Polignac conjecture is true for n=c-1 when c is odd composite >1

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Let p be the higher prime lower than c, p# its primorial, h=p#/2

Let us then consider the set of the infinite pairs of the form

[h+k(p#)-2c h+k(p#)-c]

Such pairs are good candidates to be primes with gap=c because:

- neither element in the pair could be sieved by a prime up to p
- every odd in between needs to be sieved by at least a prime up to p

Not all the gaps c need to belong to such set, nor all the pairs in the set are both primes, but it is sufficient to prove that infinitely many of them are prime indeed.

In fact, in order not to be prime, they need to be sieved by a prime q>p.

Each of such primes q>p does sieve, in one of the 2 positions, 2 out of every q pairs, thus leaving q-2 pairs unsieved.

Thus, in order to compute how many candidate pairs are not sieved by any of such infinitely many primes greater than p, one needs to compute the product of the infinitely many fractions q-2/q over all the primes q greater than p.

When q tends to the infinity, both the numerator and the denominator of such product tend to the infinity with the same strength (even if their fraction tends to be infinitesimal towards zero, quite slowly indeed, because q-2/q tends to increase toward 1 when q increases).

Thus, when the denominator tends to the infinity, also the numerator does, proving that there are infinitely many pairs of primes with gap n not sieved by any lower prime, then proving the conjecture.