find applications in the geometrization and the unification of the physical interactions. This is so remarkable that there are lot of those, who ask whether we will find enough mathematics to describe the world, whether some gap has been created between the mathematics (language of description) and the objective reality. Are we approaching the limits of our capability of creating formal models? Should this be the case, then perhaps our existing approaches and methodology are in the need of change. Gnosis from Princeton (a leading scientific center in the USA) is one such answer to these questions. The role of philosophy at this moment is growing, and the attention paid by the outstanding physicists to the philosophies of the East is by no means accidental. They are looking there for inspiration, going to as they themselves are saying, to translate the mysticism (idealism of the East) onto the mathematical formalism of the West. A lot of distinguished physicists have come to embrace the gnosticism claiming that the unification and the fundamental questions in the area of elementary particles are "beyond our minds". One has to note that the problem of creating the geometrical unification of physical interactions is an extremely important cognitive problem. Its importance could only be contrasted with the effort aimed at the creation of the great philosophical systems of antiquity, middle ages, or modern times. Trying to prolong this metaphor further, we might say that at present we are in the period preceding the great system-elaborating phase, where the role of great thinkers of antiquity is played by theoretical physicists. Contemporary Aristotle or Averroes would certainly be a theoretical physicist by profession and would occupy himself with the problems of the unification of the physical interactions.

Finally, I would like to return, for a while, to issue of Gnosis from Princeton. Using here exclusively the philosophical systems of the East seems not to be necessary. The parallels between the views of Averroists and the geometrization and unification of the physical interactions sketched by us above, speak for themselves. Let us note that there is a similar correspondence between the geometrical unification mentioned here and the philosophical synthesis of St. Thomas the Aquinas. I mean mere chiefly the Thomistic ontology. The adopting of any particular interpretation appears of course a matter of taste. The only criterion would be the one of the interpretation's coherence over the widest possible application range.

The same we say in the case of Bonaventure (Giovanni di Fidanza) and Albertus the Great (Albertus Magnus, Albert of Cologne). Contemporary, after the year 1911 the Roman Catholic Church (Congregation of Studies) came back to the original philosophy of St. Thomas the Aquinas (known now as Neothomistic philosophy) and considered the Spanish Jesuit F. Suarez as an authority only in canonic law (not in philosophy and theology). In this way we can consider neothomistic philosophy as a place to develop a geometrical unification of physical interactions. Neothomism is now under rapid developments in many philosophical faculties on catholic universities in all around the world, in particular at the Catholic University in Lublin (e.g. *Structure of a being*, *Metaphysics* (in Polish) by M. Krąpiec) (Krąpiec 1963, 1985).

In our further investigations in philosophy of physics we can develop holism and geometrization and unification of fundamental physical interactions within Neothomism, according to *Metaphysics* by M. Krąpiec (or see (Gilson 1994)). This can be achieved according to the ideas from this section. In this way an *arche* of physics geometry will be purely idealistic. Moreover, going according to the ideas advocated by W. Krajewski the interpretation of a fundamental *arche* of physics—geometry—will be materialistic. Geometry will be here material (an extension of a definition of matter). Moreover, according to Neothomism matter disappears as an independent (self-contained) being (there is only a geometry behind physical interactions). This is in some sense a priority of a spirit over a matter. R. Ingarden's phenomenology after some important improvements gives us something in between. Matter and geometry behind it have a realistic existence. Geometry means really a matter (with a realistic existence).

7 Geometrization Criterion as a Practical One. The Role of Praxis in Physics

In this chapter we shall be presenting the geometrization criterion as one dealing with practice. We might say that the scheme of geometrization and the geometrical unification could be a very convenient way for the creation of easily verifiable models and consequently — a very good physical methodology. In this sense it is a practically very convenient idea in the domain of the unification of physical interactions, obtained also as an outcome of the scientific practice in the domain of elementary particles. Let us note that there are in physics a kind of two notable different types of notions for the concept of praxis, working on two distinct levels. One of them works at the level of constructing the theoretical models and according to the viewpoints represented here, the methodology of geometrization of physical interactions constitutes the outcome of the mentioned *praxis*. This is a practice *praxis* of the theoretical work, applying the mathematical methods.

The experimental practice, leading to the rejection (or acceptance) of a given theoretical model — constitutes the other type of practice. I am expressively using the notion of praxis, and not the simple experimental verification, since this is much more complicated than it usually is believed. Let us reflect for a while how we usually accept or reject a certain model in the physics of elementary particles. For example let us consider the so called standard model. This model is a generally accepted for the weak, electromagnetic and strong interactions. It includes Glashow–Salam–Weinberg model the electro-weak interactions and QCD (Quantum Chromodynamics, a model for strong interactions). The standard model comprises lot of phenomenological (not determined by a theory) parameters and confronts us with a lot of questions about the number of fermion generations, certain internal relationships and the consistency issue. These features are not yet understood by the theoreticians. Because of that, lot of competing models was created, which in spite of having the same basic structure, bear in this or another way on these questions and sometimes give definite values for the phenomenological parameters.

Sometimes the competing models do introduce additional structures or elements missing in Glashow–Salam–Weinberg model or QCD. These new elements might be translated into particular experimental predictions concerning definite physical processes, e.g. cross-sections, particle lifetimes, their mass, etc. From this point of view it appears practically useful the creation of the so called standard supermodel or more particularly Glashow–Salam–Weinberg supermodel (which of course does not constitute a supersymmetric extension of Glashow–Salam–Weinberg model). This new model consists of the following: all its phenomenological parameters are arbitrarily independent and create a certain (multi-dimensional) abstract space of parameters. Every model (in particular, the standard one, too) has its point (one could better say: a region in this space). The experimental practice from many years determines the appropriate region in the space of parameters and thus eliminates the subsequent models, the progress in the reduction (narrowing) of the admissible region or shifting of it as a result of discovering the systematic errors. Let us note also that a very similar procedure has been employed for the case of testing the alternative theories of gravity. This is the so called PPN formalism (Parametrized Post Newtonian) put forward by C. Will. An interesting outcome of this practice has been ruling out of on theories for gravity except GRT and the Nonsymmetric Gravitation Theory.

Finally, let us reflect for a while upon the experimental practice verifying the standard model e.g. in Geneva (CERN). For one, there are large groups of experts in many areas, from physicists to technicians, computer scientists and mechanicians. Secondly, there are theories of measuring devices, e.g. detectors, accelerators with the independent practices for these instruments. At the same time we have the theory and practice of processing the experimental data on computers with the possibility of the so called computer artefacts. The latter appear lately quite often. Let us notice that we are checking the standard model modulo theories of instruments. In the case of accelerator it is the classical electrodynamics, whereas for the testing of the accelerator, even the quantum electrodynamics will be necessary. Simultaneously in the case of so numerous research teams there shall be considerable psychological effects, some reports from the theory of small groups (here we mean the groups of people in the sociological sense), sociology of management, sociology of science etc. All of these elements might become sources of the systematic errors. Such errors in turn might lead to the acceptance or rejection of the improper theory. Such accidents had already happened, e.g. the affair of the monojets in high energy physics. The question of more general nature becomes important at this moment. Namely, how much in the adopted theory of interactions is there of the following:

1) description of objective nature
2) experimental apparatus
3) our own (as cognizing subjects) cognitive categories.

All these three elements as we have seen, must appear and the only way to eliminate the remaining two, leads through the scientific practice understood as a social, activity of a general cultural character. In accordance with what was said above, this will be a hierarchically understood practice; from the theoretical one, down to the praxis of an enterprize like CERN. In this way the practice becomes the criterion for the truthfulness of the unification theories and the geometrization criterion is a very convenient tool for choosing the appropriate phenomenology in the area of fundamental physical interactions.

It is very interesting to notice that teams of international collaboration (ATLAS, CMS, etc.) are very numerous. They are publishing many huge papers every year. Such teams are multinational and multicultural which makes some interpersonal interactions very hard to manage. In such numerous teams some sociological (or socio-psychological) problems are very important and because of this the role of practice is even rising and will be rising.

8 Paul Feyerabend and the Geometrization of Physics

This chapter is devoted to a discussion of Paul Feyerabend's views. Paul Feyerabend has an established place in contemporary philosophy of science and methodology (Feyerabend 1984). He has been labeled with terms such as "anarchist" and the "debunker" of science. His claim that "anything goes" and the idea of non-commensurability of concepts in many theories are widely known and appreciated. Feyerabend uses many case studies from the history of science, and especially physics. His way of proceeding in "Against Method" seems justified, since they are rooted in the realm of genuine physical and astronomical concepts. Feyerabend refers to a great deal of notions still operational in physics, he does not invent examples in order to prove the *a priori* enacted thesis. This explains why his considerations appear to have such the convincing thrust. We are well aware: for a contemporary man, science constitutes one of many domains of activity like: common knowledge, politics, ideology, religion, art, philosophy. We are similarly aware that one cannot separate these domains completely. Naturally this is not to say that we do not know what constitutes science in its typical subvarieties nor that we are unable to develop something like philosophy of science and methodology. The very fact what Feyerabend is doing, speaks for itself. His ideas make one to reflect how really the things are: what forces us for accepting a new theory and abandoning the old one. Are the concepts of one theory translatable into the ones present in another theory, or perhaps they are completely incommensurable? May be Newton's theory and GRT are like the "monads without windows" of Leibniz? Could it be that the words described by the two theories are without any common denominator? Let us try to delve a bit deeper into all of this. Let us notice at the outset that science as one of the social fields of human activity is subject to the historical development and constitutes one of the cultural elements of the given historical period. Feyerabend seems in a sense to understand this, although stops short of saying it *expressis verbis.* In the above sense, the concepts used in any scientific theory are in a variety of ways intertwined with the notions ruling in the given historical period in either ideology or common knowledge. They are impossible to extract from one another. Writing about the incommensurability between the "impetus" and a momentum or the kinetic energy, Feyerabend is in fact writing about the incommensurability of the medieval and modern cultures. Once we have agreed that the cultures are like "monads without windows" then the incommensurability between the principle of inertia in the mechanics of "impetus" and Galilean law of inertia becomes apparent. A. Gurevich (Gurevich 1985) in his magnificent book *Categories of Medieval Culture* writes about the concepts of time and space in the Middle Ages in such a way that the creation of their modern counterparts seems very strange phenomenon indeed. The medieval and modern cultures become mutually non-translatable. Each one of them requires own set of notions, appropriate for investigating it. Should we start from the assumption of the interdependence of all cultural phenomena in the given epoch, and accepting the claim of non-translatability of the various cultures (that it is impossible to study one culture with the help of notions proper to another one), we would easily come to the conclusion that also the physical concepts in different theories are incommensurable. However, there is a kind of framework, limiting on the one hand the arbitrarity of the scientific notions — this famous "anything goes" and on the other hand introduce a commensurability of some sort. Experimental facts and the cognizing subject, creating a theory — constitute this framework. Investigating various concepts we always experience the illusion to the effect that there are exclusively the differences, we very often overlook the things which are common. Feyerabend also suffered from this illusion, and like his predecessors had looked everywhere for "discontinuities", which he termed incommensurability. Kuhn (Kuhn 1970) has called this discontinuity "the scientific revolution" — a paradigm change, whereas M. Foucault (Foucault 2002) "change of discourse". They all were overlooking that "paradigm", "discourse" constitute parts of the developing social being and the discontinuity detectable at one level might easily reveal itself as continuity upon a closer and more detailed scrutiny. Let us note that according to a model put forward by Z. Cackowski we have discontinuity at the level of experimental facts and the theories aim at explaining them, provided that we consider these two levels separately, but once we combine these levels together — the discontinuity would disappear. Together we get a continuous development; the discontinuity at one level gets compensated at the other one. Of course, Feyerabend could have immediately claim that his idea concerns solely the physical theories, where undoubtedly one could notice "incommensurability" or even jumps. Let us try to challenge his notion of incommensurability exactly there, where he appears to be the strongest, when he passes from one mechanics to another. Namely, let us ask whether the mechanics of Aristotle, Galileo and Einstein are in fact completely incommensurable? Feyerabend writes that momentum in Newtonian (Galilean) mechanics and the relativistic one represents completely different concepts, having nothing in common except the name. In fact, the notion of absolute rest is employed in Aristotle mechanics, in Newton's one we have an absolute time, while in relativistic mechanics the time and space become relative. The mechanical concepts preserve their names — we speak about force, momentum, energy, velocity, the coordinate system. All of this in Feyerabend's opinion is not reflected in a real meaning of these concepts, when they become relativized to a given theory. According to him this is nothing more than a mere linguistic convention, confirming at best the power of our habit, providing thus a proof of our conservatism. This is not my aim to diminish the impact and importance of a definite conservative attitude in the society of scientists, defending them against the change of notions, nor to deny the comfortability of using such convention. Nonetheless, I think this alone would constitute too weak a justification for retention of the names. Namely even the superficial analysis is able to reveal that there is after all something in common in these concepts, in addition to the simple fact that they constitute the adequate description of a physical reality in some respects. There is something more in common. All those dealing with the correspondence principle perceive that and try to formalize it, aware of the continuous passage between theories at the level of their numerical outcomes. This perfectly fits the observations, since these theories, in spite of being different, are claiming something about the reality after all, and enable to predict. Feyerabend questions the claim to the effect that the predicting means the understanding of the process. Nonetheless one could not agree with him in this regard.

Now I will embark on describing the theories of Aristotle, Galileo and Einstein in a language proper to them, encompassing them as special cases of the more general notions. These theories in this language are becoming in a certain sense equivalent, but at the same time they create a chain of theories interrelated closely and mutually. Simultaneously, the corresponding mechanical concepts within this framework, get related as well. This relationship establishes in a proper way the principle of correspondence. Namely, geometry constitutes this language, and it is the geometrization procedure responsible for bringing these three theories to a common denominator. All mechanical quantities have to become geometrical ones of a similar nature, although in different geometries. The relationship between the geometries will be visible and easy to grasp. The transition from one mechanics to another will be occurring via the change of a space-time symmetry group. Within the idea of geometrization, we introduce the space-time, which is a four-dimensional manifold and on this manifold we define certain structures, associated with the given mechanics (kinematics). First of all — the symmetry group. The four-dimensional manifold, a space-time constitutes an idealization of a set of all possible point-events, that is such which take very short time and occupy very small space. It is the role of the symmetry group to tell us how to obtain the mutually equivalent events. For the case of Aristotle mechanics, on the four-dimensional manifold there acts a group

$O(3)$ of rotations, leaving one axis at rest. The absolute time constitutes this axis. There is no mixing between time and space. Absolute rest is possible — the absolute coordinate system. Clearly, Aristotle never formulated his physics in this way. Nonetheless it is an instructive thing to express this in the language of geometry. In the case of Newtonian physics we also have a four-dimensional space-time, and within it Galileo group transforming the inertial systems into itself. Here we allow mixing of space-like and time-like coordinates. There remains a certain absolute element — time. Newton was very much fascinated with the absoluteness of time and space. The relation between the group O(3) of rotations and Galileo group is very simple. Namely — O(3) is a subgroup of Galileo group. Let us come now to the relativistic mechanics: Here we have Minkowski space-time with Poincaré group acting in it. This time both absolute time and space vanish and there remains an absolute space-time — the background of all events. The relationship between the Newtonian mechanics and relativity is quite simple in this language. Lie algebra of a Galileo group constitutes namely the contraction of a Lorentz group's Lie algebra. This procedure is unique and it establishes for us the correspondence principle between the relativistic mechanics and that of Newton. In all the cases brought in here, and more particularly in the two last ones, all mechanical quantities, that is momentum, energy, force, velocity and acceleration become the geometrical ones, defined on a space-time and related to each other respectively. They are nonetheless not incommensurable, in spite of being not identical. This is evident, since there are here different space-time symmetry groups. Nevertheless the space-time groups are not arbitrary even from time theoretical point of view. The relationships between them are natural from the mathematical — that is geometrical standpoint — and could be obtained in a certain sense in the unique way. At the same time, the space-time concepts lying at the basis of each of these mechanics constitute something which binds them very closely. It is worth reaching at this point that Newton–Leibniz dispute concerning the nature of the space-time is decided by the relativistic mechanics, in favour of Newton. Let us remind that this dispute, conducted by Clark was devoted to the relativeness of the space. Namely, Leibniz held that the space is relative-associated with the body, whereas Newton (according to Clark) favoured the idea of absolute space. Now in the relativistic mechanics (Special Theory of Relativity) it is the space-time continuum which plays the role of this absolute element. In the preceding chapters of this essay one might find more details on these subjects. This absolute element permeates all the mechanics mentioned here, and causes that they are not incommensurable, contrary to what Feyerabend has claimed. The second example brought by Feyerabend as justification for the incommensurability of various physical theories is one contrasting the quantum and classical mechanics. The quantum concepts are seemingly incommensurable with the classical ones, in fact, the cursory analysis would show that it has be that way. Hence for instance momentum in classical mechanics is a number (vector), whereas in the quantum one — a Hermitean operator, acting in a Hilbert space. Both these concepts are completely incommensurable from the mathematical point of view. The incommensurability of both world-views follows in fact from this. This is an apparent truth from superficial point of view. Namely the investigations of the axiomatic foundations for both mechanics at the level of the so called logic of experimental questions reveal that the difference between the classical and quantum mechanics is associated only with the shutter of such logic: quantum logic is a non-distributive lattice, whereas the classical one — is a Boolean algebra. The evidence that the classical quantum concepts are nonetheless commensurable relies on the occurring of the so called classical fragments in quantum logic. These fragments constitute the sub-lattices of the quantum logic and are being Boolean algebras as well. Thus we see that the claim to the effect, that the quantum and classical mechanics are incommensurable follows exclusively from the imprecise taking into account the analysis of the foundations for both theories. It is worth noticing here that there are a lot of possibilities for choosing other logics than the quantum and classical one. This might lead, as some maintain, toward the generalization of quantum mechanics and the introduction of a nonlinear quantum mechanics.

Finally I am going to offer an example of the extreme "leveling" of the incommensurability between concepts. Namely, let us bring together the gravities of: Aristotle, Newton and General Relativity Theory. Everyone seems to be convinced that the difference between the theory of a curved space-time of Einstein and Newton's gravity is so large that it might be the excellent case for the incommensurability of concepts. This is however not so. It turns out that the gravities of Newton and Aristotle are associated with curvature of Galileo's and Aristotle's space-time within the geometrical language of both theories. Thus we see, from the above brief critical appraisal that the ideas of "anything goes" and of incommensurability could not be defended any more, and this is true not only with regard to the external formulation of both.

In this way, the geometrization could become a very convenient tool, removing the discontinuities between the pictures brought by various physical theories. It seems that due to living in a cultural quasi-reality (as R. Ingarden used to qualify our world), we need this type of approach at least for the sake of psychological comfort.

9 Symmetries in the Theory of Elementary Particles. Attempts at Unifying of the Internal Symmetries with the Space-time Ones

Problems of combining the internal and the space-time symmetries are to be discussed in this chapter.

Super-symmetries constitute the extension of ordinary symmetries known in the theory of elementary particles. These symmetries, such as the SU(2), SU(3), SU(6) and the newer ones SU(4), SU(5) served and are still being used for the classification of elementary particles, and more precisely the strongly interacting ones, or hadrons. Let us recall first what rules govern these classifications. The Hamiltonian of the interactions responsible for the hadronic interactions (and their structure) is invariant with respect to a certain symmetry group. This means that it commutes with the generator of Lie algebra for this group. In this way to every generator of a group there corresponds a conservation law. The conserved quantities obtained in this way create a Lie algebra identical with the symmetry group Lie algebra. Of course, in a real situation, these symmetries are broken (the higher ones are more strongly broken). The breaking of these symmetries is responsible for the hadronic mass spectra. The irreducible representations of the symmetry (partly broken) groups serve the purposes of particle classification, whereas the corresponding mass formulas e.g. those of Gell-Mann–Okubo carry the magnitudes of mass spectra dispersions. In the case of an exact symmetry the particle masses belonging to the same irreducible representation would be identical. Let us note one characteristic thing. As a matter of fact the symmetry groups are not necessary for hadron classification, since their Lie algebras will do. After all we are using the irreducible representations of Lie algebras, not of Lie groups. This is very important, since this fact allows us to abandon the analogy of the Hamiltonian invariant with respect to a Lie group. It will suffice to say that a Hamiltonian is commuting with the elements of a certain Lie algebra. The irreducible representations of this algebra will enable us to classify for us the elementary particles. We know that to every Lie group there corresponds one and only one Lie algebra. On the other hand, to a single Lie algebra there could correspond several Lie groups. These groups are of course locally isomorphic.

Now, since in the classification, the essential role is being played by algebras and not by groups, it is difficult to say which one is concerned. Such a decision seems to be arbitrary. Hence we have a Lie algebra, whose irreducible representations classify the elementary particles. We “integrate” this structure to a Lie group. The choice of a specific group is arbitrary up to a discrete subgroup. One might make it a little more unique by demanding that it should be a connected one. There are however no physical indications to perform this choice. Therefore the structure of a Lie group will be fundamental for us, together with a Lie bracket (commutator) appearing in it. The fact that only commutators occur in such structures disturbs a lot of people. As a matter of fact, in quantum mechanics and quantum field theory there appear quantities composed of commutators as well as of anticommutators. As we know, the boson creation and annihilation operators are endowed with commutation rules, whereas the fermionic creation and annihilation operators satisfy the anti-commutation rules. The appearance of either the commutation or anticommutation rules is associated with the boson or fermionic statistics. Let us note that the two fundamental brackets of quantum mechanics and quantum field theory do not occur on equal footing in the algebraic structures related to symmetries. In addition to the symmetries of elementary particles mentioned above, there appear in physics the symmetries tightly associated with the space-time.

Among the space-time symmetries we have those of Lorentz, Poincaré and Galileo. To them there correspond the Lie groups of Lorentz, Poincaré and Galileo. The appropriate Lie algebras correspond to these groups. The combining of the space-time symmetries with those of elementary particles constituted one of the most interesting questions in physics. The efforts to achieve this failed. Only the results without physical sense or trivial ones were obtained. Let me recall what was the stake involved in these efforts. Namely, one had looked for a certain wider group (Lie algebra) so that Poincaré group and a group $\mathrm{SU}(n)$ (which could be one of the internal symmetry groups) would constitute its subgroup (Lie algebra). The wider symmetry group (algebra) of the said type was presumably able to reflect more deeply the structure of a physical world picture. We will for instance describe a certain extension procedure (of finding a wider symmetry group), which has managed to find applications in physics. This procedure is called an extension of a group A with respect to a group B. In this case we are looking for a certain group G for which A would be an invariant subgroup (normal divisor) whereas B would be a quotient of G by A, $B = G/A$. This problem is not unique as one might see from examples. Now, under the assumption that the group A is Abelian, we could obtain the classification of all the extensions with A and B fixed. The most simple case there would be one of projective symmetry, that is, $A = \mathrm{U}(1)$, the group B here is arbitrary. We get here the projective representations of the group B occurring in quantum mechanics. Group B acts in a Hilbert space along the radius-vectors. This corresponds to the extending of the group $G = \mathrm{U}(1) \otimes B$. In crystallography and solid state physics we encounter a more complicated situation. In this case the group A is a group of translations (a discrete one) whereas the group B is a crystallographic group of the lattice symmetry constructed by the action of a group A. The extension G in this case may not be a trivial one, that is to say, not always $G = A \otimes B$. In certain cases the group G is wider than $A \otimes B$. The elements of the group, which are not the product of a translation belonging to A and of a rotation or reflection belonging to B, constitute just this non-trivial addition. In the case of crystallography these additional elements represent the sliding planes or the symmetry screw lines. The crystals of this type do occur in nature and are called the symmorphic ones. In addition to this, there is an interesting classification of these symmetries which uses the methods of cohomology. Namely, it turns out that to every non-isomorphic extension there corresponds a cohomology group $H^2(B, A)$ element. The unit element of this group corresponds to a trivial extension $G = A \otimes B$. The classification shown enables us to find all the extensions and to check whether there are any non-trivial extensions. It is just due to this method, which enables us to find in a very elegant way all the symmorphic crystals. For our purposes the most interesting is the case when one of the groups is Lorentz and the second is SU(2), SU(3). In a particular case we take a connected component of a unity from Lorentz group and more specifically its universal covering $\mathrm{SL}(2, \mathbb{C})$. It turns out that there are only trivial extensions here, that is semi-simple products or simple ones $\mathrm{SL}(2, \mathbb{C}) \otimes \mathrm{SU}(2)$, $\mathrm{SL}(2, \mathbb{C}) \otimes \mathrm{SU}(3)$. This demonstrates that such a programme is going to fail. In a similar way one might extend Poincaré group by examining higher jet-extensions of that group. As a solution we obtain here a semi-simple product of Poincaré group and a certain solvable group. This result shows that this way leads nowhere. It is not only trivial but non-physical, since we do not have solvable groups as internal symmetries in the theory of elementary particles. On the other hand, we do not meet a failure in theory of elementary particles in the case when we extend the remaining symmetries. In the case of a symmetry SU(2) (isospin one) and a hypercharge U(1), we first have $\mathrm{SU}(2) \otimes \mathrm{U}(1)$. We extend easily that symmetry to a known unitary symmetry SU(3). Similar is the situation for the case of the SU(3) symmetry and the group SU(2) (non-relativistic spin). We obtain first $\mathrm{SU}(3) \otimes \mathrm{SU}(2)$ and later SU(6). Of course this symmetry is very strongly broken, but offers interesting indications fitting quite well with the experiment. The symmetry SU(6) constitutes a symmetry of 3 non-relativistic quarks. It assumes the mixing of spin and of the unitary spins. Its multiples contain the particles with different half-integer and integer spins. Let us recall that, in the case of SU(2) and SU(3), each of the multiples contained solely the particles with one value of spin. Therefore the symmetry SU(6) mixes among themselves in a certain specific manner the space-time and internal degrees of freedom. This is of course a non-relativistic symmetry, which constitutes its (not isolated, by the way) drawback.

The symmetry $SL(6)$ constitutes a relativistic counterpart to the SU(6) symmetry. This symmetry corresponds to the three relativistic quarks. The group $\mathrm{SL}(6,\mathbb{C})$ similarly as $\mathrm{SL}(2,\mathbb{C})$ is non-compact and this causes that the theory of representations of such groups does not have an agreeable interpretation. All its unitary representations are infinitely-dimensional. In the case of $\mathrm{SL}(2,\mathbb{C})$ which is a purely space-time symmetry this was not an obstacle. It meant only this that all values of spins (relativistic ones) are possible. In the case of $\mathrm{SL}(6,\mathbb{C})$ the issue is more difficult to interpret. Here the multiples corresponding to the irreducible representations of $\mathrm{SL}(6,\mathbb{C})$ shall comprise infinitely many particles. We are not observing these particles in nature and surely it is unlikely that we will detect them at all. One sees also that this approach fails to reach our goal of extending the symmetry with the simultaneous linking of the space-time symmetries with the internal ones. Extending the internal symmetries (broken each time more and more) presents no problems. We know the transition from SU(3) to SU(4) by introducing a new internal degree of freedom, the so called "charm" and further from SU(4) to SU(5) via introducing of "beauty". Both the procedures are standard and proceed in a similar manner as the one described for transition from SU(2) to SU(3). The introduction of new internal degrees of freedom was dictated by experimental facts, namely, the discovery of the particles ψ, ψ', ψ'' not fitting the SU(3) classification scheme, and also of the particles B not able to fit the scheme SU(4). The new classification schemes do predict new particles. Some of them are observed, which presents an excellent confirmation of the theory. For the time being, one is not able to see the limits to the approach of this type. Even the next scheme has managed to appear: SU(6), which is not to be confused with SU(6) analyzed before. The previous symmetry SU(6) was one mixing the spin SU(3) of Gell-Mann and the non-relativistic spin SU(2). It contained 3 quarks from SU(3) symmetry, u, d, s in both spin states. Assuming their equivalence we obtain a SU(6) symmetry of the old type. In the ideal situation the quark states are invariant with respect to rotations in a 6-dimensional complex space. It is out of this that we obtain the new SU(6).

Summing up — in the first case we have the unitary spin degree of freedom, that is isospin, hyper charge and ordinary spin, whereas in the second isospin, hypercharge or strangeness, charm, beauty (bottom), truth (top). In this way SU(6) becomes a group of six quark flavours. Clearly such a procedure seems to be without end. There is no reason whatsoever against the appearing of the symmetries SU(7), SU(8) etc. Happily there is an elimination of a sort, obtainable from the asymptotic freedom and from the magnitude of the so called Weinberg angle. Weinberg angle determines the mixing between the electromagnetic and weak interactions in the so called Glashow–Salam–Weinberg model. An interesting feature of it is the fact that this model associates the number of leptons with the number of quarks. At present after the discovery of lepton τ and its neutrino ν_τ we have six leptons aligned in three sequences: (e, v_e), (μ, v_μ), (τ, v_τ). Therefore the minimal number of quarks shall be 6 and hence we have SU(6) with bottom, b, and top, t, quarks.

Let us remind also a concept of the asymptotic freedom. Namely the theories with the non-Abelian gauge have an interesting feature. The coupling constant (a charge) decreases at small distances (large momentum transfers). Therefore, should we use as the theory of interquark interactions the QCD, that is gauge field theory with the gauge group $\mathrm{SU}_c(3)$ (this group has nothing to do with the group SU(3) from Gell-Mann classification), then we will obtain an asymptotic freedom provided that the number of flavours of the interacting quarks will not be excessively large. Namely, this matter has to be smaller than 16, which is also in accordance with the estimates for Weinberg angle. Therefore the maximum number of quarks is 16 (the asymptotic freedom has a very good experimental confirmation). Because of the relationship linking the number of quarks and the number of leptons mentioned above, we shall also have the same number of leptons. These 16 leptons have to be combined into 8 sequences, out of which we have thus far obtained the 3 nearest ones. Current experimental data indicate that the number of quarks should not be greater than six. The sixth quark has been discovered, therefore one could already clearly see that the symmetry groups classifying the elementary particles SU(2), SU(3), SU(4) etc., are not of too fundamental nature. They are simply related to the number of quark flavours. This fact could explain a number of failures encountered while attempting to combine these symmetries with the space-time symmetries. The fact that these are all broken symmetries also ceases to be intriguing. Simply the quark masses are not equal and that is why the symmetries are only approximate. One could only wonder that they still give quite good experimental predictions. According to the opinions currently prevailing, the good predicting power of the SU(2), SU(3), SU(4) etc. theories is due not so much to the insignificant differences of the quark masses, as rather to the fact that they are not much greater than zero. This fact has still other consequences associated with the possibility of extending the hadron classification symmetries. Namely, with the masses of quarks tending to zero, the u and d symmetry of SU(2) could be extended to a chirial symmetry of $\mathrm{SU}(2) \otimes \mathrm{SU}(2)$. This one in turn leads to the currents algebra and to the so called partial conservation of axial currents. The axial currents via the so called Noether theorem are linked with a second item of the SU(2) group and are fully conserved provided that the masses of quarks u and d are equal to zero. The magnitude of the four-divergence for these currents could be associated with the masses of the π, ρ, η mesons. Within the limits of symmetry accuracy, they have masses equal to zero. The chirial symmetry $\mathrm{SU}(2) \otimes \mathrm{SU}(2)$ could be extended to $\mathrm{SU}(3) \otimes \mathrm{SU}(3)$ or $\mathrm{SU}(4) \otimes \mathrm{SU}(4)$ or even to $\mathrm{SU}(5) \otimes \mathrm{SU}(5)$ and $\mathrm{SU}(6) \otimes \mathrm{SU}(6)$. The latter ones are however very strongly broken. Let us observe that all the symmetries of elementary particles mentioned above, represent, within the limits of accuracy, the so called global symmetries. They implement the gauge transformation of the first kind, and could not be extended to the second kind gauges — local symmetries. This fact follows from these discussed above, concerning the nature of these symmetries (the number of quark types) and also from the fact that there are no interactions associated with flavours. The interactions in question are implemented by Yang–Mills field with a gauge group $\mathrm{SU}_c(3)$. Colour symmetry group SU(3) might be extended to a gauge group of the second kind.

All the attempts to combine the internal and space-time symmetries ended in a failure. This might be understood thanks to Coleman–Mandula theorem (one of the so called “no-go theorems”). Namely, under very natural and quite general assumptions about the S matrix in the quantum field theory we get that the most general Lie algebra of the S-matrix symmetries containing the momenta (generators of translations in Minkowski space) and the generators of Lorentz rotation is a simple sum of Poincaré group Lie algebra and a certain compact Lie algebra, such that the generators of the this last algebra are Lorentzian scalars. Hence one might clearly see that in order to combine the space-time symmetries and the internal ones, one has to extend the concept of a symmetry that is — of Lie algebra.

10 Hidden Symmetries and Supersymmetric Algebras. Bosonic Strings and Strings with a Spin

Questions of hidden symmetries and of supersymmetric algebras are the subject of this chapter.

Let us return to space-time symmetries, that is to Lorentz group $\mathrm{SO}(1,3)$ and Poincaré group. These symmetries are exact ones. There were attempts of extending them toward certain higher symmetries called dynamical. The groups implementing dynamical symmetries no longer operate in the space-time but in phase space. In the case of Lorentz or Poincaré group there occurs an independent operation of these groups on the space-time coordinates and momenta. This is hardly of any interest. The interesting thing there could be finding of symmetries mixing the space-time coordinates and momenta. Such symmetries could constitute the non-trivial, proper dynamical symmetries. The symmetries of this kind exist. There are de Sitter groups $\mathrm{SO}(1,4)$ or $\mathrm{SO}(2,3)$ and a conformal symmetry $\mathrm{SO}(2,4)$ $[\mathrm{SU}(2,2)]$. These groups are linearly embedded (implemented) in the five- and six-dimensional spaces, and are associated with the so called hidden symmetries. For example we describe Kepler problem. As we know this problem has the obvious symmetry $\mathrm{O}(3)$. This is an orthogonal symmetry in a 3-dimensional space. There are the conserved quantities associated with it, the components of the moment of momentum vector. They create a Lie algebra $\mathrm{SO}(3)$. As we recall, Kepler problem has three independent solutions. Here belong: the elliptic case, the parabolic and the hyperbolic one. Hidden symmetries of various kinds reveal themselves in each of these 3 cases. In the elliptic orbit case (the energy is here negative) there appears an additional symmetry $\mathrm{O}(3)$. A conserved vector, the so called Runge–Lenz vector is associated with it. Components of this vector form an additional Lie algebra $\mathrm{SO}(3)$. In effect the final symmetry is $\mathrm{O}(4) = \mathrm{O}(3) \otimes \mathrm{O}(3)$. Let us note that we have now 6, (and not only 3) integrals of motion (3 associated with the angular momentum and 3 with Lenz vector). This enables us to determine the motion without solving the equations of motion, solely on the basis of algebraic relations. It turns out that the case of parabolic orbits (energy equals zero) has a symmetry of a non-homogeneous orthogonal group in a three-dimensional space (it is a group of rotations and translations in this space). The hyperbolic case (positive energy) in turn has a symmetry group $\mathrm{O}(1,3)$. In both cases like in the elliptic one, it is possible to find the motion without the need to solve the equations of motion. The first integrals are sufficient to that goal, and the question reduces itself to the algebraic problem. The groups $\mathrm{O}(4)$, inhomogeneous $\mathrm{O}(3)$, $\mathrm{O}(1,3)$ are here the dynamical ones. The coordinates and momenta here are manifestly mixed. One might ask: what is the smallest group containing the group $O(4)$, $O(1,3)$ and the non-homogeneous one $O(3)$? The group $O(2,4)$ has this property and it is called the concealed (hidden) symmetry of the problem. It is of course a dynamical symmetry. The knowledge of such a symmetry enables one to find the maximal number of first integrals (conservation laws) and in this way to simplify the solution of the equation of motion (sometimes down to pure algebraic problem). The group $O(2,4)$ operates in a phase space — a 6-dimensional one, mixes among themselves the momenta and the coordinates. In a phase space there are 3 regions (the 3 submanifolds) for negative energy, that equal to zero, and the positive one. The groups $O(4,2) : O(4)$, the non-homogeneous one $O(3)$ and $SO(1,3)$ are operating on these submanifolds. The group $O(2,4)$ obtained as a hidden symmetry of Kepler problem constitutes one of the implementations of a conformal group in Minkowski space-time. The conformal groups acts in a nonlinear manner on the coordinates of the Minkowski space. This is a significantly wider group than that of Poincaré. In addition to the relativistic rotations and translations there appear the transformations of acceleration and dilation. This group turns out to be a Lie group. This is interesting, because in a 2-dimensional space such a group is infinite. The holomorphic functions of one complex variable implement it. The conformal group constitutes a very significant extension of the relativistic symmetry. It turns out that Maxwell equations in vacuum are invariant with respect to this wider group. The vacuum Yang–Mills fields, massless Klein–Gordon and Dirac equation also has this symmetry. The relation among the group $O(2,4)$ (which is locally isomorphic to a conformal group) with a Poincaré group is very remarkable. Namely, Poincaré group constitutes a small group of $O(2,4)$ group. By considering the cone in a 6-dimensional space, we obtain a very simple and natural relationship between Minkowski space and a 6-dimensional space, where the group $O(2,4)$ is operating. It turns out that this cone if treated as a projective space (the space of directions of the cone), could be identified with Minkowski space. Then the contraction of $O(2,4)$ to this cone turns out to be Poincaré group. This is very interesting property. The relation between de Sitter group $O(1,4)$, Poincaré group and $O(2,4)$ is not so simple as between Poincaré group and $O(2,4)$. Namely by considering the implementation of de Sitter group on a hypersphere with a "radius" R and performing the contraction with $R \to \infty$, we obtain Minkowski space-time with Poincaré group acting on it. On the other hand, de Sitter group is a subgroup of $SO(2,4)$. $SU(2,2)$ constitutes the universal covering of the group $SO(2,4)$. With this last group ($SU(2,2)$) there are related the so called twistors, playing ever larger role in the gravitational theory and in the so called Penrose programme. Roughly speaking twistors constitute spinors of a conformal group. The group SU$(2,2)$ operates in a four-dimensional complex space, which might be considered as a complexified space-time. Now, due to the fact that such a complexified space-time could be in turn considered as a model of physical phase space, we are able to obtain very strong results concerning the space-time itself. This is associated with the using of Kähler geometry, which has very strong properties.

