Atomic, Nuclear and Gravitational Binding Energy in a Unified Model

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Atomic electron binding energies, nuclear binding energies and the gravitational binding energies of planetary bodies are all connected to Planck scale within a coherent model based on the geometry of a standard string theory background.

Introduction

Atomic electron and nuclear binding energies are shown to coincide with energy levels and sublevels[1] that descend from Planck scale in geometric sequence. (Four-dimensional) gravitational binding energies correspond, through the '10D/4D Correspondence' [1], to trans-Planckian scales ($E > E_{Planck}$) of ten-dimensional origin that coincide with energy levels and sublevels that ascend from Planck scale in geometric sequence, mirroring the cis-Planckian ($E < E_{Planck}$) levels. Each of the three sequences of levels crosses the Planck divide.

Previously, we have shown that all subatomic particles, including atomic nuclei, occupy the mass levels and sublevels of three cis-Planckian sequences that descend from Planck scale with common ratio 1/π, 2/π and 1/e, respectively [2]. We have argued that the sequences derive from the geometry of the ten-dimensional spacetime $AdS_6 \times S^4$ in which two extra dimensions are compactified on intervals of length π and π/2 in Planck units [3]. In the four-dimensional world, particles acquire mass by coupling with the Higgs field; the coupling constants derive from the extra-dimensional geometry [4]. Recently, a correspondence was discovered between particle mass scales and the (four-dimensional) length, and time, scales at which specific, correlated, processes have occurred during the expansion of the universe [1]:

$$2{m_{1,10}}^{-5} = {l_{1,4}}^2 \tag{1}$$

where $m_1 < m_{Planck}$ and $l_1 > l_{Planck}$. The subscripts 10 and 4 refer to the dimensions of the respective spacetimes of the model. Here and throughout the paper c = G = ħ = 1; i.e. we will be working in Planck units. Conjecturing that a related correspondence to (1) applies on trans-Planckian scales, we found that the (four-dimensional) masses of specific gravitationally bound celestial objects, including black holes and planetary bodies, correspond to specific trans-Planckian length scales of ten-dimensional origin [5, 6]:

$$2{l_{2,10}}^{-5} = {m_{2,4}}^2 \tag{2}$$

where $l_2 < l_{Planck}$ and $m_2 > m_{Planck}$. That is, the trans-Planckian length scales calculated from (2) for specific celestial objects occupy levels of length scale that descend from Planck scale with common ratio 1/π, 2/π and 1/e, respectively. The associated ten-dimensional trans-Planckian mass/energy scales coincide with the levels and sublevels of three sequences, of common ratio π, π/2 and e respectively, that ascend from Planck scale and oppose the cis-Planckian mass/energy sequences. Essentially, there are three sequences in all, each extending from cis-Planckian scales to trans-Planckian scales. Levels in the three sequences are numbered n_1, n_2 and n_3 sequentially from Planck scale ($n = 0$) on cis-Planckian scales and ν_1, ν_2 and ν_3 sequentially from Planck scale ($\nu = 0$) on trans-Planckian scales.

[1] Half-levels, quarter-levels, eighth-levels etc

We will show that atomic electron binding energies coincide with the mass/energy levels and sublevels of Sequences 1 and 3 on cis-Planckian scales. Nuclear binding energies will also be shown to coincide with the levels and sublevels of Sequences 1 and 3. The ten-dimensional trans-Planckian scales corresponding to the gravitational binding energies of the planets of the Solar System and the Moon will then be shown to coincide with the levels and sublevels of Sequences 1 and 3.

Electron binding energy

K-shell binding energies of atoms with Z = 1 to 9 [7] are shown in Figure 1. There is a clear tendency for binding energy in these elements of low Z to take the value of a principal (integer level-number) energy level or low order (e.g. half-integer or quarter-integer level-number) sublevel. Where the binding energy does not coincide precisely with the energy of a low order level, e.g. in the case of hydrogen, it coincides with a higher order level, as shown in Figure 2. Similar behaviour is shown for the ionisation potentials of atoms with Z = 1 to 10 [8] in Figure 3.

Five scales pertaining to the hydrogen atom and molecule are shown to lie close to the levels of Sequences 1, 2 and 3 in Figure 4. These scales are the proton and electron masses, the K-shell binding energy (13.6 eV) or hydrogen ionisation energy, the Bohr radius (5.29 x 10^{-11} m), which is a measure of the size of the atom, and the H_2 dissociation energy, 4.478 eV [9].

