Crush-down of One World Trade Center: Conditions in the Building from Roof-line Motion Data

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We analyze the crush-down collapse of One World Trade Center (1 WTC, North Tower) in the framework of the National Institute of Standards and Technology (NIST, 2005) collapse hypothesis. The main feature of crush-down is that a moving part of the building - the top section - falls onto the stationary base, and absorbs the mass in the way. We extend the Bažant-Verdure-Seffen (BVS) model of crush-down (Bažant and Verdure, 2002; Seffen, 2008), where we split the crushing front in two, one at the core and to other at the perimeter of the building. We fit the BVS and the split-front crush-down model to recently published roofline motion data (MacQueen and Szamboti, 2009), to find detailed variation of crushing force $F^C(Z)$ in the storeys 97 and 96, and the average crushing force $\langle F^C \rangle$ in the remainder of the impact zone (storeys 95 to 93) and in the base below for the remainder of roofline data (storeys 92 through 87).

We show how within the NIST hypothesis and the BVS model ϵ^C, defined as $\epsilon^C = \langle F^C \rangle / F^C(0)$, requires a correction factor of 1/6 to match the data. We construct a Controlled Demolition (CD) hypothesis which avoids this and other correction factors through two assumptions: (*i*) the top section is twice as massive as what it appears to be, where its core stretches initially down to the 75-th storey; and (*ii*), the collapse starts with the wave of massive destruction which annihilates the core below the 75-th storey and separates the top section from the base below the impact zone, following which the top section falls to the ground opposed mostly by the perimeter columns, which strength is approximately a third of the total strength. Within the CD hypothesis we achieve excellent agreement between Bažant-Verdure model of crushing force and the data.

CONTENTS

LIST OF SYMBOLS AND TERMS

The Base	Stationary part of a collapsing high-rise building	Sec. 2
Top Section	Moving part of a collapsing high-rise building	Sec. 2
Crush-down	Collapse in a high-rise building, in which the top section falls onto the base and adsorbs its entire mass	Sec. 2
$Z = Z(t)$	Displacement of the roof-line of the *top section*, $\dot{Z} = dZ/dt$ is the velocity and $\ddot{Z} = d^2Z/dt^2$ the acceleration	Sec. 2
m_0	Initial mass of the *top section*. In constant acceleration model of collapse equal to the mass of the *top section*	Sec. 2.2
$\mathcal{R}$	Resistive force that opposes the weight of the *top section* throughout the collapse	Sec. 2.2
F^C	Momentary crushing force. $\langle F^C \rangle$ is its average value, while $f^C = F^C/(M\,g)$ its scaled value	Sec. 2.3
η	Scalar constant that determines conserved quantity in collisions between the top section and the base: $\eta \equiv 0$ for momentum, and $\eta \equiv 1$ for energy	Sec. 2.3
M	Total mass of 1 WTC	Sec. 2.2
H	Height of 1 WTC. Also the position from which all distances in crush-down collapse are measured	Sec. 2.3
ρ_0	Longitudinal density of 1 WTC in 1-D models of collapse, $\rho_0 = M/H$	Sec. 2.3
$Z_a = Z_a(t)$	Position of the perimeter crushing-front	Sec. 2.3
$Z_b = Z_b(t)$	Position of the core crushing-front	Sec. 2.3
κ	Average compaction ratio	Sec. 2.3
L_a	Crush-down initiation point in the BVS model. Same but at the perimeter only in the split-front crush-down model	Sec. 2.3
L_b	Distance from the top of the building in the split-front crush-down model where the core crush-down starts	Sec. 2.3
H_1	Single storey height, equal to 3.66 m (12 ft)	Sec. 2.3
M_1	Single storey mass	Sec. 2.3
χ	Additional mass of the *top section* in the split-front crush-down model in terms of the storey masses M_1, $\chi = b\,(L_b - L_a)/H_1$	Sec. 2.3
a, b, c	Fractions of masses of perimeter columns, core columns, and dwelling surfaces in the total mass of the building, in that order, $a + b + c = 1$. In principle, one could differentiate the same fractions in the top section from those in the base, so to allow voids	Sec. 2.3

$\mathcal{P}$	Peak strength of vertical columns during crushing. Same as F^C at the beginning of crushing of one segment .... Sec. 2.4
g_{BV}	Periodic unity-normalized crushing force after Bažant and Verdure[1]. g_{pBV} is a modification that includes catastrophic failures .... Sec. 2.4
ϵ^C_{BV}	Mean-value of g_{BV}, it is used to extract peak strengh from the average value of crushing force .... Sec. 2.4
L_g	Distance over which $g = g(Z)$ is periodic. Length of a single crushing segment in the building .... Sec. 2.4
$r,\ s$	Parameters of the linear model of peak crushing force $\mathcal{P}$, while $\bar{r}, \bar{s}$ are the same parameters for $\left\langle F^C \right\rangle$ .... Sec. 2.4

1. INTRODUCTION

The World Trade Centers North Tower (One World Trade Center, or 1 WTC) was destroyed on September 11, 2001, as a result of terrorist attacks on the World Trade Center (WTC) complex in New York City, US.

Following their investigation into the collapse of the building, the Federal Emergency Management Agency (FEMA) concluded that the building has been built to the code [2]. Investigators from the National Institute of Standards and Technology (NIST) followed, and by 2005 they had helped to formulate, what we refer to as the NIST hypothesis, namely, a chain of events that they propose may have led to the collapse of 1 WTC [3]. According to the investigators, the impact of the airplane, the explosion of jet fuel and the subsequent fires heavily damaged the building between the storeys 93 and 99, thus creating an impact zone while leaving the building below and above undamaged. After one hour the vertical columns in the impact zone had experienced sufficient damage to yield to the weight of the building above the impact zone. The crush-down started spontaneously at the weakest point of the impact zone - the bottom of the 98-th storey. The 13-storeys-tall top section started to fall to the ground, and in so doing first crushed and absorbed five, presumably, heavily damaged storeys (93-97) of the impact zone, and then continued downwards to crush the rest of the, presumably, undamaged building (storeys 92 and below). Around the same time, a physical model of a progressive collapse in a high-rise structure was proposed by Z. P. Bažant, M. Verdure and their colleagues [1, 4, 5], and applied to the South Tower (Two World Trade Center, or 2 WTC) in support of the NIST hypothesis. Their work was later complemented by Seffen [6]. In their model the moving top section falls onto the stationary base, and absorbs all of the base mass, with resistance coming mostly from the load-bearing vertical columns in the crushing zone (interface between the moving and the stationary part of the building). We refer to this collection of works and results as the BVS Crush-down Model. As part of their work, Bažant and Verdure also proposed a model of crushing force that allows one to relate the peak strength of the vertical columns to their average crushing force. We refer to this as the BV Crushing Force Model.

In this report we use variety of crush-down models to extract the peak and the average crushing force over a distance of 11 storeys (97-87) from the high resolution roofline motion data [7]. We use the BV Crushing Force Model to estimate the peak strength of the columns from their averages, and *vice versa*, and so check the consistency between the predicted and actual values.