Now, let us return to the origins of supersymmetry (Fayet & Ferrara 1977; Salam & Strathdee 1978; van Nieuwenhuizen 1981; Levy & Deser (eds.) 1980; Freedman & van Nieuwenhuizen (eds.) 1981; Hawking & Roček (eds.) 1981; Ferrara & Taylor (eds.) 1982; Ferrara et al. (eds.) 1983; Gates et al. 1983; Wess & Bagger 1983; DeWitt 1984; Furlan et al. (eds.) 1986). Supersymmetric transformations have taken their roots from dual models in the theory of elementary particles and the so called strings in the theory of strong interactions. We are not going to discuss here the dual models. We will occupy ourselves only with strings. The string model uses an interesting concept of hadron, baryon and meson. We assume in it that hadron constitutes an infinitely thin relativistic string. The density of a Lagrangian in this model is the surface element spanned by the string during the motion. The strings move in conformance with the least action principle, obtained with the help of a Lagrangian with a given density. The string coordinates are described by $X_\mu(\sigma,\tau)$ where σ is a parameter along the string and τ is a proper time. We therefore have a certain possibility of changes, some gauges. By selecting specific gauges we could reduce the number of variables to a minimum of independent ones. There will appear here the infinite number of the quantities L_n, which will identically vanish provided that a system is chosen with a minimal, independent number of coordinates. The quantities L_n form a certain Lie algebra. In addition to a structure considered here we could analyze also a more complicated one. It will be a string with a continuous distribution of spin. New quantities S_n would appear here, also vanishing in a suitable coordinate system. The quantities S_n together with the preceding ones — L_n — constitute Lie algebra. There are however quite significant differences. The system L_n constituted a Lie algebra. We take now the commutators

$$[L_n, L_m] = C_{n+m} L_{n+m} \tag{10.1}$$

In the case of objects S, we take anticommutators

$$\{S_n, S_m\} = D_{n+m} L_{n+m}. \tag{10.2}$$

In the mixed case we take commutators

$$[L_n, S_m] = E_{n+m} S_{n+m}. \tag{10.3}$$

The algebra is closed with respect to commutators and anticommutators. Let us note that this algebra has two types of elements, L_n and S_n, and two types of brackets: commutator and anticommutator. This algebra represents a generalization and the extension of Lie algebra of the elements L_n. This is an example of a supersymmetric algebra, the so called supersymmetric Virasoro algebra. Let us return for a while to the spineless strings. In the system of minimal coordinates such a string has a conformal symmetry. Lie algebra of the conformal group — the conformal algebra corresponds to this system. In the case of the string with a spin this algebra undergoes the extension via the inclusion of the anticommuting elements associated with the spin. The algebra thus obtained is to be called a supersymmetric one; more precisely — the supersymmetric extension of a two-dimensional conformal symmetry. Here are two types of elements. The first ones are usual generators of a conformal algebra Lie group. The second type of elements constitutes a set of additional supersymmetric generators. In this case they are Majorana spinors.

Majorana spinors anticommute with the elements of conformal group Lie algebra, whereas the Majorana spinor and the generator of usual Lie group are anticommuting with Majorana spinors. Hence we have obtained an algebra, with commutators and anticommutators occurring in it. This algebra has a closure property and constitutes an extension of the conformal group algebra. We could obtain supersymmetric extensions of Poincaré and de Sitter groups' Lie algebras in a similar way. The supersymmetric extensions of Poincaré group Lie algebra constitutes a subalgebra of a conformal group supersymmetric algebra in four dimensions. Nonetheless we could not obtain it in such a simple way from the conformal supersymmetry as before. We have to perform some non-trivial transformations on elements of the conformal supersymmetry and only then to choose a closed subalgebra. In this subalgebra there occur the elements of both types: the commutators and anticommutators. Let us recollect now all the properties of supersymmetric algebra known from the examples above. The elements of such an algebra are quantities of two types. We will call them even and odd. Even elements define Lie subalgebra in this algebra. In this subalgebra the role of the bracket is played by a usual commutator. The other types of elements — the odd ones — do not form the subalgebra. Its product, the operation defined for two elements, is an anticommutator. This operation gives as a result the element of the first type, the even one. Between the elements of different types we employ the operation — a commutator. The odd element will constitute the result of this operation. The whole structure is complete with respect to operations given. Let us think what advantage there could arise from such an algebra in the theory of elementary particles in addition to the ones given above. We were using Lie algebras in the theory of elementary particles; more precisely, their irreducible representations were used for the classification of elementary particles. Here we proceed in a similar manner. We look for all irreducible representations of a supersymmetric algebra at hand. With a representation selected we associate the multiplet of particles. In such a multiplet we will have the fermions as well as bosons. This is a very interesting fact, not encountered thus far. In this manner the supersymmetry becomes Fermi–Bose symmetry. The generators of a supersymmetric algebra transform the bosonic states, with the integer spin, into fermionic states with a spin of $1/2$. In the previous classifications of elementary particles there were in a single multiplet the particles with either integer spin or that equal to $1/2$.

Lie algebra generators for given symmetries of elementary particles were not changing bosons into fermions and vice versa. They were operating exclusively within the realm of bosons and fermions. This situation seems to be to such a degree interesting that one should find some procedures leading to the extension of the known Lie algebras to the supersymmetric ones. Such procedures are known and lead to graded Lie algebras (algebras with gradation). We could obtain in this way graded Lie algebras $\mathrm{gsu}(n)$, $\mathrm{gsp}(n,n)$, $\mathrm{gso}(n)$, $\mathrm{gso}(p,q)$ and a lot of others. These could be used for finding new elementary particles classification schemes. The multiplets obtained in this way will turn out to be very large. It will not be possible to find enough particles in nature, to fill up the multiplet obtained. This constitutes one of the drawbacks of supersymmetry. Even in the lowest irreducible representations the number of particles is far in excess. There is an entire extensive branch in mathematics, dealing with graded Lie algebras. In the case of these types of Lie algebras, a set of elements of the algebra is divided into classes. In the examples presented, they were two classes. In general there might be an infinite but countable number of them. With every such class we associate the element of a certain Abelian group. In the case given above this group has two elements and consists of 0 and 1. Therefore it is the so called two-element cyclic group Z_2. In all examples it will be either the finite-element cyclic groups or additive groups of the integers. An element of a group, corresponding to every class to which the element belongs, will be called index of a class. All of the above has been introduced in order to define the bracket for the elements belonging to different classes. Should the two elements have the indexes m_1, m_2, then the index of the bracket's result will be the m_3 such that $m_3 = m_1 \otimes m_2$ where $\otimes$ is an operation in the group of indexes. The elements m_1, m_2, m_3 are integer numbers; therefore one could define for them the numbers. We will call them signs $(-1)^{m_1}, (-1)^{m_2}, (-1)^{m_3}$. They will be necessary for defining the symmetry of the bracket introduced in this algebra. Consequently, if $a_1, a_2 \in A$ and m_1, m_2 are their indexes, then

$$[a_1, a_2] = -(-1)^{f(m_1,m_2)}[a_2, a_1] \tag{10.4}$$

where $f(\cdot, \cdot)$ is a certain function of m_1, m_2.

One could prove that in the case when Z_2 is a group of indexes, then the bracket taken on the elements corresponding to the index 0 is a commutator (has the features of a commutator). When the index is 1, the bracket is an anticommutator and is symmetric with respect to the exchanging of components. The outcome belongs to a class with index 0, since $1 \otimes 1 = 0$ in Z_2. In the first case, $0 \otimes 0 = 0$ the result belong to a first class, with an index 0. This agrees with the postulate that the zeroth class constitutes a subalgebra. In the case when one of the elements has an index 1 and the other 0, the result belongs to the class with index 1, $0 \otimes 1 = 1$. The bracket here has the symmetry of a commutator and property of antisymmetry with respect to the exchange of the components. In the structure just obtained, there occurs one more property, namely the generalized Jacobi identity. For the case of normal Lie algebras we find in this identity only commutators. In the case of Z_2-group of indexes we find commutators as well as anticommutators, depending whether there shall appear elements of first or second class. In a more general case, when the group of indexes is not Z_2 the situation is going to be more complicated, since there are more indexes. Now, there arises quite a natural question whether one is able to classify somehow the graded Lie algebras. We mean here the classification similar to Cartan classification for normal Lie algebras. It turns out that for the gradation of the Z_2 type gradation (the group of indexes of a graded algebra is often called the gradation) this works and we get a classification similar to that of Cartan. A lot of theorems from the theory of representation of Lie algebras, are satisfied for the case of graded ones. Among others, we could introduce matrix elements for the tensor operators. We could also define the so called tensor operator's reduced matrix element and prove Wigner–Eckart theorem for the graded Lie algebras with the gradation Z_2. Let us recall that this theorem is of great significance for both atomic and nuclear spectroscopy. Namely, it enables one (on the basis of information about the problem symmetry) to separate the operator-theoretic properties of matrix elements from the purely physical ones. The physics of the problem rests in the reduced matrix elements whereas Gordan–Clebsch (that is — Wigner's in general case) coefficients known from group theory, enable us to find all the matrix elements.

In this fashion a problem of finding all the matrix elements for the operator e.g. for Hamiltonian (and hence the spectrum of energy and also other quantities and the probability of transfer between the two states) reduces to two problems. The first one is purely mathematical and consists of finding Gordon–Clebsch coefficients for a given group. One could consider it as solved. The second problem is a physical one and consists in finding all the reduced matrix elements. This problem is of a different nature and requires a detailed knowledge of interactions between the particles constituting the investigated system. The symmetry alone is not sufficient for finding these quantities. An interesting feature here is that this theorem extends on supersymmetric algebras and in this way the question of finding matrix elements for tensor operators leads to two similar problems. There still remains the problem of finding the reduced matrix element, and therefore from the algebraic considerations alone (for the case of usual symmetries as well as for the case of supersymmetries), one is not able to solve the physical problem. In either case one needs information about physical interactions. This is very indicative and interesting. It indicates the great limitations inherent in group theoretic methods, since even the introduction of completely new algebraic objects failed to improve the situation in this respect. In known examples of symmetry, that is for Poincaré graded algebra, conformal group graded algebra the rules were found which link the irreducible representations. In particular, a scheme was given for the factorization of the tensor product of the two irreducible representations into the irreducible representations. Gordan–Clebsch (in general Wigner's) coefficients were found. In particular it became possible to find decay branching for the particles appearing in the multiples, and also the conditional probabilities of these decays. Similarly as for the group-theoretic symmetries, one could give a chain of more broken supersymmetries. This chain could aid in classifying the elementary particles spectra. There appear also the applications in nuclear and atomic physics, and even in solid state physics.

In the case of Poincaré supersymmetry, there is no breaking and it could describe the massless particles like photon or photino. It is very interesting and often happens in the case of supersymmetry that to a known particle, e.g. boson, there is being added in supersymmetry a new particle, the fermion. In this way, we always get pairs like the one presented below. Photino constitutes a counterpart to photon among the fermions. Similarly we could have gravitino for graviton. These particles are waiting for the experimental discovery. There is an interesting question to ask whether one could "integrate" the supersymmetric algebras down to certain objects of the type like Lie groups. It turns out that it is not a trivial question and the answer is quite complicated. Let us concentrate exclusively on algebras with the Z_2

gradation. In the structure of such an algebra, we will have commutators as well as anticommutators. Should there exist the structure of Lie group type, capable of giving us later on the supersymmetric algebra, the two types of parameters would have to be present — the commuting and the anticommuting ones. And also two types of coordinates — bosonic and fermionic ones. The generators corresponding to the bosonic coordinates — real or complex numbers — would constitute the generators of Lie subalgebra of the supersymmetric algebra.

The generators corresponding to the anticommuting coordinates would constitute supersymmetric extension of Lie algebra. In applications they turn out to be Majorana spinors. The fact of occurring of two kinds of coordinates, namely the *c*-numbers and *q*-numbers has far reaching consequences. In a certain sense, due to the fact that a spin of 1/2 constitutes a completely quantum effect, one could consider the manifold with the anticommuting coordinates as the structure of quantum type, vanishing in the classical limit. The anticommuting coordinates — the fermionic ones represent the elements of Grassmann algebra. We shall deal with the manifolds endowed with Grassmannian coordinates further on in this essay.

11 Anticommuting Coordinates. Lie Supergroups

This chapter deals with problems of Lie supergroups and the supermanifolds. Let us return to the problem of "integrating" the supersymmetry down to the manifold-like structures. It turns out that it is a very rare situation. For gs(n) it is only possible in the case $n = 2$, and the resulting structure bears the name of graded Lie group GSU(2). In few cases one could find groups of this type acting on power series built from the elements of Grassmann algebra. It is however rare. There is a certain formal way of defining a structure possessing the given features. However, that is ineffective, since this is not able to result in structures like Lie groups. Because of that, this is not so important for an arbitrary graded algebra, as GSU(2) is a graded SU(2). Hence, the interesting question has resulted whether the new coordinates called fermionic ones constitute a generalization of the classical coordinates. These new coordinates could be treated as ones of the quantum mechanical nature. One might say that they appear in a manner similar to the occurrence of q-number in quantum mechanics. The quantization in fact consists of substituting the c-numbers from classical mechanics with q-numbers of quantum mechanics. The new quantities — q-numbers enjoy the commutation properties — different from always commuting c-numbers. Nonetheless, in classical physics (constituting a limit case in the sense of the quantum mechanical correspondence principle) we will not have spin 1/2 particles; the q-numbers corresponding to fermions have simply to disappear. In the case of q-numbers corresponding to bosons, they shall in the limit become classical c-numbers. No wonder therefore that fermionic coordinates could not have classical partners. Thus, onto a theory with fermionic coordinates one should look upon as at least partly quantum one. After all, the creators of this theory are interpreting it just that way. It seems however that this is premature. As a matter of fact, one has no idea what will happen with ordinary bosonic coordinates in such a case. Should there be the equality of rights between the two types of coordinates, then the bosonic ones would be q-numbers also, like the fermionic coordinates. At the classical limit the bosonic coordinates would transform into c-numbers, whereas the fermionic ones, as not possessing the classical limit, would simply disappear. Unfortunately in all theories there is a kind of this asymmetry. The fermionic coordinates fit the scheme here presented. But the bosonic coordinates are not q-numbers, they are ordinary numbers. Therefore, in accordance with the idea here presented, the theory has to be quantum in one part and classical in the other one. This asymmetry indicates that one has to quantize the resulting theory like any other, unless some mathematical scheme will be invented, in which bosonic coordinates would become operators. The bosonic coordinates are space-time ones, thus sometimes also the space-like coordinates. They are the so called exterior coordinates. In addition to them, there are also the internal coordinates, associated with the internal symmetries, that is associated with the internal degrees of freedom. Here belong — isospin, strangeness, charge, colour, charm and other quantities, which originated in the theory of elementary particles. These coordinates are defined on the manifolds which are Lie groups of the corresponding symmetries. For example, to the electric charge and the electromagnetic field, there corresponds an additional coordinate associated with the parameter of a group U(1). This is a global gauge group and via Noether theorem, the charge conservation law follows from it. The extension of this group to a local gauge group, leads to the appearance of the electromagnetic field. All these coordinates are real numbers. Substituting of them with q-numbers might lead to a theory which from the outset would be a quantum one. This could mean the elimination of some troubles associated with quantizing of the theories with gauge symmetries. In the case of substituting of the space-time coordinates with operators, this might lead to the creation of the quantum theory of space-time and thus quantize the gravity. There are already attempts of this type and lead to a fine-grained structure of space-time. The interesting thing in this type of approach is the obtaining of the indeterminacy principle for measurements of time and space. This leads further towards the appearance of an elementary length and of an elementary time. The quantized space-time becomes here a discrete structure. In the case of theories other than with the space-time coordinates (that is in the case of some internal bosonic coordinates) nothing similar has been created as yet. The only existing example of approaches in this category is the theory of quantum groups. They constitute quantum deformations of groups. There are even playing for them the role of the counterparts of Yang–Mills fields. Note also that the bosonic coordinates corresponding to the space-time like ones can be linked to quants of the gravitational field with the aid of gravitons. Probably with a similar bosonic coordinate in the fifth dimension, one could associate a quant of an electromagnetic field — the photon. In this way we have to comfort ourselves with the asymmetric situation thus obtained, where the bosonic coordinates are numbers, and the fermionic ones are q-numbers. This may mean an unsatisfactory state of affairs, but nevertheless worth considering. To that end, following the suggestions of Salam and Strathdee, let us consider the so called superfield. The superfield assumes complex values, but nonetheless is a function of both the space-time as well as of the additional ones — the fermionic coordinates. Therefore there appear two types of independent coordinates — numbers and the elements of Grassmann algebra. Expanding this field into a series with respect of fermionic coordinates, we could obtain, due to the anticommutation properties of these quantities — the finite number of fields. They are to be equal to the coefficients of the expansion with respect to subsequent powers of the Grassmannian elements. In this way we get the scalar, vector, spinor and other fields. These fields are therefore contained in the superfield as its components. Now, constructing the supersymmetric Lagrangian to the same field and writing down the superfield equations on the basis of the least action principle, we could obtain the equations for the fields enumerated above. They will be: Klein–Gordon equation for the scalar held, Proca's for the vector field and Dirac–Weyl for the spinor one. The superfield represents an interesting concept aiming to unify some quantities occurring in the field theory. Because of this, we have in one quantity several independent fields. Thus from a single variational principle for a single field quantity we get the equation of the superfield. From this equation the equations for other fields follow. Let us embark now on generalization of the supersymmetry from a global case to a local one. To this purpose, let us recall what the global and local symmetries are all about in the case of ordinary symmetries implemented with the aid of Lie groups. The local symmetry of the U(1) type via Noether theorem leads to the charge conservation law. This conservation law is expressed by a continuity equation. This charge is a real quantity. In principle we do not use the properties of Lie groups in the proof of Noether theorem. We use Lie algebra of this group. This is dictated by the fact of using the infinitesimal transformations of the fields and of coordinates. It is sufficient to have Lie algebra and the infinitesimal changes of group parameters to define these transformations. Thus formulation of Noether theorem for supersymmetry does not require a structure corresponding to Lie group. This is significant, because of troubles in defining this type of differential structure. Hence we could formulate a counterpart of Noether theorem for supersymmetry. The conservation law in the form of continuity equation will appear in it. The spinor current will be a quantity occurring and being conserved in the continuity equation. This is not going to be a real-valued current. The charge associated with the supersymmetry will be spinor as well. All the charges together, the conservation of which follows from Noether theorem for a definite graded (supersymmetric) algebra will constitute the mentioned algebra. Hence, the anticommutation rules will follow here, in compliance with the rules for such an algebra. The interpretation of such spinorial charges or currents is not a straightforward matter. The vacuum case, where some of these currents assume non-zero magnitudes will be very interesting. Here one could speak about the spontaneous symmetry breaking. In the case of a supersymmetric extension of Poincaré group (or conformal group) algebra this is possible to obtain. Let us recall that for Poincaré group or for the conformal one there is no possibility of having non-zero momentum or energy in the vacuum state. In the case of a supersymmetric extension of these algebras, the spinor charges could not vanish in the state of vacuum. In such an instance this vacuum state undergoes the degeneration, and supersymmetry gets spontaneously broken down to a subalgebra which does not change the value of a supersymmetric (spinor) charge obtained in the vacuum state. Thus, completely new possibilities appear, which were missing in the case of classical symmetries. All of the above appears at the global symmetry level. Let us embark now to the local gauge symmetries, the so called gauges of the second kind. In the case of an electromagnetic field, we get the gauges of the second kind by substituting the phase transformation in global gauges with a function dependent on the point in the superspace-time. In this manner we extend the gauge transformation of the first kind. By Noether theorem we have the charge conservation law, which follows from the invariance with respect to gauges of the first kind. The introduction of the second kind gauges will bring about a significant change. Namely, there will appear an additional field, called a compensating one. The whole procedure of this sort was described in the preceding chapters. For the supersymmetric case we will have a similar situation, which is to be described below.

A very interesting approach has been designed by A. Rogers in *Supermanifolds. Theory and Applications* (Rogers 2007). In this approach we have a supersymmetric manifold with geometrical quantities consisting of a "soul" and a "body", where the "body" is connected to ordinary (real or complex) manifold and the "soul" to anticommuting coordinates.

12 Supersymmetric Gauge Transformations. Supersymmetric Gauge Fields

The supersymmetric gauge fields and transformations constitute the topic of this chapter.

Let us start from a symmetry with gradation. In accordance with all what we have said above, the global infinitesimal transformations exist for such algebras and via Noether theorem we could obtain the conservation laws of certain spinorial quantities. Hence we have the infinitesimal gauges of the first kind. We could extend them to the gauge transformations of the second kind requesting that infinitesimal parameters of these transformations should depend on space-time points. In this fashion these transformations cease to be rigid and become dependent on points. Local gauges similar to these of the second kind in the electromagnetic case or Yang–Mills ones are thus become obtainable in this way. Much alike it was in ordinary Lie algebra case, the introduction of these local gauges will have significant consequences. This time they will be of a different nature, but complementary to these of Yang–Mills fields. Namely, they are going to describe the interactions between the charges whose behaviour follows from the generalization of Noether theorem for the supersymmetric algebras. As we recall, these were spinor charges. The role of the intermediate particles in such interactions is to be played by fermions. Therefore we have obtained the compensating field with the intermediate fermions. This gives us a very elegant and symmetric situation for the theory in question. We will have to substitute in the Lagrangian of the original fields (similarly as was the case of Yang–Mills fields) the ordinary derivative with gauge derivative corresponding to the fermionic gauge field. At the same time we will also add to the said Lagrangian the Lagrangian of this gauge field. This is to be Dirac Lagrangian of that field, since this will be a spinor field with spin of $1/2$ or $3/2$. Similarly as in the bosonic case, this field is massless, thus Dirac–Pauli equation will become its equation of motion. It is to be at the same time conformal group invariant equation. For the case of fermionic gauge fields we can also try to find the geometric interpretation of the quantities obtained. For Yang–Mills fields the fibre bundle with the structural group G, which was gauge group, had served this purpose. Here it will be harder, since the structures of the graded Lie group are not well defined, that is the group manifolds with the anticommuting parameters. Nonetheless we have mentioned that there are formal structures for which a graded Lie algebra constitutes a tangent space at the unit element. Proceeding in a manner similar as was the case for Yang–Mills fields, we construct a fibre bundle whose base is a space-time for which the graded group serves as its structural group. The bundle manifold obtained in this fashion has the coordinates which partially are numbers, and partially Majorana spinors. These last ones are anticommuting parameters, and thus fermionic coordinates. The connection on such a bundle will have the interpretation of a compensating fermion field, and connection's curvature — the interpretation of this field strength. The cross-section of this bundle will provide the choice of gauge, while the covariant derivative is going to describe the minimal coupling between the compensating field and a field carrying the spinor charge resulting from Noether theorem for supersymmetry. In the case when we have more than one parameter, both the fermions as well as the intermediate bosons will appear in a graded group. The compensating field here has as its carriers the particles of both types. When the supersymmetric (graded) algebra is non-commutative, the equations for this type of fields are nonlinear and we have the self-interactions between fermions and bosons. The number of intermediate fermions of different types equals the number of anticommuting parameters, whereas the number of intermediate bosons, as before, is equal to the number of numerical parameters. Hence we see that the situation is very symmetric and elegant. There are attempts of linking the known fermions with gauge fields. Regretfully however theories described here could only be used for massless intermediate bosons and fermions. The introduction of rest masses to the Lagrangians will break the gauge symmetry. There is yet another way of obtaining the rest masses. Namely, it is possible to find this on a dynamical way due to interactions with other fields, or via self-interaction. Spontaneous symmetry breaking constitutes a very elegant method here as does Higgs mechanism, too. We introduce in this case the additional, hypothetical fields, the so called Higgs fields. The number of these fields depends on theory in which we want to get masses for bosons and intermediate fermions. We require that these fields interact among themselves in a nonlinear way. The self-interaction potential could not be arbitrary; it has to be such that Higgs field in vacuum possess a non-zero average value. Often in such a case the bi-quadratic potential is being taken. At the same time we require that Higgs fields interact with the gauge fields via the minimal coupling scheme. Since in the vacuum state Higgs fields has non-vanishing value, the symmetry will be broken spontaneously, to such one which would leave this value unchanged. The vacuum will be degenerated. Once we choose one of them and start to build the excited states over it, the symmetry will break and all the carriers of the compensating (gauge) bosons and intermediate fermions are going to obtain rest mass, except those corresponding to the generators of that subgroup, with respect to which the symmetry has not been spontaneously broken. Only the latter particles will remain massless. All others will assume masses.

Thus the part of gauge field will have finite range. Nonetheless the whole Lagrangian is still to be invariant with respect to a complete gauge group of the second kind, since the symmetry has been broken solely due to the choice of the vacuum state. This state in spite of the fact that it was arbitrarily chosen, has caused breaking of the symmetry and consequently the appearance of rest masses. Spontaneous symmetry breaking and Higgs mechanism endow bosons and intermediate fermions with rest masses. The remarkable thing is here that as the matter of fact the symmetry is not being broken in this case. The Lagrangian remains invariant with respect to the second kind gauge group. The massive terms appearing here are not breaking the symmetry, since they constitute a certain effective description of fields self-interaction. Vacuum degeneration occurring here makes applicability of the perturbation calculus questionable in some cases. Namely, so long as we stay near a vacuum state chosen, everything is O.K. The situation is here very much the same as in the non-degenerated vacuum case. We are in a position to construct subsequent excited states over the one chosen. Should we come too far away from the state of vacuum, then the following situation would become possible. Contribution from the remaining vacuum states, in particular from the nearest ones, would become greater than the one coming from lower order term in perturbation calculus. The influence of the nearby vacuum states might have been caused by the tunneling effects, which increase with energy. Effects of this type make one to enquire, from which place onwards the applicability of the perturbation calculus gets lost. Once the applicability of it becomes either impossible or troublesome, one has to think about using of other methods. Here belong mainly the exact ones, like exact solutions. These exact solutions in the form of instantons or solitons find ever wider application. They might lead to a very substantial changes of the quantum field theory's formalism. Thus far the attempts of obtaining exact solutions were directed to gauge fields in the case of quantum chromodynamics. As of today, there are no exact solutions for fields, with the intermediate fermions, although we have exact solutions in the supersymmetric theories such as supersymmetric extension of the gauge field models known in time theory of elementary particles. There are also supersymmetric extensions for models like e.g. Glashow–Salam–Weinberg (GSW), and even for the entire standard model. (Standard model as we know, includes Quantum Chromodynamics (QCD), the theory of strong interactions, GSW model — that is the theory of weak-electromagnetic interactions.) Its supersymmetric extension comprises, in addition to particles appearing in these theories, also their supersymmetric partners. Thus, in addition to gluons from QCD we have gluinos, which are the spin 1/2 fermions; in addition to quarks we have also squarks (scalar particles), for $W^{\pm}$ and Z^{0} bosons $W^{\pm}$-ino and Z^{0}-ino, photon has photino, whereas for the leptons — the so called sleptons; Higgs particles have higgsinos. As yet, the supersymmetric partners have not been experimentally confirmed. Such extension of the Standard Model is known as Minimal Supersymmetric Standard Model (MSSM) and considered to be a step to get a supersymmetric GUT.

Let us come now to supergravity. In order to understand the essence of this theory, let us return to Kaluza–Klein theory in its classical casting. We have the five-dimensional manifold with a metric defined on it, together with a metric connection. In the ordinary formulation it is being assumed that the connection is symmetric (Riemannian). We assume at the same time that there is Killing vector for this metric. Then in a suitable coordinate system, the quantities involved do not depend on the fifth dimension. This dimension will be associated with the electromagnetic field gauge. The metric tensor will decompose into the symmetric space-time tensor and the four-potential of the electromagnetic field. On this 5-dimensional manifold, the changes of the coordinate system are going to decompose into the changes of the coordinate system on the space-time plus the electromagnetic field gauge. One could extend this theory onto the case of an arbitrary gauge group, not only the U(1). This generalization is particularly natural and elegant, when fibre bundle formalism is employed. In the preceding chapters this has been described in considerable details. Generalization of this theory to more dimensions and other gauge groups, gives also a unified theory of the bosonic fields and gravitation. The question arises, whether getting of a similar theory would be possible such, where the fermionic fields could appear together with the bosonic ones. It turns out to be in fact possible. The theory of this type bears the name of supergravity. We start from a manifold endowed also with the fermionic degrees of freedom. Consequently the quantities defined on this manifold depend on ordinary coordinates and the fermionic ones. The latter are elements of Grassmann algebra and therefore are anticommuting. The quantity defined on a manifold of this kind will thus constitute a superfield, referred to above. It would be an interesting thing to have a metric tensor introduced on such a manifold, and further also the connection generated by this tensor. The quantities of various types would constitute the components of this tensor. The space-time part of this tensor will be the usual space-time metric tensor. The anticommuting Majorana spinors for the spin 3/2 spinorial field will constitute the mixed components here. Due to the fact that a metric tensor represents a quantity whose part belongs to Grassmann algebra, the definitions of Christoffel coefficients will have to change. The fact of anticommutation will be taken into account in this definition. In the case when the quantities appearing here will happen to be numbers, this definition will reduce itself to the one generally known.

This is going to be the case e.g. of the space-time components of the metric tensor. The coefficients thus obtained with would enable us to construct the curvature tensor. Here also a certain modification of a definition is necessary, to comprise the anticommuting coordinates case. Further, on the basis of this tensor we build Ricci tensor, curvature scalar and Einstein tensor. All of the above subject to modification of the type as before. Now, writing down Einstein equations in vacuum for this type of quantities, we could obtain field equations. These will be ordinary Einstein equations for a spin 3/2 spinorial field, energy-momentum tensor and Majorana equation for a spin 3/2 spinorial field. Hence, this theory becomes the one unifying spin 3/2 spinorial field with gravitation. We will find in it spin 3/2 fermions and spin 2 gravitons. A spin 3/2 particle is often called gravitino; this is a fermionic partner for graviton. We might also try to generalize the five-dimensional theory, that is one of Kaluza–Klein, onto the case of fermionic variables. Like in the preceding cases, we will consider a metric tensor on this manifold. It will break into a space-time tensor, the electromagnetic field four-potentials, a spin 3/2 Majorana spinor and a spin 1/2 spinor. Once we complete the construction of all the required quantities, that is curvature tensor, that of Ricci, curvature scalar and Einstein tensor, and also write down Einstein equations, it will turn out that they are taken into the following equations: Einstein equations with the electromagnetic + spinorial fields tensors as sources, Dirac equations for spin 1/2 spinorial field, Majorana equation for spin 3/2 spinorial field, and Maxwell equations. Regretfully however, this theory could only describe uncharged fields. Namely, the fields involved here fail to be an electron. Similarly as in the preceding case, we may join the following particles. Graviton with gravitino and photon with a spin 1/2 particle, which we might call photino. This theory without difficulties can be extended onto the case of a generalized Kaluza–Klein theory with an arbitrary gauge group. Here, the supersymmetric partner for a given Yang–Mills field intermediate boson will become a spinor (spin 1/2) multiplet, belonging to a representation of the adjoint gauge group. These particles will not couple to Yang–Mills field. Their "colour" charges will vanish similarly as in the electromagnetic case, "photino" has the charge equal to 0. The theories obtained here could be extended onto the case of non-vanishing torsion. In such a case we will get the nonsymmetric connection on the multi-dimensional manifold.

"Einstein equations" will be similar to the ones from the previous cases, but the torsion will not vanish. This is very important difference between the purely bosonic case and the fermion-boson one. In the case when only the bosonic coordinates appear, the commuting ones, the torsion vanishes. When the anticommuting coordinates appear, the torsion might be different from zero. Let us notice that we could fit the supersymmetric theories here presented into a general unification scheme presented in Chapter 4. We have there two theories T_1 and T_2 which were geometrized and later combined into the geometrical theory T_3. In the theory T_3 all the quantities had obtained the geometrical interpretations. They are the quantities like connections, metric tensors, torsions and the remaining geometrical ones. Under the assumption about the existence of supersymmetry, one could extend the notions of geometrical quantities by introducing manifolds with anticommuting parameters (the so called supermanifolds) and bundles with graded Lie algebra. Let us notice for instance that the gauge field is described by a connection on a principal bundle with a structural group G. This connection constitutes a form with values in Lie algebra of this group. In the case of considering the counterpart to a supersymmetric gauge field, that is to a field whose carriers are the intermediate fermions, we will have a bundle for which a graded Lie group constitutes its structural group. This field will be described by a connection on such a bundle. In turn this connection will be a form with values in the graded Lie algebra. In this manner, as we see, the geometrization is also possible in the fermion sector, not only for bosonic fields. In both cases the geometrization is associated with the unification of the fundamental interactions.

13 Physical Determinism and Holism

The relationship between an all-encompassing holism and physical determinism is to be presented in this chapter. We are going to show that statistical determinism and a holistic world-view imply each other.

What is at present understood by determinism in physics and more precisely — the deterministic laws — what are they? They are the laws expressed in the form of differential equations, integral ones or appropriate variational principles. At this moment we have to perform certain specification, since these laws might employ two different types of variables: dynamical and probabilistic ones. The former will be ordinary functions, while the latter-random variables. The first ones give us (after solving the equations) the change in time of such characteristics like: momentum, energy, field strength, potentials, entropy, enthalpy, etc. The other will bring the change in time of the statistical distributions of the same characteristics.

The laws of the first type constitute the realm of such branches of physics like: Newtonian mechanics, thermodynamics of irreversible processes (the linear of Onsager and nonlinear one of van Kampen), microscopic and macroscopic electrodynamics, classical field theory, Special and General Relativity. The laws of the second kind occur in statistical mechanics (classical), quantum mechanics, quantum field theory and all applications of the last two, and also the theory of elementary particles in the widest possible sense.

Contrasting of these two kinds of laws, particularly laws of Newton's mechanics and of quantum mechanics (under the assumption that anything which appears as deterministic is exactly predictable, fatalistic, while anything which is indeterministic, is unpredictable) markedly leads towards the recognition of laws defining the change of random variables as indeterministic. The viewpoint of this sort, deeply rooted and many times emulated results from the misunderstanding due to only seemingly non-probabilistic nature of quantum mechanical laws. This view has also found support in a wide-spread slogan about the reducibility of the statistical and thermodynamical laws to the laws of Newtonian mechanics.

Let us first analyze this view. What does it mean? It means that Boltzmann equation, entropy increase law, Onsager relations, gas law could be derived from Newton laws. Thus the law describing the random variables, the relationships between the average values of certain distributions could supposedly be deduced in a logical sense from Newton's laws.

How do the followers of the reducibility of the motion from the thermal level to the mechanical one justify this claim? We consider they say, not every single particle — (due to the impossibility of hacking the paths of all the gas particles in a vessel) separately when it e.g. strikes at the walls of the vessel, but an average amount of them in a unit of time per unit of area. Now, by computing the average change of momenta for these particles, we obtain the pressure, and further by associating mutually the remaining macroscopic features (like the volume and temperature) we arrive at the ideal gas law. Of course, we relate before that the temperature with the particles' average kinetic energy. This is a very well known procedure, found in all the undergraduate manuals of physics and taught to students of the first course.

Now let us think how such a reduction is to be carried through at somewhat higher level. Thus we have a certain number of mutually interacting particles, acting also onto the walls of the vessel. The motion of the every one of them is described by Newton equation (the advanced methods employ the Hamiltonian formalism) where on the side one finds the gradients for the potentials of particle interactions between a selected particle and the remaining ones plus the gradient of the external potential (equal for all the particles). We could associate this last potential with e.g. gravity force, electromagnetic field (if the particles are charged) and above all — with the vessel's walls. In other words, we are concerned with taking into account of all the forces acting onto a given particle, either the external or the internal ones (originating at the particles themselves). If we now try to solve these differential equations, this would turn impossible from the practical point of view (too large number of variables) — but this will not become the greatest trouble. Namely, one needs to know initial conditions for solving the differential equations. This constitutes an extremely interesting feature of Newtonian mechanics, that is the separating of the two things: the equations of motion (the laws of motion) from initial conditions of the motion. Wigner considered this to be extremely interesting — and challenging. It was believed earlier that it is not appropriate for physics to deal with the initial conditions for the equations; physics was only to postulate the equations themselves. The issue of initial conditions was to be left for other sciences, such as geology, astronomy etc. This point of view is acceptable everywhere except the statistical mechanics — here the initial conditions are of extreme importance — equally significant as the equations of motion. After this digression let us return to our example.

Due to the impossibility of tackling analytically or numerically of this problem, one has to consider it in the statistical terms and hence to introduce the density of the statistical distribution, describing all the particles having definite momenta and coordinates. We define this probability function over a phase space of momenta and coordinates of all the particles.

Further, we may adopt certain idealizing assumptions about the distribution in question, namely to reduce it to the random variables defined on less-dimensional phase spaces. At this moment we are in a position, by using the equations of motion, to write down the equation of time evolution of our probability distribution; here a very significant moment is going to occur. We have to assume something about time initial distribution of momenta and coordinates. The assumption about initial distribution is very important, simply a crucial one, since it conditions significantly the form of the distribution after solving the distribution equations of motion.

One might perform this in variety of ways, all expressing the fact that the initial distribution of momenta and coordinates is equiprobable in the phase space. Often this fact is being expressed by a postulate of vanishing of certain correlation functions. With the aid of distribution function, we could find all statistical averages of the dynamical quantities and obtain out of them the macroscopic characteristics: the pressure, the temperature, internal energy, entropy etc. Using the relationships between these average values, we could derive lot of identities known in the phenomenological thermodynamics and e.g. entropy increase law. We might also formulate the distributions for the fluctuations of the thermodynamical quantities. Depending upon the idealization assumptions adopted with respect to:

- 1. interactions between the particles
- 2. potential of the external forces
- 3. form of the global distribution function for all the particles (e.g. by expressing it via the three-, two- or one-particle distributions)
- 4. ordinary or asymptotic ergodicity
- 5. thermodynamic equilibrium,

we will obtain the ideal gas law, that of van der Waals, Maxwell and Boltzmann distribution, thermodynamics of reversible and irreversible processes, nonlinear van Kampen thermodynamics etc. The assumption about the initial distribution of momenta and coordinates is going to occur everywhere here. Summing up we see that in order to “derive” the thermodynamics from Newton’s theory, we had to: firstly, introduce the probabilistic notions; secondly, assume some initial distribution. Hence we have introduced the concepts not present in Newtonian mechanics of the material point, and have adopted a crucial assumption about the character of the possible initial conditions. It will be important to emphasize here that by adopting a different assumption about the distribution of initial conditions, we might fail to obtain the entropy increase law, which indicates the significant nature of this assumption and its relations with experiment. Therefore we see that there is no reducibility form the thermal motion level to the mechanical one. In spite of the dynamical character of the thermodynamical laws (gas laws), they are in essence of a statistical nature, derivable from the probabilistic concepts and laws, which in turn are not reducible completely to the dynamical laws of Newton's theory. On the other hand, the fluctuations of the macroscopic quantities, such as: pressure, average number of particles in the unit volume constitute form the point of view of gas laws (that is laws of the phenomenological thermodynamics) something random, not predictable and hence indeterministic. From the statistical mechanics point of view (that is one of distribution of momenta and coordinates of all the particles), there is even no need to ask about this, since the statistical distributions of these discrepancies are known, and this is exactly what we are going to know and already know. One might say that the requirement posed in statistical mechanics is a minimalistic one. The idea of finding the motion of all the particles of gas contained in a vessel, e.g. with the volume of 1 cubic cm is thinkable. Due to the advancement of electronic computers, we could solve this problem, if not analytically, then in any case numerically.

