Searching for Prime

AUTHOR: Francois Zinserling
29 August 2014

Abstract:

Classical primality testing of large numbers requires a number to be rigorously divided by all prime numbers, up to the square root of the number to be tested. This method is time- and resource-consuming for large numbers. Some time is gained by only dividing by prime numbers to determine the factors, but this too falls short where large numbers are tested for which not all the required lower primes are known. The problem becomes no easier when the very next higher prime number is sought, because the entire rigorous process has to be repeated for every number, and the number of calculations increase as the numbers get bigger.

A prime number is a positive integer that can only be fully divided by 1 and itself. Regarding primality, a positive integer can only have 2 states: prime or non-prime. It is one or the other, no in-betweens. If it can be proven that a number is not non-prime, it is inherently proven that it is prime. This is also the basis upon which the Sieve of Eratosthenes works [1].

A method is presented for finding prime numbers, "from the bottom up" thereby allowing the sieve to be expanded as new prime numbers are discovered. The number of calculations required is greatly reduced by using a quadratic relationship.

Introduction:

For centuries man has been fascinated by prime numbers, forever trying to find patterns or series that may reveal the next unknown prime number.

In 1772 Euler identified a string of 40 prime numbers described by the equation [2]

$$y = 41 + n^2 + n \; (for\; n = 1 \ldots 40) \qquad (1)$$

I have tried to better this and so far have failed.

The Sieve of Eratosthenes is an ancient method by which all possible non-primes are identified, by "quickly" testing for integer factors. Numbers left behind have no factors, and are therefore accepted to be prime. The test is enhanced by testing only for prime factors, since all composite factors are also products of prime numbers. To test large numbers, all primes below the square root of the highest checked number must be known, or else it needs to be checked for every odd number, from the last known prime, up to the said square root. This method becomes cumbersome when large sets of numbers are checked, due to the increased number of calculations required.

A single number can be checked for primality by testing for integer factors. Dividing by every prime number lower than the square root of the tested number, with no integer result, proves the number to be prime. Once again this becomes more cumbersome if all the prime numbers below the square root of the tested number is not known.

A multitude of other specialized methods have been developed to determine very large "special" primes. By these I mean numbers that answer to a specific set of rules, e.g. a^n-1 and other structures for which there are many excellent references available [3]. The most recent great successes have been by the GIMPS project, in the detection and proving of largest prime numbers as Mersenne Primes. [4]. Mersenne primes are of the form 2^p-1. Other special primes with unique probability tests have been found [5], however, even putting all these primes together still leaves huge gaps of unidentified primes. It becomes more and more difficult to identify and prove these unknown primes, since not all lower order primes are known.

There is a saying in cycling terms that if you focus too much on the rock in the path, you will hit the rock and fall off your bicycle. I am a poor cyclist, before anyone asks, but I have decided to look where prime is not.

I present a new method of finding prime numbers. I believe I have found a practical, provable method of identifying and eliminating all non-prime numbers, therefore by default proving remaining numbers to be prime. If it is not non-prime, it is prime!

Theorem:

There exist a series of equations $y_m = n^2 + m * n$ for any n ϵ (1, 2, 3...) and any m ϵ (0, 1, 2, 3…) where all values of y is non-prime, except for values at n=1 for which the value of y might be prime.

Any value of y appearing at any n>1, for any of the equations (y_m), shows that value as being non-prime and having at least one factor of n.

Any value of y appearing at n=1, for any of the equations (y_m), that have not been identified as non-prime by any prior series (all lower values of m, expanded up to and past the value of y in question), thereby having no factors, must be prime.

Proof:

$$y_m = n^2 + mn \qquad (2)$$

$$y_m = n(n + m) \qquad (3)$$

The numbers n and m are integers, and (n+m), being the sum of 2 integers, is an integer.

y_m is always a product of two distinct positive integers and thus non-prime, except where n=1, then

$$y_m = 1(1 + m) \qquad (4)$$

which is a prime number only if (1+m) is prime.

If (1+m) is non-prime, then y_m in eq(4) is non-prime.

Method:

For example the series:

$$y_0 = n^2 + 0n \qquad (5)$$

is never prime, also not for n=1. y_0 ϵ (1, 4, 9, 16…). By definition n=1 is non-prime

The series:

$$y_1 = n^2 + 1n \qquad (6)$$

is never prime, except for n=1. y_1 ϵ (2, 6, 12, 20…). In this series, at n=1, the number 2 is prime. All other values are even and greater than 2, divisible by 2, and thus non-prime.

