Gravitational Field of Second Type, Motions of Precession and the Fourth Law of Orbital Motions

Daniele Sasso*

Abstract

Let us consider here the question relative to gravitational field of second type, to elliptical orbits and to precessional motions that happen in this type of gravitational filed. Here the cause of precessional phenomena is identified with conic motions of axes of rotation of Sun and of planets, that are not perpendicular to the plane of the ecliptic but they have an angle of inclination. These inclinations are due to reciprocal interactions of central star and of orbital planets. The paper reaches the necessity to define a new law that in the event of the Sun system is the fourth law that has to be added to Kepler's three well-known laws.

1. Introduction

From observer's viewpoint phenomenon of precession consists in a space movement of particular points (perihelion and aphelion) and of particular lines (line of the apsides and line of the equinoxes) with respect to expected positions relative to elliptical orbits supposed static. The question of planetary precession in the Sun system requires certainly a work of elaboration. In fact in scientific literature a few experimental data are present and they are incomplete with regard to different panets of the Sun system.
Generally precession of a planet is explained through the combination of both: perturbation action of other planets (in classical physics) and the further relativistic correction caused by the curvature of spacetime[1] (in modern physics) but those explanations are unsatisfactory, incomplete and above all they don't tackle the question systematically.
In the order of the Theory of Reference Frames a new interpretation of phenomenon will be given so that it doesn't have inconsistencies that are present in other interpretations.
Let us start from the consideration that the Sun has an axis of rotation that isn't exactly perpendicular to the plane of the ecliptic (orbital plane of planets), but it has an inclination of about 7°15' because of systemic interactions of planets. The consequence of that inclination is that also the axis of rotation of every planet of the Sun system isn't perpendicular to the plane of the ecliptic but it has an inclination that changes relative to every planet. In our theory therefore the inclination of the Sun axis and of planet axes is the physical cause of motions of precession.

* e_mail: dgsasso@alice.it

It is suitable to make the following considerations[2][3]:

a. if the Sun was fixed in the centre of the Sun system then planetary orbits would be perfectly circular and the Sun would be in the centre of all concentric orbits
b. the Sun isn't fixed because for the Principle of Action and Reaction it is subjected to an attraction force by every planet that generates a complex motion of the Sun and elliptical planetary orbits in which the Sun occupies one focus
c. the axis of rotation of the Sun isn't perpendicular to the plane of the ecliptic because of interaction of planets and it has a motion of conical rotation
d. axis of rotation of every planet isn't perpendicular to the plane of the ecliptic because of the inclination of the Sun axis and itself has a motion of conical rotation.

2. Circular orbits

Circular orbit is characterized by notable properties of symmetry: constant radius r, constant angular velocity ω, constant tangential velocity v, constant revolution period T. If this symmetry was present in the reality it would generate a symmetrical behavior of moving planets along a circular orbit (fig.1).