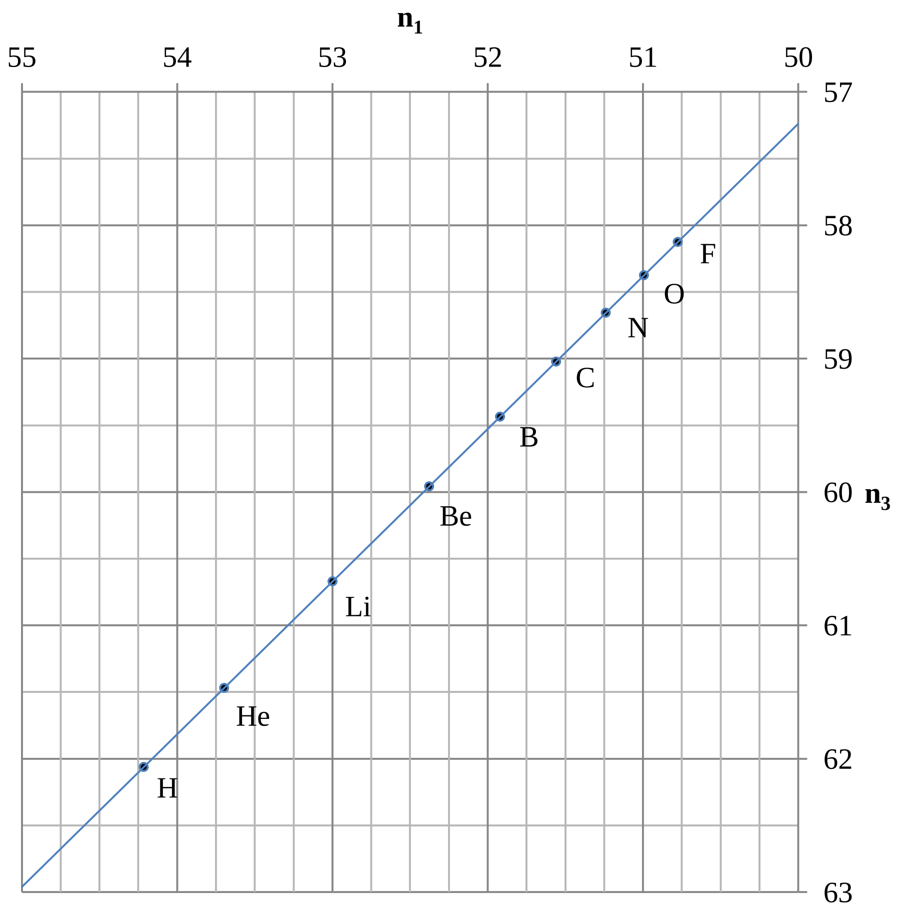


Figure 1: Electron K (1s) binding energies for the elements in their natural forms [6], in Sequences 1 and 3. The values of binding energy lie on a straight line since the level-numbers in Sequences 1 and 3 are in constant ratio.

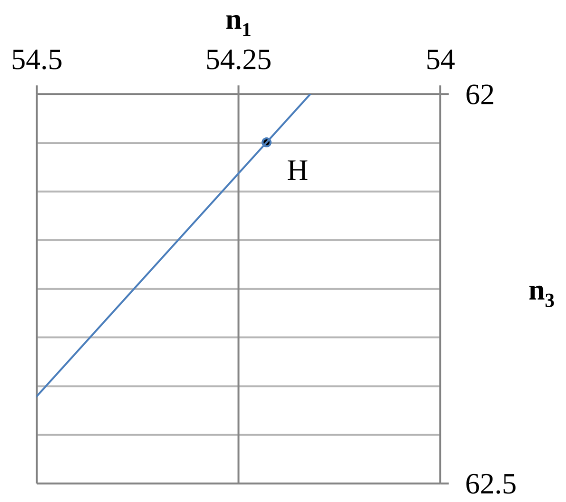


Figure 2: Detail of the hydrogen K (1s) binding energy, in Sequences 1 and 3.