We have organized the report as follows. In Sec. 2 we start with a discussion of optimization method for goodness-of-fit, and present the roofline motion data and the one-dimensional models of crush-down that they they try to explain. We finish with a presentation of the BV crushing force model. In Sec. 3 we cast the NIST hypothesis in terms of the model parameters. In Sec. 4 we show the best-fit results in the context of the NIST hypothesis. We follow with discussion in Sec. 5 and present our conclusions in Sec. 6.

2. ONE-DIMENSIONAL MODELS OF CRUSH-DOWN COLLAPSE

We limit our analysis to the so-called "crush-down" collapse of a high rise building. In crush-down collapse the moving part of the building (the top section), falls onto the stationary part of the building (the base) and in the process absorbs some, if not all, of the base mass. Under the circumstances of the 1 WTC collapse, namely, (*i*) throughout the crush-down the top section is rigid and fairly easily distinguished from the base, and (*ii*) the building collapses almost perfectly to its footprint, we can accurately describe the motion of the top section through its displacement from the initial position, which we call $Z = Z(t)$.

We start with the fitting method, as it provides a criterion for goodness-of-fit of models parameters to the roofline motion data.

2.1. Fitting Method

The following discussion is based on the assumption that the pixelization, or binning, of the descent curve by a recording device is the only source of uncertainty. Let u_0 be the smallest detectable displacement of the object by the recording device. What we call the descent curve, is in fact $\{t_i, \bar{Z}_i\}_{i=1,N}$, where $\bar{Z}_i$ are the pixel centers. The true descent curve $\{t_i, Z_i\}_{i=1,N}$, can then be written as $Z_i = \bar{Z}_i + e_i$, where $e_i \in [-\frac{u_0}{2}, \frac{u_0}{2}]$ are the uniformly distributed positional uncertainties.

Let $Z = Z(t - t_0; \{P\}) + d_0$ be a trajectory obtained by solving a theoretical model, where $\{P\}$ is the set of values of the model parameters, and t_0 and d_0 the time and the position offset of the descent curve. We interpret the descent curve as a relative position of the roofline as a function of time $\{t_i, \bar{X}_i\}_{i=1,N}$. From $\bar{X}_i = \bar{Z}_i - \bar{Z}_0$, and by setting $\bar{Z}_0 \equiv 0$, we obtain $\bar{X}_i \equiv \bar{Z}_i$. The relative trajectory is $d_0 + Z(t - t_0; \{P\}) - e_0$, so we can choose $d_0 \equiv e_0$, with a benefit of not having to increase the number of model parameters, but at the expense of uncertainty in the absolute position of the descent curve.

The fitting objective function should be a sum of absolute residuals $\sum_i |\Delta Z_i|$, when the errors $\{e_i^*\}_{i=1,N}$ are all identically uniformly distributed [8]. Let the j-th residual be $\Delta Z_j = \bar{Z}_j - \hat{Z}_j$, where $\hat{Z}_j = Z(t_j - t_0; \{P\})$ is the model prediction. In an effort to minimize $\sum_i |\Delta Z_i|$, only the points such that $|\bar{Z}_j - \hat{Z}_j| > u_0$ are relevant. Conversely, if for some j^* we have $|\bar{Z}_{j*} - \hat{Z}_{j*}| < u_0$ then the minimization goal for that point has been reached, as e_{j*} is unknown. This argument leads us to the objective function that we use throughout this report,

$$S_{pa}(t_0; \{P\}) = \sum_{i=1}^{N} \theta(|\Delta Z_i| - u_0) \cdot (|\Delta Z_i| - u_0), \tag{2.1}$$

where $\theta = \theta(x)$ is a Heaviside function, $\theta(x) = 1$ for $x > 0$, and 0 otherwise.

2.2. Roofline Motion Data and Constant-Acceleration Model

Recently, a two-part analysis was published [7] of a video recording showing the first 3.2 seconds-long of collapse of 1 WTC. In the first, the authors extracted the roofline motion as a function of time, which we show in Table I, and estimated its uncertainty to be uniform in nature and of the size of a single pixel $u_0 = 0.27$ m (0.88 ft). In the second part, the authors showed good fit between the roofline motion and a motion with constant acceleration $\hat{a}$ that starts from rest,

$$Z(t) = \frac{\hat{a}}{2} (t - t_0)^2, \tag{2.2}$$

with t_0 being the offset.

We use S_{pa} to fit the roofline motion data from Table I to model (2.2) and find,

$$\hat{a} \simeq 6.9 \text{ m s}^{-2}, \tag{2.3}$$

that is achieved for $S_{pa} \simeq 20.4$ cm. In Fig. 1 we show the best-fit trajectory, where we can see that over a distance of 35 meters (some 10 stories of the building) the constant-acceleration model fits the data quite well. We notice that all contributions to S_{pa} come from the initial moments of collapse.

The authors interpret Eq. (2.2) as that the top section of fixed mass m_0 falls under the action of two constant forces, the weight $m_0\, g$ and some average resistive force $\mathcal{R} > 0$,

$$m_0\, \hat{a} = m_0\, g - \mathcal{R}. \tag{2.4}$$

We find the resistive force as,

$$\bar{r} = \frac{\mathcal{R}}{M\,g} = \frac{m_0}{M} \cdot \left(1 - \frac{\hat{a}}{g}\right), \tag{2.5}$$

where M is the total mass of the building. The authors of the two-part analysis [7] continue that if the NIST collapse hypothesis is correct, then (i) the top section mass cannot be constant (over the length of their data set the top section should double in weight), and (ii) there should be a strong decrease in $\hat{a}$ once the top section moves from crushing the impact zone to crushing of the (presumably undamaged) base. That is, if the NIST hypothesis holds, then the constant-acceleration model should be a poor fit. Based on the fit being good, the authors conclude that the damage in the building extended beyond the five-storey impact zone for at least the length of their dataset.

2.3. Split-front Crush-down Model of Collapse

Bažant and Verdure [1] introduced a crush-down model to describe collapse of 2 WTC. The kinetic and gravitational parts of their model belong to a class of "falling chain" physics problems featuring variable mass [9]. Seffen [6] rearranged Bažant and Verdure's model in the framework of a cumulative instability propagating through a continuous structure, where the instability is a highly compacted material in the wake of the crushing front. The resulting model is the BVS Crush-down model, which in a uniform high-rise structure reads,

$$\ddot{Z} = g - \frac{1}{Z_a} \left(\frac{1 - \frac{\eta}{2}}{1 - \kappa}\, \dot{Z}^2 + \frac{1}{\rho_0}\, \mathcal{R}(Z) \right), \tag{2.6}$$

where $\rho_0 = M/H$ is the structure's longitudinal density (with M and H being its mass and height), and $Z_a = L_a + (1 - \kappa)^{-1} \cdot Z$ is the position of a crushing front. At the crushing front the mass is compacted from ρ_0, its density in the base, to ρ_1, its density after the crushing, where we assume that the compaction ratio $\kappa = \rho_0/\rho_1 \ll 1$ is a constant. We introduce collision parameter η [10], which determines whether the collisions between the top section and the base are momentum-conserving ($\eta \equiv 0$, after BV) or energy-conserving ($\eta \equiv 1$, after Seffen). Both 1 and 2 WTC comprised two distinct load-bearing structures, the perimeter columns (PC) and the core columns (CC). A reticent feature of the model in Eq. (2.6) is that it assumes that the structures' crushing fronts, at Z_a at the perimeter and Z_b at the core, are collocated ($Z_a \equiv Z_b$) so the initial mass of the top section is $m_0 = \rho_0 \cdot L_a$. The authors identify as the strongest contributors to resistive force $\mathcal{R}$ the crushing forces coming from the two vertical column groups, so that $\mathcal{R}(Z) = F_a^C(Z_a(Z)) + F_b^C(Z_a(Z))$.