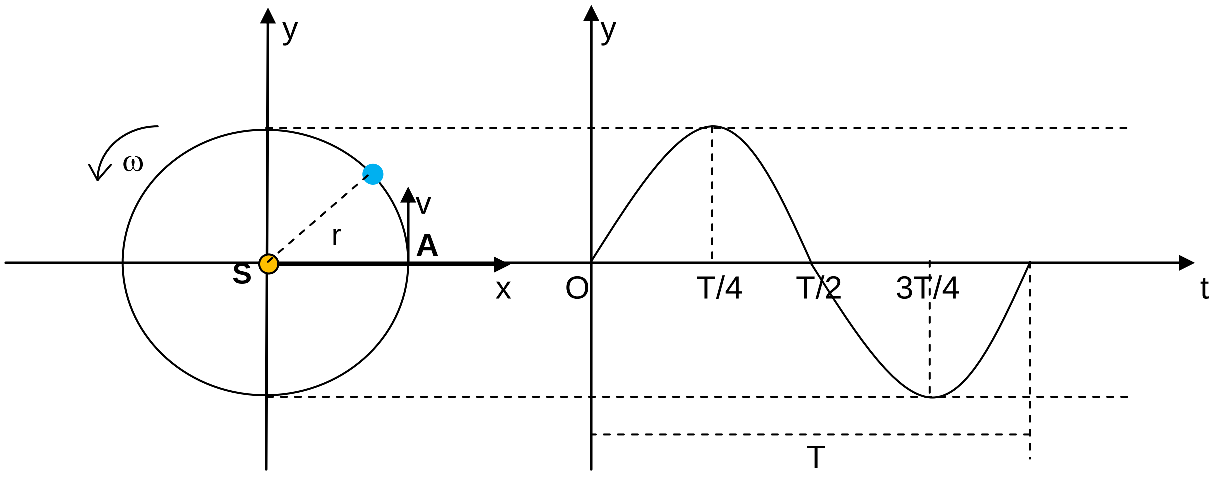

Fig.1 Graphic representation of circular orbit in the domain (x,y) and time representation in the domain (y,t). The point A is supposed to be the starting point of the planetary orbit.

In circular orbit a prospective advance of the point A and a prospective rotation of orbital trajectory would have no physical significance because of the perfect symmetry of orbit. Graphic representation in the time domain (x,t) of the circular orbit is symmetrical with respect to the representation in the domain (y,t) with a phase shift of $\pi/2$ (fig.2).
Circular orbit is characterized by the following relationship

$$\omega = \frac{2\pi}{T} \qquad (1)$$

where ω is the angular velocity of planet and T is the orbital period. Besides we have

$$v = \omega r \qquad (2)$$

in which v is the tangential velocity and r is the orbital radius.

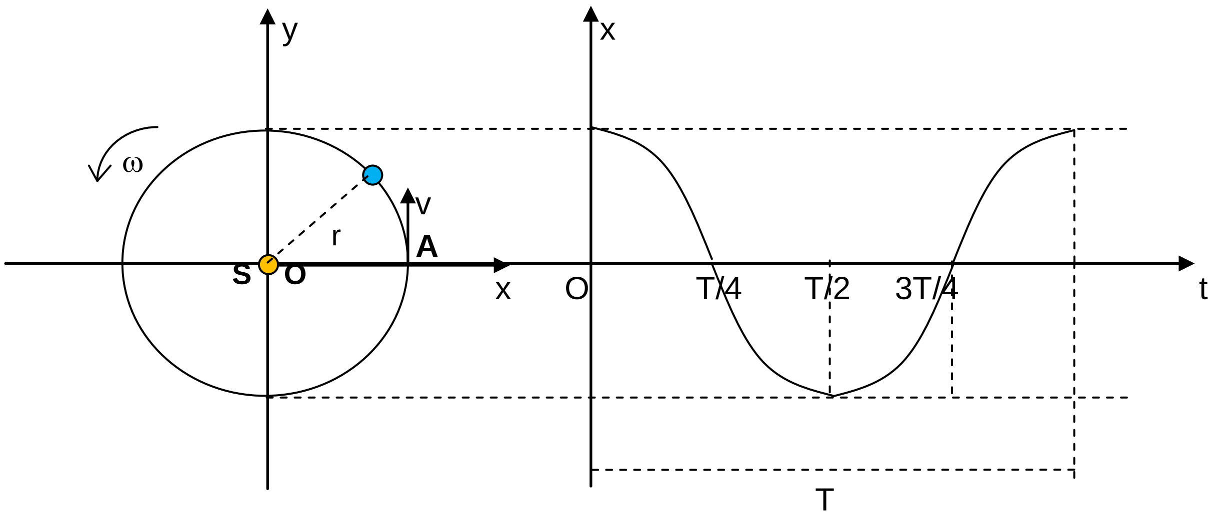

Fig.2 Graphic representation of the circular orbit in the domain (x,y) and time representation in the domain (x,t). Suppose that the planetary orbit starts from the point A.

In the two graphic representations we observe only a phase shift.
Orbit is caused by the perfect balancing, in the gravitational field of second type, of the gravitational force of attraction F_g , due to the gravitational field, and of the centrifugal force F_c , due to the circular motion, where

$$F_g = \frac{GMm}{r^2} \qquad F_c = \frac{mv^2}{r} \qquad (3)$$

in which $G=6.67 \times 10^{-11} Nm^2/kg^2$ is the gravitational constant, M is the pole mass (Sun in our case), m is the mass of the planet in revolution and v is the tangential orbital speed.
From the equality of the two forces we derive the non-quantum relationship between speed and radius in order to have a circular orbital trajectory

$$v = \sqrt{\frac{GM}{r}} \qquad (4)$$

In circular orbits, as already we noted, because of symmetry of orbit prospective phenomena of precession not would be revealing and above all they not would be detectable. We know nevertheless planetary orbits aren't circular but they are elliptical because of the Principle of Action and Reaction for which also planets exercise a reaction on the Sun pole generating like this a complex motion of the Sun and above all an inclination of its axis of rotation with respect to the plane of the ecliptic that in its turn generates an inclination of axes of rotation of single planets.