Let us use here the interesting example found by Borel. As is known, the initial conditions are indispensable for the problem solution. Now the error of the order of 10^{-100} in the determination of the coordinates and momenta of particles results in the impossibility of finding the path of a single particle already after 1 milionth part of second. After this amount of time, the error in the determining of that particle's path (this path's blurring) becomes so large that it fills out the entire phase-space. And such a change occurs already under the influence of transferring 1 g of mass in the star Sirius on the path of 1 cm. Consequently, in order to have the motion of all the particles of this gas tracked, enabling one to derive the equations for the changes of pressure, number of gas particles contained in the unit of volume, for the temperature and for the other quantities — one would have in principle to take into the account all the interactions directed onto a given particle from within the entire Universe. Thus it means that the whole matter from Sirius were to be taken into account, if we would like to consider the motion of one particle during 1 milionth of second that way. Greater the duration of the motion, smaller the magnitude of an initial disturbance capable of influencing the motion afterwards; in addition to this, these disturbances will begin to accumulate themselves. Predicting the motion of all gas particles in the volume of 1 cm^3 found on the Earth surface and consequently — the ability to write down the dynamical equations for the pressure and temperature (provided that this gas stays in a thermodynamical equilibrium with environment) calls for the introduction of a "Laplace's cosmic mind"[1] which would manage to take into account all the interactions.

Thus the "cosmic mind" of Laplace design is needed not only in order to predict exactly the Universe fate from its inception, but even to predict the fluctuations of the pressure in a small vessel filled with gas, on the Earth surface during the time of two hours. This is an extremely paradoxical conclusion, which strongly corroborates the probabilistic point of view adopted in statistical mechanics.

We have taken up as example only Newtonian mechanics and the thermodynamics with statistical physics. Nonetheless one might see that the same applies to a microscopic and macroscopic electrodynamics, where the material constants: magnetic susceptibility of a medium and the dielectric constant constitute also moments of the statistical distributions and could undergo fluctuations. We have in this case to deal with the so called stochastic differential equations.

The same conclusions could be drawn after considering the physics of elastic media, plastic and thermoplastic media, the entire macroscopic physics. All of them are dynamical theories only when viewed upon superficially — as a matter of fact they describe solely the behaviour of statistical averages. Of course we could, acting in accordance with the principles of statistical physics to find the distributions of fluctuations for the quantities involved. At the very moment however, when we would like to formulate the dynamical laws of change for the quantities appearing in the equations of these theories (I am thinking at this place about such changes of physical quantities, which are caused by their probabilistic nature; the equations of a theory contain not the functions, but the random variables) — we immediately encounter the ghost of Laplace's "cosmic mind", which like *Deus ex machina* is necessary at least for mentally solving the problem. At this moment we plainly see that in order to proceed further, one has to reflect

[1]Under the term of "Laplace's cosmic mind" we are going to understand here an intelligent being, capable of investigating the Universe in the way we are investigating the vessel filled with gas. Hence, this hypothetical being would assume, in a certain sense, a position external to the Universe. In the example of an ideal one-dimensional Universe, presented below, this hypothetical creature would play the role of an observer, trying to discern the two micro-states of the Universe staying in the given macro state. This creature would try to predict the fate of a particle via discovering the microscopic information about the Universe at the given moment and using this information in the particle's equations of motion. We assume that the said creature would be — once given the above information — capable of solving the equations of motion and find particle's trajectory. It seems that in the case of a system with a finite degree of freedom, there should be no problem (in the ideal situation) in discerning of the two micro-states corresponding to the same macro-state (refer to the example with gas, analyzed below).

upon the capabilities of the said “cosmic mind”. To this end, let us ponder from the very beginning upon the motion of a single Newtonian particle in the Universe.

The Universe is acting onto a particle and we have to take this fact into account, in order to find the theory of motion. Since we are going to find it for arbitrary time span, hence we have to take into account absolutely all the interactions. We assume that without trouble we can identify the two Universe states occupying two different macro levels, that is in different states described by macroscopic parameters (in the scale of Universe). In any case, if not necessarily we ourselves, then the “cosmic mind”, should it exist, for sure could perform this identification in quite a similar way as we were identifying the macro states of a vessel with gas. But the thermodynamical, macroscopic information is absolutely not sufficient for the identification and taking into account of all the interactions directed onto the particle, since to a single macro state of the Universe, there correspond lot of different micro-states, whose ways of interacting with this single particle could be different. Thus, one has to obtain this information.

In order to understand better this problem, let us return once more to the example of a vessel filled with gas. This time however let us consider it as a system completely isolated from the world’s external influences and think what would happen, if we first put all the gas particles into a volume smaller than the original volume of the vessel, and afterwards would allow the gas particles to fill out this original volume again. Let us follow the motion of a system in the part of the phase space accessible to it. At the beginning, before the decompression, the system (gas has occupied a certain part of the phase space). In the statistical physics there is an extremely useful concept — the probability liquid. It describes very well time notion of a statistical system and, what is very important, the liquid is incompressible (the so called Liouville theorem). The liquid in the original state (before the decompression) fills out in a homogenous manner exactly the entire part of the phase space accessible to it. Associating now the information with the position of this liquid (since we know in which place of the phase space it stays) we observe that after a sufficiently long time this information will disappear. The liquid fills up all the phase space accessible to it in a uniform way. This is true only the point of view of such volumes, which are comparable with the originally accessible part of phase space, without the change of density. Anyhow, let us divide now the whole phase space accessible in this problem into cells having volume very small in comparison with the original volume of the phase space. At this level we could observe without any difficulty the inhomogeneities and we see that in spite the fact that the average liquid density was constant in regions comparable with the original volume of phase space, now it is noticeably variable on coming from cell to cell.

Let us now translate this what has happened above, into the language of thermodynamics and information theory. Let us notice that the entropy of the system (measure of its disorder) has increased to a maximum possible value, while the information has decreased down to zero (the uniform filling of the phase space by the liquid). This could however be concluded only from the macroscopic point of view. Looking onto this phenomenon from a microscopic point of view (very small cells) we nonetheless see (this could be precisely proven) that the macroscopic information has not disappeared, but had turned into a microscopic one (the amount of liquid in the subsequent, very small cells). Notice that the information about time macroscopic state constitutes a thermodynamical information (minus entropy up to a constant) whereas the microscopic information in principle equivalent to finding the coordinates and momenta of all the particles of a gas in question at the given moment (it is sufficient to reduce accordingly to the sizes of cells). Hence it is equivalent to solving the particles' equations of motion under the assumption that the initial conditions fit the original phase space volume. Notice further that the macroscopic information (definition of entropy in statistical physics) constitutes a statistical distribution (more precisely — a logarithm of a distribution) whereas the microscopic information is a mean of implementing the said distribution (and thus represents the *post factum* distribution).

We see therefore that there will be quite natural to break down the information about the isolated system (our vessel with gas constituted just the instance of such a system) into the macro and micro ones. The first of them informs us about the statistical distribution, the second about its particular implementation (we deliberately omit the distinction between the information and its measure, in order not to complicate the presentation). Clearly, the above subdivision makes only sense after reaching thermodynamical equilibrium. Let us notice that in principle, in our case it is possible to give both information. To put it in another way: in the given macro state we are able to distinguish the two micro states corresponding to it. This is however the case of the finite, isolated system with a finite degrees of freedom. The question arises that since the Universe constitutes an isolated system — whether in this case it might be possible to distinguish between the two micro states corresponding to the give macro state.

In order to answer this question, let us consider the idealized example of a one-dimensional Universe — the straight line with the countable amount of points marked on it (they are to correspond to the particles). Let us assume that this line does not have any region which could be distinguished with regard to statistical properties e.g. the average density (the number of points divided by a length of a segment they occupy) in sufficiently large segments does not depend on the position of this region on this line. This is going to express the first, strong (statistical) cosmological principle. Let us think, whether we are able to distinguish within such one-dimensional Universe between the two micro states, which correspond to the same macro state.

Returning back to the preceding considerations we ask whether Laplace's "cosmic mind" would distinguish between the two states. We have to give the method of identification. The most simple one would be just the positioning of the two such straight lines, corresponding to the two states and check, whether the corresponding points contain the particles. This is however not possible, for this simple reason that we do not know which point correspond to each other (in principle it would be enough to show a single one on every line). This fact reflects the isotropicity of the Universe. Thus there remains only one, unique method, namely we take a sufficiently large segment on the first line, of known length and divide it into smaller ones, of equal size. We count the number of particles which fell into every smaller segment. This gives us a certain sequence of natural numbers. Now we cut out the longer segment on the second straight line, starting from an arbitrary, but fixed point (to the left and right). We divide both segments thus obtained into the smaller ones of equal size, as was the case with the first line and count the number of particles contained in each of them. In this way we will obtain the two infinite sequences of finite sequences of integral numbers. In this case the law of great numbers predicts that after a finite number of comparisons between the first sequence obtained (the one from the first line and the one from the second), we will find the identical finite sequence. Let me recall that macroscopic information (that is, the statistical distribution) is identical in both cases. Thus, we are not able to distinguish two micro states corresponding to the same macro state.

The example of a line without a distinguished point, illustrates properly the situation we encounter when comparing two microscopic states of the infinite — Universe, being in the same macroscopic state. Hence we see that we shall not be able to distinguish two micro states in a given macro state. In other words, we could only find the distribution of some characteristics of the Universe, we will only be able to predict the changes of these quantities exclusively in the probabilistic sense. On the other hand, it is not known whether the Universe is in the state of the thermodynamical equilibrium.

Should it not be in equilibrium, then this would imply that distributions of its characteristics are variable in time. Nonetheless, the very impossibility of distinguishing between its two micro states would still remain in force. One has only to divide the Universe into regions in local thermodynamical equilibrium (the fact of the very existence of such regions we conclude from the observations). For each one of these regions we could find the macro information. Here, the macro state of the Universe would be constituted by a set of local macro states, whereas the micro state would still be defined as before, and hence the whole argument would apply, too.

Let us return to the "cosmic mind" of Laplace, discussed earlier. Even this one would not be able to distinguish the two micro states of the Universe corresponding to a single macro state. Consequently, this creature would not be able to find all possible interactions directed at a given moment onto a Newtonian particle. This "mind" could only give a distribution of these interactions and thus enter them into the equations of motion in the forum of Langevin forces of some kind. In this manner the subsequent momenta and coordinates of a particle could not be known with the infinite precision. After solving the equations of motion (which in this case would involve random variables), we could at best find the distribution of the possible paths of a particle. If the solutions of these equations admit Poincaré breakdown of the trajectories, and if the distribution of trajectories thus obtained, fails to vanish on both sides of a critical point, then even the prediction of the characteristics of the particle motion could at best be made in a probabilistic sense. In the case when the equation would provide several solutions of different topological nature, the selection of any among them would depend on a randomly changing right side of the particle's equation of motion arid hence finding out, which one of them is just being implemented in the given moment, could also make sense only in probabilistic terms. The occurrence of the possible bifurcation points would still further complicate the picture of the motion. The situation of entering into the chaotic region of equation would also be possible. Classical, dynamical determinism would become completely impossible. This is not to say that the particle behaviour is indeterministic that we are not able to predict precisely the fate of a particle. We could however offer the distribution of trajectories, coordinates, momenta. This constitutes a statistical determinism, the only one possible. Of course, while considering small time periods, we are observing the most probable trajectory. If one takes the motion in larger time periods, then with the presence of critical points, one might observe cases of abrupt changes in the particle motion. This is perfectly understandable. Let us summarize the hitherto obtained results based on the classical mechanics.

Under proper treatment, even the question of a single particle motion gives the predictions of only a statistical nature. The statistical mechanics is not reducible to Newton's equations of motion in a logical sense. All the deterministic laws express the statistical determinism, as motion's distribution laws of pressure, momenta, coordinates, that is with the aid of differential equations, integral equations and integro-differential ones. One could plainly see that at this point occurs the dialectical unity between the causality and necessity. The statistical laws are necessary as necessary are the given statistical distributions and their way of change in time, that is via means like differential equations, integral and integro-differential ones, variational principles, etc.

Causality reveals itself in the fact of existence of random variables and only the random variables. The symmetry laws of the physical systems, e.g. Galileo invariance, spatial translational invariance become the symmetries for the statistical distributions. The conservation principles are defined for the statistical averages of the conserved quantities. The fluctuations of the conserved quantities are caused by the interactions of the systems in question with the whole Universe. Let us come now to quantum mechanics.

We notice from the outset that the significant difference between the theory of motion for the single Newtonian particle being analyzed above, and the theory of motion for the quantum mechanical particle does not consist in the statistical nature of predictions characterizing the latter. Both theories give the statistical predictions. The former due to the impossibility of taking into account of all the interactions directed onto the particle (and this in a fundamental way, when the motion takes place against the infinite Universe) while with respect to the latter we might conclude that for the very same reason. Nonetheless, the viewpoint is always possible to the effect that all this represents the reflection of a fundamental principle of nature and could be hardly justified in a similar way, as in the first case. Such an origin of the statistical character of quantum mechanical predictions is however worth taking it into account and we will return to this soon. As to the differences, the only one would be the possibility of probability interference. Of course this was not the case in classical mechanics. Probabilities in classical mechanics would only be additive. The difference follows from the very method of obtaining the probability density, for instance of locating a single quantum particle. This density consists of wave function, which could undergo interfere.

In order to make the two theories maximally similar, we may try to look upon Schrödinger equation as an equation describing the time evolution of a certain probability distribution. Let us consider further the conservation laws. Here they are also defined for statistical averages, e.g. energy, momentum etc. Only the charge conservation principle is not cast for the statistical average, which resembles of the classical theories. Heisenberg relations are in turn taking place among the mean deviations of the fluctuations and it is not their occurrence which seems strange, but just the fact that they are binding mutually the fluctuations of quantities which are canonically conjugated. Let us note that the previously obtained impossibility to distinguish two micro states of the Universe being in the same macro state could be considered as a "principle of indeterminacy" of some sort. One might even present some evaluation of this indeterminacy, with respect to e.g. density of matter.

Quantum mechanical symmetry principles deal with the wave function and hence constitute the probability distribution symmetries. Schrödinger equation written in terms of a wave function's phase and modulus, becomes extremely similar to Fokker–Planck equation known for diffusion (compare the probability of liquid diffusion in classical theory). All this testifies once again that the statistical predictions do not constitute an exclusive domain of quantum mechanics, but they find their counterparts in classical mechanics.

It is worth to find out and stress all the contact points between the classical and quantum mechanics, since lately some attempts have appeared in physics, aimed at applying the nonlinear classical field theory for describing the quantum phenomena. Namely, it turns out that there are certain stable solutions for nonlinear field equations, the so called solitons, which behave quite similarly like quantum particles. Lot of phenomena hitherto exclusively of quantum nature, has won in this fashion a certain new model on the basis of classical field theory. The models were obtained which could be considered as elementary particles. Some of them possess quantum statistics, electric charges and spins, even the magnetic charges (t'Hooft monopoles). Characteristics of these solutions are linked to some topological properties and invariants. This does not mean that these methods managed to supersede quantum mechanics — as a matter of fact, there are no signs of that. Nonetheless it turns out that certain properties hitherto considered as immanent quantum mechanical features, could be modelled in terms of classical field theory, which seems extremely intriguing. Simultaneously interesting investigations are under way devoted to classical methods even in such exclusively a quantum domain like hydrogen atom spectrum. The spectrum of hydrogen atom was obtained via the methods of classical electrodynamics (under the assumption about the existence of electron magnetic moment) as a difference between the stable orbits. It turns out that the electron with a magnetic moment, moving in the field of a nucleus will not radiate energy form certain orbits (they accidentally agree with these of Bohr) and will not fall down onto the nucleus. All this testifies once again that the relationship between the classical physics (not quantum one) and a quantum one is far from final explanation.

After all — quantization — what does it constitute? It is a kind of heuristic procedure, which puts into a correspondence to a classical systems the quantum one in a definite way. For systems with finite number of the degrees of freedom, the situation is quite simple and has even found its formalization in the Kirillov–Kostant form (geometrical quantization), but even here the quantization of nonlinear systems seems to be of dubious nature. One might here also take advantage of Poisson bracket algebra deformation and consider Moyal brackets – counterpart of commutators in algebra of differentiable functions.

One of many interesting problems in geometrization of physics is a programme of geometrization of quantum mechanics. We mean here ordinary known quantum mechanics without any generalizations. It means it will be a geometric formulation of ordinary Hilbert space quantum mechanics. Moreover, Hilbert space is a vector space (this is important because of interference principle in quantum mechanics). Thus we should go from Hilbert space to Hilbert manifolds (locally Hilbert space). We should define a tangent space, a metric, a symplectic form and a connection. Simultaneously we should define Kählerian structure and before it a complex structure. This is possible to do. Afterwards (if someone wants) we can go to more general structure to nonlinear quantum mechanics. Moreover, a typical example of Hilbert manifold is a manifold of coherent states. In quantum mechanics we work with infinite-dimensional Hilbert spaces. Thus our Hilbert manifold will be infinite-dimensional. It means it is an interplay between differential geometry and functional analysis. In quantum mechanics we work with unbounded operators. Thus we face a problem with domains quite obscure in differential geometry. In future generalization, maybe in order to quantize nonlinear field theories, we should consider infinite-dimensional tensors as unbounded operators and so on. This geometrization programme works quite well in the case of finite dimension, i.e. for $H \simeq \mathbb{C}^n$.

There is a completely different situation in field theory, where in principle we could only quantize the fields described by linear equations. Attempts at quantizing of the nonlinear field equations are confronted with great and principal difficulties due to inability of representing Poisson bracket of dynamical quantities by the commutators of the operators for these quantities in a quantum field theory (in a canonical quantization). We get better results by quantizing via Feynman path integral method; Yang–Mills fields with non-Abelian gauge groups are quantized this way and also attempts are being made for quantizing General Relativity Theory. This method is not quite satisfactory, since it leads to the appearance of the non-observable particles, the so called ghosts (Faddeev–Popov ghosts) associated with the choice of gauge. (Moreover, those ghosts can be exorcized.) This situation is so much unsatisfactory, that the question arises whether one should look for stable soliton solutions and view them as states of the field after the second quantization. The solutions have lot of common features with the elementary particles. In t'Hooft model for instance, there is a stable solution, which behaves asymptotically like a monopole — the magnetic charge, endowed with a mass, able also to move in the space-time. Its path, due to the fact that this solution fills the entire space-time, is undetermined. This object therefore has some properties of a quantum particle. That model deserves particular attention since the field equation for which it constitutes a solution, is just like the gauge field equation for SU(2) group, coupled with Higgs field, which spontaneously breaks the symmetry SU(2). There are at present such models of quantum field theories, which do not have corresponding classical limits. They are the so called topological field theories. Their Lagrangians constitute the topological invariants. Therefore, the classical equations are trivial. May be this represents a proper approach towards quantizing e.g. the gravity. There are some approaches using non-local field theories to get renormalizable or even finite theories.

Finally, let us think for a while about the relationship between the all-encompassing statistical determinism and holism. One could see that they are intrinsically linked. The impossibility of separating of the part from the whole could lie, it seems, at the root of an exclusively probabilistic character of physical laws. At the higher level of the structure of matter it is often hard to notice, but under deeper scrutiny it could be grasped. Let us refer for instance to our discussion of the thermodynamics of the infinite Universe and the motion of the Newtonian particle in it. More deeper we get immersed into the structure of matter, more pronouncely this phenomenon reveals itself. Maybe the mathematical formalism of quantum mechanics represents only a model capable in a certain fashion of taking into account the all-union of phenomena within the framework of all-encompassing whole — the Universe. For the case of models of elementary particles based on nonlinear field equation, the relationship between the probabilistic character of predictions and a holistic approach to their internal structure seems very akin. In the case of discrete space-time structures such a relationship would be absolutely a must. Of course, it is difficult to foresee whether in fact physics is going to develop along this path, but judging after the present trends, one might state with a high probability that the holistic approach together with the accompanying statistical determinism will be dominant.

Let us notice the following fact. For in statistical mechanics we have to do with a system of differential equations describing a movement of point particles, it seems to be natural to apply a theory of deterministic chaos. This application of deterministic chaos theory seems to be very natural for we need to get some probability measure needed in statistical mechanics. We mean a continuous probability measure which is absolutely continuous with respect to a Lebesgue measure on $\mathbb{R}^n$. This is possible only in very special situations for a very low dimensional systems, i.e. Lorenz system, Hénon system. In this case we have to do with so called "strange attractors". They are a Cartesian product of a Cantor set and an ordinary manifold. (A Cantor set is defined on the real axis.) These sets are so called fractals. They have non-integer Hausdorff dimensions. Moreover, we have also to do with some kind of universality connected to a transition to deterministic chaos described by M. Feigenbaum. The third approach is connected to the so called intermittency. All of these three approaches are very valuable in order to get in deterministic systems a continuous probability measure. If such a transition is present in some systems then small changes in initial conditions of equations of motion for particles can go to non-predictability of their movement. These phenomena can be described as a repulsion of trajectories.

This is exactly what we are talking about. In this way we solve an old Boltzmann problem. However, practical application of a deterministic chaos theory in statistical mechanics seems to be very tedious. Moreover, from philosophical point of view the problem seems to be solved and justifies any probabilistic considerations in statistical mechanics. In this way we get our statistical determinism.

Let us notice that in the theory of chaos in a system of dynamical ordinary differential equations there are really four approaches:

1. strange attractors,
2. universality by Feigenbaum,
3. intermittency,
4. bifurcations on bifurcations, e.g. L. D. Landau theory.

The second approach seems to be the most natural to apply it as a foundation of statistical physics and of a kinetic-molecular theory of matter. Working with a Poincaré map we face also interesting phenomena as Misiurewicz critical parameters which complicate a problem of holism and reductionism and statistical determinism (Ruelle 1989a, 1989b; Mandelbrot 1977; Cvitanović 1989; Gutzwiller 1990; Dorfman 1999).

I am not pretending in this chapter for a complete resolution of the question of a relation between the statistical determinism and the anti-reductionistic, holistic approach to physics. I am however of the impression that the two are always co-appearing and in number of cases one might observe that they are mutually conditioned.

A philosophical role of a deterministic chaos has been advocated by M. Tempczyk in several publications *Theory of chaos and philosophy* (in Polish) (Tempczyk 1998). Let us notice that in pure mathematics we have to do with indeterministic chaos which is not covered in Tempczyk's publications.

14 Holism and Reductionism in Contemporary Physics

The relationship between holism and reductionism in physics constitutes the topic of this chapter.

Holism represents the philosophical option (it hardly might be called system) which considers the "whole" to precede, be more basic than the "parts". We shall try to develop this general assertion, referring first to instances from outside the realm of physics. Let us begin with the position held by English empiricists. It is common parlance to say that scientific theories consist from theorems. This is an assertion reducing theories to theorems. But one could say also: it is not true, the theorems are such and nothing else, because they are following from a theory. This last statement expresses the holistic viewpoint. It is namely assuming that a theory is more primitive than the theorems which, as it seems — compose together the theory. But we could go further and ask whether the theories enter as components into the intellectual currents of a given period? According to reductionist the answer is "Yes" whereas the holistic standpoint replies "No"; there are precisely the intellectual currents of a given period, which cause that the scientific theories assume the form as they do, not the other way round. Proceeding further along this path, we ask whether the intellectual currents fit together into a whole of intellectual life in a given period? The answer "yes" reduces the intellectual life of a given period to the intellectual currents. The negative answer constitutes the holistic thesis, to the effect that the given period's overall intellectual atmosphere gives rise to the creation of this or other sort of intellectual currents.

One could prolong the reasoning here presented by introducing the concepts from ever higher and higher levels. Two extreme viewpoints will always be possible: the first one reducing the units of a higher level to ones of lower levels, and the other, deriving the properties of objects at lower level from ones at the higher level.

The reductionist standpoint and an opposite to it, the holistic one, we could formulate in the following way:

The reductionist standpoint:

1. The whole consists in a block-like fashion from the parts, e.g. the unit at higher level consists of lower level units.
2. The properties of a whole largely follow from the properties of the parts, e.g. one could deduce in logical sense the properties of lower level units from the properties of higher level units.

3. The part constitutes a cause for a whole, which is to say that the units at lower level are at the cause of higher level ones.

The holistic viewpoint negates the above claims:

1. The parts are not autonomous with respect to the whole that is the lower level unit exists solely within the context of the higher level ones.
2. The properties of the parts follow logically from the properties of the whole.
3. As the cause of lower level unit there is a unit of the higher level.

In the sequel we will develop the holistic viewpoint and try to prove its methodological superiority in contemporary physics. It is interesting to notice that structuralism in physics has been advocated by M. Tempczyk in *Structuralism in contemporary physics* (in Polish) (Tempczyk 1976).

Before we embark onto the presentation of the contemporary physical viewpoints, we will present the holistic approach in biology, linguistics, etnography, sociology and psychology. The holistic viewpoints in these disciplines often bear the name of structuralism. The structuralism, known from the works of Piaget, Levi-Strauss and Chomsky represents a certain holistic viewpoint (also at this point I withstand from calling it a philosophical system or a philosophical direction, but rather a standpoint, method or viewpoint, in accordance with the intention of its creators) in etnography and anthropology ("structural anthropology" of Levi-Strauss and Piaget's "structuralism") (Levi-Strauss 1963; Piaget 1971). Structuralism employs the concept of a structure, which constitutes undoubtedly a holistic motion. The structure is akin to the notion of a whole not reducible into parts. It has however some further properties, which qualify it a bit closer. Namely, there is a set of functions transforming a structure into itself. In this way the structure, in spite of transformations preserves its identity, that is to say, remains the being of the same type, which is to be found with the help of certain criteria. Below we will give these criteria for the particular cases to be discussed. As — the outcome of arbitrary transformations, in spite of preserving the whole-structure, and their parts may change. They undergo transformation, destruction, substitution in this fashion could not be autonomous with respect to the structure. The primary and basic is the structure, its "building" parts are less fundamental, their properties are derivative of a structure. The very concept of a structure does not exactly mean a notion of an irreducible whole. It constitutes in a certain sense the specification, on the one hand, of such a concept, but on the other hand is a certain class of abstraction of an irreducible (to its parts) whole, with respect to the admissible transformations. Let us use for instance the notion of "*avunculate*" occurring in *Structural Anthropology* by Levi-Strauss. In author's view, this structure occurs in all ahistoric societies and the traces of it appear in the historical society. Nonetheless, in spite of this being a unique structure, in its particular implementations it might undergo changes, depending on a tribe where it occurs. These changes are tightly correlated among themselves, and have been formulated with the help of a fixed law of the relationship between husband-wife, brother-sister, father-son, uncle-nephew. It is exactly this law, which determines the possible transformations occurring within the structure, without violating it. An extended family in a tribe would appear in this context as an irreducible whole. The functionalism of B. Malinowski offers also an example of the holistic standpoint. Malinowski claims (Malinowski 1987 — Introduction) that all rituals, myths, behaviours or the institutions are to be analyzed in their interrelationships among themselves, in the light of functions they are bound to play in the life of a tribe. In this fashion B. Malinowski tries, like Couvier, to recover some objects from the other ones on the basis of the whole of the tribe's life. His method constitutes transfer of Couvier's convergence from paleontology to etnography and anthropology. Couvier's method works towards the recreating of the complete dinosaur's skeleton on the basis of few fossils of it. Couvier could guess the shape of unknown skeleton's parts by assuming that the reptile's organism had been accommodated to fulfil vegetative functions (and hence it constituted a whole whose properties are irreducible to the properties of its parts) making some measurements and fittings. Malinowski's functional method was very much alike. Assuming that the tribe exists, and hence the basic functions and needs are being assured by various institutions, Malinowski had tried to reconstruct whole of the tribe's life just by examining the single subject, e.g. the sexual life. In both cases, these of Malinowski and Couvier, certain investigation methods were being used, but holistic assumptions were to be found at their roots. In Malinowski case there was a thesis to the effect that a primary thing is the whole of the tribe's life, out of which there follow all the institutions, cults, etc. Precisely here the whole of tribe's life becomes an irreducible whole, the components become derivatives, fulfil various functions — they are by no means oddities nor interesting accidental features — he claimed.

In the case of Couvier's convergence, the fundamental holistic assumption is one of non-reducibility of an organism to its organs (skeleton parts). These organs have to be such and no different, since they have to cooperate together, in order to assure the organism's existence as a whole.

Let us come to holism in biology. Holism in biology is a wide-spread standpoint and appears at different levels of structure. Let us begin from the cellular level. The reductionist viewpoint states that a cell consists from organella and its properties follow from the properties of organella. Holism contends that there are organella which have such and such properties exactly because they enter as components of a cell. At the level of the whole organism, "consisting of" cells, the holistic standpoint asserts, that it is just the organism, which constitutes an irreducible whole. Tissues, cells and organs, being elements of an organism, constitute the derivatives with regard to it. Their properties are such that they enable the organism to fulfil its vegetative functions. This is associated with the organism's homeostasis. Some cells perish, still other undergo mutation, specialization, etc., while the organism as a whole preserves its identity. Its parts are non-autonomous with respect to organism, undergo changes or even destruction so as to allow the organism to function in homeostasis. We could distinguish in biological sciences still other wholes, at the higher levels. Here belong: the species, population, biotop, biocenosis, biosphere. The holistic approach operating on these wholes is particularly common in population genetics, ecology, phytosociology, sociobiology by E. O. Wilson (Wilson 1980), etc.

Since the ancient times a question was posed whether one could subdivide the matter into infinity or whether after a finite number of divisions we will approach a limit in the form of the further indivisible constituents. During the ages of philosophical development, and later on of physics, different answers were being offered here. Democritos from Abdera replied to this question by creating the idea of an atom — a further indivisible particle. Aristotle held the view that one could divide matter *ad infinitum.* Epicur followed the path of Democritos. Scholastics rejected the existence of atoms. Dalton laid foundations under the modern atomic theory. Mach and Ostwald held atoms to be a fiction. Now due to STM (Scanning Tunneling Microscopy) we can "see" single atoms on a surface of graphite or gold.

There is another question associated with the matter divisibility problem, a derivative one: whether from the properties of the whole's components — the whole would follow? Democritos replied to this question this way. The atoms have different shapes and magnitudes. Their various combinations bring about the variety of the bodies being observed. All the changes detectable in nature are the outcome of the atomic configuration changes. The beings observed are variable and destructible, but the atoms themselves are invariable and non-destructible. Therefore the properties of bodies follow from the properties of atoms. Democritos' theory was an instance of a typical reductionist theory in its pure form. The creators of the kinetic-molecular theory 20 centuries later, by formulating the problem of reducing the properties of gases, liquid and solid bodies to the properties of atoms, have not in fact changed the problem. They have only formulated it more correctly and due to having at their disposal the advanced physico-mathematical methods, managed to prove the reduction of thermodynamical law plus that of the gas mechanics to the laws of Newton's mechanics. Have they in fact been successful in reducing the thermodynamics and other macroscopic theories to Newton's mechanics? In textbooks of physics one often finds the example of a vessel with an ideal gas, covered with a moving piston. The cluster of particles represents a model of the ideal gas. By performing the uncomplicated calculations one could "derive" with the aid of Newtonian mechanics the gas laws, simply by identifying the pressure with the average number of particles per second, striking the unit area of the vessel's walls. Similarly the temperature could be identified with the average energy of a particle and so on. In spite of the far reaching simplifications, even in this case we cannot accept the identification of e.g. the temperature and the average kinetic energy of the particles. We have to deal here with the two completely different quantities. The empirical temperature is a certain physical quantity, whose way of measurement is given. Also its properties are definite, e.g. the zeroth law of thermodynamics, the relationship with Carnot cycle. The particle's average energy is something quite different. The equality of the two quantities is nothing else as misunderstanding, since by this way the reducibility of a temperature to the average kinetic energy is only apparent. The properties of a gas as a whole — its temperature — are not reducible to properties of its components — the particles with kinetic energies. If we extend this problem and confront the question whether the thermodynamics is reducible to Newtonian mechanics, plus consider a whole this issue at somewhat higher level, it would turn out that the topic is much more complicated. After all, what is to be understood under the claim of reducibility of the thermodynamical laws to these of Newton's mechanics? It means namely that on the basis of a certain model, referred to as a kinetic-molecular theory, we deduce in logical sense from the laws of mechanics (in e.g. Hamilton's formulation) the thermodynamical ones. In this fashion we get properties of gas from the properties of particles. This is grossly exaggerated and could be overthrown by the following counter-example. The laws of Hamilton's mechanics are reversible. This is to say that the motion in one direction and that reversed to it would be equally probable. Reversibility in thermodynamics is being expressed by the law of entropy increase. Processes resulting in the decrease of entropy of the isolated systems are impossible. But according to the reductionist picture, every thermodynamical process represents a mechanical motion of particles, composing the gas. Every component motion is reversible. Therefore, how it could happen that the resulting mechanical motion is not reversible? The paradox lies in the assumption about the reducibility of system's properties to the properties of its parts. Hence, one cannot deduce the laws of thermodynamics form the mechanical laws alone. The laws of mechanics are written in the form of differential equations. It is exactly to these equations that one wanted to reduce the thermodynamical laws, e.g. the entropy increase law. One often forgets that in addition to the differential equations — laws of motion, one needs also the initial data for all the gas particles, before specifying their trajectories. Therefore, specifying the initial data for all the gas particles in the vessel and solving the equations of motion, we will find the motion of all the gas particles. The problem of this sort is in general impossible to solve for computational reasons. We see nevertheless that obtaining radically different results is possible by adopting different collections of initial data for the equations of motion. It is always possible to reverse the system of particles from every point in the phase space. Within this picture there is even no room for any kind of irreversibility. The whole problem changes in the very moment once we introduce the probabilistic concepts, and begin to deal with the particles' probabilities of trajectories and positions. At this moment the initial conditions for all the particles' equations of motion will not interest us any more, but rather their positions' probability distribution. But even this will not suffice, since not every probability distribution leads to the irreversibility observed in thermodynamics. In order to obtain the entropy increase law, we shall assume the equal probabilities of all the initial configurations in the phase space. Should we assume anything else, we would obtain completely different results. Hence, to summarize, in order to recover the laws of thermodynamics, the mechanical laws — the equations of motion alone were not enough. We had to introduce the probabilistic concepts and adopt a principal assumption about the distribution of initial conditions. In this fashion the laws of thermodynamics are not reducible to mechanical ones (more about this in the preceding chapter).

The laws concerning the whole, the properties of a whole could be reduced to the laws and properties of parts; the assumptions are to be adjoined to them, which are not appearing in the particles' equations of motion. Particularly the assumption about the equiprobable distribution of the initial conditions bears markedly holistic flavour. The adoption of this assumption on the ground of a kinetic-molecular theory is dictated by the entropy increase law, the irreversibility requirement. This constitutes a kind of limitation on the initial conditions, which in no way follows from the laws of mechanics. It seems that the probabilistic laws have certain advantage over the deterministic ones in physics, whenever the issue of particle motion in Universe enter under the consideration. The deterministic description of a particle motion with the infinite number of the degrees of freedom in the

Universe is impossible. Assertions about the particle behaviour could only be made in probabilistic terms. Even the introducing of a cosmic Laplace mind will not help in the deterministic description of a particle motion, due to the impossibility of distinguishing the two "micro" states corresponding to the same "macro" state. Let us recall that the "macro" state is characterized by the thermodynamical quantities whereas the state "micro" is defined by the positions and momenta of all the particles which fill the Universe. The probabilistic behaviour of a particle is caused by the interaction with the entire material Universe. Therefore the type of particle determinism (the statistical one) is due to the effects of a Universe as a whole (the infinite number of the degrees of freedom). This represents very remarkable instance of holism already at the quantum mechanical level (see Chapter 13).

As a foundation for a theory, we have in quantum mechanics the probabilistic nature of the laws governing particle motion, expressed by a wave function. The model would be possible which assumes that the probabilistic nature of motion constitutes a mathematical reflection of a necessity of taking into account the particle's interactions with the whole Universe. Intuitionally it is clear here that smaller the particle, less factors are able to disturb its motion. This would explain the probabilistic nature of the quantum mechanical laws. Of course in quantum mechanics we have the interference of the probabilities, a factor absent in classical physics. This is due to the fact that the probability distribution includes a wave function.

Yet another argument on behalf of analogy between the quantum mechanical particle and the one moving in a Universe and possessing the infinite number of degrees of freedom is the fact that in both cases the particle state is being described with the aid of infinite number of parameters. In the case of quantum mechanics they are grouped in the wave function — the quantum state. In the classical particle case within the infinite Universe there are the coordinates and the momenta of all the remaining particles interacting with a given one. In quantum mechanical situation we know that a wave function constitutes an element of a Hilbert space. In the second situation, we do not know the structure of a functional space, which describes the particle state. Nonetheless these analogies are striking and certainly not accidental. Let us also notice that the holistic approach (taking into account of the entire Universe) has led us to the probabilistic nature of a physical determinism in a classical case. In quantum mechanics the global character of a wave function (particle state) is associated with a probabilistic description (density of probability). In both cases the statistical determinism is linked with holism.

Let us return once again to the problem of matter divisibility and the reducibility of a theory. Let us take for instance the physics of chemical molecules and atoms entering into them. It is quite clear that one is not able to deduce the properties of atones, e.g. their magnetic moments and the remaining ones — from the properties of chemical molecules. One might instead obtain a good theory of chemical molecules (not so complicated) by using quantum mechanics and the properties of atoms. Here, the reductionism may score the veritable triumph. As a matter of fact, from the properties of the parts we get the properties of the whole (on the basis of a theory). It seems however that there is no question here of reducing one set of laws to another, of one properties to others. Both the molecules and atoms are described by a theory of the same type. One might say that these are the units of the same rank. There is no analogy in this case to the situation of thermodynamics vs. Newtonian mechanics. The theory of the atomic structure is in the analogous situation. The only holistic-type assumption occurring here is the self-dual field theory in Hartree–Fock approach or Thomas–Fermi model of an atom. The statement to the effect that atoms consist form the nuclei and the election shell is well founded, similarly like the assertion that from the properties of nuclei and electrons we are finding the properties of atoms (on the basis of quantum mechanics). The self-consistency of a field is to be considered as a computational trick, simplifying the proceedings, a kind of an assumption of pragmatic nature, associated with the investigator's instrumentarium.

Stepping down into the structure of matter we encounter the atomic nuclei and the elementary particles. In classical nuclear physics, the whole vs. parts issue is analogous as it was in the atomic physics. The nuclei consist of protons and neutrons. The properties of a nucleus follow from the properties of its composing parts. Nuclear models — the droplet, shell, superconductive, collective — are only of a pragmatic nature. The assumptions adopted in these models, which were of a holistic character, like a self-consistency of a nuclear potential in an independent particle model, the surface tension of a liquid in a droplet model and others, are of pragmatic nature. Similarly like in the atomic physics, these are not the fundamental assumptions but rather the comfortable instrumental approaches, enabling the quantitative predictions. In many cases they are evidently false (e.g. the surface tension of a nuclear liquid), nonetheless they often allow the reasonable quantitative predictions within a certain range. The application of these models is dictated by a necessity, since in the times of their inception, nothing or almost nothing was known about the forces binding the nucleons together inside a nucleus. These models constitute a reflection of our ignorance. Gradually, with the accumulation of knowledge about the nuclear forces, one has managed to obtain models out of the incomplete theories of the interactions between the nucleons. In any case it needs emphasizing that the theory of atomic nuclei construction and the theory of atomic structures are analogous. The fundamental difference between them consists in the fact that the forces between the electron shell and an atomic nucleus are well known, while the forces between the nucleons are far less investigated. The second difference, less important, lies in the fact that within the atom we have a central body — the nucleus, creating a field within which the electrons revolve, whereas in a nucleus there is nothing similar. This in turn implies that while inside atom one might consider the interactions among electrons as disturbances, in the nucleus however they are essential. This is exclusively the difference of a technical nature, although a troublesome one in the case of applications.