We extend the BVS Crush-dwon model in Eq. (2.6) where we allow for the initial top section mass to vary by splitting the crushing fronts at the core and at the perimeter of the building. We assume that the two crush-downs are independent except for the common top section that does the crushing, and which by its rigidity maintains distance between the two fronts. We call this the Split-front Crush-down model,

$$\ddot{Z} = g - \frac{1}{Z_a + \chi H_1} \left(\frac{1 - \frac{\eta}{2}}{1 - \kappa} \dot{Z}^2 + \frac{1}{\rho_0} F_a^C(Z_a) + \frac{1}{\rho_0} F_b^C(Z_b) \right). \tag{2.7}$$

The two crushing fronts are at $Z_{a,b} = L_{a,b} + (1-\kappa)^{-1} \cdot Z$, where L_b is the collapse initiation point at the core. We show the schematic of such a split crushing front in Fig. 2. Omika *et al.* [11] estimated the fractions of total mass of each structural element: perimeter columns (PC) a, core columns (CC) b, and dwelling surfaces c, where $a = 0.3$, $b = 0.6$, and $c = 0.1$. The initial mass of the top section becomes $m_0/\rho_0 = a\, L_a + b\, L_b = L_a + b \cdot (L_b - L_a) = L_a + H_1\, \chi$, with H_1 being a storey height. The parameter χ, represents the extra mass of the top section in terms of the mass of a single storey $M_1 = \rho_0\, H_1$. In Eq. (2.7), $\mathcal{R}$ is again represented by its two strongest contributors, the perimeter (core) columns' crushing force, but at the position Z_a (Z_b).

2.4. Crushing Force F^C

Based on discussion of F^C by Bažant and Verdure [1], we subdivide the base into a number of segments or sub-zones, where the length of each is $n \cdot H_1$, where $n \geq 1$ (BV sets $n \equiv 1$). Over each segment F^C strongly decreases monotonically as the segment is compacted. As the crushing front reaches $L_g = n \cdot (1 - \kappa) \cdot H_1$, the compacted segment is instantaneously absorbed and the crushing front jumps by $\kappa \cdot n \cdot H_1$ to $n \cdot H_1$. At the continuous limit, this averages such that the top section moves with the velocity $\dot{Z}$, while the crushing front moves with an average velocity $\dot{Z}_a = \dot{Z}/(1 - \kappa)$. In terms of the roofline motion the base structure repeats with distance L_g.

Bažant and Verdure [1] assumed that the crushing of vertical columns over one L_g-segment proceeds in two stages, namely, the longitudinal ductile compression followed by buckling, and that F^C is continuous across the two. We introduce the scaled penetration distance, $u^* = Z^*/L_g$, at which ductile deformation switches to buckling, and find unity-normalized crushing force g_{BV}

to be,

$$g_{BV}(u) = \min\left(1, \frac{\sqrt{1-(1-u^*)^2}}{\sqrt{1-(1-u)^2}}\right). \tag{2.8}$$

The two are related as $F^C(u) = F^C(0) \cdot g_{BV}(u)$, with $F^C(0) \equiv \mathcal{P}$, where $\mathcal{P}$ is the local peak strength on the segment. The mean value of g_{BV} over a single crushing segment is

$$\epsilon^C_{BV} = \int_0^1 du\; g_{BV}(u) = \pi\sqrt{\frac{u^*}{2}} - u^* + \mathcal{O}(u^{*3/2}), \tag{2.9}$$

so that the following relationship applies between the segment-averaged crushing force $\langle F^C \rangle$ and $\mathcal{P}$,

$$\langle F^C \rangle = \epsilon^C \cdot \mathcal{P} \tag{2.10}$$

Bažant and Verdure [1] proposed that ϵ^C is a constant of crush-down, that is, u^* and L_g are the same for all columns in the entire building.

Seffen [6]argued that collapse which affects the entire building, one has to consider that $\mathcal{P}$ is at least an affine function of distance, $\mathcal{P} = (M\,g)\,(r + s\frac{Z}{H})$, yielding

$$\frac{\langle F^C \rangle}{M\,g} = r\,\epsilon^C + s\,\epsilon^C\,\frac{Z}{H}. \tag{2.11}$$

For convenience, we define the barred quantities $\bar{r} = r\,\epsilon^C$ and $\bar{s} = s\,\epsilon^C$.

In fitting the crush-down model to the roofline motion data, whether to use F^C or its average $\langle F^C \rangle$ depends on the local resolution of data. As discussed by Bažant and Verdure [1], F^C can be used when the top section moves slowly, while otherwise one uses $\langle F^C \rangle$.

3. THE NIST COLLAPSE HYPOTHESIS

We cast the NIST hypothesis using the basic dimensions of 1 WTC, for which the height of one storey is $H_1 = 3.6576$ m (12 ft), and the height of the whole building is $H = 417$ m $= 114 \cdot H_1$. We use Bažant and Verdure estimate of the total mass $M \simeq 5.76 \cdot 10^8$ kg, so the mass of a single storey is $M_1 = M/114 \simeq 5 \cdot 10^6$ kg. We find the peak strength of vertical columns $p = \mathcal{P}/(M\,g)$, as a function of scaled distance $z = Z/H$ using the linear model in Eq. (2.11) and the NIST data [3]. The perimeter columns (PC) comprise 236 columns which have external dimensions were 14”-by-14”, and are made of structural steel the yield strength of which varies from 58 (36) KSI and thickness $\frac{1}{4}$" at the top of the building to 110 (100) KSI and thickness 1" at the bottom, where we list the ultimate strength followed by the nominal strength in the brackets. This yields $p_a(0) \simeq 0.12$ and $p_a(1) \simeq 1.07$, so the ultimate (peak) strength of PC's as a function of distance is,

$$p_a(z) \simeq 0.12 + 0.95 \cdot z \tag{3.1}$$

For the core columns (CC), we assume that the scaling of Omika *et al.* [11] between the masses of the perimeter and CC directly translates to their strength, so that $p_b = b/a \cdot p_a$, or, $p_b(z) \simeq 0.24 + 1.90 \cdot z$. Our working estimate for the total peak capacity is thus,

$$p(z) \simeq 0.4 + 2.9 \cdot z. \tag{3.2}$$

We proceed to quantify the NIST hypothesis as follows. The impact zone is 18.5 m in height. The collapse initiation point is at the bottom of the 98-th storey at $L_a = (110-98+1)\,H_1 = 47.5$ m for both the perimeter and the core, $L_b \equiv L_a$, or $\chi \equiv 0$. The initial mass of the top section is $m_0 = 13\,M_1$, or $m_0/M \approx L_a/H = 0.11$. The collapse is spontaneous so the total peak capacity of compromised columns at or near the collapse initiation point is approximately the weight pressing on it, $p^* \approx m_0/M \approx 0.11$.