3. Elliptical orbits

For stationary elliptical orbits of planets of the Sun system Keplero's three laws are valid:

1. Planets describe elliptical orbits around the Sun that is in one of the two foci.
2. Areas covered by the radius vector (linear segment that joins the centre of planet with the Sun centre) are proportional to times that are necessary for covering them. That is equal areas are covered in equal times.
3. Squares of revolution periods T are proportional to cubes of greatest semi-axes R of elliptical orbits: $T^2=kR^3$.

It needs to specify all main properties of symmetry that are valid in circular orbits lose meaning in elliptical orbits because radius, angular velocity and tangential velocity aren't constant. In fig.3 suppose that orbit starts still from the point A.

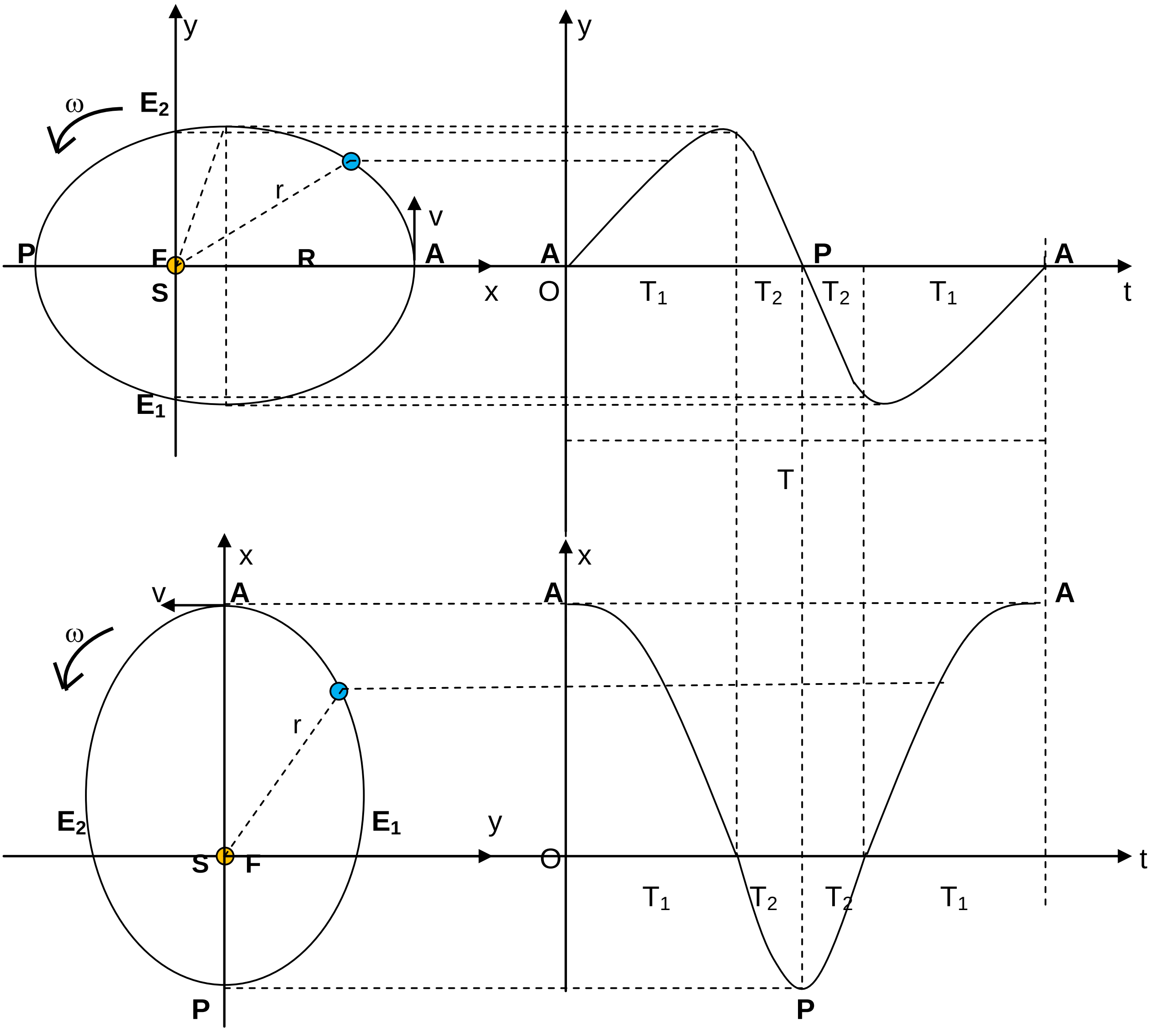

Fig.3 Graphic representation of elliptical orbit in the domain (x,y) and in the domains (y,t) and (x,t). Suppose that planetary orbit starts from the point A.
E_1E_2 is the line of the equinoxes and AP is the line of the apsides.

In figure the ellipse ovalness is greater than the real ovalness. Both lines, of the apsides and of equinoxes, go through the focal point in which the Sun is and therefore perihelion P, aphelion A and Sun are aligned like the two points of equinoxes E_1, E_2 and the Sun are aligned.
In stationary uniform circular orbit, characterized by complete symmetry, there is a perfect concurrence between the completion of an entire cycle of 2π (360°) and the revolution period T through the angular velocity ω that is constant along the entire circular path, on a par with radius and tangential velocity, i.e. $\omega=2\pi/T$.
in elliptical orbit that behavior is not warranted but anyway the symmetry of alignment of the Sun with perihelion and aphelion and with the two equinoxes is satisfied.