To conclude the above discussion, let us stress also that in atomic and nuclear physics the reductionism is predominant, while holism fits solely the pragmatic, technically-instrumental side of the problem setting. Here it is not fundamental. What concerns the nuclear and atomic physics, the fact that holism might bear here only the instrumental character, is undoubtedly due to the fact that the components of atomic nuclei and the atomic nuclei themselves are the objects of the same kind. We remember though that a proton constitutes the nucleus in the hydrogen atom, and an alpha particle — the nucleus in the atom of helium.

We will discuss in the sequel the situation taking place in the theory of elementary particles.

By the time when the theory of elementary particles was born, becoming a branch of nuclear physics, the reductionism was scoring its greatest triumph. There were no indications of a radical change which was to take place soon afterwards. At this time only proton, neutron and electron were known. The existence of neutrino and meson was predicted. The situation was roughly the following. Several elementary particles constitute the foundation for the edifice of the entire Universe. All atomic nuclei, atones, molecules are constructed out from the few basic components. The whole variety of nature is caused by various combinations among these components. In a sense this constituted the development and mathematization of Democritos' idea. All laws of motion would presumably reduce to the laws of motion of these few elementary particles. The reality turned out to be more complicated. Within the few years a lot of the "elementary particles" were discovered: their number was so great that the very notion of the elementarity of these objects became questionable. Since hadrons, the particles taking part in strong interactions constituted the majority among the newly discovered particles, this has given rise for suspicion, whether they are really elementary or are composed of lower rank objects. There was no indication to the effect that some hadrons would be more elementary than others. The attempted construction of a model, envisaging hadrons as composed form neutron, proton and hyperon Λ^0 resulted in a failure. This was labelled as Sakata model, and failed to confront lot of difficulties. At the same time other concepts of elementary particle composition has begun to develop. This was mainly the idea of a *bootstrap* — democracy of particles. It asserted that all hadrons are equally elementary or equally non-elementary. Every hadron was in a certain sense to be composed of all the remaining ones. The relationship between them identifies all the particles existing in nature and their parameters, like: rest masses, magnetical moments, spins, isospins, hypercharges, etc. The character of this relationship was at issue here. Chew and his followers have maintained that the assumption of S-matrix analyticity constitutes this global provision, giving rise to the relationship in question. S-matrix or scattering matrix defines all the possible inter-particle reaction channels. In this fashion the assumptions pertaining to that matrix give rise to the appearance of such and not other channels, and consequently of the particles with definite properties. In its extremal casting, the S-matrix theory rejected even the meaningfulness of space-time concepts for elementary particles. It was claimed that upon satisfying of all the assumptions for S-matrix and by adopting only one parameter with a dimension of length, one should be able to recover the entire spectrum of elementary particles. Here belonged various S-matrix symmetries like SU(2), SU(3), its unitarity and parity conservation, etc. The data concerning the elementary particle spectrum were obtained from the approximate symmetries. Predictions of the masses of these particles were obtained on the basis how the symmetries SU(2), SU(3) were broken. The prediction and experimental confirmation of Ω^- particle within the framework of SU(3) constituted a great success for this line of argument. Let us add that the introduction of the so called Regge poles into the S-matrix theory fuelled further development and subsequent successes.

Let us notice now that the S-matrix theory and the theory of its symmetries constitutes a holistic theory, and non-parametric one as was the shell or droplet model. This theory does not mention any hadronic "elementary particles". Neither does it refer to interactions between the particles. This theory assumes that the nature of these interactions is different from the ones hitherto encountered. They are not of space-time character, and consequently there is no question of motion, even in quantum mechanical sense. The whole and unique information about the particles and their interactions resides in S-matrix. The global, holistic assumption concerning the properties of that matrix results in obtaining of all the particle's properties.

This extremely holistic programme was however not fully implemented. At the very outset, it had to be incoherent with respect to the electromagnetic interactions, which are long-range ones and exhibit the space-time nature in macrophysics: the same was with regard to gravitational interaction, encompassing all particles. Due to that, this programme had to be weakened while the democracy postulate — that of equal rights for all the particles — has been retained. This assumption have found a place in Heisenberg's theory of prematter (*Urmateriegleichungen*). Heisenberg revived in his idea the concept of Aristotle to the effect that every being consists of substance and form. At the same time he had retained the assumption that there could be no matter without form. He put forward a nonlinear equation of quantum field theory for a spin 1/2 spinor field. This was the equation for prematter. Various kind of this field's states constituted the elementary particles to be observed in nature. The field itself could not be directly observed in nature. In a certain sense, this was the kind of Aristotle's first substance, whereas the state of this field constituted the form of such a first substance. Together they gave an elementary particle.

This was a holistic idea and its ultimate formulation is to be found in Heisenberg's books: *Physics and Philosophy* and *Unified Theory of Elementary Particles* (Heisenberg 2000, 1966). W. Heisenberg, who managed in his undertakings to employ ideas of ancient philosophers, has left not only the one taken from Aristotle. He took over also the holistic concepts dating back to Democritos and Plato. Namely, Democritos atoms were often represented as equilateral or regular polyhedra. The equilateral polygon, constituting the said polyhedra were not existing independently, since they were not atoms; dividing atoms into fully flat elements was not possible. Heisenberg claimed that there is a complete analogy between the said polygons and his prematter idea. In a manner similar as these polygons, the prematter could not be observed directly, although it could be mathematically described. In both cases, he maintained — this was a remarkable thing. The "part" was not autonomous with respect to the "whole". The "whole" could not be subdivided into parts. Any being could not be broken down into the substance and form in spite that it consisted of them. The polyhedron — the atom, could not be subdivided into polygons, since these were flat, not spatial. The elementary particle could not be broken into prematter and form, although it consisted of them. The "parts" appearing in these concepts were not autonomous with respect to the "whole" and could not be obtained out of it as the result of division. In this fashion Heisenberg managed to obtain in his theory the bootstrap — "a democracy of particles" and at the same time explained in theoretical terms definite features of these particles. Due to the considerable computational difficulties, this theory has not developed much.

Nonetheless, it scored some successes in explaining hadronic mass spectra. At present there are occurring some references to the concepts advocated for above, but from quite a different angle which we will discuss below.

Holistic theories were developed in parallel with the reductionist ones. Attempts were undertaken in them at explaining the properties of elementary particles by reference to some from among their building elements — the quarks. Quark models pioneered by Gell-Mann and Zweig had assumed; that there are elementary building blocks of matter, namely the quarks. One is speaking at present about at least five of them, while some time ago only three were being mentioned. Each baryon consisted of three quarks, each meson from a quark and an antiquark. Assuming appropriate binding forces between quarks and antiquaries enables one to recover the spectrum of elementary particles by using three quarks and three antiquarks. Quark models to be encountered in these theories had much in common with models of atoms and atomic nuclei. Everything was in perfect agreement with the experimental data, except the fact that quarks have never been observed in nature. In addition to this, quarks had fractional electric charges and baryonic numbers. At the same time the quarks had to fit the so called para-statistics. Namely, there is an elementary particle, the so called Δ^{++} resonance with the isospin 3/2 and the spin of 3/2. This resonance consists of three identical quarks assuming the same quantum state. If quarks are fermions, then it would violate Pauli principle; on the other hand they could not belong to bosons, due to their spin of 1/2. Consequently they would appear as the so called para-fermions of rank 3. This is to say that in every quantum state one might have 3 particles at once (not only one as is the case for fermions). This solution seemed to be not too appropriate due to the connection between spin and the statistics of the particles in quantum field theory. Hence the idea of the so called colour of quarks was born. It was assumed that the quarks have an additional quantum number, the so called colour. This quantum number does not characterize hadrons; the quarks therefore have to saturate in this number. On time other hand, we save Pauli exclusion principle, quarks differ in colour. In this fashion each one from among the known quarks could appear in any of the three colour states: red, green and blue. Hadron as a whole is white. The quantum number were saturated in a manner similar to chemical valence or bonds. The assumption about the existence of colour was very rich in consequences. All elementary particles observed were "white", no unsaturated colour was encountered. This has created a ground for quark non-observability (confinement) hypothesis. The quarks therefore, appearing as hadronic components could not be directly observable. The role of the "part" and "whole" in this case could have turned to be quite different than in nuclear or atomic, physics. On the other hand, quarks were not to be taken as pure fiction. They could indirectly be observed in experiments involving deep inelastic electron scattering on hadrons. Here hadron appeared as a cluster of loosely related point particles. One could even experimentally determine the average electric charge of such a charged particle. The particles observed in deep inelastic scattering were called partons and it became customary to identify them with quarks and the so called gluons. Gluons constituted the hypothetical particles gluing the quarks into hadrons; they were quanta of inter-quark interactions. The uncharged partons were identified precisely with gluons. On the other hand, the scattering experiments have led one to believe that quarks have small rest masses. All this has led to the conclusion that quarks are hadronic components in yet another sense, not to be found anywhere outside the realm of the theory of elementary particles. They were supposedly not able to appear independently. This remains in sharp contrast to our intuitions about the notions of divisibility and of a component. In this way hadron appears to us as something indivisible, in spite of being composed of lower rank elements. The hadronic part — a quark makes sense only within the context of a whole hadron, and outside of this one, it looses any possibility of material existence. This reminds us the atom of Democritos — the polyhedron composed of polygons or the substance and the form of it. In a sense it is akin to Heisenberg idea, which will become still clearer below. In the approach of this kind we find the dialectics between the whole and the part. They become somewhat like the two sides of the same medal; they become inseparable.

The next step in the development of this theory would consist of finding a mathematical model for this theory. This was already accomplished — quantum chromodynamics is such a model. It shows the interactions among the quarks. This is the so called gauge field with $\mathrm{SU}(3)_c$ gauge group. The gauge fields, introduced by Yang and Mills (and therefore labelled as Yang–Mills ones) are very much similar to the electromagnetic field, but have different gauge groups. One might express this in popular parlance by saying that there here appear more "photons", that is interaction-carrying particles. The most significant difference consists in the fact that these "photons" could be charged, which is impossible in electrodynamics. The charges being carried over by gluons are colour ones, not electric charges. These charges are responsible for the forces keeping the hadron together. The gauge field (with a non-Abelian gauge group) e.g. $\mathrm{SU}(3)_c$ exhibits a very interesting property. At the very small distances (which correspond to large momentum transfers — big energy) the coupling constants — the charges of colour quarks — diminish very quickly. In this fashion at small distances the hadron appears as a cluster of very weakly interacting particles. This situation is being asymptotic freedom. This agrees with an experiment, the parton model. The fundamental problem of quantum chromodynamics consists in proving that at large distances the coupling constants grow sufficiently quickly, in such a manner that quarks could not escape. The theory assumes that the quarks are massless and obtain their masses in the outcome of interaction with the nonlinear gauge field. According to this approach, the quarks first are endowed with masses, then the chirial symmetry is being broken and finally the quarks get confined. This confinement is called infrared-confinement. This model illustrates the field-theoretic instance of an approach within a theory of strong interactions with hadrons. In this theory, the hadrons are composed of the fields, which are not independently observable: quark fields and a gauge field. This model is very similar conceptually to Heisenberg's prematter model, but here the prematter is more complicated, it is not solely the single spinorial field. We find here quark fields and gauge fields (gluonic ones). We would like to emphasize that this is a field theoretic and holistic theory at the same time. The underlying fields here are not directly observable. The hadron consisting of quarks and gluons constitutes a whole to be understood holistically. The parts are not independent with respect to the whole, they are endowed meaning in the context of the whole.

The question of composition of elementary particles in the light of the theories here considered, looks much more complicated than was the case for atomic nuclei or atoms. The inability of obtaining the components (quarks, gluons) points to the fact that the old concept of a part and a whole has to be changed. The only idea, which can be retained here, is the holistic concept. Naturally, this is not an extreme idea. As a matter of fact, the properties of elementary particles are in a sense reducible to the properties of quark and gauge fields. However this is a holistic or structuralist kind of reduction. The state of art of reductionism in contemporary physics has been profoundly described in note 3 (page 205).

15 Holism in Physics of Nonlinear Phenomena and in Cosmology

This chapter deals with holism in physics of nonlinear phenomena and in cosmology.

In contemporary physics one might distinguish to an ever growing extent the important role of nonlinearity, The nonlinear phenomena begin to exert still greater influence onto the way of understanding the fundamental processes. The linear equations, due to the superposition principle were deemed most important thus far. Let us recall that linear are Maxwell's equations in electrodynamics, Dirac equation, Schrödinger and Klein–Gordon ones. Nonlinear are the equations of Einstein, Euler, Navier–Stokes and these of magnetohydrodynamics. Hitherto the nonlinear equations were solved either approximately or by linearization. From some time already one began to look after the exact solutions of such equations. Lot of solutions was found, and thus an entirely new domain of mathematical physics came into being. The solutions of such equations with stable properties are called solitons. It seems that soliton is a model for an individuum, and therefore it well might be that such an approach is able to overcome the difficulties of elementary particle theory, nonlinear optics, hydrodynamics, solid state physics and others. The application of this approach in quantum field theory and in the theory of elementary particles appears to be particularly interesting. Lot of solutions for nonlinear field equations has been found by now, which behave in a way analogous to particles — the individua, e.g. sine-Gordon solution and Korteweg–de Vries one (historically this was the first soliton solution), Schrödinger nonlinear equation with logarithmic or cubic type of nonlinearity, and — what seems to be the most interesting — the solutions of gauge fields coupled to Higgs fields. In certain cases these solutions model the behaviour of quantum particles. Namely their trajectory is blurred (fills the entire space). The counterparts of baryons and their mesonic excitations could appear, too. A certain approach is also possible to "bare" particles and the "dressed" ones.

Let us take for instance sine-Gordon equation in two-dimensional space-time.

This is a nonlinear equation, which for small values of the fields transforms into Klein–Gordon equation. From the very form of this equation it is clear that a factorization of the field into oscillators does not make much sense. Let us recall that in order to quantize the field e.g. described by Klein–Gordon equation, one has to apply Fourier transform to it, that is has to factor it out into the harmonic oscillators. Then we introduce the creation and annihilation operators for elementary excitations of such oscillators. Instead the breakdown of this field into the mathematical pendulae is possible and very natural (there is beautiful mechanical model of this system). Due to the nonlinearity of the equation one could have tried to find soliton solutions. Such solutions exist and one might interpret them as a collective motion of the mathematical pendulae distributed in a continuous way on a thin string performing the screw-like vibrations. This solution behaves in a manner similar to a "dressed particle" in quantum field theory. Here the mathematical pendulum is a "bare particle". Two- and more-soliton systems are also possible as well as the counterparts of the bound states like Cooper's pairs. One might go still further and consider the excited states of a soliton. This is done in the following way. First linearize the fields in the neighbourhood of a soliton solution. This gives a harmonic oscillator vibrating around the stable solution. The states of this oscillator define the excitation of a soliton. Now it could be approached as a meson — excited baryon. Of course this is a model situation. On the other hand let us note that the linearization has broken the state of the excited soliton into the soliton in ground state and the oscillatory excitation. Finding of the soliton solution for the equation, which would correspond in linear approximation to a state of the form: soliton + oscillatory excitation would give e.g. "dressed baryon" with a mesonic excitation as a collective motion of the "bare" field states — the particles. In the case of sine-Gordon equation this would be a collective motion of the mathematical pendulae. Notice also what type of philosophical conclusions such a method is inviting. Namely, in the sense of the field equation, the excited state of a "dressed particle" is not a sum of a particle ground state and its exciton (meson). In spite of that, the ground state constitutes a solution of the field equations, but the "meson excitation" is such only in linear approximation. In this manner, in spite of the fact that the excited state of a particle is "composed of ground state" and the excitation it is impossible to separate the excitation from the excited state. The whole in spite of "being composed" from roughly speaking two parts, as a matter of fact is not reducible to its components. The situation of this type occurs for instance when real baryon "consists of" quarks do not have such a theory as we find in sine-Gordon for strong interactions. Some simplifications of the quantum chromodynamics lead under suitable assumptions to this type of situations. In such a case the hadrons would become exact solutions of chromodynamical field equations. At the same time the non-observability of quarks and gluons would be explainable in a strictly mathematical way. The "whole" — hadron would not be reducible to the composing parts (quarks with nest masses), although it would consist of them. Quarks could not be solutions for the field equations. They would only exist as a certain description of a solution (for example with large momentum transfers, hadron would appear as a cluster of almost point-like particles). In practice this would reveal itself through the increase of the interactions between the quarks, whenever one would try to separate them, that is — would result in quark confinement. Such a situation has been described from quite another side in Chapter 14. To summarize, let us notice that there is an anti-reductionist trend as follows: the objects from the microworld — elementary particles endowed with an internal structure — are not approached as the "wholes" reducible to their "parts". In fact they constitute the exact solutions for the field equations. Using the phrases like "they consist of" or "the successive levels of a structure" assumes a completely new holistic meaning. This occurs only in the context of a whole.

Let us observe that the approach treating the particles as the solutions of field equations is due to A. Einstein himself. In the book of A. Einstein and L. Infeld we read about the competition between the particle- and field-theoretic approaches in physics. The authors say that the motion of a stone in Earth's gravitational field could also be treated in the field-theoretic manner. In such an approach instead of a real stone's position we have a very strong concentration of a field; such strong and at the same time stable, so the stone preserves its identity during motion. This whole motion is a solution of the field equations. The authors fail to say more clearly what these equations are and what field do they mean. Probably they meant here the equations of a unified field theory. Let us note that the situation referred to in their book is very akin to modern approach to be found in the soliton theory. The soliton constitutes a solution for the field equations, e.g. sine-Gordon ones. It preserves its identity during motion. One could find a region where the field density is very high and take this as a location of an individuum described by the soliton. A. Einstein's programme maintaining that all the particles (in the first place the so called elementary ones) constitute the solutions for the equations of a unitary field theory, had failed for a variety of reasons. First of all, he was looking for equations which would comprise, in a non-trivial way, the electromagnetism and gravity. There was no room here for the strong and weak interactions. On the other hand, he was not successful in the construction of stable models for the particles — solutions of field equations. Nonetheless, the direction which he has established, seems appropriate for several reasons. Firstly, the unification programme aims at combining all the interactions into a unified geometrical theory. This is to say that the physical interactions obtain, like gravitation, the interpretation of geometrical quantities. Yang–Mills field theory is just this kind of a theory. It was successfully employed within the theory of weak and electromagnetic interactions in the so called Glashow–Salam–Weinberg model (which was discussed in detail above). At present we might speak about the electromagnetic-weak interactions. Similarly as in Maxwell's theory the electricity and magnetism constitute the sides of the same medal, hence also in this case the separation of the electromagnetic and weak interactions is only possible under special circumstances, the two interactions are inseparable, they represent the two sides of the same medal. In this sense one might say that electromagnetic and weak interactions constitute the part of the broader, electro-weak interactions. These interactions are components of Glashow–Salam–Weinberg ones exclusively in the holistic sense. They could exist only upon the simplifying assumptions, concerning globally the whole electro-weak interaction. The entire unification represents after all the holistic programme somewhat analogous to B. Malinowski's functionalism or Couvier's method (which we have described before). The programme of geometrizing physics together with its unification, put forward by A. Einstein represents only a certain precise statement of the unification proper. It offers a kind of indication as to how one should unify the theories of physical interactions. In the preceding chapters it was advocated for a viewpoint asserting that unification of physical interactions at present is tightly connected with the geometrization of physics. The geometrization of the electro-weak and strong interactions is a fact. In chromodynamics we use Yang–Mills field with the $\mathrm{SU}(3)_c$ gauge group, in Glashow–Salam–Weinberg theory — Yang–Mills fields with $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ group. Yang–Mills fields represent the connections in the suitable fibre bundles. The postulate of physical interactions' geometrizability with the simultaneous unification of them, provides a certain selection criterion among various non-trivial combinations of given theories. The original theory has to be the geometrical one. In practice one might obtain in this manner Kaluza–Klein theory — joining in it the geometrical and gravitational interactions. One could generalize this approach to arbitrary Yang–Mills field. Should one become successful in geometrizing of a spontaneous symmetry breaking, one might attempt constructing of the geometrical theory unifying the gravitational and electro-weak interactions, and perhaps consequently the strong ones, to. This programme is now under way. It aims at constructing of a field theory comprising in a unified way all the known fundamental physical interactions, that is the gravitational, weak, electromagnetic and the strong ones. This theory has to be geometrical, which is to say that the unified field is to be a geometrical quantity, for instance some connection. The field equations are to follow from a certain least action principle. Of course these equations will be nonlinear. All known elementary particles would constitute stable solutions for these equations. The spectra of elementary particles, as well as their charges, magnetic moments, spins — all should follow from the parameters of a solution. These solutions ought to have much in common with solitons, referred to above. The quantization of this theory is to be similar to one we have in sine-Gordon case. This programme is inherently holistic, combining the advantages of all the holistic approaches in the theory of elementary particles. Therefore the elementary particles “are constructed”; they consist of the “elementary interactions”. These interactions exist only within the context of a whole — the particle constituting a solution of the field equations. The particle energy is the energy of a field constituting the particle. As the model cases used to show, such a field most often fills out the entire space. Energy therefore represents a global feature of the whole space. Of course, such a field quickly vanishes except the relatively small region (a particle is localized). Nonetheless, the complete energy of a particle includes also these distant contributions. Similar situation occurs with the fixed couplings, for instance in electric or magnetic charges. This charges are usually topological, that is they are determined by the topological — global properties of the entire space-time or other multi-dimensional manifold. A model situation known from the solution called t’Hooft monopole, constitutes an exact solution of the nonlinear field equations. This field is gauge one with a SU(2) gauge group and Higgs field. This solution asymptotically behaves like a magnetic charge. Consequently its magnetic field is asymptotically Coulomb-like. The magnitude of the magnetic charge could be determined from the theory. But the solution itself has no singular point. In fact in the vicinity of a point, where the above mentioned monopole would have to be located, the field remains regular, changing its character in a function of a distance. We may easily compute the mass of such a particle — the monopole by determining the decomposition energy of the fields associated with this solution. Hence we see that in this approach all the properties of a particle follow from the global properties of solutions for the equations, among others, from the properties of the space-time. This represents the most holistic picture for elementary particles composition. In addition to t’Hooft monopole referred to above, we have other examples of “kink-type” and various kinds of solitons. There are also some models of such particles, obtained from nonlinear field equations by numerical means. This type of models are also known for the real particles, such as proton and neutron. The remarkable fact being here that the mathematical models and theories employed in this type of theories by itself imply the holistic viewpoint. These theories cast globally the properties of the topological manifolds and spaces. These manifolds could be the solutions of the field equations. Global features of such mathematical entities could be identified with the “local” properties of the particles, such as energy, mass, charge etc. There are also known some solutions of the nonsymmetric Kaluza–Klein theory endowed with finite energy electric charge and without singularity. They constitute models of charged particles.

Let us turn our attention to the question of holism in cosmology.

Cosmology is a relatively young discipline (Bondi 1960; Weinberg 1972; Ehlers & Schafer (eds.) 1992; Weinberg 2008; Liddle & Lyth 2000; Mukhanov 2005; Martinez et al. (eds.) 1992; Dirac 1973; Abbott & Pi (eds.) 1985). In a matter of fact, first scientifically sound notions about Universe have emerged only in XIX Century. About this time cosmology began to gain independence from physics and astronomy. The very problem of determining the properties of the Universe appears to be ideally suited for the holistic treatment. In fact, from the very definition of Universe it follows that it constitutes an ultimate whole. In the chain of subsequent components of a structure hierarchically positioned, from the elementary particles, through atomic nuclei, atoms and chemical particles to planets, stars and galaxies, the Universe constitutes a final element. There is still another element, which substantially distinguishes the Universe from galaxy or elementary particle. Universe is unique. We have no opportunity for comparing it to whatever entity of the same category. We are not able to investigate many Universes, as is the case when we examine lot of galaxies or stars. In this manner we could not speak about the typical Universe, as we used to say about the typical star or a molecule. We have no possibility of comparisons. This is very important circumstance. At the same time we are a part of Universe, observing it somewhat like "from within". We could not look at it as we are doing when looking via telescopes at the stars or galaxies. All these circumstances contribute to the fact that the cosmology itself is holistic in majority of its concepts. We have two approaches to cosmology: according to the first, we assume that the laws of physics discovered in our most immediate neighbourhood will also remain valid for the Universe as a whole. For instance, we assume the validity of Newton's gravitation law or of general relativity. Using these laws and also the so called cosmological principles, we construct the model for the Universe. Cosmological principles constitute certain assumptions about the symmetry of the Universe. Thus we assume for instance that there are no distinguished points in the Universe. The newly constructed model is to undergo observational checking. This means that we draw conclusions of the type liable for testing via the observational means. Within this approach, we deduce from Universe's global properties — its local properties, e.g. the escape of galaxies. The second approach assumes some global properties of the Universe and tries to extract from them its local features, particularly the locally observed relationships. The first approach might be termed as one proceeding “from physics to cosmology”, while the second as “from cosmology to physics”. This second approach represents an extremely holistic attitude. Let us try to explain this on the example of Milne’s theory. In this theory certain space-time metric for the Universe is being assumed. From the form of this metric and some quite natural assumptions Milne obtains Newton’s gravitation law for the two nearby material points. This is a surprising result. For here out of the Universe’s global properties, we get laws about the behaviour of its parts. This is not the only approach. Attempts of associating the properties of the Universe as a whole with properties of elementary particles are very characteristic in cosmology. This applies to the properties like e.g. masses, charges and radii. One might ask way to relate them just with the properties of elementary particles, not the ones of stars, planets or the Solar system. The answer is quite simple. The parameters of stars or galaxies are distributed over a considerable region and it is thus hard to determine some of them as the most characteristic. In the case of atoms, their nuclei or in the case of molecules, there also occurs large dispersion of parameter values, and in addition to that we are rightly convinced about the complexity of these objects. Elementary particles seem to be the ultimate stage of dividing matter in a usual sense. The question of their construction out of quarks was discussed earlier. Lot of researchers hold the view, that it is just this lowest level of structure associated with the elementary particles, which should be linked with the global properties of the Universe. The entire problem in turn had originated from the so called large number hypothesis of P. A. Dirac (Dirac 1973). This hypothesis states that any two great dimensionless numbers in nature have to be related together in a simple arithmetic way. What are these great dimensionless numbers? They are, to name the most significant ones:

$$
\begin{gathered}
\frac{q^2}{4\pi\varepsilon_0 G_N m_p m_e} = 0.23\cdot 10^{40}\\
\frac{R}{\frac{q^2}{4\pi\varepsilon_0 m_e c^2}} = 4\cdot 10^{40}\\
\frac{\rho R^3}{m_p} = 10^{80}
\end{gathered}
\tag{15.1}
$$

where:

G_N — Newton's constant

q — elementary charge

$4\pi\varepsilon_0$ is the permittivity factor

m_e, m_p — mass of electron, proton

c — velocity of light in a vacuum

R — radius of the Universe

ρ — average density of matter in the Universe.

The first of these numbers determines the relationship between the electromagnetic and gravitational forces inside the hydrogen atom. The second represents the ratio of Universe's radius to the so called classical radius of electron. The third constitutes the number of nucleons (baryons) in the Universe. The remarkable thing is that the first two numbers are of the same order. The probability for the occurrence of such a relationship at random is negligibly small. Therefore it is believed that they imply some deeper relationships between the global properties of the Universe and of the elementary particles. The third number is approximately a square of the two first ones. This relationship seems also to be intriguing. One might form more relationships which would include dimensionless atomic constants on the one side and the dimensionless cosmological constants on the other. All will include the numbers of the order of 10^{40} or its squares or third powers. Together these relationships are quite strange provided that we assume an entirely random cause. There are lot of attempts aimed at explaining these correlations. Among others it was assumed that these numbers remain constant. Due to the expansion of the Universe, this would mean the expanding of elementary particles, for instance of the electron. This hypothesis leads therefore to the conclusion that the elementary particles change their properties in conformance with changing properties of the Universe. Thus, out of the global properties of the Universe, the properties of its parts would follow. P. A. Dirac tried to explain the assumed constancy of the numbers obtained by the variability of a gravitational constant. In fact, one could observe in following relationship (constituting a corollary from the ones written above)

$$G_N \rho R^2 = c^2. \tag{15.2}$$

This condition implies the variability of the gravitational constant. It is of course very slow variability, but it is probably possible for observational detection in parallel with the enhancements of the investigatory techniques within the next decade (if this phenomenon really is taking place). There are still other attempts aimed toward relating the universal constants e.g. of an elementary charge (of the fine structure constant), or constants for the weak and strong interactions in formulas with cosmological parameters. They lead to the predictions about the variability of these constants in agreement with the expansion of the Universe as a whole. Attempts are being made to confirm experimentally these predictions. The said changes can have and could already have had great influence onto the Solar system structure or the interior of the stars. That is why even the traces of these changes are being sought at distant geological epochs. In addition to the hypothesis about great numbers theories are being put forward, which would enable us to infer them. Here belong the theories of Hayakawa and Eddington. Hayakawa has assumed that the entire Universe constitutes such a whole, where the particles are the distinguished dynamical states. The particles are stable and localized, but the conditions of their stability are determined by the properties of the said whole. This is very much like the vibrations of a string. The wave's nodes and the antinodes, plus also the wavelengths are locally observable, but the standing wavelength depends on the distance. This theory provides an instance of an extremely holistic one. It features the absolute dominance of the whole over parts. The properties of parts are being completely determined by the properties of a whole. The solution put forward by Hayakawa resembles Mach's principle for the properties other than the inertial mass. Let us recall that Mach postulated the following principle: inertial mass is being determined by the gravitational interactions of distant masses. In other words, the inertia constitutes a global feature of the entire system of bodies. There is no inertia in an empty space, where only one body is present. Already at the times of Mach the theories were created, which conformed to his principle. They represented the modification of Newton's gravity law. The gravitational potential in these theories depend on velocity. The inertia of an entire body found its origin at the gravitational interactions with distant bodies. In a similar way the electrical charges and masses of elementary particles were determined by global properties of the whole Universe. Eddington's theory belongs to this type of theories. It predicts the magnitude of the fine structure constant, number of nucleons in the Universe and a lot of other quantities. The most interesting thing here is the prediction of the value for the fine structure constant, which determines the magnitude of the electromagnetic forces and thus the value of an electric charge. Here, the dominance of the whole over the part is complete.

Finally, I would like to note that the inflationary models for Universe, developed recently and cosmic string models are very holistic in their essence. This applies primarily to the inflationary model. There are several inflationary scenarios for the Universe. Here belong the first, the second supersymmetric and the chaotic inflationary scenarios. In their construction, the theory of elementary particles is being used, gauge field and the spontaneous symmetry breaking with Higgs mechanism. Due to Higgs mechanism, the Universe starts from the state without matter. Here, de Sitter model with a cosmological constant is being followed. Because of spontaneous symmetry breaking, corresponding to the phase transition of the second kind after inflating (the exponential Universe expansion in de Sitter model) of the Universe, there follows the phase transition of the first kind, leading to Friedmann model. From this moment onwards the hitherto known normal evolution of the Universe takes place. The Universe in the phase of de Sitter expansion starts in the state of the so called meta-stable vacuum (false one). After completing the phase transition of the second kind, corresponding to the transfer from the state of meta-stable to the stable vacuum (the true one), there follows the creation of matter out from the vacuum (from the energy corresponding to the state of meta-stable vacuum — the cosmological constant having the interpretation of negative pressure). This gives rise to the possibility of creationist interpretations. It is possible (likely) that the Universe in its initial state was divided into many mutually non-communicating (causally) parts, all being in the state of meta-stable vacuum. At the outcome of the phase transition mentioned above, the coalescence of these parts would occur, with the subsequent entry into the expansion within the framework of Friedmann model. It is also likely that the Universe began the evolution as a solely one part. The differences between the subsequent inflationary scenarios for the Universe development, which were referred to above, are purely technical. In the first scenario we have quantum transition (tunnel effect) between the state of a meta-stable vacuum and a stable one. In the second scenario we have the transition of a classical type. In the supersymmetrical scenario we have to deal with taking into account of the supersymmetry (fermions). Inflationary models are capable of explaining some intriguing properties of the Universe. Namely, its very large spatial flatness (the fact that average matter density is very near to a critical density value). This coincidence had always appeared to be very mysterious. The inflationary models introduce new problems, e.g. of magnetic monopoles etc. These problems could be solved by the extended inflationary models, too. It seems that the inflationary models are capable of solving also a lot of other problems associated with the fluctuations of matter density, responsible for proper size of galaxy clusters and the observed inhomogeneity of matter in the Universe. There are several inflationary scenarios. The most important is Linde chaotic inflation with (supersymmetric) slow-roll inflation. The first inflation theory constructed by Guth using Weinberg–Coleman theory promises us a strict connection between cosmology and particle physics.

The relationship between cosmology and physics of elementary particles within these models places them inside a series of exceptionally holistic models. It seems that the inflationary models could be derived from the theories of Kaluza–Klein type, complemented with the dimensional reduction. The application of chaotical dynamical methods could probably explain the existence of superclusters and superholes between them (e.g. the Great Attractor). In such a case one might try to explain successfully also the fractal structure of the distribution of matter in the Universe at the level of clusters and superclusters. Let us notice here that the distribution of superclusters observed could not be explained by the accretion of the galaxy clusters due to their mutual gravitational interaction, since the relaxation time of such a big system is far greater than the age of the Universe. Hence, this structure had to be created already before the galaxies were born. Consequently it looks like the observed fractal (or multi-fractal) matter distribution pattern in the Universe ought to have its origin in the evolution from the primordial Universe, that is to say — in the inflationary age. This is the extremely holistic viewpoint. The interesting problem is to deduce a proper theory of gravitation from cosmological observations.

Recently we found from an observation that an inflationary scenario in cosmology is correct. Simultaneously it happens that the Universe is spatially flat. The Universe is also filled by a dark matter which acts only gravitationally. This matter is a cold dark matter (a dispersion of velocities of the matter is very small). In the budget of a full matter content of the Universe we have 4.9% of an ordinary matter (the so called barionic matter), 26.8% of a dark matter and 68.3% of the so called dark energy. The Universe is spatially flat thus a total density of matter in the Universe must be equal to a critical energy density. In this way a paradigm of cosmology is ΛCDM model. It means Λ — cosmological constant, CDM — Cold Dark Matter. According to modern ideas a dark matter (a cold one) could consist of supersymmetric particles (sparticles). There are some hypotheses: gravitino, WIMP (Weakly Interacting Massive Particle), also axions and scalarons. Someone calls an additional dark energy (e.g. a cosmological constant) a quintessence, which could be described by a scalar field from e.g. Nonsymmetric Jordan–Thiry Theory. In this way we have in the Universe five types of matter:

1. an ordinary barionic matter, which is visible,
2. a radiation,
3. a hot dark matter (e.g. neutrinos),
4. a cold dark matter,
5. a dark energy — a quintessence.

This is similar to Aristotle four elements theory plus *la quinta essentia*, the fifth element, i.e. five elements in his Universe. Due to the observations (Cobe, WMAP, Planck's satellite) of CMB (Cosmic Microwave Background) radiation (it means a correlation of its fluctuations) we can prove a ΛCDM model of the Universe in a very great probability. It happens that probably we need also an additional dark matter, so called "warm dark matter". The theory of "dark matter" has been recently highly developed. We have the so-called "dark strong interaction" with "dark quarks" and "dark gluons". In some sense this is similar to a "mirror world" known in $E8 \times E8$ heterotic string theory (moreover quite different). Simultaneously a geometrization and unification of fundamental interactions connected to a holistic approach seems to be a right track in contemporary cosmology and physics. Thus an *arche* of the world seems to be a geometry.

An interesting approach to cosmology has been designed by R. Penrose. He considers the so-called cycles of time and in his cosmology a singularity has been removed by a coordinate transformation (it is an apparent singularity as an apparent singularity in the case of horizon singularity for a black hole). In this way we are not forced to consider quantum cosmology, classical is enough. His cosmology is conformal and the Universe interacts infinite number of times with the future time-like infinity. It is possible to test empirically his CCC (conformal cyclic cosmology) theory observing microwave background radiation looking for rings. He claims that he does not need dark energy to explain cosmic acceleration (in his approach it is a gravitational radiation coming from past Universe).

Let us give some details of R. Penrose's theory (Penrose 2010). We have an infinite (countable) number of universes with Friedmann metrics. Every copy of a Friedmann universe can be attached through conformal boundary. In this way we get a new solution of Einstein equations which is considered as the entire Universe. R. Penrose called the existing sectors *aenos*. The concentric circles which can be observed on a sky (WMAP) can be considered as confirmation of the theory. However we can also get such circles (rings) from the scattering of many universes in Multiverse.

After the Planck mission we corrected cosmological model parameters and a composition of the Universe (we give the last results above). The paradigm of a contemporary cosmology is of course ΛCDM model. Moreover, Starobinsky nonlinear gravity (R^2) works very well for an inflation. There is almost flat spectrum for a primordial matter fluctuation $n_s = 0.9652 \pm 0.0062$, for a Hubble constant we get $H_0 = 67.3 \pm 1.0$ and the age of the Universe is 13.8 Billion years. Simultaneously a discovery of a polarization of a microwave background radian due primordial gravitational waves was not confirmed.