Apart from the abovementioned number 2 as exception at m=1, we can exclude all:

$$y_m = n^2 + mn\ ;\ m\ \epsilon\ (3,5,7,9\ \ldots .) \qquad (7)$$

as non-prime for all other odd values of m.

n	n^2+1n	n^2+3n	n^2+5n	n^2+7n	n^2+9n	
1	2	4	6	8	10	…
2	6	10	14	18	22	
3	12	18	24	30	36	
4	20	28	36	44	52	
5	30	40	50	60	70	
6	42	54	66	78	90	

Table 1: Series of odd (m) numbered equations

The number 2 need not be shown again in our method further. We have identified it as prime and eliminated all other even numbers as non-prime. It is not our purpose to prove primality of numbers, but merely to distinguish the prime from the non-prime.

We are thus further only concerned with the sets of series with all even numbers of m.

Each consecutive m ϵ (0, 2, 4, 6, 8….) will reveal a new equation, headed by a unique consecutive odd number, to be tested for primality.

These sets of equations contain all odd numbers to infinity, and no possible number (prime or not) is therefore excluded from testing.

The first few sets of equations in the series are shown in columns below:

I have highlighted square numbers in red, prime numbers in orange, and non-prime numbers in blue (light blues are all even numbers from an earlier elimination)

n	$y=n^2$	n^2+2n	n^2+4n	n^2+6n	n^2+8n	n^2+10n	n^2+12n	n^2+14n	n^2+16n	….		
1	1	3	5	7	9	11	13	15	17	19	21	…
2	4	8	12	16	20	24	28	32	36			
3	9	15	21	27	33	39	45	51	57			
4	16	24	32	40	48	56	64	72	80			
5	25	35	45	55	65	75	85	95	105			
6	36	48	60	72	84	96	108	120	132			
7	49	63	77	91	105	119	133	147	161			
8	64	80	96	112	128	144	160	176	192			
9	81	99	117	135	153	171	189	207	225			
10	100	120	140	160	180	200	220	240	260			

Table 2: Series of odd numbered equations

If we expand the set of series from left to right with m, and top to bottom with n, for however many values of m or n we need to at that time, we observe the following:

All even numbers of n return even numbers for y. We know even numbers are non-prime, and we will thus not include even numbers for n in future exercises.

Looking from the top left; Any non-prime value that appears at n=1 will already have been revealed by an earlier series (earlier means to the left) as non-prime, e.g. number 9 at n=3 of y=n^2; or e.g. number 15 at n=3 of y=n^2+2n, provided we have expanded sufficient numbers of n in each previous series to test the primality of the value in question.

All non-prime numbers for n return duplicates of earlier factors. We will thus only include prime numbers for n in calculations.

We can thus eliminate a host of unnecessary n values, and already improve the series, and reduce the number of calculations required as such:

n	$y=n^2$	n^2+2n	n^2+4n	n^2+6n	n^2+8n	n^2+10n	n^2+12n	n^2+14n	n^2+16n	....		
1	1	3	5	7	9	11	13	15	17	19	21	...
3	9	15	21	27	33	39	45	51	57			
5	25	35	45	55	65	75	85	95	105			
7	49	63	77	91	105	119	133	147	161			

Table 3: Improved (n) Series of Equations

Looking ahead; even in the incomplete series above, we find no exclusion for 19, the next prime number, but we do find 21. This is explained further in the document.

Comment or footnote: A benefit of expanding the series like this is that it reveals n as one of the factors of the non-prime numbers. Not all factors are revealed though, as a result of the reduced calculations required. The square method eliminates some duplicates.

For the values y=3, 5, 7, 11, 13, at n=1 of their respective series, we find no evidence in earlier series at any value of n. I call these values "excluded" because they have not been identified and eliminated as non-prime. Excluded values are therefore prime.

We can therefore say, any value of y at n=1 for any series, which has not been eliminated as a non-prime number by any of the above rules, by any prior series, is therefore excluded from being non-prime, is therefore proven to be prime, and need not be tested further for primality.

One may still expect to see somewhat rigorous calculations to prove that a number is prime but the sequence of series reveals primality (or not) for each number with few calculations. If one allows expansion of the series to "follow" the numbers being tested, numbers are isolated as prime long before the series reaches the number in question at n=1.