The presence of planets generates an average inclination $\alpha=7°15'=7.25°$ of the axis of rotation of the Sun with respect to the perpendicular axis of the plane of the ecliptic. In the absence of reaction of planets the axis of rotation would be perfectly perpendicular to that plane. This inclination of the Sun axis generates in its turn an inclination of the axis of rotation of every planet with respect to the plane of the ecliptic and its conical rotation. Consequently because of reactions of planets on the Sun the axis of rotation of the Sun executes a conical motion in anticlockwise direction like the revolution direction of planets around the Sun. At the same time also axes of rotation of planets execute a conical motion. It involves a rotation of the elliptical orbit in the plane of the ecliptic, a precession of the line of the equinoxes and of the line of the apsides, an advance Δ_p of the perihielion of planets and of all orbital points so that perihelion, aphelion and focus are always aligned.
In modern physics advance of elliptical orbits in the Sun system, with precession of the lines of the apsides and of equinoxes is due in large part to the perturbation action of other planets and in small part to the curvature of spacetime theorized in General Relativity. Now we know it is due entirely to the conical motion of rotation of axes of planets inclined on the plane of the ecliptic. The inclination of the axis of rotation of every planet with respect to the plane of the ecliptic is generated by the inclination α of the Sun axis with respect to the same plane (fig.4).
In the event of the Earth we know the total precession of the line of the equinoxes is $\Delta_T=50.26''$ for Earth sidereal year.

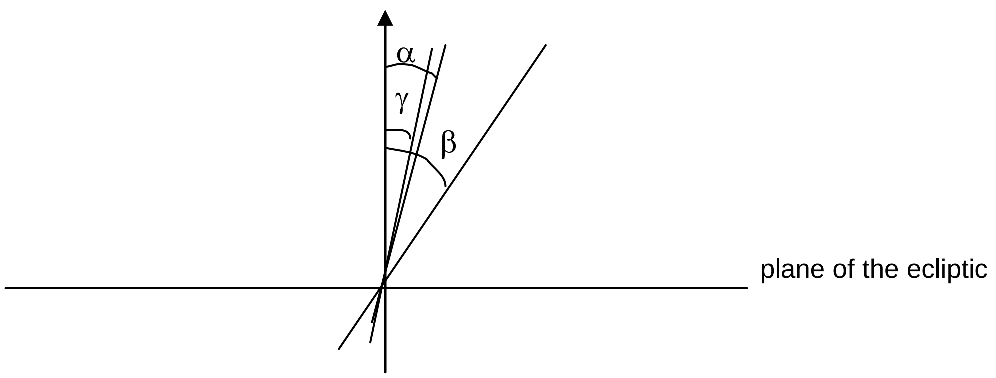

Fig.4 Inclination of the axes of conical rotation of the Sun (α), of the Earth (β) and of Mercury (γ) on the plane of the ecliptic.