Conclusions, Remarks and Prospects for Further Research

Ideas put forward here are meant to illustrate a certain approach leading towards the unification of physics. This is a geometrization approach. The postulate of geometrizing physics constitutes a kind of methodological doctrine, which might be defined as one of using the geometrical methods in physics whenever possible, his somewhat imprecise definition needs some additional comments and further elaboration. The essay discusses examples of geometrization in: theory of relativity, mechanics, gravitation theory, electrodynamics. In many cases the methods of joining two geometrized theories into a single, unified one were also demonstrated. Supersymmetry and supergravity calls for the introducing of many hitherto unknown structures into mathematical physics. Here belong graded Lie algebras originally created specifically for the second quantization problems, and developed independently afterwards. But this is not all. Problems of geometrization and unification in physics call for the application of quite different mathematical apparatus. It is due to this need that quaternions and octonions entered into the theoretical physics, this is to say the non-commutative and non-associative structures. The usage of manifolds, not only complex ones, but also quaternionic and octonionic became wide-spread. These manifolds provide the example of the varieties with non-commuting coordinates, which were referred to above. It is being hoped that such an approach might result in joining many areas of elementary particles theories. This applies to the case of colour and its non-observability, known as "confinement". With non-commutative algebras are related in a peculiar way the algebras exceptional in Cartan classification: G2, F4, E6, E7, E8. These algebras and Lie groups corresponding to them are candidates for general gauge group of the fundamental interactions within the so called grand unification. Grand unification aims at unifying the electromagnetic, weak and strong interactions with the aid of a single gauge group. At the same time this group is to provide classification of quarks and leptons. In contrast to G.U.T. the so called small unification strives to unify only the electromagnetic interactions with the weak ones, viewing the strong interactions as independent, described by quantum chromodynamics. Simultaneously this scheme will comprise lepton classification, and hence it aims at quite modest goals. The programme of small unification seems to be near completion due to Glashow–Salam–Weinberg model. The programme of grand and small unification is partly geometrized due to Yang–Mills fields occurring there, or the connection on fibre bundles. These programmes use heavily also other methods, namely

Higgs mechanism and the spontaneous symmetry breaking. Both these interesting and important mechanisms are already partly geometrized. One might consider this to be a success of the geometrization programme, in spite that they still do include lot of phenomenology. There is a hope for the gradual removal of the remnants of the phenomenological inconvenience mentioned, mainly one associated within specific form of self-interaction potential for Higgs fields. All that could similarly be applied to the case of the spontaneous supersymmetry breaking and the supersymmetric Higgs mechanism. Here, also lot of phenomenology lurks through, even more than in the case of ordinary symmetry. Let us observe that the contemporary physics begins to follow the path set forth years ago by Albert Einstein in his idea of a unified geometrical field theory. He has devoted to it almost half of his life, without any significant outcomes. In fact it was only he, who worked in this domain, since after his death the idea was abandoned. The desire of having a grand synthesis in physics, of obtaining its unification via geometrizing the fundamental interactions has been thus abandoned. Now we see that Albert Einstein was right, he was on correct path. His programme was formulated prematurely; in his times it could not have been implemented. Nonetheless, some basic facts established within his research approach have remained for good. Among others was he who formatted the ultimate Kaluza–Klein theory unifying gravitational and electromagnetic interactions. The idea of geometrizing physics with, simultaneous unification of this science represents the holistic concept. The future geometric theory has to include in itself as special cases the already existing theories of interactions, e.g. general relativity and electrodynamics. This is mandated by the correspondence principle. This future theory will not be the mere compilation of the ones referred to above. Due to the fact that these have to be geometrical theories, some interference effects are expected to appear between interactions of various kinds. It is in precisely this fact, where pronounced holistic moment is contained. According to Einstein's programme, the particles have to cease playing the role of field's singular points. They are to become the solutions of the field equations; only in the approximate description will they become singular points. Albert Einstein and Leopold Infeld in their book (Einstein & Infeld 1938) *Evolution of Physics* write the following: *let us imagine the motion of a stone inside Earth's gravitational field from the purely field-theoretic point of view. This is not going to be the motion of a particle-body any more. Rather it will be the motion of the fields, which due to mutual interactions preserve the shape of a stone in an unaltered form. The energy of this field very quickly vanishes beyond the geometrical limits of the stone. The motion of these fields could be well described as the motion of the stone's centre of energy (the centre of gravity) in constant gravitational field.* It could be clearly seen that this idea and the notion of elementary particles as the solution of the field equations constitute conceptually identical approaches. They differ only in this that they have originated in different historical periods, and this explains why the more recent one is more precise. Also — we have already certain models capable of implementing this idea. Here belong the soliton solutions to nonlinear differential equations in various branches of physics. It seems that some of them could quite proficiently describe certain properties of elementary particles, including also quantum features. This kind of attitude towards the elementary particles is very holistic. One could even say that extremely holistic. The particles are here the solutions of the field equations. Precisely this constitutes the continuation of Albert Einstein's programme, and once again confirms his genial intuition.

The main thesis of this essay asserts that physics abandons the additive-analytic viewpoints on behalf of the holistic ones, and that this trend is going to continue. Out of this claim certain practical corollaries follow, bearing onto the way of cultivating physics, and also the type of mathematical apparatus used in it. The claim was advanced to the effect that it is the machinery of differential geometry, algebraic topology and algebraic geometry, which is going to play the role of principal mathematical techniques applicable. There appears that just these mathematical theories are particularly useful in theoretical physics, whenever the holistic standpoint enters into the play. It is likely that these theories are best capable of grasping the "part vs. whole" dialectics in contemporary physics. Judging after the attitude exhibited in works advancing them, contemporary physics approaches the situation, whereupon the basic components of matter — elementary particles — are going to be described by holistic methods. The structure of elementary particles (these particles do possess the internal structure) could not be explained by additive methods. This is going to reveal itself in the fact that it will not be possible to extract the hypothetical components of these particles. In this way the subdivision of a "whole" into "parts" is going to become a matter of convention, will exist due to convenience, a procedure without any counterpart in reality. In fact the ontological status of "parts" will be completely different than that of a "whole". The viewpoint advocated for above, bears also still other repercussions, associated this time with way of doing physics as such. Namely, the idea is being propagated recently to the effect that the search for the elementary components of matter could be prolonged into infinity. According to some views, quarks are going to be isolated, one day, and afterwards their components are going to be discovered, e.g. the subquarks, etc. Should this process of subdividing be continued into infinity, then even assuming the eternal existence of civilization on Earth, this will find no limit. The matter is going to assume the hierarchical structure, and this will be the structure without any boundary, when viewed in the direction of ever simpler components. Supposedly in this fashion the infinite variation of matter is going to reveal itself. This viewpoint represents the prolongation of the situation encountered in molecular, atomic and nuclear physics of today far into the future of physics.

The examples demonstrated above and considerations presented question such a thesis. The infinite variety of matter could reveal itself at any level of its structure; it is not necessary to introduce the infinite number of these levels.

The conviction expressed above brings about some particle consequences. Namely, since we are never to reach the matter's basic components, is it worth investigating all this, has the spending of such a tremendous amount of effort on searching for these any value? At present, when the science itself and its methods are being assaulted from the irrational standpoints, it is important to show that the fundamental natural science — physics — is not heading in the agnostic direction, leading nowhere.

One should note that the links the holistic programme exhibits with geometrization, are by no means accidental. Application of the methods of differential geometry, as well of these of algebraic topology and algebraic geometry is associated with global approaches toward such quantities like: mass and charge of a particle, its magnetic moment, spin and a lot of others. The above properties characterize the whole space-time. In this fashion a particle fills up the whole space, although in practice it is sufficient to consider only the small region of space. This feature determines the correspondence between the purely field-theoretic picture and the field-particle one. At the limit, the region in question could be considered to be a point, and the particles — material points, the singular points of a field.

The superstring model is a very holistic one, seems to be highly promising in its prediction power and the unification of fundamental interactions in the area of elementary particles. This model, or rather a collection of models has linked together the concepts of Kaluza–Klein, supersymmetry, supergravity, of dual models and that-of strings. The latter ones has resulted out from the modified hadronic strings, referred to earlier in this essay, in this case some among their weaknesses in the physics of hadrons turned into advantages, at the different level of structure, namely at Planck's lengths (10^{-33} cm). This theory is capable of creating the quantum theory of gravity (a finite one) corresponding to GTR (General Relativity Theory) in the classical limit.

Such a deep level of structure (distances of the order 10^{-33} cm — Planck's scale) would call for the introduction of a discrete theory for space-time, e.g.

of the cristall-lattice type. Perhaps in such a case, the role of gravity in the micro-world, of GRT would find proper explanation. The attempts of creating a discrete theory for space-time have already been undertaken many times in the past and are still being made. It is postulated that GRT within such an approach would become a macroscopic theory, phenomenological thermodynamics of a sort, obtainable after taking the averages over the regions big enough in comparison to Planck's length. From this point of view, the quantization of GRT in its present shape would be devoid of any merit, due to the absence of quantization of thermodynamics. Superstring model leads toward viewing GRT from yet another side.

Let us notice the following fact in quantum gravity approaches. It is possible to construct a consistent Hamilton formulation of General Relativity using new type of canonical variables going to be the so called Quantum Geometry (Rovelli 2004; Thiemann 2007). In this way we can quantize an area, a length and a volume. Some approaches to the theory of quantum black holes have been made getting the second law of black holes dynamics and Hawking radiation. It was also possible to resolve cosmological singularity using this approach (see Bojowald models). In some sense Quantum Geometry is in line of geometrization of physics. Moreover, there are a lot of problems to be solved in order to get a unification and a holistic picture of the Nature.

Important attitude within philosophy of physics, very much related to holism is a viewpoint taking the physical process to be the most fundamental one. In this manner the world gets represented as a dynamical process, devoid of any fundamental bricks. Only the process as a whole enjoys the absolute property of existence, with everything else being relatively separate part there of, in suitable conditions undergoing transformation or destruction. That is why in this approach the notion of vacuum (or even emptiness) assumes greater importance. The vacuum is not empty, it is full of processes leading toward creation or annihilation of the virtual particles. Exhibits lot of properties characteristic of any material medium, is polarized, could be viewed upon as a semiconductor or even a superconductor, has non-vanishing dielectric and magnetic properties, could even exert pressure. The vacuum is therefore an extremely complex being, far more complicated than the'mechanical model of aether put forward in his time by C. Maxwell. It is worth mentioning that there are interesting works confronting the notion of vacuum in QCD (Quantum Chromodynamics) and Maxwell's aether. The former is much more sophisticated, due to the fact that it could be degenerated. Given such an attitude toward vacuum, we have of course troubles with "elementary particles", that is their ontological status is doubtful. Are they the most basic constituents of matter or not? Are they indivisible (in the sense of Democritos) or not? The reply to both questions is in principle negative. The world assumes here the form of a process (at the most basic level) where elementary particles reveal themselves at the outcome of this very process. This is very fertile idea, to a large extent finding confirmation in the theory of superstrings. These theories (I say this in plural, since there are lot of alternative models of this kind) have a chance of becoming the theories for all the fundamental physical interactions. A continuous process of joining and separation of the vibrating, relativistic strings constitutes the model of the world within the framework of superstring theories. The motion of the strings takes place in the multi-dimensional space-time (26-dimensional and 10-dimensional one). The observed space-time corresponds to the state of vacuum, resulting at the outcome of higher dimensions' compactification. In this way the multi-dimensional space-time in the state of vacuum (its ground state) constitutes a Cartesian product of the 4-dimensional Minkowski space and 6-dimensional compact manifold endowed with a suitable topology. This represents the extension of Kaluza–Klein idea. The superstrings wind around the higher dimensions producing the spectra of the observable particles. There is presently a lot of works, trying to derive the standard model and new predictions in high energy physics. Some of them were quite successful. Summing up, the superstrings mean the imaginative fusion of Kaluza–Klein theory with dual models, supersymmetry and supergravity, theory of solitons and almost entire contemporary and past mathematics. Superstrings models are capable of avoiding anomalies, even be finite (in the sense of quantum field theory), it is possible to derive GR.T from them, corrected with scalar interactions (as is the case of Brans–Dicke theory) or even to be endowed with non-Riemannian, nonsymmetric metric. We could obtain Yang–Mills fields, quark and leptonic spectra, a number of families in the standard model, spectrum of the nonsymmetric particles, etc. The most distinguished superstring model, the so called heterotic strings, give us a single distinguished group, that is $E8 \times E8$ unifying the interactions.

Superstrings (particularly the model of heterotic strings) have been considered as a starting point for the creation of the all-encompassing theory, that is the theory of the entire physical reality. They failed to satisfy to these expectations, due to the following reasons. They were not a unique model singled out on the basis of the mathematical constituency involved.

Due to the manner of obtaining the heterotic strings model, the hope has arisen to the effect that this could be a unique model of physical reality. This has however turned to be misleading. A great deal of competing models was created, which had enjoyed similar properties. One could even hardly determine, how much of them there are. On the other side, the linking of standard model's parameters directly to some specific superstring model ended in a failure. Because of this, in principle only the so called superstring-inspired models were investigated. This has not yet provided any significant hints for experimental work. The only new predictions comprised the ones dealing with the existence of the additional bosons $Z^0(Z^{0\prime})$ and their mixing with Z^0) boson. In general no predictions about their masses were obtained. The superstring models were employing very interesting mathematical formalism within the string quantization framework. This formalism, labelled with the name of Polyakov employs the theory of Riemann, Teichmüller space and also determines for operators acting in infinite-dimensional spaces of functional analysis. The need for compactifying the superstring model has caused the development of the theory of Calabi–Yau manifolds. Solving (by Yau) of the conjecture by Calabi about these manifold served as the main argument supporting the model including E6 group within the grand unification resulting from the model of heterotic strings. It seems however that in spite of a great abundance of the string models, their importance in high energy physics is quite limited. They fail to provide (at present) easily verifiable predictions. Hence they are not good theories in the sense of Popper. The physics of Descartes and Newton constitutes a complete negation of such an approach. In spite of the fact that here the notion of vacuum is far more important than was the case in pre-Newtonian approaches, they still fail to offer adequate ontological status for vacuum. The notion of vacuum (void) is heavily substantially linked to the geometrization of physics. The geometry (that is the void, the vacuum) occurs here as the basic rule for the world, and it constitutes just this physical process mentioned above. Such an approach could also be developed within the superstring context. The possibility arises here of associating the spectra of the observable elementary particles together with their features with the cosmological model of the Universe (the so called compactification on the orbifold, linked with the cosmic string). In this manner the experimental testing of the cosmological model could be inseparably associated with the investigations into high energy physics, which is very interesting.

Cosmic strings are one-dimensional material objects. In the relativistic theory of gravitation they span a two-dimensional surface in the space-time. They (that is — strings) could be open or closed. In the model mentioned above, the cosmic string is a consequence of the common 2 dimensions for an 8-dimensional orbifold and the space-time (4-dimensional) entering into the cosmological model. In this fashion, there will appear 2-dimensional world — sheet of a string in the space-time. In the 3-dimensional space (for a fixed cosmological time) a one-dimensional object is observed as the cosmic string evolving in time.

The process-like approach might be combined with a systematic one, that is to say with the structuralistic approach. Together all this is extremely holistic. There is also an interesting combination of the string and superstring theories with black holes. This is associated with string gravity, resulting after going to the limit with $\alpha' \to 0$. The parameter α' called Regge trajectory's slope, tends here to 0 in a special way. In this manner we obtain (via the tree-like approximation of the heterotic string theory) the classical Lagrangian of GRT. Thus we could examine the corrections a' (associated with the quadratic terms in Lagrangian) to the black-hole type solution of GRT.

Taking advantage of quantum corrections to the tree-like approximation within the heterotic string model, we could obtain in a similar manner the corrections to α' (of the second order in perturbative quantum string calculus) to GRT Lagrangian. In these cases we examine of course the tree-like diagrams plus one- and two-loop corrections for spin 2 particle, which here has the interpretation of graviton. Proceeding in this fashion, we could look for quantum corrections to GRT Lagrangian (or full supergravity). This is going to bring us quantum effects in the space-time generated by black-hole-type solution.

In a similar manner we get Lagrangians for Yang–Mills fields, the scalar ones, together with the corrections α' and also the corrections of higher orders in perturbative string calculus. If the whole theory (of superstrings) is finite (there are no ultraviolet nor infrared divergencies) then this should constitute a means for obtaining the quantum theory of gravity. By investigating the effects around the exact solution (implementing the black hole) to the classical field equations, we obtain quantum effects of gravity inside the black hole. At this moment these are not semi-classical methods any more, hitherto referred in this essay, and associated with Hawking effect. (See some considerations below.) There will be instead significant effects of quantum gravity, originating from the heterotic string model.

Superstring models have recently undergone considerable development (Kaku 1999; Polchinski 1998; Green et al. 1987). We have today strings and superstrings in various space-time dimensions. The dimensions 10 and 26 critical for the quantum superstrings and strings, have turned far from being unique. The strings and superstrings have also appeared in the four-dimensional space-time. Let us recall that by a critical dimension of a superstring or string is to be understood such a dimension, at which we obtain consistent quantum theory of string or superstring. Referring to Chapter 10, we might say that for a bosonic string (without spin) in dimension 26 — relativistically invariant and the canonical quantization become equivalent. For a string with spin the same occurs in dimension 10. The critical dimensions of 10 and 26 could also be obtained at the outcome of other string quantization methods. By using Polyakov method, we may obtain them provided that the assumption about uncoupling of Liouville's theory was adopted. A connection between the formula (10.1) and Kac–Moody algebras indicates that the critical dimensions singled out, are not that much important and they change from model to model.

We mention here on superstring consistent models. There are five possible String Theories: Type IIA, Type IIB, Type I, heterotic E8 $\times$ E8 and heterotic SO(32). All of them are connected by some transformations. For this someone considered them as realizations of more fundamental theory called M-theory. M-theory is considered as 11-dimensional supergravity (all consistent String Theories are 10-dimensional). In String Theories we have expected also higher dimensional objects (higher than 1-dimensional), i.e. p-branes. String is 1-brane etc. They are living in 10-dimensional space-time. The most important are D3, D7, D5. D means that we choose Dirichlet boundary conditions on this object in 9-dimensional space (the remaining dimension is a time). This project as TOE (Theory Of Everything) seemed to be very promising. However, it is a very hard problem with compactification, i.e. a landscape problem. It seems that we have to do with a huge number of possible compactifications (some say 10^{500}). In this way we cannot choose (we have no mathematical criterion) a proper compactification to get our world we live. There is also F-theory which is closer to our contemporary physical world but still to vague to be TOE. An interesting approach to TOE is AdS/CFT. We will not consider these theories for they are far away from our considerations on holism and geometry.

Let us mention on some developments in Kaluza–Klein-like theories. In these approaches additional dimensions can help us in hierarchy problem of fundamental interactions (Randall–Sundrum model) or by introducing branes to solve some problems in cosmology (inflation).

It is worth to mention on Topological Quantum Field Theory (TQFT). There are several models of TQFT. In all of these models the amplitudes (correlation functions) do not depend on a space-time metric. They are topological invariants. Quantum gravity is considered as background independent theory. TQFT models are examples of such theories. It is hard to say if they can describe fundamental physical interactions. Moreover, they can be useful in solid state physics as effective theories (fractional quantum Hall effects). In some sense they show many holistic aspects. TQFTs are subjects of pure mathematical research not only of theoretical physics interests. Independent models have been constructed by Atiyah, Witten, Schwartz, Linker, Kontsevich. TQFTs can be considered (in some case) as non-physical models for computational approximation to string theory. Donaldson (1986), Jones (1990), Witten (1990) and Kontsevich (1998) were awarded by Fields Medals (Nobel Prize for mathematics) for works related to TQFTs. TQFTs are connected to knot theory, geometry, topology of four-manifolds.

Models of strings and superstrings were developed in other directions also. One such direction is the so called model of p-adic string. This model uses a number field other than the usually encountered field of real or complex numbers. It uses the field of p-adic numbers and also the field of complex p-adic numbers. The field of p-adic numbers arises as a result of complementing the field of rational numbers with respect to the p-adic norm. p-adic norm (modulus) is defined for any prime number p in the following way:

$$\left|\frac{a}{b}\right|_p = \frac{1}{p^\alpha}, \quad b, a \in Z,\ b \neq 0,$$
$$\alpha = \mathrm{ord}_p(a) - \mathrm{ord}_p(b)$$

where

$y = \mathrm{ord}_p(x)$ is defined as the greatest number satisfying the condition $x = \pm p^y x_0, \quad x, y, x_0 \in N_0^\infty$.

One could prove that this norm agrees with the multiplication and also that it fails to satisfy the axiom of Archimedes from elementary geometry. Instead of the familiar formula

$$|x+y| \leq |x| + |y|$$

we have

$$|x+y|_p \leq \max(|x|_p, |y|_p).$$

The fundamental characteristic displayed by the field of p-adic numbers is the fact that a set $\{na\}$ is bounded in p-adic norm, where $n \in N_0^\infty$, and a is a rational number (in general p-adic). The field of complex p-adic numbers is obtained by the algebraic closure and complementation in an extended p-adic norm. It is proved that in this field the principal theorem of algebra holds, which means that every polynomial with coefficients from this field has a root also in this field.

The application of p-adic number field in the string theory has the following sense. We assume that the variables a and r (see Chapter 10) are p-adic, and $X_\mu(\sigma, \tau)$ has real values. In this way we construct an internal

p-adic space inside a string. In another approach it is assumed also that X_μ is p-adic, and only the transition amplitudes are complex. The most extreme approach assumes that even the transition amplitudes between different string configurations are p-adic. This last direction finds its continuation in the so called p-adic quantum mechanics. In such an approach one imports the entire quantum mechanical formalism into the field of p-adic numbers. Under the influence of this type of research, the entire p-adic analysis, p-adic functional analysis, p-adic geometry etc. were developed.

Complex p-adic analysis has interesting properties: Liouville theorem does not hold, and a necessary and sufficient condition for the convergence of a series is

$$\sum_{n=1}^{\infty} a_n \text{ is convergent iff } |a_n|_p \to 0.$$

In the p-adic functional analysis the Hahn–Banach theorem does not hold. The development of p-adic analysis enables us to compute the transition amplitudes for p-adic strings. The results obtained are very interesting, although quite remote from experimental checking. Following the example of using the field of p-adic numbers to physics, attempts were undertaken to use also other fields. This applies to the finite fields, too. Minkowski space over the field of p-adic numbers, over the finite field were constructed, the appropriate Lorentz group was built etc. Attempts were also made towards extending of this analysis onto Riemann geometry and GRT. The results seem to be quite interesting, but regretfully enough they remain too distant from experimental checking. The philosophical importance of this type of concepts is tremendous. Why? It is known from algebra that the only norms, defined over the field of rational numbers are the usual norm and the p-adic one, for any p prime. Of course one means here only non-trivial norms, coinciding with the structure of the field of rational numbers. Rational numbers are available to us as results of measurements. We do not get real numbers as the outcomes of measurements. This is impossible. We use the field of real numbers for the sake of pure convention, referred to by H. Poincaré (Poincaré 1905) and later by K. Ajdukiewicz.

The adoption of the field of real numbers fellows entirely due to our convenience and thus by a convention. Now the question arises here, whether one could change this convention by taking the field Q_p (that is — the field of p-adic numbers) instead of the field of reals. Could it be that this change in the manner we understand measurement would enable us to solve the fundamental problems of contemporary physics? This is interesting and already brings us some results in the form of a convention more suitable for e.g. the domain of superstrings. We should envisage this type of outcomes, provided that we adopt the conventionalist attitude within philosophy of science. We know after all that no physical measurement whatsoever is capable to check, whether the given function assumes real values. We could only assert that it assumes rational values (the precision of a measurement). The substituting of the field of p-adic numbers instead of real ones might amount simply to a convention change. Issuing from the W. Van Orman Quine's logical empirism one could interpret it as a change of a measurement number field. W. Van Orman Quine even suggested to change the logic of inferences. For instance — the adoption of multi-valued logic instead of a two-valued classical logic.

The postulate of adopting different logic is associated with the introduction of a multiple-valued logics, e.g. of Reichenbach, Łukasiewicz, J. von Neumann and Déstouches–Févriér to inferences in the quantum mechanics within the context of a wave-particle duality.

In this approach it was the experimental checking which was to guide us in deciding what kind of logic to use. Thus it is also the experiment which could dictate us what type of number field to use in measurements. Let us recall that W. Van Orman Quine (Quine 1961) claimed that in science there are no purely analytical nor synthetical sentences. It is impossible to separate knowledge originating from experience and the other one, independent from any experience. In his understanding, every scientific assertion is so deeply involved into the analytico-synthetical relationships that deciding the truth of this assertion exposes the whole science onto the verdict of experimental evidence. Therefore everything could be changed once a given scientific theory is falsified. Here might belong the changing of: logic (rules of inference), number field in which we cast the results of measurements, space-time symmetry etc. Perhaps the only criterion left would be the consistency of the results of measurements on the widest possible set of experimental outcomes. At this point we could note that a very notion of consistency might be relativized and thus also changed to another one more admissible or suitable. This constitutes a drawback of Quine's Logical Empirism and also of the whole attitude, represented by the neopositivists, which has given rise to Logical Empirism. R. Ingarden has offered (known in the literature of the subject) possibility of relativizing the ideas of Quine as the one which could get contradicted after an unfavourable outcome of experimental checking. To summarize: the idea of p-adic strings and of p-adic structures in theoretical physics brings about exceptionally hard philosophical problems — either we are forced to go to Ajdukiewicz–Poincaré conventionalism, or rather a major change in the structure of entire science *à la* Quine (or perhaps something still completely different) is to emerge.

A time is ripe at this moment for a broader philosophical summary. The ideas exposed here constitute an extension of Wheeler's geometrodynamics. Let us recall that in formulating his geometrodynamics, Wheeler wanted to obtain the properties of elementary particles, e.g. of electrons from the topological or geometrical properties of the space-time. However, the properties of a basic construct examined by Wheeler — that is of geon — are not realistic enough. Geon is unstable and undergoes decay too quickly. Making geon stable would be associated with a substantial increase of its mass. Such a large mass may in turn to rule the geon out as a model e.g. of electron. Here it is worth mentioning that a relatively stable geon would have a mass comparable with the mass of entire visible Universe. Therefore this kind of approach seems to be completely unrealistic. To a certain degree this is understandable, since after all the geometry and topology of the space-time alone seems to be too poor for an adequate casting of the elementary particle's internal degrees of freedom. On the other hand, we have to mention yet another approach, amounting to a completely contradictory viewpoint. This was the standpoint of Leibniz, who denied the space any title for an autonomous existence. He maintained that the space (and also time) do not exist beyond matter. This he held somewhat contrary to what we find in Wheeler's geometrodynamics, where matter is deemed unable to exist autonomously and originates from the geometrical and topological properties of the space-time. To conclude, we see that according to Leibniz, the space is only an appearance in contrast to Wheeler, who held that matter is an illusion, and there exist only space (empty space). Let us recall that mainly for this very reason Leibniz questioned Newton's mechanics, where absolute space and time featured very prominently. One could develop further the idea of Wheeler's geometrodynamics provided that we consider more (than 4)-dimensional space-time. It could be many-dimensional, but empty as in Wheeler's approach. Here, the exact vacuum solutions of Einstein equations will be stable, if the geometry is suitably defined. Simultaneously we have to admit also such spaces (manifold) which have the coordinates different from real or complex numbers. In the case of supergravity (and supersymmetry) there are anticommuting fermionic coordinates, e.g. Majorana spinors. In this manner, Kaluza–Klein programme after combining it with supergravity and supersymmetry becomes a continuation of Wheeler's geometrodynamics in multi-dimensional space-time. In this approach we will look after elementary particles as exact solutions of the vacuum field equations. Of course these are the geometrical field equations, like the multidimensional ones of Einstein, where on the left side only purely geometrical quantities appear, and on the right side is zero. Solving these equations gives us the geometry of the multi-dimensional space-time in non-commuting coordinates, too. Should this solution be stable — a soliton of a sort, new kind of geon, then it might serve us as a model of an individuum, e.g. of an elementary particle. This is somewhat similar like the Skyrme model.

Let us recall that Skyrme's model uses strong interactions' effective Lagrangian (in a sense derivable from quantum chromodynamics) and looks after exact solution for the field equations obtained from this Lagrangian. These solutions are interpreted as elementary particles, e.g. nucleon, Δ^{++} resonance etc. It is possible to extend this model in such a manner, which would enable us to obtain as exact solutions also the strange particles as well. With some efforts one might classify the solutions with the aid of the irreducible representations for groups SU(2), SU(3). Here, the symmetries are involved, which are instrumental in classifying the hadronic spectra. One could also obtain more information about masses, magnetic moments, form factors etc. Skyrme's model and its derivatives give, in addition to effective Lagrangian, quite satisfactory predictions (when confronted with experiment) on properties of hadrons. Recently, Skyrme model has scored a new success by explaining the distribution of spin inside a nucleon. Hence, the topological Skyrme solitons could serve as realistic models of elementary particles. This is very promising. As a matter of fact, exact solutions of the nonlinear, partial differential equations — that is solitons — could function as models of particles, like e.g. billiard balls, Cooper's pairs, excitons — only in terms of analogy without the possibility of confronting their properties with experiment and obtaining an agreement.

Therefore, returning to the main thread (that is — the issue of multi-dimensional geometry of the space-time as a model for elementary particle) we see that such an approach is able to recover all the advantages of the effective Skyrme model and also proves itself a more realistic one. This could occur due to the fact that the field equations are far more realistic upon the introduction of gauge fields and the non-commuting quantities associated with fermions. In this manner we might expect complete implementation of Wheeler's idea of geometrodynamics in the modern casting. This new programme combines together the theory of Kaluza–Klein, supergravity and supersymmetry, theory of nonlinear differential equations, Wheeler's geometrodynamics and Einstein's programme. Such a programme could put on equal footing the implementation of Einstein's programme, Wheeler's geometry as well as Skyrme's idea, without being a mere compilation of their results. In purely computational problems, the extension of a method by Hirota for solving nonlinear, partial differential equations with anti-commuting coordinates appears to be very promising. The link between that and other methods too, seems also to be very promising, since at present one might think that some formal successes of superstring models constitute solely the successes of the two-dimensional conformal field theory, and thus (presumably) would be only associated with Kac–Moody and Virasoro type algebras. On the other hand we are aware of some links between these algebras (and groups) with theories of Kaluza–Klein.

Thus we see that this programme solves the controversy between Leibniz and Newton in favour of the latter, and extends Wheeler's geometrodynamics. In its extremal formulation this approach removes matter and leaves only an empty space. Here the matter is an illusion, a Maya. The only way out from such a situation for materialism is to accept this empty space as an extension of the concept of matter, in accordance with Lenin's idea of matter. Otherwise the matter will disappear and there would remain the emptiness — nothing. We could therefore pose a question: does the contemporary physics invalidate materialism? One could easily imagine Platonic and Pythagorean interpretations of such the state of affairs, as well as yet another one, which would go back to Aristotelian tradition with its prolongation in Scholasticism and Thomism. The scientific materialism employing a cognitive-theoretic definition of matter, without any difficulty will offer its interpretation in such a way that the negative reply would follow.

An interesting thing from both the physical and also the philosophical points of view, would be to consider a problem of the so called black holes in GRT. Namely, black holes constitute exact solution for vacuum Einstein equations. We have got two principal types of black holes. These are Schwarzschild and Kerr black holes. The former are spherical-symmetric, while the latter are only axially-symmetric. Both types are stationary, but Schwarzschild one is also static. Both solutions are characterized by the existence of a singularity, and also of the event horizon. What does it mean? It means the existence of a surface, playing the role of semi-permeable membrane (this is a surface of a ball — the sphere with a radius called Schwarzschild radius). This semi-permeable membrane functions in such a way that all material bodies, radiation and particles etc. could penetrate the membrane in only one direction — that is toward the singularity of space-time generated by the solution in question. After passing through the horizon, the observer is not able to communicate with this region of space-time, which is located beyond the surface limited by the event horizon. It works that way for both types of black holes mentioned above. In the case of Kerr black hole, we have one more surface, which is the ellipsoid of revolution surface. This surface constitutes the so called surface of infrared infinite shift. This means that on such a surface the infrared Doppler shift is infinitely large. In the case of Schwarzschild black hole the two surfaces coincide. The surface of a horizon covers the so called singularity. What is singularity? This is a manifold where space-time geometry does not make any sense. The structure of this manifold differs for the two varieties of black holes. In this manner black hole absorbs all matter and thus increases its mass. This justifies the name of such an object. In the case of Kerr black hole, additional phenomena occur, due to the existence of two surfaces. Namely, in the space comprised between the two surfaces, in the so called ergosphere, the so called Penrose effect or process might occur. This process consists of extracting the energy from the black hole. Of course, this is not contradicting the very notion of black hole. One could only extract from the black hole the energy associated with its rotation and thus as a result of the said process, all black holes will become non-rotating ones, which means that they are going to become Schwarzschild-type black holes. This process has a considerable importance in the case of accretion disks around the rotating black holes. Accretion disks are formed due to the accretion of matter falling onto a black hole. Because of Penrose process, the mentioned disks could generate highly-energetic radiation and hence they shine. Violent outbursts of matter from these disks (the so called "jets") are also possible. In accordance with current views, quazars and active galactic nuclei represent the accretion disks around the rotating black holes. At present, due to the advancement of the mathematical techniques in the area of the nonlinear differential equations, we are capable of finding exact solutions of Einstein equations, which have many event horizon surfaces. They constitute thus in a sense the nonlinear superpositions of many black holes of Schwarzschild or Kerr type. Soliton methods referred to many times before, constitute the class of methods used for finding this type of solutions. The space-time structure described by them is very complicated. It seems however that the principal features of the type "being a black hole" are still retained here. There are today also other methods of investigating black holes. Here belong the so called theorems about singularities. In conformance with these theorems, under certain quite natural assumptions about matter's energy-momentum tensor the singularities and event horizons will always occur. Hence in GRT we are condemned to the appearance of singularity. This is highly unsatisfactory, since in the case of appearance of singularity, a whole of our physics ceases to make sense. This fact has given rise to a conviction that at this point GRT has an exceptional weakness, and one should look for its generalization at the level of classical field theory. The topological methods for black hole research have resulted in the creation of the classification of the notions of causality in a space-time. The causality known in Minkowski space is here the simplest one.

There are also the so called “charged black holes” (that is ones endowed with an electric charge and also the magnetic one, provided that it exists). They represent Einstein–Maxwell solutions in the spherical-symmetric and axially-symmetric cases. In the first case it is the static solution, whereas in the second — only a stationary one. The black holes named here have two event horizons. The first is an usual one, as in Schwarzschild black hole and the second for the charged particles. Spherically-symmetric black hole with the electric charge is known as Reissner–Nördström solution. Axially-symmetric “charged black hole” is often referred to as charged Kerr solution or Kerr–Newman solution. Using of the soliton methods (there is an inverse scattering method) for Einstein equations is also possible in order to obtain the superposition of several solutions of this type, as was the case for uncharged black holes. Obtaining a superposition of many uncharged and charged black holes in a curved space-time, which would correspond to a cosmological model is also possible. Such a solution has the interpretation of a Universe with black holes situated within it. Clearly, the structure of the singularities hidden beneath the event-horizons is in this case quite different. In the case of a solution situated in the background of a cosmological model, one encounters still another type of a cosmological singularity.

Black holes are thought to represent final stages of the stellar evolution. Depending on the initial mass, the stars conclude their existence as white, later on as black dwarfs, neutron stars (or pulsars) and black holes surrounded by the matter falling onto them in the form of accretion disks. There is a criterion, enabling one to distinguish a neutron star (pulsar) surrounded by the accreting matter from a black hole surrounded by such a matter. This is a criterion of mass of an object suspected to be a black hole. Object with a substantial mass could not be a neutron star. It has to be a black hole. Whenever such an object appears in a binary system, that is it has a companion star, then inspecting the relative motion of the two stars, we are able to find out on the basis of observing the mass of an object surrounded by the accretion disk created from the matter pulled out from the other star. If the mass of this object exceeds the critical mass, than it has to be a black hole. At present we are aware of several objects which probably are black holes. Two of them are located in the constellation *Cygni*, CygX-1, CygX-3. We have also binary and trinary systems of black holes. These are of course only our guesses, hopes that we in fact are observing black holes on the sky. Also we only hope that the most massive stars terminate their evolution as the supernovae of the second kind, collapsing down into a singularity surrounded by an event horizon. Moreover, we consider as black holes Active Galactic Nuclei (AGN) in all galaxies and especially in our Galaxy, i.e. *Sagitarius* A^*. It seems that such a black hole in the center of a galaxy is very important in a galaxy formation. We should consider also primordial black holes in the Big Bang. Thus we expect very massive black holes (AGN), ordinary black holes and also black holes with intermediate masses.

There appeared recently also other methods of investigating black holes. In 1974 S. Hawking put forward semi-quantum methods for black hole research. Namely, he elected to continue looking upon the black hole's gravitational field classically, in conformance with the principles of GRT. He used in turn the methods of quantum field theory to examine the radiation being absorbed and emitted by black hole. In this picture photons or other particles, for instance electrons, move in a curved space-time. The space time of a black hole includes a horizon. Due to this, the behaviour of the said particles will be quite different than the behaviour of these moving in a flat space-time. This simple, but genial idea has led to the conclusion that the black holes are not black but grey. What does it mean? — one may ask: have the theorems and conclusions mentioned thus far lost their validity? No, one has only to modify them and have them complemented a bit. To explain this, we are going to examine for a while the thermodynamics of black holes. Namely, as we know, there are the so called laws of thermodynamics. Let us have a look onto their contents.

The first principle of thermodynamics, which represents the energy conservation law states that a change of internal energy inside a physical system is equivalent to the transfer of heat and the work there by performed. The second law of thermodynamics asserts that the entropy of an isolated system may not decrease. The third law in turn (Nernst principle) might be given the form of the following postulate: in a finite number of steps it is not possible to reach the temperature of absolute zero (zero degrees Kelvin) starting from any non-zero absolute temperature. There is also the so called 0-th law of thermodynamics, to the effect that if the two bodies stay in a thermodynamical equilibrium with a third body, then they are in equilibrium among themselves, too. The concept of a temperature could be introduced due to the relationship taking place as stated in the zero-th law. This 0-th law of thermodynamics also implies that a body in a state of thermodynamical equilibrium maintains the same temperature everywhere inside it. The second law of thermodynamics introduces the notion of a certain function of state — the entropy. This function constitutes, as we know, a measure of disorder inside a given system. Entropy increase indicates that the system evolves from a state with more order (less probable) to one with less order (more probable state). After introducing the entropy we are in a position to express the flow of heat as a product of the temperature and the entropy increase. In this manner entropy increase enters into the first law of thermodynamics.

Let us notice also that the laws of thermodynamics are of the phenomenological character. They are very well confirmed experimentally. There occur some discrepancies, known as fluctuations. The fluctuations can be derived out from a kinetic-molecular theory (of statistical physics).

Now, going back to black holes, let us introduce some relationships. Namely, upon the fusion of two black holes, a surface area of a horizon of the third black hole thus obtained is always not smaller than the sum of the horizon's surface areas of the two original black holes. Hence we have a certain quantity which quite similarly like entropy — never decreases. Multiplying the area of a horizon's surface by a suitable constant, we will obtain a never-decreasing quantity, which has the dimension of entropy. That quantity has been named entropy of a black hole. The fact that horizon's surface area is non-decreasing constitutes the statement of the second law of black hole's thermodynamics. Let us notice that for the case of a single black hole surrounded with matter, there is also going to occur an entropy increase. The black hole absorbs matter. In this way its mass will increase and so its horizon's surface area is going to increase. Detailed calculations convince us that the total entropy of this system: black hole + surrounding matter will increase as well. Now it is easy to understand that the matter being absorbed by a black hole is in more chaotic state than the external matter, if we only remember that a given black hole "has no hair". What does this statement mean? Black holes are characterized by several parameters, like mass, moment of momentum, electric charge (and magnetic one, whenever it exists).

Thus, if the black hole has absorbed matter which was also being characterized by other parameters, the information about these other parameters is going to disappear. One might say that black hole implements the state of "elderly baldness". This assertion finds its counterpart in cosmology of the inflationary model (refer to Chapter 15). There, the Universe starts from a state characterized only by the parameters mentioned above. All the other parameters are created at the outcome of the exponential expansion followed by a phase transition to Friedmann model. This state could be termed as the state "of infantile baldness".

The fact of using the parameters of the type described above in the description of the black hole properties, has prompted the discussion about taking advantage of this in the physics of the elementary particles. Namely, a question was raised, whether black holes might serve as models of elementary particles. Unfortunately, in the case of GRT, the solutions describing black holes are singular and this property rules them out as models of elementary particles. The energy of black hole's gravitational field is divergent.