Bažant and Verdure [1] discussed model parameters for 2 WTC in support of the NIST hypothesis. They assumed that the top section free-falls for one storey, following which the average crushing force settles at a constant $\bar{r}_{2WTC} \approx 0.05$. For 1 WTC assuming $\chi \equiv 0$, for the duration of the observed roofline motion the top section grows in mass from $13 \cdot M_1$ to $24 \cdot M_1$, yielding an average of $(18.5 \pm 5.5) \cdot M_1$. From Eq. (2.5) we see that $\bar{r}_{1WTC} = 0.04 \pm 0.01 \approx \bar{r}_{2WTC}$, but this $\bar{r}_{1WTC}$ combines static as well as kinetic resistance.

We continue with Bažant and Verdure analysis, and assume that this $\bar{r} = \bar{r}_{1WTC}$ represents only F^C in the impact zone. As we know strength p^* there, this yields $\epsilon^C = \bar{r}/p^* \simeq 0.36$. However, at some point over the distance covered by the roofline data, the top section should enter the undamaged base, at which point $\bar{r}$ should change to, say, $\bar{r}_2$. E.g., if this occurs at the top of the 92-nd storey, we have $\bar{r}_2 \sim 0.96 \cdot 0.36 \approx 0.4 \gg \bar{r} \sim 0.04$, and this change should be clearly visible in the acceleration of the top section. The results presented in Sec. 2.2 suggest this not to be the case, reaffirming findings in [7].

4. RESULTS

Through numerical analysis of roofline motion we find $L_g = 6.4$ m, so $\kappa = 1/8$. We thus divide the 16 m ($= 7/8 \cdot 18.5$ m) impact zone into two sub-zones, where the first is 6.4 m long. In the first sub-zone we choose the mesh $\{0\ m, 1.5\ m, 6.4\ m\}$ and define linear interpolant over the mesh with values for the crushing force of $\{f^C_{0,0}, f^C_{0,1}, f^C_{0,2}\}$. We use $\bar{r}_2, \bar{r}_3$, for the average crushing force on the second sub-zone of the impact zone (distance between 6.4 m and 16 m), and in the rest of the base (above 16 m). The remaining parameters is t_0, the time offset in the reference frame of the data, which we drop for brevity. We always use $\eta \equiv 1$, after Seffen [6].

As we use linear interpolant for the f^C on the first sub-zone (crushing segment) of length L_g, we can find an approximate value for ϵ^C through Simpson's integration formula, as

$$\epsilon^C = \frac{1}{f^C_{0,0}\, L_g} \left(\frac{f^C_{0,0} + f^C_{0,1}}{2} \cdot 1.5 \text{ m} + \frac{f^C_{0,1} + f^C_{0,2}}{2} \cdot 4.9 \text{ m} \right). \tag{4.1}$$

Once ϵ^C is known, we use it to derive the approximate peak strengths in other sub-zones $f^C_{2,3} = \bar{r}_{2,3}/\epsilon^C$. We also use ϵ^C to find the average crushing force in the first sub-zone, $\bar{r}_1 = \epsilon^C \cdot f^C_{0,0}$. We call these *the derived quantities.*

We use Monte Carlo sensitivity analysis with respect to initial conditions to find the approximate values of the model parameters: We create a large number (few thousands) of uniformly distributed acceptable initial conditions, and then through optimization collect their best-fit values. We keep only those solutions for which $S_{pa} \leq 0.2$ m, that is, which are a better fit than the constant-acceleration motion. For solving the ordinary differential equations we use a 8-th order Runge-Kutta Prince-Dormand method, while for the optimization we use a simplex method of Nelder and Mead, as the latter does not require Jacobian computation.[12]

For the NIST hypothesis we fit the data to the BVS model from Eq. (2.6), which is equivalent to the split-front crush-down model of Eq. (2.7) with $\chi \equiv 0$. Through Monte Carlo analysis we find that the best-fit solutions achieve mostly a perfect fit $S_{pa} = 0.01 \pm 0.00$ $(0.00, 0.16)$ m, representing the mean value and its standard deviation (with minimum and maximum values in brackets), all rounded to two decimal places. The best-fit parameters are,

Best-fit parameters	Derived quantities
$f_{0,0}^C = 0.087 \pm 0.009\ (0.069, 0.111)$	$\epsilon^C = 0.225 \pm 0.042$
$f_{0,1}^C = 0.016 \pm 0.006\ (0.000, 0.030)$	
$f_{0,3}^C = 0.004 \pm 0.006\ (0.000, 0.033)$	$\bar{r}_1 = 0.020 \pm 0.004$
$\bar{r}_2 = 0.033 \pm 0.002\ (0.026, 0.046)$	$f_2^C = 0.148 \pm 0.029$
$\bar{r}_3 = 0.015 \pm 0.003\ (0.000, 0.019)$	$f_3^C = 0.065 \pm 0.019$

We plot these values in Fig. 3, and find that $f_{0,0}^C$ is consistent with the onset of collapse being spontaneous. We find $\epsilon^C \simeq 0.23 \pm 0.04$ on the first segment, and remark that this is m_0-independent quantity.

From the plot in Fig. 3 we see that ϵ^C derived from the first sub-zone is too great for the other sub-zones. We calculate the correction factors for ϵ^C in each sub-zone as follows. We find the expected nominal values $f_{2,0} = 0.88 \pm 0.03$ in the second sub-zone (storeys 96 through 94), and $f_{3,0} = 1.01 \pm 0.09$ in the third sub-zone (storeys 93 through 87). Their ϵ^C's are $\epsilon_2^C = \bar{r}_2 / f_{2,0} = 0.038 \pm 0.003$ and $\epsilon_3^C = 0.015 \pm 0.003$, or $\epsilon^C / \epsilon_2^C \sim 6 \pm 1$ and $\epsilon^C / \epsilon_3^C \sim 15 \pm 3$. Now, in the NIST hypothesis the damage to the base decreases with increasing distance. We combine the two and conclude that the corrected ϵ^C as a function of presumed damage in the building decreases with decreasing damage, i.e., ϵ^C is the greatest in the most damaged part, and the smallest in the least damaged part.

First consequence of this trend in ϵ^C in the framework of the NIST hypothesis is that the Bažant and Verdure proposition about ϵ^C being a constant of collapse does not hold.

Secondly, this behavior of ϵ^C is an example of correction factors that needs to be introduced *a posteriori*, so that the predictions of the BVS model in context of the NIST hypothesis would match the data. As an extreme example we mention Seffen's conjecture by which all ϵ^C's have correction factors which are randomly varying quantities. Obviously, this cannot be the case as we have just shown that these correction factors are progressively smaller quantities.

Lastly, we are interested in a collapse scenario, which with minimal number of assumptions does not require correction factors at all.