One only needs to expand each series to an n value where y values for all prior series are equal or greater than the number to be tested.

I will demonstrate:

Step1: We start with a new set of series, each expanded only to n=1. At row n=1 we test and eliminate for primality

n	n^2	+2n	+4n	+6n	+8n	+10n	+12n	+14n
1	1	3	5	7	9	11	13	15
3								
5								
7								
9								
11								

1	2	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31	33
35	37	39	41	43	45	47	49	51

Table 4: Applying the method - step1

The top table shows the expanded sets, and the bottom table shows results; odd numbers from 1 to 51, to which we will assign prime status as we encounter them. Ordinarily we would highlight in row n=1, but it is impractical to expand row n=1 as far as required on the page.

Square numbers will be highlighted in red. Other numbers are not highlighted yet since we do not "know" their status.

As per previous discussion, we know 2 to be prime and mark it orange to indicate prime number found.

Step2: We expand the rows at n=3 and n=5 so that all y values, for each row are equal or higher than 15. Row n=1 is already expanded, but will also need to be dynamic in an automated checker. Here we want to test the remaining numbers 3,5,7,9,11,13,15 for primality

n	n^2	+2n	+4n	+6n	+8n	+10n	+12n	+14n
1	1	3	5	7	9	11	13	15
3	9	15	21	27	33	39	45	
5	25							
7								
9								
11								

1	2	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31	33
35	37	39	41	43	45	47	49	51

Table 5: Mark the non-prime numbers

Since all equations in the series have been expanded to y≥15 we can now test what has been shown as non-prime.

By our theorem, all rows for n>1 return non-prime values. So 9 and 15 are non-prime, and should also be marked so in row n=1. The other numbers 21, 27, 33, 39, 45 should all be marked non-prime, even though they're not relevant to our current testing of primality for the numbers ≤ 15. Suffice to know that all the equations in the series have been expanded to ≥ 15.

Since 9 and 15 have been revealed as non-prime, we can mark them as such in row n=1. Any number shown to be non-prime by expanding the series should be marked immediately.

Since 9 is a non-prime number, it is also removed from column n. (It will only produce duplicate results). Building the table up dynamically means it need not be added in the first place. Future expansion of n will also exclude 15, 21, 27, 33, 39, 45 and all other non-primes we find.

Step3: Marking the primes

n	n^2	+2n	+4n	+6n	+8n	+10n	+12n	+14n
1	1	3	5	7	9	11	13	15
3	9	15	21	27	33	39	45	
5	25							
7								
11								
13								

1	2	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31	33
35	37	39	41	43	45	47	49	51

Table 6: Marking the primes

Since the series have all been expanded to y ≥ 15, there will be no more non-prime values below 15.

We can mark all remaining unmarked values, smaller than 15, as prime. Since the numbers 3, 5, 7, 11, 13 have been excluded from the non-prime list, they have been shown to be prime.

Any number found at n>1 is non-prime according to the theorem. Expanding the series reveals all non-prime numbers. Any number not found at n>1, provided the rules have been followed, is therefore prime.

The table can be further expanded, but the reader has to see this for himself to experience it. In the table above we can already see that 17, 19, and 23 will be excluded from the non-prime list. 21 is already shown as non-prime, and 25 is the next square at n=5 for m=0. The values 27, 33, 39, 45 have already been shown as non-prime. We need to only expand the series slightly to see if 29 and 31 will be excluded.

Author's Note:

- Duplicate calculations reveal multiple factors for composite numbers. If factors are not important, the calculations may be further reduced by using smart dynamic algorithms. However, checking a number against multiple references adds hugely to the work of a processor, especially for large numbers.
- It is possible to use this method to find a list of primes within a certain range, where the series are expanded from below the lower testing limit to above the upper testing limit.
- The exclusion method is economical for making up lists of prime numbers. However, it is not economical to find single prime numbers.

REFERENCES

1. http://mathworld.wolfram.com/SieveofEratosthenes.html
2. Euler 1772; Nagell 1951
3. a. http://primes.utm.edu/index.html
   b. http://mathworld.wolfram.com/PrimeNumber.html
   c. http://en.wikipedia.org/wiki/Prime_number
4. http://www.mersenne.org/
5. http://primes.utm.edu/prove/index.html