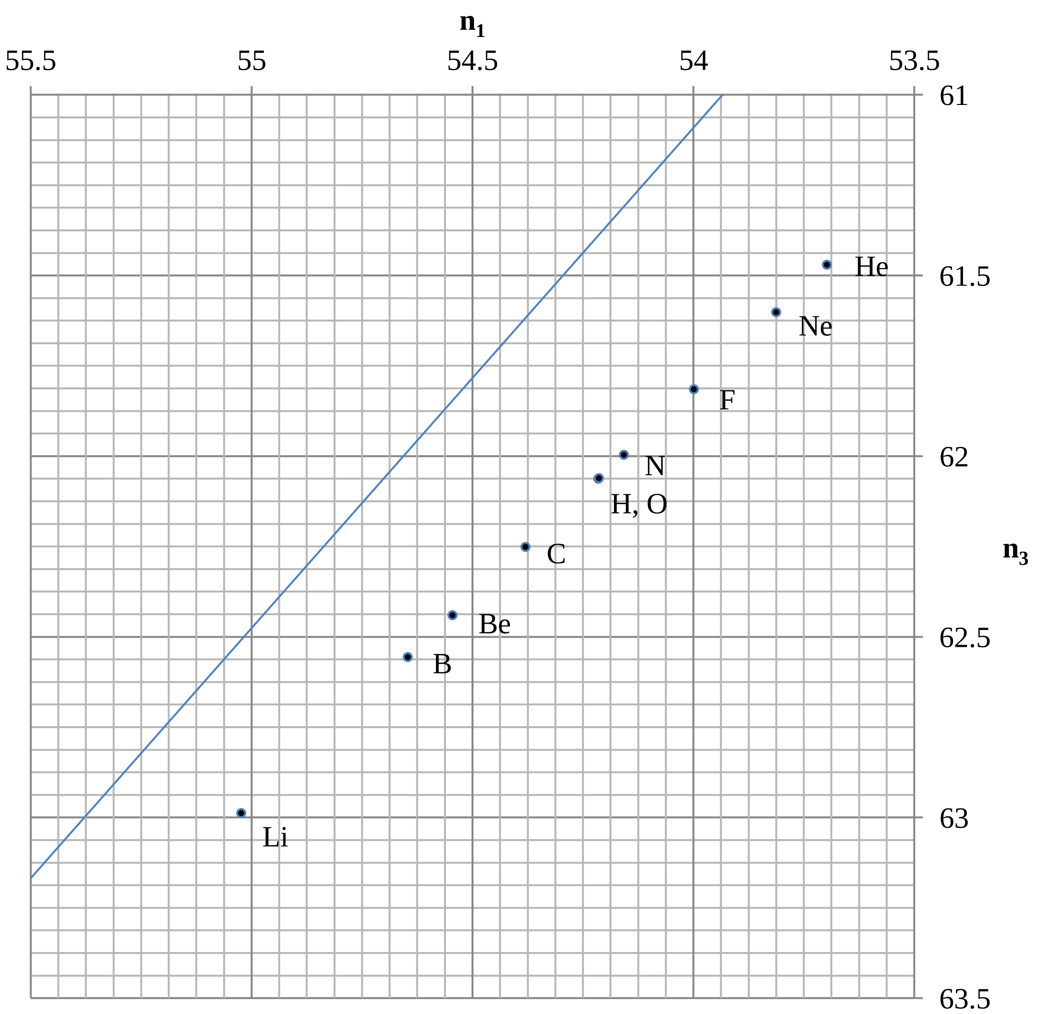
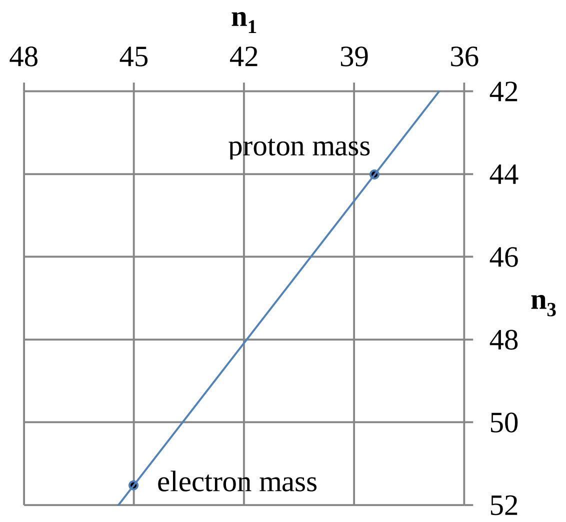


Figure 3: Ionisation potentials of the neutral atoms with Z = 1 to 10 [7], in Sequences 1 and 3.