5. DISCUSSION

We focus on the second sub-zone of the impact zone, in which $\epsilon^C / \epsilon_2^C \sim 6 \pm 1$. We remark that the impact zone only suffers damage from the airplane impact and its aftermath.

We propose an explanation comprising two assumptions:
Firstly, let us assume that the core crushing front forms at the 75-th, so called, mechanical storey. The initial mass of the top section is then the NIST's 13 storeys plus $\chi = b\cdot(97-75+1) = 13.8 \approx 14$, or $m_0/M \approx 27/114 = 0.24$, i.e., double its NIST value.
Secondly, let us assume that the collapse starts when the strength of the core below the 75th storey is totally compromised in an event we call "the wave of massive destruction" (WMD). We model the WMD after Bažant and Verdure analysis of 2 WTC crush-down, in which they propose yet another correction factor in form of a *heat wave* (BV heat wave, or BVHW) that the crushing front pushes in front of itself, which locally halves the columns peak strength. Because the crushing front (at the bottom of the top section) can be displaced from the hinge of the buckling column by as much as $\frac{H_1}{2}$, if the crushing front radiates heat, then most of the crushed column has an elevated temperature. If ΔT is the average temperature increase of the column caused by the BVHW, then this requires energy $W = C_v\,\rho_0\,Z\,\Delta T$, where $C_v = 480$ J/(kg K) is the specific heat of structural steel. Were this energy part of the split-front crush-down model (2.7) it would appear as an additional resistive force $r_{HW} = R_{HW}/(M\,g) = -C_v\,\Delta T/(H\,g)$. We need $\Delta T \sim 600$ ^{o}C to halve the strength of structural steel, which yields $r_{HW} \sim 70 \ggg \bar{r} \sim 0.04 \pm 0.01$. The argument can also be reversed, where $\partial\bar{r}/\partial\Delta T = -C_v/(H\,g) = 0.11\cdot\alpha$, which gives an increase in $\bar{r}$ per 1 ^{o}C increase in the temperature of the crushed structural steel whose fraction of the total mass is α. The WMD we propose inverts the cause and effect of the BVHW, where in the wake of the WMD the top section is left mainly supported by the perimeter columns. As the perimeter columns in the impact zone are at best undamaged, their peak strength is $p_a^*(L_a/H) \le p_a(L_a/H) \simeq 0.22 < m_0/M$. We thus see that in absence of the core and with the connections between the top section and the base below the impact zone compromised or severed, the PC cannot support the top section and the crush-down may start "spontaneously."

As the PC strength is approximately 1/3 of the total building strength, this combined with doubling of m_0 produces desired factor of 6. As in this scenario the damage to the building extends beyond the impact zone, we refer to it as the controlled demolition (CD) hypothesis.

We test our CD hypothesis through Monte Carlo analysis, where we use split-front crush-down model (2.7) with a fixed $\chi \equiv 14$. We find that the best-fit solutions found through sensitivity analysis achieve mostly perfect fit with $S_{pa} = 0.00 \pm 0.00$ $(0.00, 0.13)$ m, where

Best-fit parameters	Derived quantities
$f^C_{0,0} = 0.187 \pm 0.012\ (0.151, 0.234)$	$\epsilon^C = 0.228 \pm 0.031$
$f^C_{0,1} = 0.035 \pm 0.008\ (0.015, 0.057)$	
$f^C_{0,2} = 0.009 \pm 0.008\ (0.000, 0.029)$	$\bar{r}_1 = 0.043 \pm 0.007$
$\bar{r}_2 = 0.082 \pm 0.005\ (0.068, 0.099)$	$f^C_2 = 0.360 \pm 0.054$
$\bar{r}_3 = 0.049 \pm 0.008\ (0.021, 0.062)$	$f^C_3 = 0.217 \pm 0.046$

We plot these values in Fig. 4, and find an excellent agreement between $f^C_{0,0}$, and f^C_2, f^C_3 on one side, and the estimated strengths of the perimeter columns. In particular, in the first sub-zone best-fit f^C is consistent with the partially damaged PC from the airplane wing cutting them from their support.

We next argue that ϵ^C need not be a constant of collapse providing that the healthier the columns, the greater the ϵ^C. The best fit data confirmed that $L_g \approx 2\,(1-\kappa)\,H_1$ in the most damaged part of the impact zone. We examine what happens if in the rest of the impact zone, L_g switches to the value initially proposed by Bažant and Verdure, namely, $L_g \approx (1-\kappa)\,H_1$. We start from the estimate for $u^* \simeq 2 - 2\,\sqrt{1 - 4\,(d/L_g)^2}$, i.e., that the compression is replaced by buckling when the center of the column moves two thicknesses sideways. For the perimeter columns we find $u^* \simeq 0.01$, for $d = 14$" (transverse dimension of perimeter column) and $L_g = 24$ ft, and $u^{**} \simeq 0.04$ for $L_g = 12$ ft and the same d. From Eq. (2.9) we find $\epsilon^C\big|_{2\,H_1} = \epsilon^C_{BV}(u^*) \simeq 0.21$, which is very close to the best-fit value $\epsilon^C \simeq 0.23$. For the second sub-zone we have $\epsilon^C\big|_{H_1} = \epsilon^C_{BV}(u^{**}) \simeq 0.39$ that should be used with $\bar{r}_2$, and this drops the estimated peak strength to $f^C_2 \sim 0.21$, which is within error margin from p_a. So, by increasing ϵ^C (decreasing the size of crushing segment) we produce almost constant p_a that fully agrees with the undamaged PC in the second sub-zone. As for the third sub-zone below the impact zone, using u^* and u^{**} suggest its PC's to be mildly (20%) to severely (50%) compromised. Our discussion suggests that in the contex of the CD hypothesis a way to indirectly compromise the PC's is to severe them from the dwelling surfaces rather then directly cutting through them.

Finally, let us discuss implications of the CD hypothesis on the Bažant and Verdure Crushing Force model. We notice $f^C_{0,2} = 0.009 \pm 0.008 \approx 0.00$ at the end of the first L_g-segment, which differs from $\lim_{u \to 1} g_{BV}(u) \sim \sqrt{2\,u^*} \gg u^* > 0$, which for $u^* = 0.01$ yields, $g_{BV}(1) \simeq 0.15$. We propose a following modification. We think that g_{BV} is probably a reasonable approximation when the crushing is slow, so the column can maintain its integrity for the entire duration of buckling. However, during rapid crushing the hinges in a buckling column may fail prematurely, for example, when cracks develop that quickly spread and split the column in two. We simplify this scenario and posit that an individual column may suffer random total catastrophic failure, following which its strength drops to zero, and all participating columns fail by the end of a crushing segment. The total crushing force f_Y^C of a group "Y" of N_Y identical columns, is

$$f_Y^C = \sum_{i \in \text{ group } Y} f_1^C(u) \cdot \theta(u_i^\dagger - u) \approx N_Y \cdot f_1^C(u)\,(1 - C(u)), \tag{5.1}$$