Nonetheless, the concept is not fully without a foundation. Exact solutions in nonsymmetric Kaluza–Klein theory could be non-singular with a finite magnitude of energy and of the field. Thus, they might serve as models of elementary particles. Having completed this digression, let us come back to the thermodynamics of black holes.

We have thus a counterpart of the black hole's entropy — its horizon's surface area.

One would like to know what might play the role of the black hole's absolute temperature. A gravitational acceleration on the surface of the black hole's horizon is the likely candidate here. The zero-th law of black hole thermodynamics might assume the following wording: the gravitational acceleration is constant on a black hole's horizon. Multiplication of this acceleration on the horizon by the suitable physical constants like Boltzmann constant, Planck constant and the velocity of light gives us the quantity with a dimension of an absolute temperature. In this fashion it turns out that a black hole with a mass of the order of one Solar mass has a temperature of the order of 10^{-7} K. Black holes believed to reside in the nuclei of active galaxies and quazars will be still colder, with a temperature about 10^{-14} K. Heavier the black hole, lower its temperature (which of course is to be calculated in a manner sketched above).

In the case of a very light black hole with a mass of an order of 10^{18} g, this temperature would be about 10 million degrees (Kelvin). This would be therefore a substantial magnitude. Let us recall also the first and the third laws of the black hole thermodynamics. The first law of thermodynamics for black holes states that a change of the mass of the rotating black hole is equal to the product of change of the horizon's surface area by the gravitational acceleration on the horizon's surface (multiplied by a certain constant) plus a product of a change of the black hole's moment of momentum and black hole's angular velocity (again multiplied by a suitable constant). The first component of the above sum has the interpretation of heat flow, while a second — the interpretation of the work performed. The mass of a black hole constitutes its external energy. Thus this first law is in fact formally equivalent to the first law of ordinary thermodynamics. The third law of black hole thermodynamics states that it is impossible in a finite number of steps to reach a temperature of absolute zero (thus converting a black hole into naked singularity) by simply increasing its moment of momentum. Here again, the temperature of black hole is to be calculated in a manner described above. It is easy to notice that the third law of black hole thermodynamics is equivalent to the existence of Cosmic Censorship. It namely prohibits the very creation and the existence of naked singularities.

The laws of black hole thermodynamics formulated above, for quite a long time were considered as a mere curiosity, having nothing in common with genuine thermodynamics. The interpretation of the acceleration on the horizon's surface as a temperature has proved most controversial. This temperature could — according to this wording — be always positive. But on the other hand we know that a non-rotating black hole fails to emit any radiation whatsoever (hence its name: black hole). At the same time it is generally known that any body with a temperature above zero degree Kelvin emits radiation. This results in a contradiction of a kind. On the one side, we have to associate with any black hole a temperature of 0 degree Kelvin, while on the other side we have a non-zero quantity, with a property of a black hole temperature. The adoption of this second option results in black holes ceasing to be black. They become grey. After such a choice — they will become sources of radiation which would be proportional to the 4-th power of their absolute temperature. The strength of this radiation is going to be very small for heavy black holes. For the light tones it will become a noticeable quantity. Due to the radiation, black hole mass will decrease and thus it will cause the increase of the strength of its radiation. This process is going to accelerate with time and finally, upon emitting last portion of energy will become an explosive one. What is going to remain after such an explosion — we do not know. Perhaps nothing — or may be only a naked singularity? We see thus that the adoption of the possibility here considered leads to the unusual conclusions. One has therefore to find out whether such a situation could be possible. To that end, one has to examine the black hole behaviour in the field of its radiation. Exactly this was done by S. Hawking with the resulting discovery of the so called Hawking effect or process. He had considered the phenomena associated with black holes by employing semi-classical methods. He viewed a black hole's gravitational field classically (in a non-quantum way) but interpreted electromagnetic radiation quantum-mechanically. He investigated the possibility of creating photons near the event horizon of a non-rotating black hole. It turned out that a black hole radiates like an ideally black body with a temperature greater than zero. This temperature is equal to a magnitude derived from laws of the black hole thermodynamics. It has also turned out that not only a total black hole radiation coincides with a formula for radiation of an ideally black body, but also a spectral distribution of this radiation agrees with Planck's distribution. Minor differences occurring here are rather insignificant. The so called Bogolyubov transformation applies here, and the radiated photons could be interpreted as quasi-particles (ref. Introduction).

Similar to Hawking process is the so-called Unruh process which appears in uniformly accelerated Minkowski space. We get an electromagnetic radiation of an accelerated observer.

Let us describe now what this mysterious Hawking (or Unruh) process, capable of prompting such interesting consequences is all about. In quantum mechanics, as we know, there occur the so called Heisenberg relations. One of them dealing with energy and time states that for a certain very short time period the energy conservation law might be violated. This occurs for such an energy and such a time that their product is smaller than the so called quantum of action (Planck's constant divided by 2π). In this fashion the so called virtual particles could appear and could be immediately annihilated. Now, if we supply enough energy from the outside, these virtual particles could become real. Let us imagine now these virtual processes of photon creation near the event horizon. Let us assume that there are two virtual photons, which are to be created that way near the event horizon. One with negative energy, the other with positive energy. The virtual photon with negative energy could, due to a quantum effect, penetrate the event horizon. Beneath the horizon, according to GRT, particles with negative energy are admissible. Thus a virtual photon with negative energy, once it enters beneath event horizon — could become a real particle there. Under favourable circumstances, a virtual photon with positive energy could become real, too; it might escape to infinity and thus become detectable. The real photon with negative energy, by penetrating beneath event horizon, will cause the mass of a black hole to decrease. One can interpret the decrease of this mass as an outcome of radiating the photons out from the black hole, which we could observe at infinity. Let us notice also that larger the gravitational acceleration on horizon's surface, the easier could a virtual photon with a negative energy penetrate into the interior of a horizon. Thus, higher the black hole temperature, stronger the radiative effect will be. Let us notice that Hawking's effect and the radiation from black holes contradict the second and third laws of black hole thermodynamics. Because of emitting out the energy, the surface of black hole's horizon gradually decreases and after a certain time the black hole will disappear in an explosive-like manner. It could be that the violent outbursts of gamma radiation observed on the sky constitute the evidence of this process. Hawking effect, very interesting in its own right, raises some hopes for the future, associated with a possibility of formulating quantum theory of gravitation. Let us imagine for a while that GRT represents a type of macroscopic theory, resulting from the averaging of a certain unknown microscopic theory. In this sense, this unknown microscopic theory would correspond to statistical physics, whereas GRT would correspond to phenomenological thermodynamics. As we know from the history of physics, the deviations from the second law of thermodynamics, the so called fluctuations had provided the principal evidence in favour of a kinetic-molecular theory and of statistical physics. Perhaps Hawking's effect as a deviation from second law of black hole thermodynamics — will play the role of fluctuations in statistical physics and is going thus to contribute to the creation of quantum gravity. Hawking himself shares this view, too. Finally, let us turn our attention to a certain historical precedence. Namely, the quantum theory has originated from solving some difficulties related to the ideally black body radiation problem. Perhaps quantum gravity will take shape due to investigations aimed at explaining black hole radiation?

Of course, in this case we could only refer to the so called old quantum theory of Bohr and Sommerfeld. A quantum gravity proper, corresponding to quantum mechanics should start from the extended theory of gravitation viewed upon as an instance of a classical field theory. One might try to employ here the gravity as used in superstring theory, plus also quantum space-time, quantum groups etc. Neither Hawking's approach, using the concept of a mini-superspace, nor the recent advances in a canonical quantization do not bring anything original in this regard, though they are very interesting from both formal and cognitive points of view. Particle-like type of solutions, admissible within the nonsymmetric theory of Kaluza–Klein (Jordan–Thiry) provide for the occurrence of a horizon, but without singularity. In this kind of a theory, creation of a quantum formalism for small vibration around a stable, particle-like solution is possible. Such a formalism has already been elaborated for an arbitrary field theory which admits the solutions of this type. In the case of nonsymmetric Kaluza–Klein theory one should also use a mini-superspace formalism, put forward by S. Hawking. Black hole type of solutions are also possible for the multi-dimensional theories. They have similar properties and drawbacks as in the four-dimensional case. It seems that this is a drawback in a theory with Riemann geometry and thus one should substitute it with another one, that is non-Riemannian geometry. The programme of nonsymmetric Kaluza–Klein theory implements this, by using the geometry of Einstein–Kaufman type, known from the Nonsymmetric Field Theory. Let us return again for a while to the problem of a cosmology based on GRT. All the cosmological solutions are singular here. This is unsatisfactory even in the inflationary theories, since with a singularity, a whole physics (and not only gravitational physics) becomes meaningless. There is no place for the notions of time, space, causality etc. That explains why a search for the alternative theories of gravity, which would be free from this drawback is so important. The theory of Einstein–Cartan is capable of providing the solutions without a cosmological singularity. Similar is the case of Nonsymmetric Theory of Gravitation. Also the theory of Kaluza–Klein (Jordan–Thiry) exhibits comparable properties. Here, too, non-singular solutions are possible.

The very interesting problem is to consider Kerr–Newman solution as a model of an electron, a fermion. The solution describes a gravitational and an electromagnetic field in a stationary and axially-symmetric case being a solution of Einstein–Maxwell equations. The solution is characterized by the mass m, the electric charge q and the angular momentum per unit mass a. A magnetic field of the solution is asymptotically dipole-like and we can calculate a gyromagnetic ratio of this field. A dipole moment is $\mu = qa$, and angular momentum $J = ma$. Thus a gyromagnetic ratio equals $\varrho = \frac{q}{m}$. It is a gyromagnetic ratio for an electron, i.e. $g = 2$ ($\varrho_{\rm Dirac} = g\varrho_{\rm class}$, $\varrho_{\rm class} = \frac{q}{2m}$). There is a problem to consider the solution as a model of an electron because of singularity and because of a quadrupole electric moment associated with the solution. It seems that an exact solution in Nonsymmetric Kaluza–Klein Theory[2] (stationary and axially-symmetric) can cure the drawbacks, for we

[2]One hundred years ago three men: A. Einstein, D. Hilbert and O. Klein were discussing what should be a lagrangian for a gravitational field. They decided it should be a scalar curvature R for Levi-Civita connection defined on 4-dimensional manifold (a space-time) and compatible with a symmetric metric tensor defined on this manifold. In that time and additional lagrangian was known. It was a lagrangian for an electromagnetic field $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ coming from Maxwell equations which were known since about fifty years. Some years later T. Kaluza designed a theory where a lagrangian on 5-dimensional manifold R_5 with additional symmetries is equal to $R_4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. Now, 100 years of General Relativity, almost 150 years of of Maxwell equations and almost 100 years of Kaluza idea we came to the conclusion that only a scalar curvature can be a lagrangian for unified field theory of all physical interactions. This will be a scalar curvature of a connection defined on many-dimensional manifold with some symmetries and additional degrees of freedom induced by a nonsymmetric (e.g. hermitian) metric. Let us give some elements of the Nonsymmetric Kaluza–Klein Theory.

We consider the Nonsymmetric Kaluza–Klein Theory in a non-Abelian case and the Nonsymmetric Kaluza–Klein Theory with Higgs' mechanism and spontaneous symmetry breaking in a new setting. We give a comprehensive review of a subject with many new features.

The subject is specialized of course, but it could be very interesting for a wide audience because geometrization and unification of fundamental physical interactions are very interesting. This idea gives a justification for some phenomenological theories which are completely arbitrary. There is no physics without mathematics, especially without geometry—differential geometry. Even Maxwell–Lorentz electrodynamics happens *post factum* geometrized in fibre bundle formalism. In the case of ordinary Kaluza–Klein Theory the geometrization and unification have been achieved. Unfortunately, without "interference effects". We consider some additional versions of the Nonsymmetric Kaluza–Klein Theory. In particular, except of a real version we consider also Nonsymmetric Hermitian Theory in two realizations, complex and hypercomplex. They are natural (Hermitian) metrization of a fiber bundle over a space-time. The nonsymmetric Kaluza–Klein (Jordan–Thiry) Theory (a real version) has been developed in the past. The theory unifies gravitational theory described by NGT (Nonsymmetric Gravitational Theory) and Yang–Mills' fields (also electromagnetic field). In the case of the Nonsymmetric Jordan–Thiry Theory this theory includes scalar field. The Nonsymmetric Kaluza–Klein Theory can be obtained from the Nonsymmetric Jordan–Thiry Theory by simply putting this scalar field to zero. In this way it is a limit of the Nonsymmetric Jordan–Thiry Theory.

The Nonsymmetric Jordan–Thiry Theory has several physical applications in cosmology, e.g.: (1) cosmological constant, (2) inflation, (3) quintessence, and some possible relations to the dark matter problem. There is also a possibility to apply this theory to an anomalous acceleration problem of Pioneer 10/11.

A scalar field $\Psi = 0$ ($\rho = 1$). Moreover, the extension to Jordan–Thiry Theory in any nonsymmetric version is still possible and will be done elsewhere. The scalar field can play a role as a dark matter—quintessence with weak interactions with ordinary matter. On the classical level, this is only a gravitational interaction with the possibility to change a strength of gravitational interaction via a change of gravitational constant. On a quantum level due to an excitation of a quantum vacuum a very weak nongravitational interaction with ordinary matter is possible, i.e. a scattering of scalarons with ordinary matter particles and also a scattering of skewons with those particles.

The theory unifies gravity with gauge fields in a nontrivial way via geometrical unifications of two fundamental invariance principles in Physics: (1) the coordinate invariance principle, (2) the gauge invariance principle. Unification on the level of invariance principles is more important and deeper than on the level of interactions for from invariance principles we get conservation laws (via the Noether theorem). In some sense Kaluza–Klein theory unifies the energy-momentum conservation law with the conservation of a color (isotopic) charge (an electric charge in an electromagnetic case).

Let us notice that an idea of geometrization and simultaneously unification of fundamental interactions is quite old. GR is 100 years old and Kaluza–Klein Theory is almost 100 years old. Both ideas: a geometrization of physical interactions and a unification are well established contemporarily.

This unification has been achieved in higher than four-dimensional world, i.e. $(n+4)$-dimensional, where $n = \dim G$, G is a gauge group for a Yang–Mills' field, which is a semisimple Lie group (non-Abelian). In an electromagnetic case we have $G = \mathrm{U}(1)$ and a unification is in 5-dimensional world. The unification has been achieved via a natural nonsymmetric metrization of a fiber bundle. This metrization is right-invariant with respect to an action of a group G. We consider also an Hermitian metrization of a fiber bundle in two versions: complex and hypercomplex. The connection on a fiber bundle of frames over a manifold P (a bundle manifold) is compatible with a metric tensor (nonsymmetric or Hermitian in complex or hypercomplex version). In the case of $G = \mathrm{U}(1)$ the geometrical structure is biinvariant with respect to an action of $\mathrm{U}(1)$, in a general non-Abelian case this is only right-invariant.

Let us notice the following fact. We use a notion of a nonsymmetric metric as an abuse of nomination for a metric is always symmetric. This will not cause any misunderstanding. It is similar to an abuse of nomination in the case of Minkowski metric in Special Relativity for a metric is always positive definite.

The unification is nontrivial for we can get some additional effects unknown in conventional theories of gravity and gauge fields (Yang–Mills' or electromagnetic field). All of these effects, which we call *interference effects* between gravity and gauge fields are testable in principle in experiment or an observation. The formalism of this unification has been described.

The theory considered here is non-Abelian and even if there are some formulations similar to those from electromagnetic case one should remember that the theory described is an Abelian theory with U(1) group. The difference is profound not only because a higher level of mathematical calculations but also because of completely new features which appear in a non-Abelian theory. If we can use similar formulations as in electromagnetic case it means that a geometrical language is correct to describe a physical reality.

It is possible to extend the Nonsymmetric (non-Abelian) Kaluza–Klein Theory to the case of a spontaneous symmetry breaking and Higgs' mechanism by a nontrivial combination of Kaluza principle (Kaluza miracle) with dimensional reduction procedure. This consists in an extension of a base manifold of a principal fiber bundle from E (a space-time) to $V = E \times M$, where $M = G/G_0$ is a manifold of classical vacuum states.

We consider a condition for a color confinement in the theory. We solve the constraints in the case of non-Abelian Nonsymmetric Kaluza–Klein Theory getting an exact form of an induction tensor for Yang–Mills' fields in the theory. We find a formula for a non-Abelian charge in the theory in comparison to 4-momentum in gravitation theory. We derive the Lagrangian for Yang–Mills' field and an energy-momentum tensor in terms of $H^a_{\mu\nu}$ only. We consider also Nonsymmetric Kaluza–Klein Theory with Higgs' fields and spontaneous symmetry breaking. We solve constraints in the theory getting Lagrangian for Yang–Mills' field, kinetic energy Lagrangian for a Higgs field and Higgs potential in terms of gauge fields and Higgs fields only. We derive pattern of masses for a massive intermediate bosons and Higgs' particles. We derive also a generalization of Kerner–Wong–Kopczyński equation for a test particle. In such an equation there is a new charge for a test particle which couples a Higgs' field to the particle. This is similar to a Lorentz force term in an electromagnetic case. This term is also similar to a new term coupled a Yang–Mills' field to a test particle via a color (isotopic) charge in ordinary Kerner–Wong–Kopczyński equation.

The Nonsymmetric Kaluza–Klein Theory is an example of the geometrization of fundamental interaction (described by Yang–Mills' and Higgs' fields) and gravitation according to the Einstein program for geometrization of gravitational and electromagnetic interactions. It means an example to create a Unified Field Theory. In the Einstein program we have to do with electromagnetism and gravity only. Now we have to do with more degrees of freedom, unknown in Einstein times, i.e. GSW (Glashow–Salam–Weinberg) model, QCD, Higgs' fields, GUT (Grand Unified Theories). Moreover, the program seems to be the same.

We can paraphrase the definition from *McGraw–Hill Dictionary of Scientific and Technical Terms*: *Unified Field Theory* (*McGraw ...* 1989)*: any theory which attemps to express gravitational theory and fundamental interactions theories within a single unified framework. Usually an attempt to generalize Einstein's general theory of relativity alone to a theory of gravity and classical theories describing fundamental interactions*. In our case this single unified framework is a multidimensional analogue of geometry from Einstein Unified Field Theory (treated as generalized gravity) defined on principal fiber bundles with base manifolds: E or $E \times V$ and structural groups G or H. Thus the definition from an old dictionary (paraphrased by us) is still valid.

Summing up, Nonsymmetric Kaluza–Klein Theory connects old ideas of unitary field theories with modern applications. This is a geometrization and unification of a bosonic part of four fundamental interactions.

Let us give some mathematical details of the Nonsymmetric Kaluza–Klein Theory.

Elements of the Nonsymmetric Kaluza–Klein Theory in general non-Abelian case and with spontaneous symmetry breaking and Higgs' mechanism. GSW (Glashow–Salam–Weinberg) model in the Nonsymmetric Kaluza–Klein Theory

Let $\underline{P}$ be a principal fiber bundle over a space-time E with a structural group G which is a semisimple Lie group. On a space-time E we define a nonsymmetric tensor $g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]}$ such that

$$\begin{aligned} g &= \det(g_{\mu\nu}) \neq 0 \\ \widetilde{g} &= \det(g_{(\mu\nu)}) \neq 0. \end{aligned} \tag{1}$$

$g_{[\mu\nu]}$ is called as usual a skewon field (e.g. in NGT). We define on E a nonsymmetric connection compatible with $g_{\mu\nu}$ such that

$$\overline{D}g_{\alpha\beta} = g_{\alpha\delta}\overline{Q}^{\delta}{}_{\beta\gamma}(\overline{\Gamma})\overline{\theta}^{\gamma} \tag{2}$$

where $\overline{D}$ is an exterior covariant derivative for a connection $\overline{\omega}^{\alpha}{}_{\beta} = \overline{\Gamma}^{\alpha}{}_{\beta\gamma}\overline{\theta}^{\gamma}$ and $\overline{Q}^{\alpha}{}_{\beta\delta}$ is its torsion. We suppose also

$$\overline{Q}^{\alpha}{}_{\beta\alpha}(\overline{\Gamma}) = 0. \tag{3}$$

We introduce on E a second connection

$$\overline{W}^{\alpha}{}_{\beta} = \overline{W}^{\alpha}{}_{\beta\gamma}\overline{\theta}^{\gamma} \tag{4}$$

such that

$$\overline{W}^{\alpha}{}_{\beta} = \overline{\omega}^{\alpha}{}_{\beta} - \tfrac{2}{3}\,\delta^{\alpha}{}_{\beta}\overline{W} \tag{5}$$

$$\overline{W} = \overline{W}_{\gamma}\overline{\theta}^{\gamma} = \tfrac{1}{2}(\overline{W}^{\sigma}{}_{\gamma\sigma} - \overline{W}^{\sigma}{}_{\sigma\gamma})\overline{\theta}^{\gamma}. \tag{6}$$

Now we turn to nonsymmetric metrization of a bundle $\underline{P}$. We define a nonsymmetric tensor γ on a bundle manifold P such that

$$\gamma = \pi^{*}g \oplus \ell_{ab}\theta^{\alpha} \otimes \theta^{b} \tag{7}$$

where π is a projection from P to E. On $\underline{P}$ we define a connection ω (a 1-form with values in a Lie algebra $\mathfrak{g}$ of G). In this way we can introduce on P (a bundle manifold) a frame $\theta^{A} = (\pi^{*}(\overline{\theta}^{\alpha}), \theta^{a})$ such that

$$\theta^{a} = \lambda\omega^{a}, \quad \omega = \omega^{a}X_{a}, \quad a = 5, 6, \ldots, n+4, \quad n = \dim G = \dim\mathfrak{g}, \quad \lambda = \text{const}.$$

Thus our nonsymmetric tensor looks like

$$\gamma = \gamma_{AB}\theta^{A} \otimes \theta^{B}, \quad A, B = 1, 2, \ldots, n+4, \tag{8}$$

$$\ell_{ab} = h_{ab} + \mu k_{ab}, \tag{9}$$

where h_{ab} is a biinvariant Killing–Cartan tensor on G and k_{ab} is a right-invariant skew-symmetric tensor on G, $\mu = \text{const}$.

We have

$$\begin{aligned} h_{ab} &= C^{c}{}_{ad}C^{d}{}_{bc} = h_{ab} \\ k_{ab} &= -k_{ba} \end{aligned} \tag{10}$$

Thus we can write

$$\overline{\gamma}(X, Y) = \overline{g}(\pi'X, \pi'Y) + \lambda^{2}h(\omega(X), \omega(Y)) \tag{11}$$

$$\underline{\gamma}(X, Y) = \underline{g}(\pi'X, \pi'Y) + \lambda^{2}k(\omega(X), \omega(Y)) \tag{12}$$

($C^{a}{}_{bc}$ are structural constants of the Lie algebra $\mathfrak{g}$).

$\overline{\gamma}$ is the symmetric part of γ and $\underline{\gamma}$ is the antisymmetric part of γ. We have as usual

$$[X_a, X_b] = C^c{}_{ab} X_c \tag{13}$$

and

$$\Omega = \frac{1}{2} H^a{}_{\mu\nu} \theta^\mu \wedge \theta^\nu \tag{14}$$

is a curvature of the connection ω,

$$\Omega = d\omega + \frac{1}{2}[\omega, \omega]. \tag{15}$$

The frame θ^A on P is partially nonholonomic. We have

$$d\theta^a = \frac{\lambda}{2}\Big(H^a{}_{\mu\nu}\theta^\mu \wedge \theta^\nu - \frac{1}{\lambda^2}\, C^a{}_{bc}\theta^b \wedge \theta^c\Big) \neq 0 \tag{16}$$

even if the bundle $\underline{P}$ is trivial, i.e. for $\Omega = 0$. This is different than in an electromagnetic case. Our nonsymmetric metrization of a principal fiber bundle gives us a right-invariant structure on P with respect to an action of a group G on P. Having P nonsymmetrically metrized one defines two connections on P right-invariant with respect to an action of a group G on P. We have

$$\gamma_{AB} = \left(\begin{array}{c|c} g_{\alpha\beta} & 0 \\ \hline 0 & \ell_{ab} \end{array}\right) \tag{17}$$

in our left horizontal frame θ^A.

$$D\gamma_{AB} = \gamma_{AD} Q^D{}_{BC}(\Gamma)\theta^C \tag{18}$$

$$Q^D{}_{BD}(\Gamma) = 0 \tag{19}$$

where D is an exterior covariant derivative with respect to a connection $\omega^A{}_B = \Gamma^A{}_{BC}\theta^C$ on P and $Q^A{}_{BC}(\Gamma)$ its torsion. One can solve Eqs (18)–(19) getting the following results

$$\omega^A{}_B = \left(\begin{array}{c|c} \pi^*(\overline{\omega}^\alpha{}_\beta) - \ell_{db} g^{\mu\alpha} L^d{}_{\mu\beta}\theta^b & L^a{}_{\beta\gamma}\theta^\gamma \\ \hline \ell_{bd} g^{\alpha\beta}(2H^d{}_{\gamma\beta} - L^d{}_{\gamma\beta})\theta^\gamma & \widetilde{\omega}^a{}_b \end{array}\right) \tag{20}$$

where $g^{\mu\alpha}$ is an inverse tensor of $g_{\alpha\beta}$

$$g_{\alpha\beta} g^{\gamma\beta} = g_{\beta\alpha} g^{\beta\gamma} = \delta^\gamma_\alpha, \tag{21}$$

$L^d{}_{\gamma\beta} = -L^a{}_{\beta\gamma}$ is an Ad-type tensor on P such that

$$\ell_{dc} g_{\mu\beta} g^{\gamma\mu} L^d{}_{\gamma\alpha} + \ell_{cd} g_{\alpha\mu} g^{\mu\gamma} L^d{}_{\beta\gamma} = 2\ell_{cd} g_{\alpha\mu} g^{\mu\gamma} H^d{}_{\beta\gamma}, \tag{22}$$

$\widetilde{\omega}^a{}_b = \widetilde{\Gamma}^a{}_{bc}\theta^c$ is a connection on an internal space (typical fiber) compatible with a metric ℓ_{ab} such that

$$\ell_{db}\widetilde{\Gamma}^d{}_{ac} + \ell_{ad}\widetilde{\Gamma}^d{}_{cb} = -\ell_{db} C^d{}_{ac} \tag{23}$$

$$\widetilde{\Gamma}^a{}_{ba} = 0, \quad \widetilde{\Gamma}^a{}_{bc} = -\widetilde{\Gamma}^a{}_{cb} \tag{24}$$

and of course $\widetilde{Q}^a{}_{ba}(\widetilde{\Gamma}) = 0$ where $\widetilde{Q}^a{}_{bc}(\Gamma)$ is a torsion of the connection $\widetilde{\omega}^a{}_b$.

We also introduce an inverse tensor of $g_{(\alpha\beta)}$

$$g_{(\alpha\beta)} \widetilde{g}^{(\alpha\gamma)} = \delta^\gamma_\beta. \tag{25}$$

We introduce a second connection on P defined as

$$W^A{}_B = \omega^A{}_B - \frac{4}{3(n+2)}\,\delta^A{}_B \overline{W}. \tag{26}$$

$\overline{W}$ is a horizontal one form

$$\overline{W} = \operatorname{hor}\overline{W} \tag{27}$$

$$\overline{W} = \overline{W}_\nu \theta^\nu = \tfrac{1}{2}(\overline{W}^\sigma{}_{\nu\sigma} - \overline{W}^\sigma{}_{\sigma\nu}). \tag{28}$$

In this way we define on P all analogues of four-dimensional quantities from NGT. It means, $(n+4)$-dimensional analogues from Moffat theory of gravitation, i.e. two connections and a nonsymmetric metric γ_{AB}. Those quantities are right-invariant with respect to an action of a group G on P. One can calculate a scalar curvature of a connection $W^A{}_B$ getting the following result:

$$R(W) = \overline{R}(\overline{W}) - \frac{\lambda^2}{4}\bigl(2\ell_{cd}H^c H^d - \ell_{cd}L^{c\mu\nu}H^d{}_{\mu\nu}\bigr) + \widetilde{R}(\widetilde{\Gamma}) \tag{29}$$

where

$$R(W) = \gamma^{AB}\bigl(R^C{}_{ABC}(W) + \tfrac{1}{2}\,R^C{}_{CAB}(W)\bigr) \tag{30}$$

is a Moffat–Ricci curvature scalar for the connection $W^A{}_B$, $\overline{R}(\overline{W})$ is a Moffat–Ricci curvature scalar for the connection $\overline{W}^\alpha{}_\beta$, and $\widetilde{R}(\widetilde{\Gamma})$ is a Moffat–Ricci curvature scalar for the connection $\widetilde{\omega}^a{}_b$,

$$H^a = g^{[\mu\nu]}H^a{}_{\mu\nu} \tag{31}$$

$$L^{a\mu\nu} = g^{\alpha\mu}g^{\beta\nu}L^a{}_{\alpha\beta}. \tag{32}$$

Usually in ordinary (symmetric) Kaluza–Klein Theory one has $\lambda = 2\frac{\sqrt{G_N}}{c^2}$, where G_N is a Newtonian gravitational constant and c is the speed of light. In our system of units $G_N = c = 1$ and $\lambda = 2$. This is the same as in Nonsymmetric Kaluza–Klein Theory in an electromagnetiic case. In the non-Abelian Kaluza–Klein Theory which unifies GR and Yang–Mills field theory we have a Yang–Mills lagrangian and a cosmological term. Here we have

$$\mathcal{L}_{\rm YM} = -\frac{1}{8\pi}\,\ell_{cd}\bigl(2H^c H^d - L^{c\mu\nu}H^d{}_{\mu\nu}\bigr) \tag{33}$$

and $\widetilde{R}(\widetilde{\Gamma})$ plays a role of a cosmological term.

In order to incorporate a spontaneous symmetry breaking and Higgs' mechanism in our geometrical unification of gravitation and Yang–Mills' fields we consider a fiber bundle $\underline{P}$ over a base manifold $E \times G/G_0$, where E is a space-time, $G_0 \subset G$, G_0, G are semisimple Lie groups. Thus we are going to combine a Kaluza–Klein theory with a dimensional reduction procedure.

Let $\underline{P}$ be a principal fiber bundle over $V = E \times M$ with a structural group H and with a projection π, where $M = G/G_0$ is a homogeneous space, G is a semisimple Lie group and G_0 its semisimple Lie subgroup. Let us suppose that (V,γ) is a manifold with a nonsymmetric metric tensor

$$\gamma_{AB} = \gamma_{(AB)} + \gamma_{[AB]}. \tag{34}$$

The signature of the tensor γ is $(+---,\underbrace{---\cdots-}_{n_1})$. Let us introduce a natural frame on $\underline{P}$

$$\theta^{\tilde{A}} = (\pi^*(\theta^A), \theta^0 = \lambda\omega^a), \quad \lambda = \text{const}. \tag{35}$$

It is convenient to introduce the following notation. Capital Latin indices with tilde $\widetilde{A}, \widetilde{B}, \widetilde{C}$ run $1,2,3,\dots,m+4$, $m = \dim H + \dim M = n + \dim M = n + n_1$, $n_1 = \dim M$, $n = \dim H$. Lower Greek indices $\alpha,\beta,\gamma,\delta = 1,2,3,4$ and lower Latin indices $a,b,c,d = n_1+5, n_2+5, \dots, n_1+6, \dots, m+4$. Capital Latin indices $A,B,C = 1,2,\dots,n_1+4$. Lower Latin indices with tilde $\widetilde{a}, \widetilde{b}, \widetilde{c}$ run $5,6,\dots,n_1+4$. The symbol over θ^A and other quantities indicates that these quantities are defined on V. We have of course

$$n_1 = \dim G - \dim G_0 = n_2 - (n_2 - n_1),$$

where $\dim G = n_2$, $\dim G_0 = n_2 - n_1$, $m = n_1 + n$.

On the group H we define a bi-invariant (symmetric) Killing–Cartan tensor

$$h(A,B) = h_{ab} A^a B^b. \tag{36}$$

We suppose H is semisimple, it means $\det(h_{ab}) \neq 0$. We define a skew-symmetric right-invariant tensor on H

$$k(A,B) = k_{bc} A^b B^c, \quad k_{bc} = -k_{cb}.$$

Let us turn to the nonsymmetric metrization of $\underline{P}$.

$$\kappa(X,Y) = \gamma(X,Y) + \lambda^2 \ell_{ab} \omega^a(X) \omega^b(Y) \tag{37}$$

where

$$\ell_{ab} = h_{ab} + \xi k_{ab} \tag{38}$$

is a nonsymmetric right-invariant tensor on H. One gets in a matrix form (in the natural frame (35))

$$\kappa_{\tilde{A}\tilde{B}} = \left(\begin{array}{c|c} \gamma_{AB} & 0 \\ \hline 0 & \ell_{ab} \end{array} \right), \tag{39}$$

$\det(\ell_{ab}) \neq 0$, $\xi = \text{const}$ and real, then

$$\ell_{ab}\ell^{ac} = \ell_{ba}\ell^{ca} = \delta^c{}_b. \tag{40}$$

The signature of the tensor κ is $(+,---,\underbrace{-\cdots-}_{n_1},\underbrace{--\cdots-}_{n})$. As usual, we have commutation relations for Lie algebra of H, $\mathfrak{h}$

$$[X_a, X_b] = C^c{}_{ab} X_c. \tag{41}$$

This metrization of $\underline{P}$ is right-invariant with respect to an action of H on P.

Now we should nonsymmetrically metrize $M = G/G_0$. M is a homogeneous space for G (with left action of group G). Let us suppose that the Lie algebra of G, $\mathfrak{g}$ has the following reductive decomposition

$$\mathfrak{g} = \mathfrak{g}_0 \dot{+} \mathfrak{m} \tag{42}$$

where $\mathfrak{g}_0$ is a Lie algebra of G_0 (a subalgebra of $\mathfrak{g}$) and $\mathfrak{m}$ (the complement to the subalgebra $\mathfrak{g}_0$) is $\operatorname{Ad} G_0$ invariant, $\dot{+}$ means a direct sum. Such a decomposition might be not unique, but we assume that one has been chosen. Sometimes one assumes a stronger condition for $\mathfrak{m}$, the so called symmetry requirement,

$$[\mathfrak{m}, \mathfrak{m}] \subset \mathfrak{g}_0. \tag{43}$$

Let us introduce the following notation for generators of $\mathfrak{g}$:

$$Y_i \in \mathfrak{g}, \quad Y_{\tilde{\imath}} \in \mathfrak{m}, \quad Y_{\hat{a}} \in \mathfrak{g}_0. \tag{44}$$

This is a decomposition of a basis of $\mathfrak{g}$ according to (42). We define a symmetric metric on M using a Killing–Cartan form on G in a classical way. We call this tensor h_0.

Let us define a tensor field $h^0(x)$ on G/G_0, $x \in G/G_0$, using tensor field h on G. Moreover, if we suppose that h is a biinvariant metric on G (a Killing–Cartan tensor) we have a simpler construction.

The complement $\mathfrak{m}$ is a tangent space to the point $\{\varepsilon G_0\}$ of M, ε is a unit element of G. We restrict h to the space $\mathfrak{m}$ only. Thus we have $h^0(\{\varepsilon G_0\})$ at one point of M. Now we propagate $h^0(\{fG_0\})$ using a left action of the group G

$$h^0(\{fG_0\}) = (L_f^{-1})^*(h^0(\{\varepsilon G_0\})).$$

$h^0(\{\varepsilon G_0\})$ is of course $\operatorname{Ad} G_0$ invariant tensor defined on $\mathfrak{m}$ and $L_f^* h^0 = h^0$.

We define on M a skew-symmetric 2-form k^0. Now we introduce a natural frame on M. Let $f^i{}_{jk}$ be structure constants of the Lie algebra $\mathfrak{g}$, i.e.

$$[Y_j, Y_k] = f^i{}_{jk} Y_i. \tag{45}$$

Y_j are generators of the Lie algebra $\mathfrak{g}$. Let us take a local section $\sigma : V \to G/G_0$ of a natural bundle $G \mapsto G/G_0$ where $V \subset M = G/G_0$. The local section σ can be considered as an introduction of a coordinate system on M.

Let ω_{MC} be a left-invariant Maurer–Cartan form and let

$$\omega^\sigma{}_{MC} = \sigma^* \omega_{MC}. \tag{46}$$

Using decomposition (42) we have

$$\omega^\sigma{}_{MC} = \omega^\sigma{}_0 + \omega^\sigma{}_{\mathfrak{m}} = \theta^{\widehat{\imath}} Y_{\widehat{\imath}} + \overline{\theta}^{\tilde{a}} Y_{\tilde{a}}. \tag{47}$$

It is easy to see that $\overline{\theta}^{\tilde{a}}$ is the natural (left-invariant) frame on M and we have

$$h^0 = h^0{}_{\tilde{a}\tilde{b}} \overline{\theta}^{\tilde{a}} \otimes \overline{\theta}^{\tilde{b}} \tag{48}$$

$$k^0 = k^0{}_{\tilde{a}\tilde{b}} \overline{\theta}^{\tilde{a}} \wedge \overline{\theta}^{\tilde{b}}. \tag{49}$$

According to our notation $\widetilde{a}, \widetilde{b} = 5, 6, \dots, n_1 + 4$.

Thus we have a nonsymmetric metric on M

$$\gamma_{\tilde{a}\tilde{b}} = r^2 \left(h^0{}_{\tilde{a}\tilde{b}} + \zeta k^0{}_{\tilde{a}\tilde{b}} \right) = r^2 g_{\tilde{a}\tilde{b}}. \tag{50}$$

Thus we are able to write down the nonsymmetric metric on $V = E \times M = E \times G/G_0$

$$\gamma_{AB} = \left(\begin{array}{c|c} g_{\alpha\beta} & 0 \\ \hline 0 & r^2 g_{\tilde{a}\tilde{b}} \end{array} \right) \tag{51}$$

where

$$\begin{aligned} g_{\alpha\beta} &= g_{(\alpha\beta)} + g_{[\alpha\beta]} \\ g_{\tilde{a}\tilde{b}} &= h^0{}_{\tilde{a}\tilde{b}} + \zeta k^0{}_{\tilde{a}\tilde{b}} \\ k^0{}_{\tilde{a}\tilde{b}} &= -k^0{}_{\tilde{b}\tilde{a}} \\ h^0{}_{\tilde{a}\tilde{b}} &= h^0{}_{\tilde{b}\tilde{a}}, \end{aligned}$$

$\alpha, \beta = 1, 2, 3, 4$, $\widetilde{a}, \widetilde{b} = 5, 6, \dots, n_1 + 4 = \dim M + 4 = \dim G - \dim G_0 + 4$. The frame $\overline{\theta}^{\tilde{a}}$ is unholonomic:

$$d\overline{\theta}^{\tilde{a}} = \frac{1}{2} \kappa^{\tilde{a}}{}_{\tilde{b}\tilde{c}} \overline{\theta}^{\tilde{b}} \wedge \overline{\theta}^{\tilde{c}} \tag{52}$$

where $\kappa^{\tilde{a}}{}_{\tilde{b}\tilde{c}}$ are coefficients of nonholonomicity and depend on the point of the manifold $M = G/G_0$ (they are not constant in general). They depend on the section σ and on the constants $f^{\tilde{a}}{}_{\tilde{b}\tilde{c}}$.