4. Mercury' s precession

Mercury is the closest planet to the Sun. Mercury's precession of perihelion[4][5] has been measured with good precision by U. Le Verrier, but also by other astronomers recently, and it is equal to Δ_{ME}=5600" arcseconds for Earth's century, i.e. Δ_{ME}=56" arcseconds for Earth's year and those measurements are valid with respect to the Earth reference frame. As we wrote, that precession is caused by the conical rotation of the Mercury axis, that has an average inclination γ=7° with respect to the perpendicular of the plane of the ecliptic and it consists in a time and space advance of the perihelion.
Because Earth's year is T_T =365.256g (g=1day) and Mercury's year is T_{ME}=87.97g=T_T/4.15 it follows that Mercury's advance of perihelion and the rotation of the line of the apsides Δ'_{ME}, for Mercury's year T_{ME}, calculated with respect to Mercury's reference frame, is

$$\Delta'_{ME} = 56\,\frac{T_{ME}}{T_T} = \frac{56}{4.15} = 13.49\text{'' (arcseconds)} \qquad (5)$$

Δ'_{ME} represents the precession of Mercury's perihelion for Mercury's year, calculated with respect to Mercury's reference frame.

5. Earth's precession

For the Earth the average inclination of the axis of rotation with respect to the plane of the ecliptic is β=23°27'=23.44°. The action of the Sun and of other planets produces a precession for Earth's year that can be calculated considering the inclination of Earth's axis with respect to the plane of the ecliptic:

$$\Delta_T = \frac{23.44 \times 13.49}{7} = 45.17\text{''} \qquad (6)$$

Because the measured total precession of the equinoxes for the Earth, in clockwise direction, is equal to 50.26", it follows that Moon's action Δ_{TL} on Earth's precession, for Earth's year, is

$$\Delta_{TL} = 50.26\text{''} - \Delta_T = 5.09\text{''} \qquad (7)$$

Relative to Earth's precession of the equinoxes there are therefore two components

a. the component due to the Sun and to other planets that is equal to Δ_T=45.17"
b. the component due to the Moon that is equal to Δ_{TL} = 5.09"

Earth's total precession for Earth's year is $\Delta_{Tt}=\Delta_T+\Delta_{TL}$=50.26".
In fig.5 the precession of the equinozes and the precession of perihelion-aphelion for the Earth are represented.

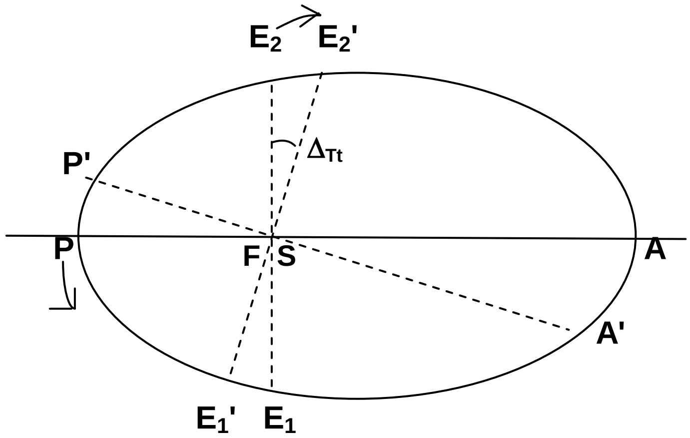

Fig.5 Graphic representation of Earth's precession. **AP** is the line of the apsides at the initial instant of observation and **A'P'** is the same line after an earth sidereal year. **E_1E_2** is the line of the equinoxes at the initial instant and **$E_1'E_2'$** is the same line after an Earth sidereal year. In figure the scale of distances is not respected.