where $C(u)$, is the cumulative probability distribution function of distances $u^\dagger$ at which the catastrophic failure occurs, while $f_1^C(u)$ is the crushing force of a single column. We assume $\{u_i^\dagger\}_{i=1,N_y}$ to have probability distribution as, $\pi(u) = \alpha u^{\alpha-1}$, with $\alpha > 0$, so that $C(u, \alpha) = u^\alpha$. This modifies the g_{BV} given in Eq. (2.8) to

$$g_{pBV}(u, u^*, \alpha) = (1 - u^\alpha) \cdot g_{BV}(u, u^*), \tag{5.2}$$

which we refer to as the *probabilistic-BV* (pBV) model of F^C. Its mean value, $\epsilon_{pBV}^C(u^*, \alpha) = \int_0^1 du\ g_{pBV}(u, u^*, \alpha)$, is most easily found numerically, but a closed expression exists.
We now apply the probabilistic model from Eq. (5.2) to the first segment of crushing and solve $\epsilon_{pBV}^C(u^*, \alpha) = \epsilon^C = 0.23$ with $u^* = 0.01$ to find $\alpha \simeq 30$. This tells us that F^C is almost everywhere BV-like except toward the end of the crushing segment. In Fig. 5 we plot the best-fit linear interpolant for F^C, BV model for g_{BV} with $u^* = 0.01$, and g_{pBV} with $u^* = 0.01$ and $\alpha = 30$ we propose, and find an excellent agreement.

6. CONCLUSION

We conclude that in the context of the CD hypothesis the observed roofline motion is fully consistent with the theoretical models for crushing force and the crush-down collapse, with that "caveat" that it is the perimeter columns that resist the crush-down and that the top section is twice as massive as what it appears to be. Furthermore, we see that the CD hypothesis provides consistent and flexible theoretical framework for interpretation of the existing and new evidence when such become available.

Conversely, we have shown that the NIST hypothesis does not provide a consistent framework in which the collapse can be studied, as demonstrated by the various correction factors we have examined.

[1] Z. P. Bažant and M. Verdure, J. Eng. Mech. - ASCE **133**, 308 (2007), see also separate *Discussions* by G. Szuladzinski, and J. R. Gourley and the *Closure* by Z. P. Bažant and J.-L. Le, J. Eng. Mech. - ASCE 133 (2007) 308-319.

[2] FEMA 403/2002, *World Trade Center Building Performance Study: Data Collection, Preliminary Observations, and Recommendations* (Federal Emergency Management Administration, Washington D.C., 2002).

[3] NIST National Construction Safety Team, *NIST NCSTAR 1 - Final Report on the Collapse of the World Trade Center Towers* (U. S. Government Printing Office, Washington D.C., 2005).

[4] Z. P. Bažant and Y. Zhou, J. Eng. Mech. - ASCE **128**, 2 (2002), see also *Addendum*, in J. Eng. Mech. - ASCE 128(2002)369, and *Closure*, in J. Eng. Mech. - ASCE 129(2003)839, both by Z. P. Bažant and Y. Zhou.

[5] Z. P. Bažant, J.-L. Le, F. R. Greening, and D. B. Benson, J. Eng. Mech. - ASCE **134**, 892 (2008), see also *Closure* by J.-L. Le and Z. P. Baant, J. Eng. Mech. 136, 934 (2010).

[6] K. A. Seffen, J. Eng. Mech. ASCE **134**, 125 (2008).

[7] G. MacQueen and T. Szamboti, Journal of 9-11 Studies **24**, 1 (2009), the data set is based on a frame-by-frame analysis of the video recording of the collapse of 1 WTC that was recorded by the French film-maker Etienne Sauret and posted on `http://www.youtube.com/watch?v=xGAofwkAOlo`.

[8] Y. Bard, *Nonlinear Parameter Estimation* (Academic Press, New York and London, 1974).

[9] See e.g., S.T. Thornton and J.B. Marion, "Classical Dynamics of Particles and Systems," 5th Ed., Brooks/Cole, Belmont CA, 2004, from problem 9-15. onward.

[10] C. P. Pesce, J. Appl. Mech. ASME **70**, 751 (2003).

[11] Y. Omika, E. Fukuzawa, N. Koshika, H. Morikawa, and R. Fukuda, Journal of Structural Engineering - ASCE **131**, 6 (2005).

[12] M. Galassi and J. Davies and J. Theiler and B. Gough and G. Jungman and M. Booth and F. Rossi, GNU Scientific Library, Reference Manual, Version 1.3, 2002, `http://www.gnu.org/software/gsl`.

7. FIGURES AND CAPTIONS

index j	relative time t_j (s)	displacement (pixels)	displacement (ft)	displacement $\hat{Z}_j$ (m)
0	0.000	0	0.00	0.00
1	0.167	1	0.88	0.27
2	0.333	2	1.76	0.54
3	0.500	4	3.52	1.07
4	0.667	6	5.28	1.61
5	0.833	9	7.92	2.41
6	1.000	13	11.44	3.49
7	1.167	17	14.96	4.56
8	1.333	23	20.24	6.17
9	1.500	29	25.52	7.78
10	1.667	37	32.56	9.92
11	1.833	44	38.72	11.80
12	2.000	52	45.76	13.95
13	2.167	61	53.68	16.36
14	2.333	71	62.48	19.04
15	2.500	81	71.28	21.73
16	2.667	92	80.96	24.68
17	2.833	104	91.52	27.90
18	3.000	117	102.96	31.38
19	3.167	130	114.40	34.87

TABLE I. Roof-line motion during the initial moments of collapse of 1 World Trade Center [7]. The authors measure the displacements by counting the pixels, where the size of one pixel is $u_0 = 0.27$ m (0.88 ft).