We have here three groups H, G, G_0. Let us suppose that there exists a homomorphism μ between G_0 and H,

$$\mu : G_0 \to H \tag{53}$$

such that a centralizer of $\mu(G_0)$ in H, C^μ is isomorphic to G. C^μ, a centralizer of $\mu(G_0)$ in H, is a set of all elements of H which commute with elements of $\mu(G_0)$, which is a subgroup of H. This means that H has the following structure, $C^\mu = G$.

$$\mu(G_0) \otimes G \subset H. \tag{54}$$

If μ is a isomorphism between G_0 and $\mu(G_0)$ one gets

$$G_0 \otimes G \subset H. \tag{55}$$

Let us denote by μ' a tangent map to μ at a unit element. Thus μ' is a differential of μ acting on the Lie algebra elements. Let us suppose that the connection ω on the fiber bundle $\underline{P}$ is invariant under group action of G on the manifold $V = E \times G/G_0$. This means the following.

Let e be a local section of $\underline{P}$, $e : V \subset U \to P$ and $A = e^*\omega$. Then for every $g \in G$ there exists a gauge transformation ρ_g such that

$$f^*(g)A = \operatorname{Ad}_{\rho_g^{-1}} A + \rho_g^{-1}\, dg_g, \tag{56}$$

f^* means a pull-back of the action f of the group G on the manifold V. We are able to write a general form for such an ω. We have

$$\omega = \widetilde{\omega}_E + \mu' \circ \omega^\sigma{}_0 + \varPhi \circ \omega^\sigma{}_{\mathfrak{m}}. \tag{57}$$

(An action of a group G on $V = E \times G/G_0$ means left multiplication on a homogeneous space $M = G/G_0$.) where $\omega^\sigma{}_0 + \omega^\sigma{}_{\mathfrak{m}} = \omega^\sigma{}_{MC}$ are components of the pull-back of the Maurer–Cartan form from the decomposition (47), $\widetilde{\omega}_E$ is a connection defined on a fiber bundle Q over a space-time E with structural group C^μ and a projection π_E. Moreover, $C^\mu = G$ and $\widetilde{\omega}_E$ is a 1-form with values in the Lie algebra $\mathfrak{g}$. This connection describes an ordinary Yang–Mills' field gauge group $G = C^\mu$ on the space-time E. $\varPhi$ is a function on E with values in the space $\widetilde{S}$ of linear maps

$$\varPhi : \mathfrak{m} \to \mathfrak{h} \tag{58}$$

satisfying

$$\varPhi([X_0, X]) = [\mu' X_0, \varPhi(X)], \quad X_0 \in \mathfrak{g}_0. \tag{59}$$

Thus

$$\begin{aligned} \widetilde{\omega}_E &= \widetilde{\omega}^i{}_E Y_i, \quad Y_i \in \mathfrak{g}, \\ \omega^\sigma{}_0 &= \theta^{\widehat{\imath}} Y_{\widehat{\imath}}, \quad Y_{\widehat{\imath}} \in \mathfrak{g}_0, \\ \omega^\sigma{}_{\mathfrak{m}} &= \overline{\theta}^{\tilde{a}} Y_{\tilde{a}}, \quad Y_{\tilde{a}} \in \mathfrak{m}. \end{aligned} \tag{60}$$

Let us write condition (57) in the base of left-invariant form $\theta^{\widehat{\imath}}, \overline{\theta}^{\tilde{a}}$, which span respectively dual spaces to $\mathfrak{g}_0$ and $\mathfrak{m}$. It is easy to see that

$$\varPhi \circ \omega^\sigma{}_{\mathfrak{m}} = \varPhi^a{}_{\tilde{a}}(x) \overline{\theta}^{\tilde{a}} X_a, \quad X_a \in \mathfrak{h} \tag{61}$$

and

$$\mu' = \mu^a{}_{\hat\imath}\,\widehat{\theta^\imath} X_a. \tag{62}$$

From (59) one gets

$$\Phi^c_{\tilde b}(x) f^{\tilde b}_{\hat\imath\tilde a} = \mu^a_{\hat\imath}\Phi^b_{\tilde a}(x) C^c{}_{ab} \tag{63}$$

where $f^{\tilde b}_{\hat\imath\tilde a}$ are structure constants of the Lie algebra $\mathfrak{g}$ and $C^c{}_{ab}$ are structure constants of the Lie algebra $\mathfrak{h}$. Eq. (63) is a constraint on the scalar field $\Phi^a_{\tilde a}(x)$. For a curvature of ω one gets

$$\begin{aligned}\Omega = \frac{1}{2}\,H^C{}_{AB}\theta^A\wedge\theta^B X_C = \frac{1}{2}\,\widetilde{H}^i{}_{\mu\nu}\theta^\mu\wedge\theta^\nu\alpha^c_i X_c + \overset{\text{gauge}}{\nabla_\mu}\,\Phi^c_{\tilde a}\theta^\mu\wedge\theta^{\tilde a}X_c \\ + \frac{1}{2}\,C^c{}_{ab}\Phi^a_{\tilde a}\Phi^b_{\tilde b}\theta^{\tilde a}\wedge\theta^{\tilde b}X_c - \frac{1}{2}\,\Phi^c_{\tilde d} f^{\tilde d}_{\tilde a\tilde b}\theta^{\tilde a}\wedge\theta^{\tilde b}X_c.\end{aligned} \tag{64}$$

Thus we have

$$H^c{}_{\mu\nu} = \alpha^c{}_i\widetilde{H}^i{}_{\mu\nu} \tag{65}$$

$$H^c{}_{\mu\tilde a} = \overset{\text{gauge}}{\nabla_\mu}\,\Phi^c_{\tilde a} = -H^c{}_{\tilde a\mu} \tag{66}$$

$$H^c{}_{\tilde a\tilde b} = C^c{}_{ab}\cdot\Phi^a_{\tilde a}\Phi^b_{\tilde b} - \mu^c_{\hat\imath}\widehat{f^\imath}_{\tilde a\tilde b} - \Phi^c_{\tilde d} f^{\tilde d}{}_{\tilde a\tilde b} \tag{67}$$

where $\overset{\text{gauge}}{\nabla_\mu}$ means gauge derivative with respect to the connection $\widetilde{\omega}_E$ defined on a bundle Q over a space-time E with a structural group G

$$Y_i = \alpha^c_i X_c. \tag{68}$$

$\widetilde{H}^i{}_{\mu\nu}$ is the curvature of the connection $\overline{\omega}_E$ in the base $\{Y_i\}$, generators of the Lie algebra of the Lie group G, $\mathfrak{g}$, α^c_i is the matrix which connects $\{Y_i\}$ with $\{X_c\}$. Now we would like to remind that indices a,b,c refer to the Lie algebra $\mathfrak{h}$, $\widetilde{a},\widetilde{b},\widetilde{c}$ to the space $\mathfrak{m}$ (tangent space to M), $\widehat{\imath},\widehat{\jmath},\widehat{k}$ to the Lie algebra $\mathfrak{g}_0$ and i,j,k to the Lie algebra of the group G, $\mathfrak{g}$. The matrix α^c_i establishes a direct relation between generators of the Lie algebra of the subgroup of the group H isomorphic to the group G.

Let us come back to a construction of the Nonsymmetric Kaluza–Klein Theory on a manifold P. We should define connections. First of all, we should define a connection compatible with a nonsymmetric tensor γ_{AB}, Eq. (51),

$$\overline{\omega}^A{}_B = \overline{\Gamma}^A{}_{BC}\theta^C \tag{69}$$

$$\overline{D}\gamma_{AB} = \gamma_{AD}\overline{Q}^D{}_{BC}(\overline{\Gamma})\theta^C \tag{70}$$

$$\overline{Q}^D{}_{BD}(\overline{\Gamma}) = 0$$

where $\overline{D}$ is the exterior covariant derivative with respect to $\overline{\omega}^A{}_B$ and $\overline{Q}^D{}_{BC}(\overline{\Gamma})$ its torsion.

Using (51) one easily finds that the connection (69) has the following shape

$$\overline{\omega}^A{}_B = \left(\begin{array}{c|c}\pi^*_E(\overline{\omega}^\alpha{}_\beta) & 0 \\ \hline 0 & \widehat{\overline{\omega}}^{\tilde a}{}_{\tilde b}\end{array}\right) \tag{71}$$

where $\overline{\omega}^\alpha{}_\beta = \overline{\Gamma}^\alpha{}_{\beta\gamma}\overline{\theta}^\gamma$ is a connection on the space-time E and $\widehat{\overline{\omega}}^{\tilde a}{}_{\tilde b} = \widehat{\overline{\Gamma}}^{\tilde a}{}_{\tilde b\tilde c}\overline{\theta}^{\tilde c}$ on the manifold $M = G/G_0$ with the following properties

$$\overline{D}g_{\alpha\beta} = g_{\alpha\delta}\overline{Q}^\delta{}_{\beta\gamma}(\overline{\Gamma})\overline{\theta}^\gamma = 0 \tag{72}$$

$$\overline{Q}^{\alpha}{}_{\beta\alpha}(\overline{\Gamma}) = 0 \tag{73}$$

$$\widehat{\overline{D}} g_{\tilde{a}\tilde{b}} = g_{\tilde{a}\tilde{d}} \widehat{\overline{Q}}^{\tilde{d}}{}_{\tilde{b}\tilde{c}}(\widehat{\overline{\Gamma}}). \tag{74}$$

$$\widehat{\overline{Q}}^{\tilde{d}}{}_{\tilde{b}\tilde{d}}(\widehat{\overline{\Gamma}}) = 0$$

$\overline{D}$ is an exterior covariant derivative with respect to a connection $\overline{\omega}^{\alpha}{}_{\beta}$. $\overline{Q}^{\alpha}{}_{\beta\gamma}$ is a tensor of torsion of a connection $\overline{\omega}^{\alpha}{}_{\beta}$. $\widehat{\overline{D}}$ is an exterior covariant derivative of a connection $\widehat{\overline{\omega}}^{\tilde{a}}{}_{\tilde{b}}$ and $\widehat{\overline{Q}}^{\tilde{a}}{}_{\tilde{b}\tilde{c}}(\widehat{\overline{\Gamma}})$ its torsion.

On a space-time E we also define the second affine connection $\overline{W}^{\alpha}{}_{\beta}$ such that

$$\overline{W}^{\alpha}{}_{\beta} = \overline{\omega}^{\alpha}{}_{\beta} - \frac{2}{3}\,\delta^{\alpha}{}_{\beta}\overline{W}, \tag{75}$$

where

$$\overline{W} = \overline{W}_{\gamma}\overline{\theta}^{\gamma} = \tfrac{1}{2}(\overline{W}^{\sigma}{}_{\gamma\sigma} - \overline{W}^{\sigma}{}_{\gamma\sigma}).$$

We proceed a nonsymmetric metrization of a principal fiber bundle $\underline{P}$ according to (51). Thus we define a right-invariant connection with respect to an action of the group H compatible with a tensor $\kappa_{\tilde{A}\tilde{B}}$

$$D\kappa_{\tilde{A}\tilde{B}} = \kappa_{\tilde{A}\tilde{D}} Q^{\tilde{D}}{}_{\tilde{B}\tilde{C}}(\Gamma)\theta^{\tilde{C}} \tag{76}$$

$$Q^{\tilde{D}}{}_{\tilde{B}\tilde{D}}(\Gamma) = 0$$

where $\omega^{\tilde{A}}{}_{\tilde{B}} = \Gamma^{\tilde{A}}{}_{\tilde{B}\tilde{C}}\widetilde{\theta}^{\tilde{C}}$. D is an exterior covariant derivative with respect to the connection $\omega^{\tilde{A}}{}_{\tilde{B}}$ and $Q^{\tilde{A}}{}_{\tilde{B}\tilde{C}}$ its torsion. After some calculations one finds

$$\omega^{\tilde{A}}{}_{\tilde{B}} = \left(\begin{array}{c|c} \pi^*(\overline{\omega}^{A}{}_{B}) - \ell_{db}\gamma^{MA}L^{d}{}_{MB}\theta^{b} & L^{a}{}_{BC}\theta^{C} \\ \hline \ell_{bd}\gamma^{AB}(2H^{d}{}_{CB} - L^{d}{}_{CB})\theta^{C} & \widetilde{\omega}^{a}{}_{b} \end{array}\right) \tag{77}$$

where

$$L^{d}{}_{MB} = -L^{d}{}_{BM} \tag{78}$$

$$\ell_{dc}\gamma_{MB}\gamma^{CM}L^{d}{}_{CA} + \ell_{cd}\gamma_{AM}\gamma^{MC}L^{d}{}_{BC} = 2\ell_{cd}\gamma_{AM}\gamma^{MC}H^{d}{}_{BC}, \tag{79}$$

$L^{d}{}_{CA}$ is Ad-type tensor with respect to H (Ad-covariant on $\underline{P}$)

$$\widetilde{\omega}^{a}{}_{b} = \widetilde{\Gamma}^{a}{}_{bc}\theta^{c} \tag{80}$$

$$\ell_{db}\widetilde{\Gamma}^{d}{}_{ac} + \ell_{ad}\widetilde{\Gamma}^{d}{}_{cb} = -\ell_{db}C^{d}{}_{ac} \tag{81}$$

$$\widetilde{\Gamma}^{d}{}_{ac} = -\widetilde{\Gamma}^{d}{}_{ca}, \quad \widetilde{\Gamma}^{d}{}_{ad} = 0. \tag{82}$$

We define on P a second connection

$$W^{\tilde{A}}{}_{\tilde{B}} = \omega^{\tilde{A}}{}_{\tilde{B}} - \frac{4}{3(m+2)}\,\delta^{\tilde{A}}{}_{\tilde{B}}\overline{W}. \tag{83}$$

Thus we have on P all $(m+4)$-dimensional analogues of geometrical quantities from NGT, i.e.

$$W^{\tilde{A}}{}_{\tilde{B}}, \quad \omega^{\tilde{A}}{}_{\tilde{B}} \quad \text{and} \quad \kappa_{\tilde{A}\tilde{B}}.$$

Let $\underline{P}$ be a principal fiber bundle

$$\underline{P} = (P, V, \pi, H, H) \tag{84}$$

over the base space $V = E \times S^2$ (where E is a space-time, S^2—a two-dimensional sphere) with a projection π, a structural group H, a typical fiber H and a bundle manifold P. We suppose that H is semisimple. Let us define on P a connection ω which has values in a Lie algebra of $H, \mathfrak{h}$. Let us suppose that a group SO(3) is acting on S^2 in a natural way. We suppose that ω is invariant with respect to an action of the group SO(3) on V in such a way that this action is equivalent to SO(3) action on S^2. This is equivalent to the condition (56). If we take a section $e : E \to P$ we get

$$e^*\omega = A^a{}_A \overline{\theta}^A X_a = A_A \overline{\theta}^A \tag{85}$$

where $\overline{\theta}^A$ is a frame on V and X_a are generators of the Lie algebra $\mathfrak{h}$.

$$[X_a, X_b] = C^c{}_{ab} X_c. \tag{86}$$

We define a curvature of the connection ω

$$\Omega = d\omega + \frac{1}{2}[\omega, \omega]. \tag{87}$$

Taking a section e

$$e^*\Omega = \frac{1}{2} F^a{}_{AB} \overline{\theta}^A \wedge \overline{\theta}^B X_a = \frac{1}{2} F_{AB} \overline{\theta}^A \wedge \overline{\theta}^B \tag{88}$$

$$F^a{}_{AB} = \partial_A A^a{}_B - \partial_B A^a{}_A - C^a{}_{cb} A^c{}_A A^c{}_B. \tag{89}$$

Let us consider a local coordinate systems on V. One has $x^A = (x^\mu, \psi, \varphi)$ where x^μ are coordinate system on E, $\overline{\theta}^\mu = dx^\mu$, and ψ and φ are polar and azimuthal angles on S^2, $\theta^5 = d\psi$, $\theta^6 = d\varphi$. We have $A, B, C = 1, 2, \dots, 6$, $\mu = 1, 2, 3, 4$. Let us introduce vector fields on V corresponding to the infinitesimal action of SO(3) on V. These vector fields are called $\delta_m = (\delta^A_m)$, $\overline{m} = 1, 2, 3$, $A = 1, 2, \dots, 6$. Moreover, they are acting only on the last two dimensions ($A, B = 5, 6$, $\widetilde{a}, \widetilde{b} = 5, 6$). We get:

$$\begin{aligned} &\delta^\mu_{\bar m} = 0 && \text{and} \\ &\delta^\psi_1 = \cos\varphi, && \delta^\varphi_1 = -\cot\psi \sin\varphi, \\ &\delta^\psi_2 = -\sin\varphi, && \delta^\varphi_2 = -\cot\psi\cos\varphi, \\ &\delta^\psi_3 = 0, && \delta^\varphi_3 = 1. \end{aligned} \tag{90}$$

They satisfy commutation relation of the Lie algebra A_1 of a group SO(3),

$$\delta^A_{\bar m} \partial_A \delta^B_{\bar n} - \delta^A_{\bar n} \partial_A \delta^B_{\bar m} = \varepsilon_{\bar m \bar n \bar p} \delta^B_{\bar p}. \tag{91}$$

The gauge field A_A is spherically symmetric (invariant with respect to an action of a group SO(3)) iff for some $V_{\bar m}$—a field on V with values in the Lie algebra $\mathfrak{h}$—

$$\partial_B \delta^A_{\bar m} A_A + \delta^A_{\bar m} \partial_A A_B = \partial_B V_{\bar m} - [A_B, V_{\bar m}]. \tag{92}$$

It means that

$$\underset{\delta_{\bar m}}{\mathcal{L}} A_A = \partial_B V_{\bar m} - [A_A, V_{\bar m}], \tag{93}$$

a Lie derivative of A_A with respect to $\delta_{\bar m}$ results in a gauge transformation (see also Eq. (56)).

Eq. (93) is satisfied if

$$V_1 = \Phi_3 \frac{\sin\varphi}{\sin\psi}, \quad V_2 = \Phi_3 \frac{\cos\varphi}{\sin\psi}, \quad V_3 = 0 \tag{94}$$

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and

$$A_\mu = A_\mu(x), \quad A_\psi = -\varPhi_1(x) = A_5 = \varPhi_5, \quad A_\varphi = \varPhi_2(x)\sin\psi - \varPhi_3\cos\psi = A_6 = \varPhi_6 \tag{95}$$

with the following constraints

$$\begin{aligned} [\varPhi_3, \varPhi_1] &= -\varPhi_2, \\ [\varPhi_3, \varPhi_2] &= \varPhi_1, \\ [\varPhi_3, A_\mu] &= 0. \end{aligned} \tag{96}$$

$A_\mu, \varPhi_1, \varPhi_2$ are fields on E with values in the Lie algebra of $H(\mathfrak{h})$, $\varPhi_3$ is a constant element of Cartan subalgebra of $\mathfrak{h}$. Let us introduce some additional elements according to the Nonsymmetric Hermitian Kaluza–Klein Theory. We have on E a nonsymmetric Hermitian tensor $g_{\mu\nu}$, connections $\overline{\omega}^\alpha{}_\beta$ and $\overline{W}^\alpha{}_\beta$. On S^2 we have a nonsymmetric metric tensor

$$\gamma_{\tilde a\tilde b} = r^2 g_{\tilde a\tilde b} = r^2\left(h^0{}_{\tilde a\tilde b} + \zeta k^0{}_{\tilde a\tilde b}\right) \tag{97}$$

where r is the radius of a sphere S^2 and ζ is considered to be pure imaginary,

$$h^0{}_{\tilde a\tilde b} = \left(\begin{array}{c|c} -1 & 0 \\ \hline 0 & -\sin^2\psi \end{array}\right) \tag{98}$$

$$k^0{}_{\tilde a\tilde b} = \left(\begin{array}{c|c} 0 & \sin\psi \\ \hline -\sin\psi & 0 \end{array}\right) \tag{99}$$

and a connection compatible with this nonsymmetric metric

$$g_{\tilde a\tilde b} = \begin{array}{cc} \begin{array}{cc} 5 & 6 \end{array} & \\ \left(\begin{array}{c|c} -1 & \zeta\sin\psi \\ \hline -\zeta\sin\psi & -\sin^2\psi \end{array}\right) & \begin{array}{c} 5 \\ 6 \end{array} \end{array} \tag{100}$$

$$\widetilde{g} = \det(g_{\tilde a\tilde b}) = \sin^2\psi(1+\zeta^2) \tag{101}$$

$$g^{\tilde a\tilde b} = \frac{1}{\sin^2\psi(1+\zeta^2)} \begin{array}{cc} \begin{array}{cc} 5 & 6 \end{array} & \\ \left(\begin{array}{c|c} -\sin^2\psi & -\zeta\sin\psi \\ \hline \zeta\sin\psi & -1 \end{array}\right) & \begin{array}{c} 5 \\ 6 \end{array} \end{array}, \tag{102}$$

$\widetilde{a}, \widetilde{b} = 5, 6$. In this way we have to do with Kählerian structure on S^2 (Riemannian, symplectic and complex which are compatible). This seems to be very interesting in further research connecting unification of all fundamental interactions. On H we define a nonsymmetric metric

$$\ell_{ab} = h_{ab} + \xi k_{ab} \tag{103}$$

where k_{ab} is a right-invariant skew-symmetric 2-form on H.

One can rewrite the constraints (96) in the form

$$\begin{aligned} [\varPhi_3, \varPhi] &= i\varPhi \\ [\varPhi_3, \widetilde{\varPhi}] &= -i\widetilde{\varPhi} \\ [\varPhi_3, A_\mu] &= 0 \end{aligned} \tag{104}$$

where $\varPhi = \varPhi_1 + i\varPhi_2$, $\widetilde{\varPhi} = \varPhi_1 - i\varPhi_2$.

In this way our 6-dimensional gauge field (a connection on a fiber bundle) has been reduced to a 4-dimensional gauge one (a connection on a fiber bundle over a space-time E) and a collection of scalar fields defined on E satisfying some constraints. According to our approach there is defined on S^2 a nonsymmetric connection compatible with a nonsymmetric tensor $g_{\tilde a\tilde b}$, $\widetilde{a}, \widetilde{b} = 5, 6$,

$$\begin{aligned} \widehat{D} g_{\tilde a\tilde b} &= g_{\tilde a\tilde d} Q^{\tilde d}{}_{\tilde b\tilde c}(\widehat{\Gamma})\overline{\theta}^{\tilde c} \\ Q^{\tilde d}{}_{\tilde b\tilde d}(\widetilde{\Gamma}) &= 0 \end{aligned} \tag{105}$$

where $\widehat{D}$ is an exterior covariant derivative with respect to a connection $\widehat{\omega}^{\tilde{a}}{}_{\tilde{b}} = \widehat{\Gamma}^{\tilde{a}}{}_{\tilde{b}\tilde{c}}\overline{\theta}^{\tilde{c}}$ and $Q^{\tilde{d}}{}_{\tilde{b}\tilde{c}}(\widehat{\Gamma})$ its torsion.

Let us metrize a bundle P in a nonsymmetric way. On V we have nonsymmetric tensor

$$\gamma_{AB} = \left(\begin{array}{c|c} g_{\mu\nu} & 0 \\ \hline 0 & r^2 g_{\tilde{a}\tilde{b}} \end{array}\right) \tag{106}$$

and a nonsymmetric connection $\overline{\omega}^A{}_B = \Gamma^A{}_{BC}\theta^C$ compatible with this tensor

$$\begin{gathered} \overline{D}\gamma_{AB} = \gamma_{AD}Q^D{}_{BC}(\overline{\Gamma})\theta^C \\ Q^D{}_{BD}(\overline{\Gamma}) = 0. \end{gathered} \tag{107}$$

The form of this connection is as follows

$$\overline{\omega}^A{}_B = \left(\begin{array}{c|c} \overline{\omega}^\alpha{}_\beta & 0 \\ \hline 0 & \widehat{\omega}^{\tilde{a}}{}_{\tilde{b}} \end{array}\right) \tag{108}$$

where $\overline{D}$ is an exterior covariant derivative with respect to $\overline{\omega}^A{}_B$ and $Q^D{}_{BC}(\overline{\Gamma})$ its torsion.

Afterwards we define on P a nonsymmetric tensor

$$\kappa_{\tilde{A}\tilde{B}}\theta^{\tilde{A}} \otimes \theta^{\tilde{B}} = \pi^*(\gamma_{AB}\overline{\theta}^A \otimes \theta^B) + \ell_{ab}\theta^a \otimes \theta^b \tag{109}$$

where

$$\theta^{\tilde{A}} = (\pi^*(\overline{\theta}^A), \lambda\omega^a), \tag{110}$$

$\omega = \omega^0 X_a$ is a connection defined on P $(\tilde{A}, \tilde{B}, \tilde{C} = 1, 2, \ldots, n+6)$.

We define on P two connections $\omega^A{}_B$ and $W^A{}_B$ such that $\omega^A{}_B$ is compatible with a nonsymmetric tensor $\kappa_{\tilde{A}\tilde{B}}$,

$$\begin{gathered} D\kappa_{\tilde{A}\tilde{B}} = \kappa_{\tilde{A}\tilde{D}}Q^{\tilde{D}}{}_{\tilde{B}\tilde{C}}(\Gamma)\theta^{\tilde{C}} \\ Q^{\tilde{D}}{}_{\tilde{B}\tilde{D}}(\Gamma) = 0, \end{gathered} \tag{111}$$

where D is an exterior covariant derivative with respect to a connection $\omega^{\tilde{A}}{}_{\tilde{B}}$ and $Q^{\tilde{D}}{}_{\tilde{B}\tilde{C}}(\Gamma)$ its torsion.

The second connection

$$W^{\tilde{A}}{}_{\tilde{B}} = \omega^{\tilde{A}}{}_{\tilde{B}} - \frac{4}{3(n+4)}\,\delta^{\tilde{A}}{}_{\tilde{B}}\overline{W} \quad (n = \dim H). \tag{112}$$

In this way we have all quantities known from the Nonsymmetric non-Abelian Kaluza–Klein theory with spontaneous symmetry breaking. We calculate a scalar of curvature (Moffat–Ricci) for a connection $W^{\tilde{A}}{}_{\tilde{B}}$ and afterwards an action

$$\begin{aligned} S = -\frac{1}{V_1V_2}\int_U \sqrt{-g}\,d^4x \int_H \sqrt{|\ell|}\,d^nx \int_{S^2} \sqrt{|\widetilde{g}|}\,d\varOmega\,R(W) \\ = -\frac{1}{r^2V_1V_2}\int_U \sqrt{-g}\,d^4x \int_{S^2} \sqrt{|\widetilde{g}|}\,d\varOmega\,\Big(\overline{R}(\overline{W}) \\ + \frac{8\pi G_N}{c^4}\Big(\mathcal{L}_{\mathrm{YM}} + \frac{1}{4\pi r^2}\,\mathcal{L}_{\mathrm{kin}}(\nabla\varPhi) - \frac{1}{8\pi r^2}\,V(\varPhi) - \frac{1}{2\pi r^2}\,\mathcal{L}_{\mathrm{int}}(\varPhi, \widetilde{A})\Big) + \lambda_c\Big) \end{aligned} \tag{113}$$

where $V_1 = \int_U \sqrt{|\ell|}\,d^nx$, $V_2 = \int_{S^2} \sqrt{|\widetilde{g}|}\,d\varOmega$, $U \subset E$,

$$\lambda_c = \Big(\frac{\alpha_s^2}{\ell_{\mathrm{pl}}^2}\,\widetilde{R}(\widetilde{\Gamma}) + \frac{1}{r^2}\,\underline{\widetilde{P}}\Big) \tag{114}$$

where $\widetilde{R}(\widetilde{\Gamma})$ is a Moffat–Ricci curvature scalar on a group H.

$$\underline{\widetilde{P}} = \frac{1}{V_2}\int_{S^2}\sqrt{|\overline{g}|}\,d\varOmega\,\widehat{R}(\widehat{\Gamma}) \tag{115}$$

where $\widehat{R}(\widehat{\Gamma})$ is a Moffat–Ricci curvature scalar on S^2 for a connection $\widehat{\omega}^{\tilde{a}}{}_{\tilde{b}}$.

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{8\pi}\,\ell_{ij}\big(\widetilde{H}^{(i}\widetilde{H}^{j)} - \widetilde{L}^{i\mu\nu}\widetilde{H}^{j}{}_{\mu\nu}\big) \tag{116}$$

where

$$\ell_{ij}g_{\mu\beta}g^{\gamma\mu}\widetilde{L}^{i}{}_{\gamma\alpha} + \ell_{ji}g_{\alpha\mu}g^{\mu\gamma}\widetilde{L}^{i}{}_{\beta\gamma} = 2\ell_{ji}g_{\alpha\mu}g^{\mu\gamma}\widetilde{H}^{i}{}_{\beta\gamma} \tag{117}$$

One gets from (78)

$$L^{b}{}_{\tilde{a}\tilde{b}} = h^{bc}\ell_{cd}H^{d}{}_{\tilde{b}\tilde{a}}, \tag{118}$$

$$\begin{aligned} V(\varPhi) = -\frac{1}{V_2}\int\sqrt{|\overline{g}|}\,d\varOmega\,\big(2h_{cd}(H^{c}{}_{\tilde{a}\tilde{b}}g^{\tilde{a}\tilde{b}})(H^{d}{}_{\tilde{c}\tilde{d}}g^{\tilde{c}\tilde{d}}) - \ell_{cd}g^{\tilde{a}\tilde{m}}g^{\tilde{b}\tilde{n}}L^{c}{}_{\tilde{a}\tilde{b}}H^{d}{}_{\tilde{m}\tilde{n}}\big) \\ = \frac{1}{V_2}\,\frac{2\pi^2}{\sqrt{1+\zeta^2}}\,\kappa\big((\varepsilon_{\bar{r}\bar{s}\bar{t}}\varPhi_{\bar{t}} + [\varPhi_{\bar{r}},\varPhi_{\bar{s}}]),(\varepsilon_{\bar{r}\bar{s}\bar{t}}\varPhi_{\bar{t}} + [\varPhi_{\bar{r}},\varPhi_{\bar{s}}])\big) \end{aligned} \tag{119}$$

$$\kappa_{de} = (1-2\zeta^2)h_{de} + \xi^2 k^{c}{}_{d}k_{ce} \tag{120}$$

where

$$k^{c}{}_{d} = h^{cf}k_{fd} \tag{121}$$

$$V_2 = \int_{S^2}\sqrt{|\overline{g}|}\,d\varOmega = 4\pi\sqrt{1+\zeta^2}, \tag{122}$$

$\bar{r},\bar{s},\bar{t} = 1,2,3$, $\varepsilon_{\bar{r}\bar{s}\bar{t}}$ is a usual antisymmetric symbol $\varepsilon_{123} = 1$.

We get also from (78)

$$\ell_{dc}g_{\mu\beta}g^{\gamma\mu}L^{d}{}_{\gamma\tilde{a}} + \ell_{cd}L^{d}{}_{\beta\tilde{a}} = 2\ell_{cd}F^{c}{}_{\beta\tilde{a}}. \tag{123}$$

Using the equation

$$\begin{aligned} L^{n}{}_{\omega\tilde{m}} = \overset{\text{gauge}}{\nabla_\omega}\varPhi^{n}_{\tilde{m}} + \xi k^{n}{}_{d}\overset{\text{gauge}}{\nabla_\omega}\varPhi^{d}_{\tilde{m}} - \big(\zeta\overset{\text{gauge}}{\nabla_\omega}\varPhi^{n}_{\tilde{a}}h^{0\tilde{a}\tilde{d}}k_{0\tilde{d}\tilde{m}} + \widetilde{g}^{(\alpha\mu)}\overset{\text{gauge}}{\nabla_\alpha}\varPhi^{n}_{\tilde{m}}g_{[\mu\omega]}\big) \\ - 2\xi\zeta k^{n}{}_{d}\overset{\text{gauge}}{\nabla_\omega}\varPhi^{d}_{\tilde{d}}\widetilde{g}^{(\delta\alpha)}g_{[\alpha\omega]}h^{0\tilde{d}\tilde{a}}k^{0}{}_{\tilde{a}\tilde{m}} + \xi k^{n}{}_{d}\big(\zeta^2 h^{\tilde{d}\tilde{a}}\overset{\text{gauge}}{\nabla_\omega}\varPhi^{d}_{\tilde{a}}k^{0}{}_{\tilde{d}\tilde{b}}k^{0}{}_{\tilde{m}\tilde{c}}h^{0\tilde{c}\tilde{b}} \\ + \overset{\text{gauge}}{\nabla_\beta}\varPhi^{d}_{\tilde{m}}\widetilde{g}^{(\delta\beta)}g_{[\delta\alpha]}g_{[\omega\mu]}\widetilde{g}^{(\alpha\mu)}\big) - \xi^2 k^{nb}k_{bd}\big(\zeta\overset{\text{gauge}}{\nabla_\omega}\varPhi^{d}_{\tilde{a}}h^{0\tilde{a}\tilde{b}}k^{0}{}_{\tilde{m}\tilde{b}} + \widetilde{g}^{(\alpha\beta)}\overset{\text{gauge}}{\nabla_a}\varPhi^{d}_{\tilde{m}}g_{[\omega\beta]}\big) \end{aligned}$$

where

$$k^{nb} = h^{na}h^{bp}k_{ap},$$

one gets

$$\begin{aligned} L^{n}{}_{\omega\tilde{m}} = \overset{\text{gauge}}{\nabla_\omega}\varPhi^{n}_{\tilde{m}} + \xi k^{n}{}_{d}\overset{\text{gauge}}{\nabla_\omega}\varPhi^{d}_{\tilde{m}} - \widetilde{g}^{(\alpha\mu)}\overset{\text{gauge}}{\nabla_\alpha}\varPhi^{n}_{\tilde{m}}g_{[\mu\omega]} \\ + \xi k^{n}{}_{d}\overset{\text{gauge}}{\nabla_\beta}\varPhi^{d}_{\tilde{m}}\widetilde{g}^{(\delta\beta)}g_{[\delta\alpha]}g_{[\omega\mu]}\widetilde{g}^{(\alpha\mu)} - \xi^2 k^{nb}k_{bd}\widetilde{g}^{(\alpha\beta)}\overset{\text{gauge}}{\nabla}\varPhi^{d}_{\tilde{m}}g_{[\omega\beta]}. \end{aligned} \tag{124}$$

Moreover, now we have to do with Minkowski space $g_{\mu\nu} = \eta_{\mu\nu}$ and

$$L^{n}{}_{\omega\tilde{m}} = H^{n}{}_{\omega\tilde{m}} + \xi k^{n}{}_{d}H^{d}{}_{\omega\tilde{m}}. \tag{125}$$

We remember that $\widetilde{m} = 5, 6$ or φ, ψ and that

$$H^n{}_{\mu\tilde{m}} = \overset{\text{gauge}}{\nabla_\mu} \Phi^n_{\tilde{m}}. \tag{126}$$

We have

$$\mathcal{L}_{\text{kin}}(H^n{}_{\mu\tilde{m}}) = \frac{1}{V_2} \int \sqrt{|\widetilde{g}|}\, d\Omega\, (\ell_{ab} \eta^{\beta\mu} L^a{}_{\beta\tilde{b}} H^b{}_{\mu\tilde{a}} g^{\tilde{b}\tilde{a}}). \tag{127}$$

Finally we get

$$\mathcal{L}_{\text{kin}}(\nabla_\mu \Phi_{\bar{m}}) = \frac{2\pi^2}{V_2} \frac{\eta^{\mu\nu}}{\sqrt{1+\zeta^2}} \bar{\kappa}\big(\overset{\text{gauge}}{\nabla_\mu} \Phi_{\bar{m}}, \overset{\text{gauge}}{\nabla_\nu} \Phi_{\bar{m}}\big) \tag{128}$$

$$\overline{\kappa}_{ad} = (h_{ad} + \xi^2 k_{ab} k^b{}_d) \tag{129}$$

where

$$\overset{\text{gauge}}{\nabla_\mu} \Phi_{\bar{m}} = \partial_\mu \Phi^a{}_{\bar{m}} - [A_\mu, \Phi_{\bar{m}}]. \tag{130}$$

Now we suppose $\operatorname{rank} H = 2$ and afterwards $H = G2$. In this way our lagrangian can go to the GSW model where $\mathrm{SU}(2) \times \mathrm{U}(1)$ is a little group of Φ_3. We get also a Higgs' field complex doublet and spontaneous symmetry breaking and mass generation for intermediate bosons. For simplicity we take $\xi = 0$ and also we do not consider an influence of the nonsymmetric gravity on a Higgs' field. We get also a mixing angle θ_W (Weinberg angle). If we choose $H = G2$ we get $\theta_W = 30^\circ$. We get also some predictions of masses

$$\frac{M_H}{M_W} = \frac{1}{\cos\theta_W} \cdot \sqrt{1 - 2\zeta^2} \tag{131}$$

where ζ is an arbitrary constant

$$\frac{M_H}{M_W} = \frac{2\sqrt{1-2\zeta^2}}{\sqrt{3}}. \tag{132}$$

We take $M_H \simeq 125\,\text{GeV}$ and $M_W \simeq 80\,\text{GeV}$.

One gets

$$\zeta = \pm 0.911622 i. \tag{133}$$

Thus ζ is pure imaginary. This means we can explain mass pattern in GSW model. r gives us a scale of mass and is an arbitrary parameter.

Moreover, a scale of energy is equal to $M = \frac{\hbar c}{r\sqrt{2\pi}\sqrt{1+\zeta^2}}$ which we equal to MEW (electro-weak) energy scale, i.e. to M_W. One gets $r \simeq 2.39 \times 10^{-18}\,\text{m}$. In the original Manton model Higgs' boson is too light. We predict here masses for W, Z^0 and Higgs bosons in the theory taking two parameters, ζ (Eq. (133)) and $r \simeq 2.39 \times 10^{-18}\,\text{m}$ in order to get desired pattern of masses. The value of the Weinberg angle derived here for $H = G2$ has nothing to do with "GUT driven" value $\frac{1}{4}$ for $\frac{1}{4}$ is a value of our $\sin^2\theta_W$, not $\sin\theta_W$. A Lie group H should have a Lie algebra $\mathfrak{h}$ with rank 2. We have only three possibilities: G2, SU(3) and SO(5). The angle between two roots plays a role of a Weinberg angle. For SO(5) $\theta = 45^\circ$ and for SU(3) $\theta = 60^\circ$. Only for $G2$, $\theta = \theta_W = 30^\circ$, which is close to the experimental value. In this way a unification chooses $H = G2$.

Let us notice that $\dim G2 = 14$ and for this $\dim P = 20$.

Moreover, we have

$$M_Z = \frac{M_W}{\cos\theta} = \frac{M_W}{\cos\theta_W} = \frac{2}{\sqrt{3}} M_W \simeq 92.4 \tag{134}$$

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and we get from the theory

$$\sin^2\theta_W = 0.25 \quad (\theta_W = 30^\circ). \tag{135}$$

However from the experiment we get

$$\sin^2\theta_W = 0.2397 \pm 0.0013 \tag{136}$$

which is not 0.25.

Moreover, from theoretical point of view the value 0.25 is a value without radiation corrections and it is possible to tune it at $Q = 91.2\,\mathrm{GeV/c}$ in the $\overline{\mathrm{MS}}$ scheme to get the desired value.