6. Precession of the other planets

Considering the same reasoning and the same causal law also for the other planets of Sun's system we can calculate their precession, with respect to whether the reference frame of the planet or the reference frame of Earth's observer.

6.1 Mars

The planet Mars has an axis of conical rotation that has an inclination of 25.32° with respect to the perpendicular of the orbital plane or plane of the ecliptic. We deduce therefore that Mars's precession for Mars' year, due to the Sun and to other planets, is given by

$$\Delta_{MA} = \frac{45.17'' \times 25.32°}{23.44°} = \frac{13.49'' \times 25.32°}{7°} = 48.79'' \qquad (8)$$

Because Mars' orbital year is $T_{MA}=1.88T_T$, Mars's precession for Earth's year is then

$$\Delta'_{MA} = 48.79''\frac{T_T}{T_{MA}} = 25.95'' \text{ (arcseconds)} \qquad (9)$$

6.2 Venus

Venus's axial inclination with respect to the perpendicular of the plane of the ecliptic is 177.36°. Because Venus's motion of rotation is clockwise, it is into reverse whether to the rotation motion of other planets or to its revolution motion around the Sun. Consequently

Venus's rotation clockwise motion around its axis, inclined of 177.36° with respect to the perpendicular, is equivalent to the anticlockwise motion with respect to the axis inclined of 2.64°. It follows that Venus's precession, for Venus' year, is given by

$$\Delta_V = \frac{45.17'' \times 2.64°}{23.44°} = 5.09'' \quad (10)$$

Because Venus's orbital year is T_V=0.62T_T, Venus' precession for Earth's year is

$$\Delta'_V = 5.09 \frac{T_V}{T_T} = 8.21'' \text{ (arcseconds)} \quad (11)$$

6.3 Jupiter

Jupiter's axis of conical rotation has an inclination of 3.13° with respect to the perpendicular of the plane of the ecliptic. We deduce therefore Jupiter's precession, for Jupiter's year, due to the Sun and to other planets, is given by

$$\Delta_G = \frac{45.17'' \times 3.13°}{23.44°} = 6.03'' \quad (12)$$

Because Jupiter's orbital year is T_G=11.86T_T , Jupiter's precession for Earth's year is

$$\Delta'_G = 6.03 \frac{T_G}{T_T} = 0.51'' \text{ (arcseconds)} \quad (13)$$

6.4 Saturn

Saturn has an axis of conical rotation with an inclination of 26.73° with respect to the perpendicular of the plane of the ecliptic. Consequently Saturn's precession, for Saturn's year, due to the Sun and to other planets, is given by

$$\Delta_{SA} = \frac{45.17'' \times 26.73°}{23.44°} = 51.51'' \quad (14)$$

Because Saturn's orbital year is T_{SA}=29.45T_T , Saturn's precession for Saturn's year is

$$\Delta'_{SA} = 51.51 \frac{T_T}{T_{SA}} = 1.75'' \text{ (arcseconds)} \quad (15)$$

6.5 Uranus

Uranus's axial inclination with respect to the perpendicular of the plane of the ecliptic is 97.77° and besides its motion of rotation is clockwise like Venus. Consequently also for Uranus the clockwise real motion of rotation around its axis, inclined of 97.77° with respect to the perpendicular, is equivalent to the anticlockwise motion with respect to the axis inclined of 82.23°. It follows that Uranus's precession, for Uranus' year, is given by

$$\Delta_U = \frac{45.17'' \times 82.23°}{23.44°} = 158.46'' \qquad (16)$$

Because Uranus's orbital year is T_U=84.07T_T , Uranus's precession for Uranus's year is given by

$$\Delta'_U = 158.46 \frac{T_T}{T_U} = 1.89'' \text{ (arcseconds)} \qquad (17)$$

6.6 Neptune

Neptune has an axis of conical rotation with an inclination of 28.32° with respect to the perpendicular of the orbital plane (plane of the ecliptic). Neptune's precession, for Neptune's year, due to the Sun and to other planets, is given by

$$\Delta_N = \frac{45.17'' \times 28.32°}{23.44°} = 54.57'' \qquad (18)$$

Because Neptune's orbital year is T_N=164.88T_T, Neptune's precession for Earth's year is

$$\Delta'_N = 54.57 \frac{T_T}{T_N} = 0.33'' \text{ (arcseconds)} \qquad (19)$$

Precession values of single planets calculated whether with respect to Earth's year or with respect to the year of the single planet prove every planet experiences in the Sun system a precession determined by both, general physical properties of the Sun system and particular physical property of the single planet: mass, distance from the Sun, orbital velocity, sidereal period. Those general and particular properties determine the inclination of the axis of rotation with respect to the plane of the ecliptic that is the cause of the precession phenomenon.
There aren't experimental confirmations of calculated values unless for the Earth and Mercury, for other planets calculated theoretical values are waiting for experimental verification.