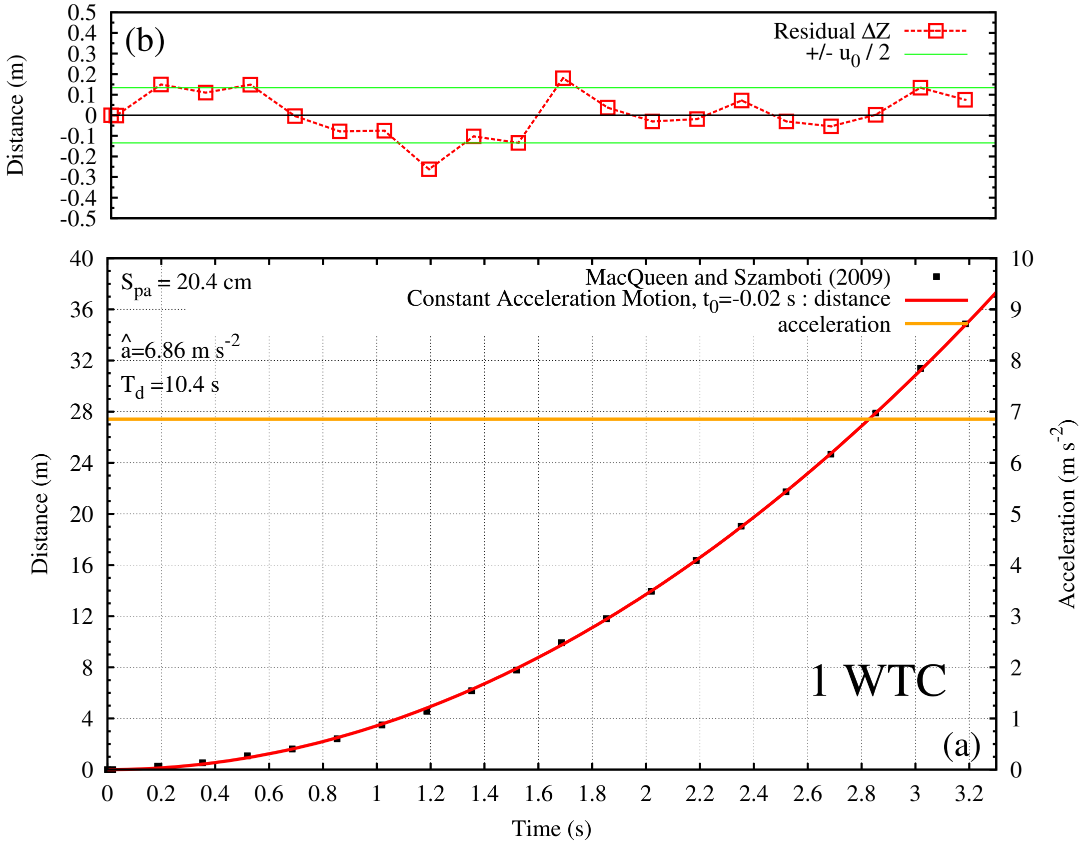

FIG. 1. The best-fit trajectory in the constant acceleration model (2.2). Panel (a) gives the roofline data from Table I (black points), and the model position (red solid line) and acceleration (orange solid line). We also list S_{pa}, the best-fit acceleration $\hat{a}$, and the time T_d it takes the crushing front to reach the ground. Panel (b) shows the residuals between the model and the data.

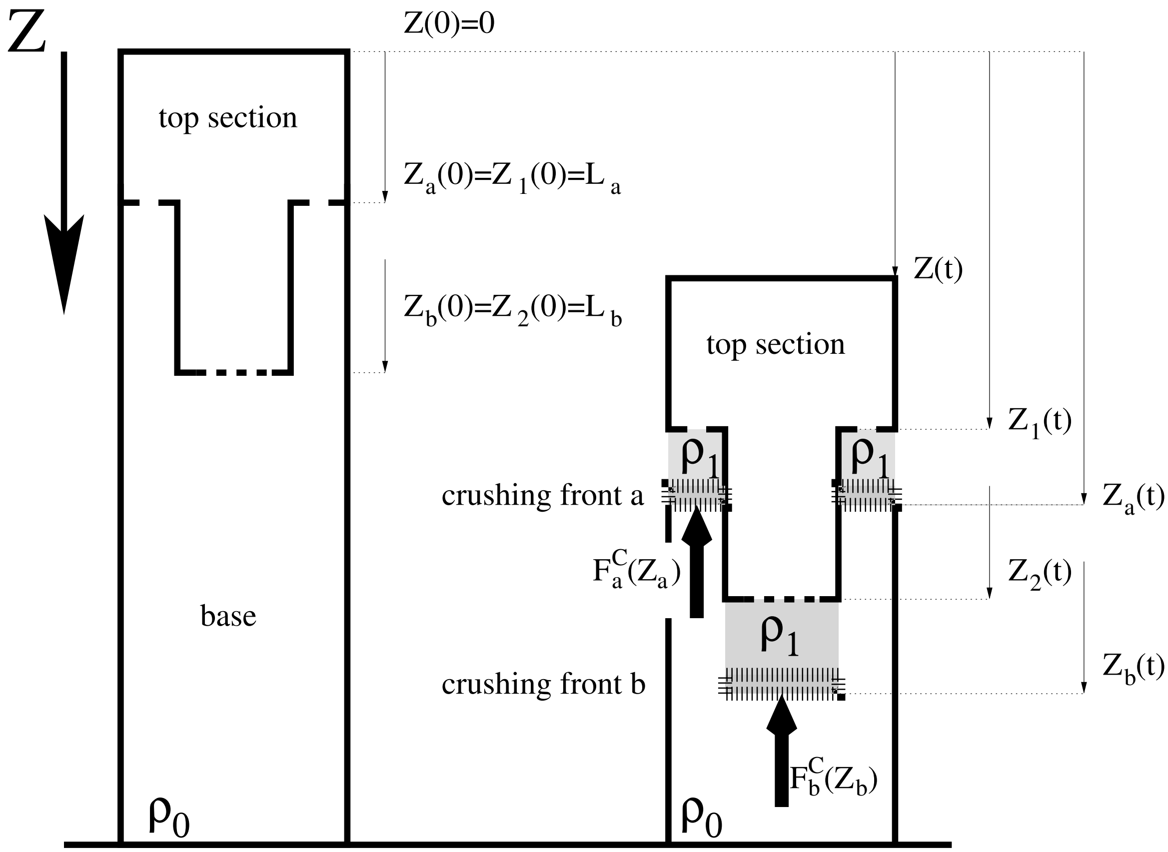

FIG. 2. Schematic of the split-front crush-down model (2.7), which main purpose is to allow us to vary the top section initial mass through separation of crushing fronts at the perimeter and the core of the building. In the NIST hypothesis the crushing fronts at the core and the perimeter are collocated, $Z_a(t) \equiv Z_b(t)$.