Let us notice the following fact. In the electroweak theory we have a Lagrangian for neutral current interaction

$$\begin{aligned}\mathcal{L}_N = qJ_\mu^{\mathrm{em}}A^\mu + \frac{g}{\cos\theta_W}(J_\mu^3 - \sin^2\theta_W J_\mu^{\mathrm{em}})Z^{0\mu}\\ = qJ_\mu^{\mathrm{em}}A^\mu + \sum_f \overline{\psi}_f\gamma_\mu(g_V^f - g_A^f\gamma^5)\psi_f Z^{0\mu}\end{aligned} \tag{137}$$

where g_V^f and g_A^f are coupling constants for vector and axial interactions for a fermion f. One gets

$$\begin{aligned} g_V^f &= \frac{2q}{\sin 2\theta_W}(T_f^3 - 2q_f\sin^2\theta_W)\\ g_A^f &= \frac{2q}{\sin 2\theta_W}\end{aligned} \tag{138}$$

where T_f^3 is the third component of a weak isospin of a fermion f and q_f is its electric charge measured in elementary charge q,

$$q_f = T_f^3 + \frac{Y_f}{2} \tag{139}$$

where Y_f is a weak hypercharge for f. It is easy to see that for an electron we get $g_V^f = 0$ if $\theta_W = 30^\circ$.

Moreover, we know from the experiment that

$$g_V^f \neq 0. \tag{140}$$

We use the following formulae

$$\Phi_5 = \frac{1}{2}(\varphi_1^* x_{-\alpha} + \varphi_2^* x_{-\beta} - \varphi_1 x_\alpha - \varphi_2 x_\beta) \tag{141}$$

$$\Phi_6 = \frac{\sin\psi}{2i}(\varphi_1 x_\alpha + \varphi_2 x_\beta + \varphi_1^* x_{-\alpha} + \varphi_2^* x_{-\beta}) - \Phi_3\cos\psi. \tag{142}$$

Φ_3 is constant and commutes with a reduced connection. $\mathrm{SU}(2)\times U(1)$ is a little group of Φ_3,

$$\Phi_3 = \frac{1}{2}\,i(2 - \langle\gamma,\alpha\rangle)^{-1}(h_\alpha + h_\beta), \tag{143}$$

x_α, $x_{-\alpha}$, x_β, $x_{-\beta}$ are elements of a Lie algebra $\mathfrak{h}$ of H corresponding to roots $\alpha, -\alpha, \beta, -\beta$, h_α and h_β are elements of Cartan subalgebra of $\mathfrak{h}$ such that

$$h_\alpha = \frac{2\alpha_i}{\alpha\cdot\alpha}\,H_i = [x_\alpha, x_{-\alpha}], \tag{144}$$

where $\alpha = (\alpha_1, \dots, \alpha_k)$, $k = \operatorname{rank}(\mathfrak{h})$, $\gamma = \alpha - \beta$, $[H_i, x_\omega] = \omega_i x_\omega$, H_i form Cartan subalgebra of $\mathfrak{h}$, $[x_\omega, x_\tau] = C_{\omega,\tau} x_{\omega+\tau}$ if $\omega + \tau$ is a root, if $\omega + \tau$ is not a root x_ω and x_τ commute. We take $k = 2$.

$$\langle \gamma, \alpha \rangle = \frac{2\gamma \cdot \alpha}{\alpha \cdot \alpha} = 2\,\frac{|\gamma|}{|\alpha|} \cos\theta. \tag{145}$$

In this way we get a Higgs' doublet $\binom{\varphi_1}{\varphi_2} = \widetilde{\varphi}$.

The $\mathrm{SU}(2) \times \mathrm{U}(1)$ generators are given by

$$t_1 = \frac{1}{2}\, i(x_\gamma + x_{-\gamma}), \qquad t_2 = \frac{1}{2}(x_\gamma - x_{-\gamma}), \qquad t_3 = \frac{1}{2}\, i h_\gamma, \qquad y = \frac{1}{2}\, i h. \tag{146}$$

h is an element of Cartan subalgebra orthogonal to h_γ with the same norm. Now everything is exactly the same as in symmetric theory except the fact that

$$\bar{k}_{ad} = h_{ad} - \xi^2 k_{ab} k^b{}_d \tag{147}$$

$$k_{ad} = (1 - 2\zeta^2) h_{ad} - \xi^2 k_{ab} k^b{}_d. \tag{148}$$

In symmetric theory

$$\bar{k}_{ad} = k_{ad} = h_{ad}. \tag{149}$$

A four-potential of Yang–Mills' field (a connection ω_E) can be written as

$$A_\mu = \sum_{i=1}^{3} A_\mu t_i + B_\mu y \tag{150}$$

$$\text{or} \quad A_\mu = \frac{1}{2}\, i(A_\mu^- x_\gamma + A_\mu^+ x_{-\gamma} + A_\mu^3 h_\gamma + B_\mu h) \tag{151}$$

$$A_\mu^\pm = A_\mu^1 \pm i A_\mu^2. \tag{152}$$

We have

$$h(t_i, t_j) = -\frac{1}{\gamma \cdot \gamma}\,\delta_{ij}$$

$$h(y, y) = -\frac{1}{\gamma \cdot \gamma}$$

$$h(t_i, y) = 0$$

$$F_{\mu\nu} = \big(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \varepsilon^a{}_{bc} A_\mu^b A_\nu^c\big) t_a + (\partial_\mu B_\nu - \partial_\nu B_\mu) y = F^a{}_{\mu\nu} t_a + B_{\mu\nu} y \tag{153}$$

$$h(F_{\mu\nu}, F_{\mu\nu}) = -\frac{\delta_{ab}}{\gamma \cdot \gamma}\, F^a{}_{\mu\nu} F^{b\mu\nu} - \frac{1}{\gamma \cdot \gamma} B_{\mu\nu} B^{\mu\nu} \tag{154}$$

$$\begin{aligned} \overset{\text{gauge}}{\nabla_\mu} \Phi = {} & \Big(\partial_\mu \varphi_1 - \frac{1}{2}\, i A_\mu^- \varphi_2 - \frac{1}{2}\, i A_\mu^3 \varphi_1 - \frac{1}{2}\, i \tan\theta B_\mu \varphi_1\Big) x_\alpha \\ & + \Big(\partial_\mu \varphi_2 - \frac{1}{2}\, i A_\mu^+ \varphi_1 + \frac{1}{2}\, i A_\mu^3 \varphi_2 - \frac{1}{2}\, i \tan\theta B_\mu \varphi_2\Big) x_\beta \end{aligned} \tag{155}$$

$$\begin{aligned} \overset{\text{gauge}}{\nabla_\mu} \widetilde{\Phi} = {} & -\Big(\partial_\mu \varphi_1^* + \frac{1}{2}\, i A_\mu^+ \varphi_2^* + \frac{1}{2}\, i A_\mu^3 \varphi_1^* + \frac{1}{2}\, i \tan\theta B_\mu \varphi_1^*\Big) x_{-\alpha} \\ & - \Big(\partial_\mu \varphi_2^* + \frac{1}{2}\, i A_\mu^- \varphi_1^* - \frac{1}{2}\, i A_\mu^3 \varphi_2^* + \frac{1}{2}\, i \tan\theta B_\mu \varphi_2^*\Big) x_{-\beta} \end{aligned} \tag{156}$$

We redefine the fields A_μ^a, B_μ and $\widetilde{\varphi}$ with some rescaling (g is a coupling constant)

$$A_\mu'^a = L_1 A_\mu^a, \quad B_\mu' = L_1 B_\mu, \quad \widetilde{\varphi}' = L_2 \widetilde{\varphi} \tag{157}$$

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where

$$L_1 = \frac{1}{g}\,\frac{1}{(\gamma\cdot\gamma)^{1/2}} \tag{158}$$

$$L_2 = \frac{1}{g}\Big(\frac{\gamma\cdot\gamma}{\alpha\cdot\alpha}\Big)^{1/2} \tag{159}$$

We proceed the following transformation

$$\begin{pmatrix} Z^0_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix}. \tag{160}$$

According to the classical results we also have $\frac{g'}{g}=\tan\theta$, assuming $q=g\sin\theta$, where q is an elementary charge and g and g' are coupling constants of A^a_μ and B_μ fields. The spontaneous symmetry breaking and Higgs' mechanism in the Manton model works classical if we take for minimum of the potential

$$\widetilde{\varphi}_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i\alpha}, \quad \alpha \text{ arbitrary phase}, \tag{161}$$

and we parametrize $\widetilde{\varphi}=\binom{\varphi_1}{\varphi_2}$ in the following way

$$\widetilde{\varphi}(x) = \exp\Big(i\,\frac{1}{2v}\,\sigma^a t^a(x)\Big)\begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix}. \tag{162}$$

For a vacuum state we take

$$\widetilde{\varphi}_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \tag{163}$$

$t^a(x)$ and $H(x)$ are real fields on E. $t^a(x)$ has been "eaten" by A^a_μ, $a=1,2$, and Z^0_μ fields making them massive. $H(x)$ is our Higgs' field. σ^a are Pauli matrices.

In the formulae (147)–(148) we take $\xi=0$. One gets in the Lagrangian mass terms:

$$M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}\,M_Z^2 Z^0_\mu Z^{0\mu} - \frac{1}{2}\,M_H^2 H^2,$$

where $W^+_\mu = A^+_\mu$, $W^-_\mu = A^-_\mu$, getting masses for $W^\pm$, Z^0 bosons and a Higgs boson (see Eqs (131)–(135)). For G2 $\langle\gamma,\alpha\rangle=3$ and $\theta=30^\circ$, θ is identified with the Weinberg angle θ_W.

In order to proceed a Higgs' mechanism and spontaneous symmetry breaking in this model we use the following gauge transformation

$$\widetilde{\varphi}(x)\mapsto U(x)\widetilde{\varphi}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix}, \tag{164}$$

where

$$v = \frac{2\sqrt{2}}{rg}\cos\theta \tag{165}$$

a vacuum value of a Higgs field

$$U(x) = \exp\Big(-\frac{1}{2v}\,t^a(x)\sigma^a\Big). \tag{166}$$

$H(x)$ is the remaining scalar field after a symmetry breaking and a Higgs' mechanism. One gets

$$A_\mu \mapsto A^u_\mu = \mathrm{ad}'_{U^{-1}(x)}A_\mu + U^{-1}(x)\partial_\mu U(x) \tag{167}$$

$$F_{\mu\nu} \mapsto F^u_{\mu\nu} = \mathrm{ad}'_{U^{-1}(x)} F_{\mu\nu}. \tag{168}$$

Using some additional fields $\varPhi_1, \varPhi_2, \varPhi_3$ and also $\varPhi$ and $\widetilde{\varPhi}$, we can write $\overset{\text{gauge}}{\nabla}_\mu \varPhi_5$ and $\overset{\text{gauge}}{\nabla}_\mu \varPhi_6$ in terms of Higgs' fields φ_1 and φ_2,

$$\begin{aligned}
\overset{\text{gauge}}{\nabla}_\mu \varPhi_5 = \frac{1}{2}\overset{\text{gauge}}{\nabla}_\mu (\varPhi + \widetilde{\varPhi}) = \frac{1}{2}\Big[&\Big(\partial_\mu\varphi_1 - \frac{1}{2}\, iA^-_\mu \varphi_2 - \frac{1}{2}\, iA^3_\mu\varphi_1 - \frac{1}{2}\, i\tan\theta B_\mu\varphi_1\Big)x_\alpha \\
&+ \Big(\partial_\mu\varphi_2 - \frac{1}{2}\, iA^+_\mu\varphi_1 + \frac{1}{2}\, iA^+_\mu\varphi_2 - \frac{1}{2}\, iB_\mu\varphi_2\tan\theta\Big)x_\beta \\
&- \Big(\partial_\mu\varphi_1^* + \frac{1}{2}\, iA^+_\mu\varphi_2^* + \frac{1}{2}\, iA^3_\mu\varphi_1^* + \frac{1}{2}\, iB_\mu\varphi_1^*\tan\theta\Big)x_{-\alpha} \\
&- \Big(\partial_\mu\varphi_2^* + \frac{1}{2}\, iA^-_\mu\varphi_1^* - \frac{1}{2}\, iA^3_\mu\varphi_2^* + \frac{1}{2}\, i\tan\theta B_\mu\varphi_2^*\Big)x_{-\beta}\Big]
\end{aligned} \tag{169}$$

$$\begin{aligned}
\overset{\text{gauge}}{\nabla}_\mu \varPhi_6 = \frac{\sin\psi}{2i}\overset{\text{gauge}}{\nabla}_\mu (\varPhi - \widetilde{\varPhi}) = \frac{\sin\psi}{2i}\Big[&\Big(\partial_\mu\varphi_1 - \frac{1}{2}\, iA^-_\mu \varphi_2 - \frac{1}{2}\, iA^3_\mu\varphi_1 - \frac{1}{2}\, i\tan\theta B_\mu\varphi_1\Big)x_\alpha \\
&+ \Big(\partial_\mu\varphi_2 - \frac{1}{2}\, iA^+_\mu\varphi_1 + \frac{1}{2}\, iA^+_\mu\varphi_2 - \frac{1}{2}\, iB_\mu\varphi_2\tan\theta\Big)x_\beta \\
&- \Big(\partial_\mu\varphi_1^* + \frac{1}{2}\, iA^+_\mu\varphi_2^* + \frac{1}{2}\, iA^3_\mu\varphi_1^* + \frac{1}{2}\, iB_\mu\varphi_1^*\tan\theta\Big)x_{-\alpha} \\
&- \Big(\partial_\mu\varphi_2^* + \frac{1}{2}\, iA^-_\mu\varphi_1^* - \frac{1}{2}\, iA^3_\mu\varphi_2^* + \frac{1}{2}\, i\tan\theta B_\mu\varphi_2^*\Big)x_{-\beta}\Big]
\end{aligned} \tag{170}$$

where

$$e^*\omega_E = A^i_\mu \bar\theta^\mu t_i + B_\mu\bar\theta^\mu y \tag{171}$$

$$e^*\omega = \alpha^c_i A^i_\mu \bar\theta^\mu \widetilde{t}_i + \varPhi^a_{\tilde a}\theta^{\tilde a} X_a, \tag{172}$$

$$\widetilde{t}_i = t_i,\ i = 1,2,3, \quad \widetilde{t}_4 = y. \tag{173}$$

Let us proceed a spontaneous symmetry breaking and Higgs' mechanism. In this way we transform

$$\overset{\text{gauge}}{\nabla}_\mu \varPhi_{\tilde a} \mapsto \mathrm{ad}'_{U^{-1}(x)} \overset{\text{gauge}}{\nabla}_\mu \varPhi_{\tilde a} = \overset{\text{gauge}}{\nabla}_\mu \varPhi^u_{\tilde a}, \quad \widetilde{a} = 5, 6, \tag{174}$$

where

$$\begin{aligned}
\overset{\text{gauge}}{\nabla}_\mu \varPhi^u_5 &= \frac{1}{2\sqrt{2}}\Big[\partial_\mu H(x)(x_\beta - x_{-\beta}) \\
&+ \frac{i}{2}(v + H(x))\big(A^{3u}_\mu(x_\beta + x_{-\beta}) + B_\mu\tan\theta(x_{-\beta} - x_\beta) - A^{+u}_\mu x_{-\alpha} + A^{-u}_\mu x_\alpha\big)\Big]
\end{aligned} \tag{175}$$

$$\begin{aligned}
\overset{\text{gauge}}{\nabla}_\mu \varPhi^u_6 &= \frac{\sin\psi}{2i}\Big[\partial_\mu H(x)(x_\beta + x_{-\beta}) \\
&+ \frac{i}{2}(v + H(x))\big(A^{+u}_\mu x_{-\alpha} - A^{-u}_\mu x_\alpha + A^{3u}_\mu(x_\beta - x_{-\beta}) + B_\mu\tan\theta(x_{-\beta} - x_\beta)\big)\Big]
\end{aligned} \tag{176}$$

where

$$H^u_{56} = -\frac{\sin\psi(v + H(x))}{2}\Big((v + H(x))\frac{\beta_i}{\beta\cdot\beta}\, H_i + \sqrt{2}\cos\psi(x_\beta + x_{-\beta})\Big) \tag{177}$$

$$H^u_{56} = -H^u_{65} \tag{178}$$

where

$$A^+_\mu \mapsto A^{+u}_\mu = \big(\mathrm{ad}'_{U^{-1}(x)} A_\mu\big)^+ + \frac{i}{2v}\,\partial_\mu t^+(x) \tag{179}$$

get nonsingular, stationary and axially-symmetric solution in this theory.

The last remarks concern canonical quantum gravity, i.e. loop quantum gravity, known also as Ashtekar–Lewandowski formalism. The theory is able to quantize surface area, length of curves and a volume (the so-called quantum geometry). This approach is able to derive Bekenstein–Hawking formula for a black hole entropy and gives some instants of quantum field theory in a curved space-time. There are also some important achievements in quantum cosmology to resolve cosmological singularity. Superstring theory has also some achievements to derive a formula for black hole entropy.

$$A_\mu^- \mapsto A_\mu^{-u} = \left(\operatorname{ad}'_{U^{-1}(x)} A_\mu\right)^- + \frac{i}{2v}\,\partial_\mu t^-(x) \tag{180}$$

$$A_\mu^3 \mapsto A_\mu^{3u} = \left(\operatorname{ad}'_{U^{-1}(x)} A_\mu\right)^3 + \frac{i}{2v}\,\partial_\mu t^3(x). \tag{181}$$

Let us suppose that $H = G2$. In this case one gets

$$\begin{gathered}
|\beta| = |\alpha| = \sqrt{2}, \quad |\gamma| = \sqrt{6}, \\
\alpha\cdot\alpha = \beta\cdot\beta = 2, \quad \gamma\cdot\gamma = 6, \\
\langle\gamma,\alpha\rangle = 3, \quad \langle\gamma,\beta\rangle = \langle\alpha,\beta\rangle = -1, \\
\frac{\gamma_1\alpha_2 - \gamma_2\alpha_1}{\gamma\cdot\gamma} = \frac{\sqrt{3}}{6}, \\
\theta = 30^\circ, \quad \cos\theta = \frac{\sqrt{3}}{2}\,, \quad \sin\theta = \frac{1}{2}\,.
\end{gathered} \tag{182}$$

Thus we have (in principle) a lagrangian for a bosonic part of all physical interactions: gravitation, electro-weak and strong. We have also "interference effects" among all of those interactions. The lagrangian of fermions can be obtained as multidimensional generalization of Dirac lagrangian with multidimensional spinor. The Yukawa coupling can be easily got after dimensional reduction. In this way we can find equations from Palatini variational principle, looking for exact solutions of obtained equations. We can expect many "interference effects" from these solutions, testable in experiments. An interesting point here is a generalized Kerner–Wong–Kopczyński equation in a case with spontaneous symmetry breaking. In particular for a GSW model we get some additional charges coupled to Higgs' field. These charges (if they are not zero) can force a test particle to move along fifth and sixth dimensions (formed a sphere of very small radius). This can be testable and implicate many physical, technological and philosophical consequences. All of these mentioned problems are classical (in a meaning of classical field theory). Moreover, we can indicate a way to quantize a theory. As we mention in the main text, there are two ways. The first is a canonical quantization in an Ashtekar–Lewandowski-like approach. The second is a non-local quantization in an Efimov–Yukawa approach coming to renormalizable theory of all mentioned interactions. Moreover, we have a possibility of hierarchy of symmetry breaking in our geometrical setting, having in mind GUT idea. Moreover, because gravitational waves seem to exist (in a linearized GR, which means in any valiable theory of gravity), our gravito-electromagnetic waves in the Nonsymmetric Kaluza–Klein can also exist in the reality.

Thus we get geometric, unified, non-linear, quantum non-local theory of physical interactions.

The problem of “dark matter” originates from the so-called flat velocity curve in our Galaxy and also in different spiral galaxies. According to these results there is missing non-luminous mass in galaxies. Moreover, it is possible to explain the flat velocity curve using Modified Newtonian Dynamics (MOND) by Milgrom. This dynamics can be derived from scalar-tensor gravitational theory by Milgrom and Bekenstein. Thus a “dark matter” can be non-necessary or almost non-necessary. Moreover, we need also a dark matter in galactic clusters outside galaxies. This problem seems to be solved also by using alternative theories of gravitation. Thus up to now we cannot settle the controversy. Probably we need both “dark matter” and an alternative theory of gravitation. The problem with “dark energy” is more subtle. We need a cosmological constant in order to explain expansion of the Universe. The origin of the cosmological constant can be dynamical. It means, a quintessence — scalar field (or multiplet of scalar fields). Probably we can merge problems with a “dark matter” with a “dark energy” getting an alternative theory of gravitation with scalar fields. The solution will be in this case really holistic. There is a possibility to avoid a “dark energy” problem (cosmological constant) if we consider inhomogeneous cosmological solutions of Einstein equations using Lemaître–Tolman–Bondi models or even anisotropic Szekeres models as Krasiński, Bolejko, Hellaby and Céléner.

It is important to mention on A. Connes’ noncommutative geometry and its Standard Model (noncommutative). In noncommutative geometry we abandon points on a differential manifold and we consider an algebra of smooth functions which is an Abelian algebra. In terms of the algebra we can describe all geometrical notions. Let us take a non-Abelian algebra, e.g. matrix algebra. Now for us an algebra is a fundamental notion (not manifold). We can formulate geometrical notions in terms of an algebra. Moreover, we cannot reduce them to some notions defined on a manifold (such a manifold does not exist). The general definition is as follows. It is a spectral triple (A, H, D) where A is a C^*-algebra with a representation in the Hilbert space H, and D is an unbounded operator on H. D has a compact resolvent and $[D, a]$ is a bounded operator on H, where a is an element of a dense subalgebra of A. It is possible to construct a Standard Model using a spectral triple getting Higgs’ field and spontaneous symmetry breaking. This model has some experimental predictions. However, some of them are inconsistent with experiment. Someone can introduce noncommutative Yang–Mills’ fields fields using Moyal brackets and noncommutative gravity. Quantum groups by S. L. Woronowicz and quantum sphere by P. Podleś are similar notions. Moreover, a quantum space-time with quantum Poincaré group can give us some experimental or observational consequences. They are so-called rainbow gravities. The curved momentum space by J. Kowalski-Glikman also results in a rainbow gravity.

Looking backwards onto the 2.5 thousand years of development of the European philosophy, we are able to see lot of different views. They were being put forward and then faded away. They were being rediscovered again, had continued for some time and again get abandoned. However, lot of them endured, which also might be witnessed for instance in our treatment concerning the geometrization and unification of the fundamental physical interactions. Pythagoras, Plato, Aristotle, Prothagoras, Kant, Leibniz, Engels or Lenin were advancing such a different philosophical standpoints. This was due to the fact that they were finding partial truths, but seemed to have dealt with them as with ultimate truths. The urge of obtaining an ultimate model of the world, prompted them to look upon their own creations as the accomplished laws, building blocks of the philosophical systems embedded around these partial truths; often presuming that these philosophical systems are capable of explaining whatever questions one might advance against them. Their followers in turn were adding the contributions of their own, developing thus these systems to such a level of sophistication, until they become mutually contradictory. In spite of all this, the partial truths endured, get refined and or complemented, have led to the advancement of yet new problems and the finding still new solutions. In this fashion, philosophy and science kept advancing and it might well be that this mode of self-transformation provides the only possible and adequate development pattern. The geometry and unification plus Einstein programme undoubtedly belong to the thread of this type, so much eager to find the basic principle of the world — world's *arche*. The relationships this approach enjoys with both theoretical and experimental physics, makes it far more scientifically-minded than was the case of treatment cultivated by Plato or Aristotle. Nonetheless this contemporary approach generously adopts the valuable ideas inherited after ancient philosophers, including Plato, Pythagoras, Aristotle plus the tradition of Scholasticism, complemented suitably even by the ones belonging to a tradition of the philosophies of the East. The programme of unification and geometrization provides an answer to a question: what does constitute a principle of the world, its *arche*? This is geometry. (It is interesting to mention that after more than 2500 years after Thales[3] we get a Standard

[3]Thales of Miletus (c. 624 – c. 546 BC) is considered as the first philosopher. Moreover, it seems that the first philosopher was Gilgamesh (*Epic of Gilgamesh*), the Second Dynasty ruler of Lagash (Uruk) (2144–2124 BC). Gilgamesh by no means is a historical personality, a Summerian king (lugal) of Uruk (Summerian Unug). In this way he is one of the first kings in the history of the human culture known from his name. Simultaneously he is a king of the first nation in human culture known from its name (Summerian — "black

Model (SM) of fundamental interactions which can be geometrized in lines described here.) One has to find it out and later it out and later be able to respond the questions which would follow, plus check them experimentally. Looking back onto the development of philosophy we see that a need of having an overall all-encompassing system was universally encountered. Only the methods and the aspirations striven for were again and again different. At present the goals one tries to achieve are tremendously ambitious, even extremely so. All this results from the fact that a situation was reached, where we possess a lot of partial truths. The urge which keeps a human being searching for such an overall synthesis, a drive toward finding that ultimate principle of the world, seems to constitute a very characteristic feature of the human mind. This property probably results from the fact that we live in the so called cultural reality, that is, as R. Ingarden (Ingarden 1998) had put it — in a quasi-reality. Just by reproducing this quasi-reality we are urged to answer still new and newer questions, extending thus the applicability range of partial truths. In this fashion the quasi-reality is evolving and becomes still closer to an objective reality. This last claim constitutes of course also a certain philosophical thesis *per se*, one which we could attribute as e.g. the viewpoint maintained by scientific materialism advocated for instance by W. Krajewski. On the other hand, we have to keep in mind, recalling again the history of the human thought, that there might occur the case of the so called "contact points of the principal oppositions". This is to be understood in such a way that any pair of philosophical systems, in spite of being in principle contradictory (or even constituting mutually exclusive solutions) could possess very similar partial solutions and partial truths. In this fashion e.g. materialism might be enriched by Thomism and *vice versa*. The passage from one system to another, with jumps from a contradiction

heads"). Due to his very important achievements during the reining he was deificated after his death. After that he became an hero of the *Epic of Gilgamesh*. In this Epic except many adventures of Gilgamesh, his friend Enkidu, a fight with Humbaba (Huwawa) and influences of gods and goddesses (Innane–Ishtar) there are a lot of philosophical reflections. In this way Gilgamesh started to be a philosopher, to be "He who Saw the Deep" and "He Sees the Unknown". Simultaneously these philosophical reflections are connected to an old dream of all mankind — to be immortal and forever young. Gilgamesh (as an hero of the Epic) is a son of the goddess Ninsun and the halfgod Lugalbanda. Thus he is not a full god, he must die. He is looking for immortality despite the fact he knows that gods want immortality only for themselves, not for humans. Eventually he failed in his efforts. He obtained some advices what to do. They are: "Be happy of all of these you have, you will never be immortal". During my classes on philosophy with my students we agreed that Gilgamesh got some kind of immortality. It passed about five thousand years and we still are taking and discussing about him. However this immortality has nothing to do with the immortality he wanted to get. In some sense we can consider him as a first philosopher with a distinction from religio-philosophical gnosis.

to yet another one, seems to fit into the pattern of philosophy development sketched above. The role of practice (praxis), to be understood either in dialectic way or quite pragmatically is therefore rightfully emphasized here.

Philosophy is apodictic and thus always takes the partial truths as the all-encompassing and the ultimate ones. This explains why the history of science and the history of philosophy seems to assume that much importance whenever we try to comprehend the sense of what is observed today in contemporary physics.

To conclude, let us notice also that physics[4] (and hence the philoso-

[4]Physics is the most fundamental of all natural sciences. Since 1928 (after the invention of the Dirac equation) chemistry started to be applied physics. On the molecular level there is no difference between chemistry and physics. There is only physics. Quantum chemistry (or even relativistic quantum chemistry) is still applied quantum mechanics. Biochemistry and molecular biology (in simulations) are also reduced to quantum or classical physics in molecular dynamics (QMD, MD).

In simulations we are using several well established methods. One of them is Car–Parinello method (Car–Parinello dynamics). The lagrangian of electrons and nuclei are given as follows:

$$\mathcal{L} = \frac{1}{2}\Big(\sum_i \dot{\vec{R}}_i^2 + m \cdot \sum_j dr\, |\psi_j(r,t)|^2\Big) - V\big(\{\dot{\psi}_j\}, \{\vec{R}_i\}\big)$$

where $V\big(\{\psi_j\}, \{\vec{R}_i\}\big)$ is a Kohn–Sham energy density functional and ψ_j are electron wave functions (orbitals). Equation of motion are obtained by a variational principle with respect to ψ_j and $\vec{R}_i$ (nuclei positions) supposing orthogonality of ψ_i. The Car–Parinello method is a generalization of Born–Oppenheimer approximation.

From this point of view on the molecular level there is no difference between physics, (bio)chemistry, biology or medicine (human biology). There is only physics. This seems to be a reduction. Moreover, this is a holistic type of reduction. Biological identities (cells, organella) can be considered as molecular machines governed by laws of quantum mechanics with some supposed interacting potentials. Such molecular machines can be simulated by using existing computer programmes. In this way we can simulate fundamental life processes on a molecular level: DNA replication, RNA synthesis and protein synthesis via simulated gene's expression. We can also simulate a full cell of existing bacterium (*Mycobacterium* (*Mycoplasma*) *genitalium*), however with smaller number of genes (100 in place of 300 of the original bacterium). Many known life processes in higher organisms can also be simulated. In the case of a single bacterium cell we can see in simulations all important life processes as: gene's expression (from a genom of bacterium), DNA to RNA translation, protein synthesis and transport, also a death or replication of a bacterium cell.

We can see in simulations a bacterium genom replication. This is of course *in silico*. If we want to prove our theory we should synthetize a bacterium cell *in vitro*. Up to now we can do it only using existing "parts" of a living cell. We can also synthetize artificial chromosomes. The reduction of life processes to physics seems to be proved. Moreover, it is a holistic reduction. In the case of an evolution on the molecular level we suspect several physical processes important in contemporary organism, e.g. an origin of chirality of proteins, which can be connected to polarization of β-decays electrons (parity breaking in GSW model). Moreover, an evolution of organisms on the Earth can be described by RNA-systems (RNA-life) afterwards DNA, RNA, also PNA, protein systems which can be simulated using molecular-like dynamics. Evolution on higher level (i.e. with living cells) can be also modeled using some elementary (physical) processes. A very important problem appears for extremophiles (e.g. *Archea*) and their metalloenzymes with heavy metals group. This problem is connected to relativistic effects (relativistic quantum chemistry). Are these relativistic effects important for a function of these enzymes? If so, an evolution (in a known sense) cannot be proceeded without relativistic effects. This is really interesting. In this way physics can (in principle) explain life processes and also evolution of organisms. There is also a very difficult and interesting problem, a problem of consciousness. According to evolutionary ideas an intelligence (a human intelligence) and also a consciousness is a product of the evolution. However, we still do not understand a consciousness from physical point of view.

This is a problem similar but more complicated to other physiological processes in higher organisms. One of the interesting solutions to this problem is a consideration of consciousness as a macroscopic quantum phenomenon. We know several macroscopic quantum phenomena: superconductivity and superfluidity. They are described by one wave function (a wave function of a superfluid or a wave function of a superconductor). These wave functions are quantum states electrical-magnetic properties of a superconductor, or fluid properties of a superfluid. Everybody sees a piece of a superconductor without properties known from the book by G. Gamow *Mr. Tompkins in Wonderland* (Gamow 1940) (I mean here billiard balls), because superconductivity does not concern mechanical properties of a piece of the superconductor. In this way consciousness described by one quantum state has nothing to do with mechanical properties of a brain. The brain need not be a billiard ball from the mentioned book by G. Gamow. The macroscopic quantum phenomenon described by a wave function (by a quantum state) — consciousness can be lost in a decoherence process. In this way we loose consciousness. In this way physics can be employed even in psychology.

Let us consider the following interesting problem: Is a Standard Model (SM) able to cover all the physical reality on the Earth or not? It seems it is. Why? We consider SM as a Quantum Field Theory (renormalizable) describing all physical interactions except gravity. Due to the Bethe–Salpeter equation we can apply SM in Atomic, Molecular, Solid State Physics and in Quantum Chemistry (via reduction of SM to Quantum Mechanics with electromagnetic interactions). We can neglect weak and strong interactions in mentioned branches of physics. This achievement has been beautifully described in Slater's book *Quantum Theory of Matter* and in full in many volume work on *Quantum Theory of Atom*, *Quantum Theory of Molecules*, *Quantum Theory of Metals, Semiconductors, Isolators, Dielectrics, Chemical Bonds*, and so on, i.e. *Quantum Theory of Atomic Structure*, *Quantum Theory of Molecules and Solids*, vol. 1: *Electronic Structures of Molecules*, vol. 2: *Symmetry and Energy Bands of Crystals*, vol. 3: *Insulators, Semiconductors and Metals*, vol. 4: *The Self-Consistent Field for Molecules and Solids, Solid-State and Molecular Theory. A Scientific Biography* (Slater 1968, 1966, 1975, 1963–74). In this way we can (in principle) obtain macroscopic properties of solid state materials, liquids, gases.

There are some experimental results from LHC which if confirmed will be a signature of new interactions of TeV scale. Moreover, it does not change our conclusions of a power of SM to describe physical reality of the Earth. This is a triumph of SM.

Using SM we can get also Nuclear Physics (in principle) from QCD and many properties of nuclear models and nuclei. The theory of quarks and gluons can (in principle) explain Nuclear Physics (with additional simplifications). Thus SM (via Quantum Mechanics) can explain several macroscopic theories: elasticity, plasticity, and also optics. The last theory is quite easily obtained from QED with ingredients from quantum solid state physics. The quantum optics is the greatest achievement of these investigations.

Thus we get from SM also such macroscopic theories as thermoelasticity, rheology, magnetic properties of macroscopic materials, magnetooptics, dynamics of polymers. Meantime we use Ehrenfest theorems to get from Schrödinger equations (or Dirac) Newtonian (or relativistic) equations of motion and also statistical mechanics. In this way we get foundations of engineering.

The applications of SM with ingredients of GR are also very important. Thus as our conclusion we can say SM+GR explains (in principle) all physical phenomena. It does not mean that science has been finished. In some sense this is not end, this really a beginning, because we go in the right direction.

Moreover, we have the following problem: our world (and also we as a mankind) is classical (from the point of view of classical physics). SM is a quantum theory. Thus we need a transition from quantum world to classical world. This gives us a decoherence. Roughly speaking, this is obtained in the following way. The whole Universe is described by one wave function (one quantum state), say $|\Psi\rangle$. This quantum state is a projector $|\Psi\rangle\langle\Psi| = \rho_u$. In this way we have ρ_u — density matrix of the whole Universe. Now we divide the degrees of freedom of the Universe into three parts:

1° environment

2° object

3° observer

with corresponding hamiltonians H_{e}, H_{obj}, H_{obs} and corresponding ρ-matrices ρ_{e}, ρ_{obj}, ρ_{obs}. We have also interaction hamiltonians H_{int1}, H_{int2}, H_{int3} and so on. Every of the considered ρ-matrices is a trace of ρ_u with respect to the remaining degrees of freedom from whole Universe. In this way after some simplification and additional assumptions concerning hamiltonians we get a master equation for ρ_{obj}, which in a locality limit (not memory) can be reduced to Lindblad–Kossakowski equation. In this equation we can see decoherence for ρ_{obj}. This means that the off-diagonal elements of ρ_{obj} go to zero. If ρ_{obj} is finite-dimensional we see also that diagonal elements of ρ_{obj} are going to the limit $\frac{1}{N}$, where N is the dimension of ρ_{obj}. Let us notice the following fact. Even if we have a decoherence we can still have entanglement (as in a classical Aspect experiment).

The problem of Lorentz invariance of decoherence is still under investigations. The same problem to include GR (or any relativistic theory of gravitation). In this way our world is classical (in the sense of non-quantum physics). Moreover, our methodology to quantize our classical theories is different. We are using classical field theory describing physical interactions and afterwards we quantize these theories using several prescriptions which we mention in the main text. The geometrization of the classical theories, it means, to use as an *arche* a geometry, can help to find a right classical theory to be quantized.

In this way the Standard Model (SM) can explain all physical phenomena (natural physical reality) on the Earth including chemistry, biochemistry, molecular biology, chemical and biological evolution and even intelligence and psychology of consciousness, which we mention above. It is interesting to mention that we should work with quantum information theory (also in the relativistic case).

Even the scheme described above seems to be a reductionist scheme, the holistic aspects are evident, because in order to get properties e.g. of solids from Quantum Theory we suppose many holistic ingredients.

This is of course a truism, but geology, climatology, meteorology, planetology etc. can be reduced to physics. We have here only some computational problems to apply microphysics to physics of clouds, rains, snows, rivers, seas and oceans. We have several aspects of phy of physics, too) plays a certain culture-advancing role. Great problems of physics become problems of an overall human culture, being involved into the process of reproducing the reservoir of the cultural quasi-reality in R. Ingarden's casting, just referred to above. In this fashion the role of the philosophy of physics within the framework of an overall human culture is substantially greater than the one played by either philosophy of science or by physics alone. In this way philosophy is the highest wisdom.

Let us give some historical remarks. In the history of philosophy we have several periods. Some of them are critical periods, some of them are periods of great philosophical systems. Now we are after a critical period. Maybe in a next future in XXI century some great philosophical systems would be created. The role of physics in such systems will be very important. Moreover, this is only a historical remark.

There are some prospects for further research. They consist in finding a place of a geometrization and unification of fundamental physical interactions with holistic ingredients in a realistic philosophical phenomenology by R. Ingarden. However this achievement should be preceded by an introduction of Quantum World to philosophical phenomenology as we sketched in the Introduction (A. Szczepański's criticism). A similarity between a construction of an aesthetic object in R. Ingarden's aesthetics and a process of measurement in Quantum Mechanics can help in the problem.

What is the ontological status of our unification and geometrization? It is easy to see that someone can say this philosophy is materialistic. It means, in the old controversy: what is first, a matter or a spirit, it says: a matter is first. Why? It is easy to see that we reduce anything to physics, to physical world, which is strictly materialistic. It means, three types of beings from the classifications of beings by W. Krajewski have been reduced to space-time beings. Moreover, this is not an end of the reduction. Our "arche" of the world is a geometry (multidimensional geometry with some holistic ingredients). A space-time is a conclusion of a geometry. All space-time beings are emerging from a geometry and also via them remaining types of beings. This means that our conception is idealistic (in the sense of Plato, matter in this philosophy is an illusion, Maya). Simultaneously this philosophy is monistic. This is a geometrical monism. Someone can call the geometry a new kind of matter, but this is not necessary.

atmospheric physics, which can cover climate changes going to physics of climate. In all of these case of applications we need many holistic assumptions. However, the geometry and unification of fundamental interactions are absent on this level of investigations. Moreover, we should remember the origin of classical physics we applied here. Thus the overall picture of all natural phenomena seems to be coming from a geometrization and unification of fundamental physical interactions with holism.

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[5] Mathematica™ is the registered mark of Wolfram Co.

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