Planet	Planet-Sun precession for Earth's year [arcseconds]		Precession for planet year [arcseconds]
Earth	45.17	Moon quota 5.09	50.26
	50.26		
Mercury	56		13.49
Mars	25.95		48.79
Venus	8.21		5.09
Jupiter	0.51		6.03
Saturn	1.75		51.51
Uranus	1.89		158.46
Neptune	0.33		54.57

Fig. 6 Precessions in arcoseconds of planets for Earth's year and for planet year in the Sun system.

7. Fourth law of orbital motions

Considerations on precession motions of planets of the Sun system and on movements of lines of the apsides, of the lines of the equinoxes and of perihelions induces to think those phenomena of precession have a systemic nature that goes beyond generic perturbations generated by direct interactions of other planets. In fact those direct interactions are casual because reciprocal positions of planets change continually and consequently also intensity of interactions changes continually and it would produce variable precessions for year. Similarly also the spacetime curvature is insufficient to explain completely those phenomena of precession, in fact General Relativity would explain only the anomaly of Mercury's precession and not the complete precession of Mercury and of other planets.

This systemic nature induces us to think the complete description of the Sun system requires a new law in addition to Kepler's three known laws. Our considerations on precession motions of planets of the Sun system involve Kepler's third law is valid only for stationary elliptical orbits, i.e. for orbits in the absence of precession. It follows that it is necessary to add a fourth law that considers precession movements of elliptical orbits.
If Δx and Δy are planetary-solar precessions of two planets, calculated with respect to the planet's year, and if α and β are inclinations of their axes of rotation with respect to the perpendicular of the plane of the ecliptic, the fourth law claims

" The ratio of precessions of the two planets is equal to the ratio of relative inclinations", i.e.

$$\frac{\Delta x}{\Delta y} = \frac{\alpha}{\beta} \qquad (20)$$

Because the ratio $\Delta x/\alpha = \Delta y/\beta = K_p$ is constant and equal to

$$K_p = \frac{\Delta x}{\alpha} = 1.93 \text{ arcseconds/degree} \qquad (21)$$

the fourth law can be formulated also like this:

> **Planetary-solar precession of a planet with respect to the planet year is proportional to the inclination of its axis of rotation with respect to the perpendicular of the plane of the ecliptic via the constant of proportionality K_p=1.93arcseconds/degree=5.36x10^{-4} .**

With the theory that here we have formulated, we think to have given the theoretical solution, complete and systemic, to the question regarding precession motions of planets in the Sun system.
A last consideration concerns the concept of physical law. In fact in the paper "Physico-Mathematical Fundamentals of the Theory of Reference Frames"[6] a difference between physical law and physical principle was established. As per those definitions Kepler's laws would be more exactly physical principles but we have preferred here to maintain the initial standard convention.

References

[1] A. Einstein, Theory of General Relativity (Annals of Physics, 1916)
[2] D. Sasso, Dynamics and Electrodynamics of Moving Real Systems in the Theory of Reference Frames, arXiv.org, 2010, id: 1001.2382
[3] D. Sasso, Physics of Gravitational Fields, viXra.org, 2014, id: 1405.0028
[4] M. Pantaleo, G. Polvani, P. Straneo, B. Finzi, F. Saveri, G. Armellini, P. Caldirola, and A. Aliotta, Fifty Years of Relativity, Giuntine and Sansoni Editors, Florence, 1955
[5] R. Rydin, The Theory of Mercury's Anomalous Precession and Correct Orbit Computations, 2009
[6] D. Sasso, Physico-Mathematical Fundamentals of the Theory of Reference Frames", viXra.org, 2013, id: 1309.0009