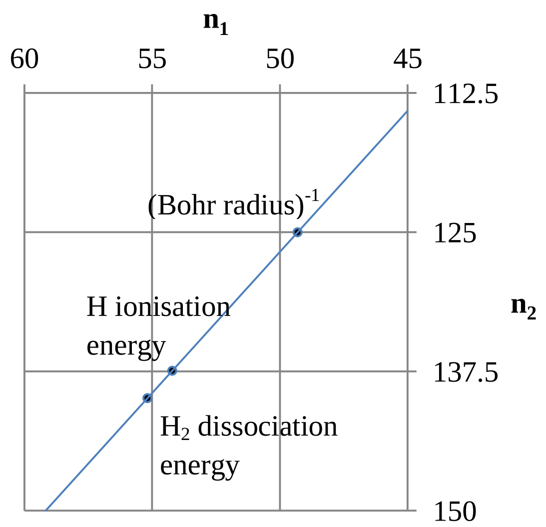


Figure 4: Hydrogen scales on the mass/energy levels of Sequences 1, 2 and 3.

Nuclear binding energy

The binding energies [10] of stable nuclei show a clear tendency to coincide with principal levels or low order sublevels in Sequences 1 and 3. The binding energies of the 'alpha-particle nuclei' ^{4}He, ^{12}C, ^{16}O and ^{20}Ne coincide with principal levels and half-levels, as shown in Figure 5. The binding energy of the unstable alpha-particle nucleus ^{8}Be does not coincide with a low order level. Also as shown in Figure 5, the binding energies of the stable alpha-particle nuclei ^{32}S and ^{40}Ca, the nucleus (^{56}Fe) with the lowest mass per nucleon and the double magic number nucleus ^{208}Pb coincide with principal levels and half-levels. The binding energies of ^{4}He, ^{12}C, ^{16}O and ^{20}Ne are related to those of ^{208}Pb, ^{56}Fe, ^{40}Ca and ^{32}S, respectively, through two-fold rotational symmetry about (39.75, 45.5). The double magic number nuclei seem to be paired through the symmetry, i.e. ^{4}He with ^{208}Pb and ^{16}O with ^{40}Ca, indicating the importance of the local geometry on stability.

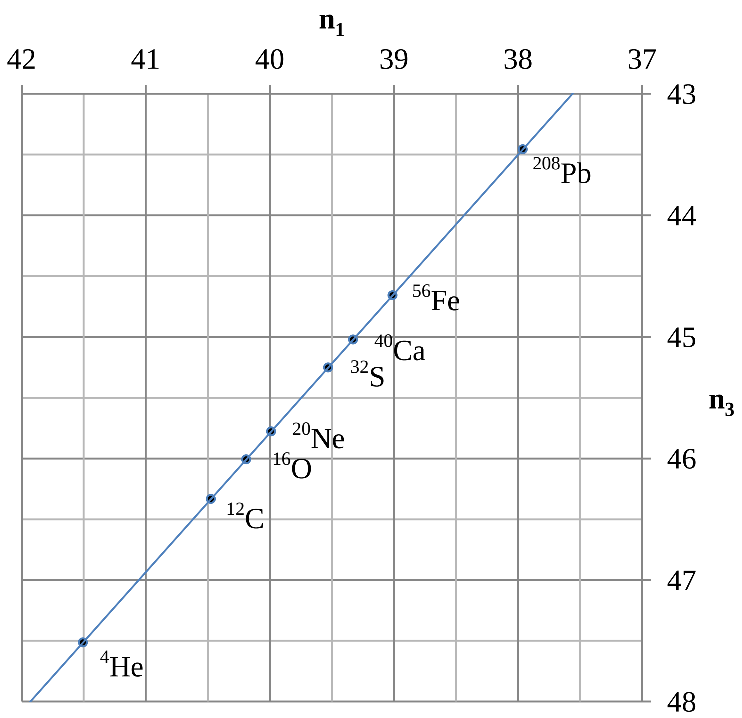


Figure 5: Binding energies of selected stable nuclei in Sequences 1 and 3.