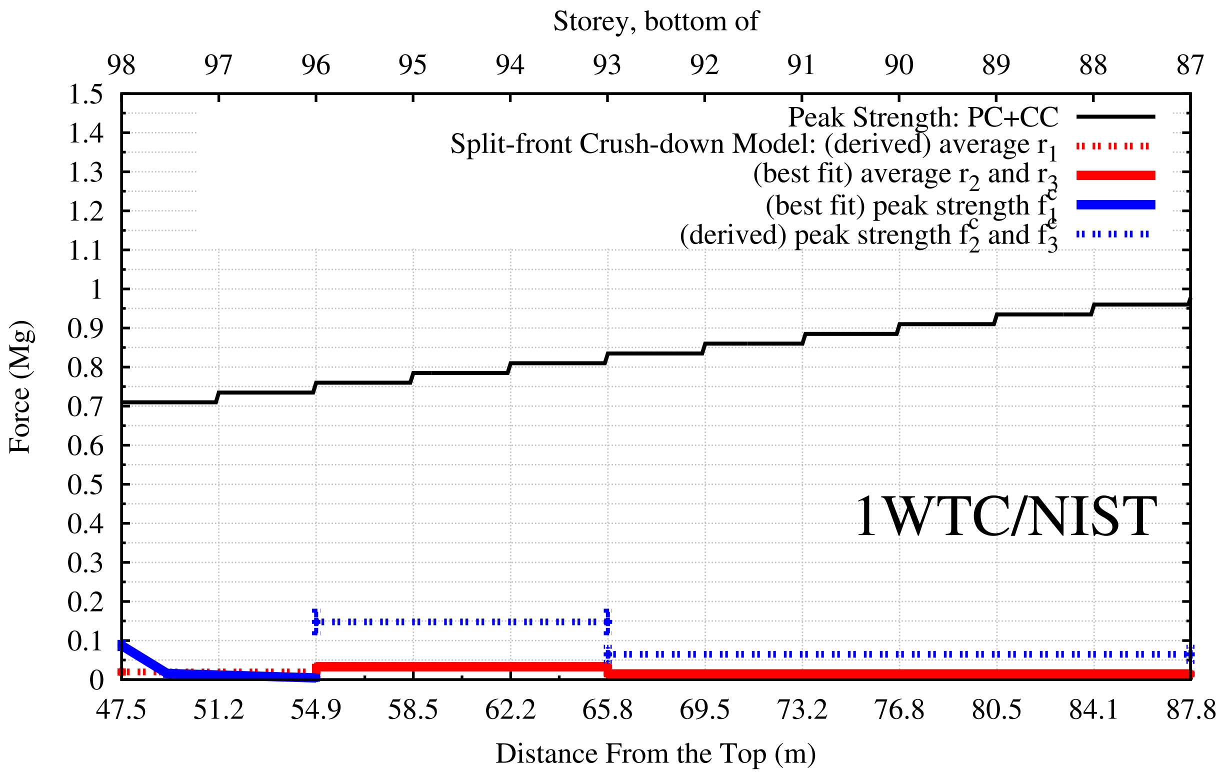

FIG. 3. The NIST hypothesis: Best-fit values (solid red and blue lines) for F^C (in red) and their averages (in blue) inside the base of 1 WTC. We use best-fit derived ϵ^C to estimate one (shown as dotted line of respective color) from the other. We also show the estimated peak strength of the core and the perimeter columns (CC and PC, in black). We see no agreement between the estimated peak strengths of the PC and CC, and their best-fit values.

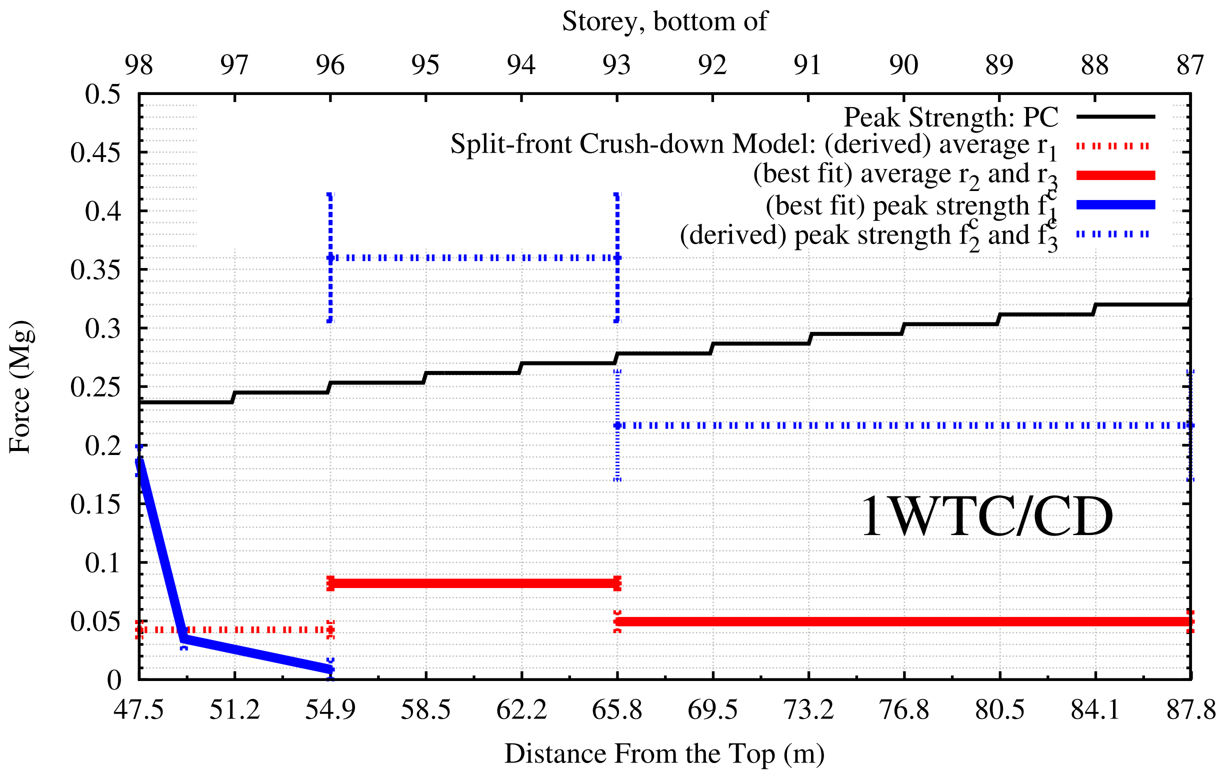

FIG. 4. Same as Fig. 3 but for the Controlled Demolition Hypothesis and the perimeter columns (PC). The agreement between the estimated peak strength of the PCs and their best-fit values is excellent.

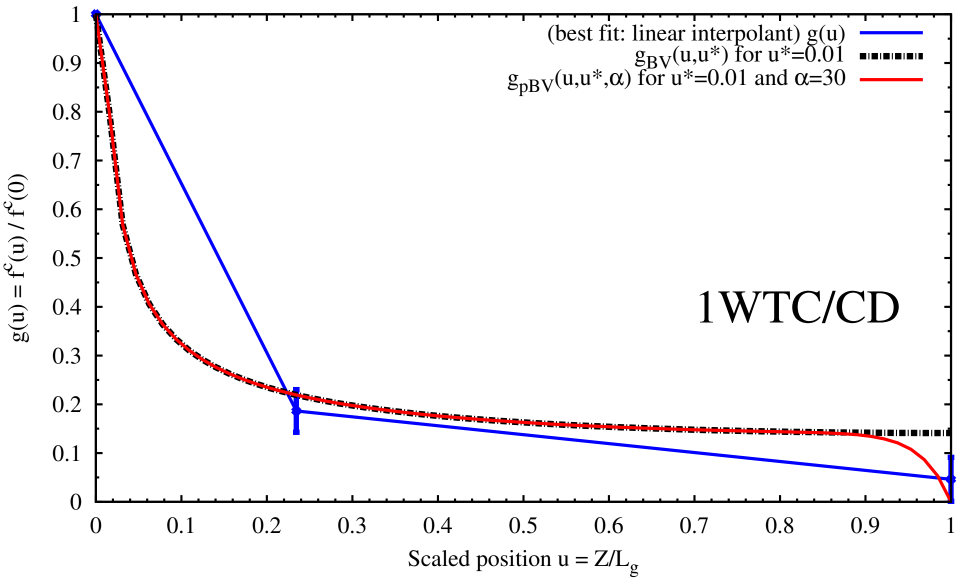

FIG. 5. The best-fit estimate for F^C (its unit-normalized linear interpolant g is given in blue, together with its error bars) and g_{BV} from Eq. (2.8) in the framework of the Controlled Demolition hypothesis. Modified g_{pBV} from Eq. (5.2) that includes catastrophic failures reproduces the best-fit derived ϵ^C quite well, see discussion in text.