The masses of the double magic number nuclei ^{4}He, ^{16}O, ^{40}Ca and ^{208}Pb are shown together with the binding energies of the nuclei on the levels and sublevels of Sequences 1 and 3 in Figure 6. Similar patterns of particle mass and binding energy occur on the mass/energy levels. The mass and binding energy of the ^{56}Fe nucleus both lie on principal levels within Sequences 1 and 3, as shown in Figure 7.

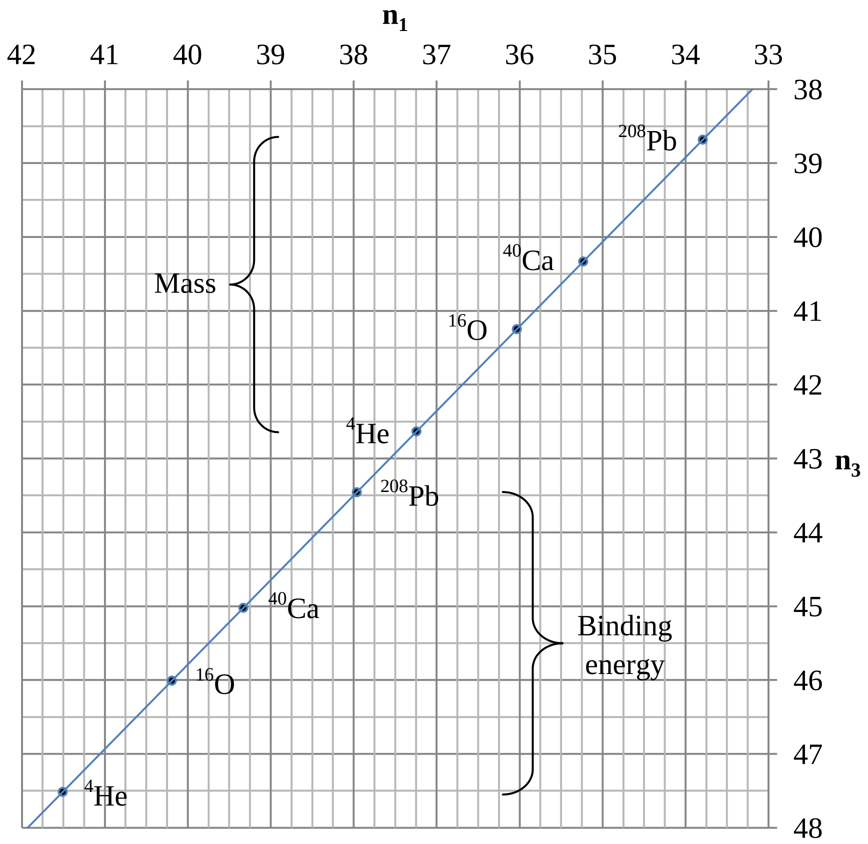


Figure 6: Masses and binding energies of four double magic number nuclei.

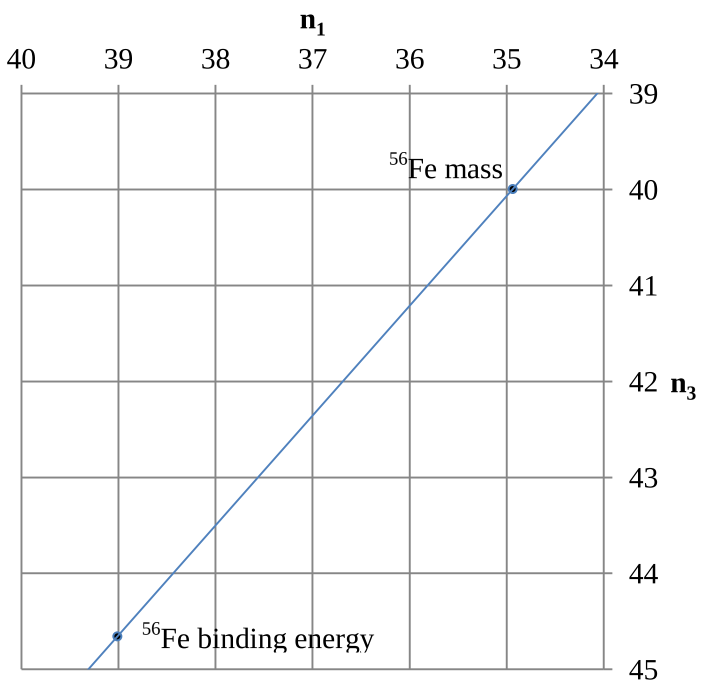


Figure 7: The mass and binding energy of the ^{56}Fe nucleus within Sequences 1 and 3.

Gravitational binding energy

The gravitational binding energy of a spherical body is $\sim m_b{}^2/r_b$, where m_b is the mass of the body and r_b is its radius. Conjecturing that the gravitational binding energy of a planetary body is precisely equal to $m_b{}^2/r_b$, we first calculated the (four-dimensional) binding energies of the planets of the Solar System, which are objects whose masses and radii are known with some precision [11], making them suitable for study. Next, by use of (2), we calculated the corresponding values of ten-dimensional trans-Planckian energy and plotted them on the levels of Sequences 1 and 3. The results of that procedure for the gas giants of the Solar System are shown in Figure 8. The resulting trans-Planckian energies coincide closely with principal levels in Sequences 1 and 3, which behaviour has also been shown to occur for the atomic electron and nuclear binding energies of the most stable atoms on cis-Planckian scales.

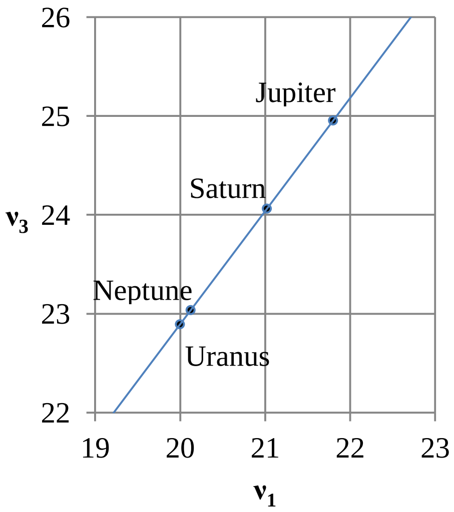


Figure 8: Gravitational binding energies of the gas giant planets of the Solar System on the trans-Planckian levels of Sequences 1 and 3.

The gravitational binding energies of the gas giants are also shown on the levels of Sequences 1 and 2 in Figure 9. The binding energies tend to take up values equal to those of coincident levels and sublevels within the three sequences.

The gravitational binding energies of the rocky planets of the Solar System, and the binding energy of the Moon, are shown on the levels of Sequences 1 and 3 in Figure 10. The binding energy of the Moon equals the energy of a principal level while the binding energy of Venus equals that of a half-level. The binding energies of the other planets equal the energies of higher order sublevels.

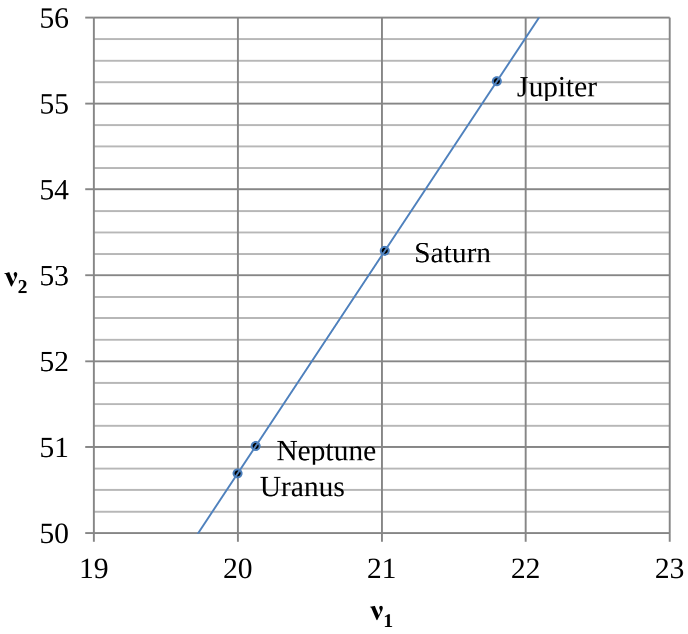


Figure 9: Gravitational binding energies of the gas giant planets of the Solar System on the trans-Planckian levels of Sequences 1 and 2.

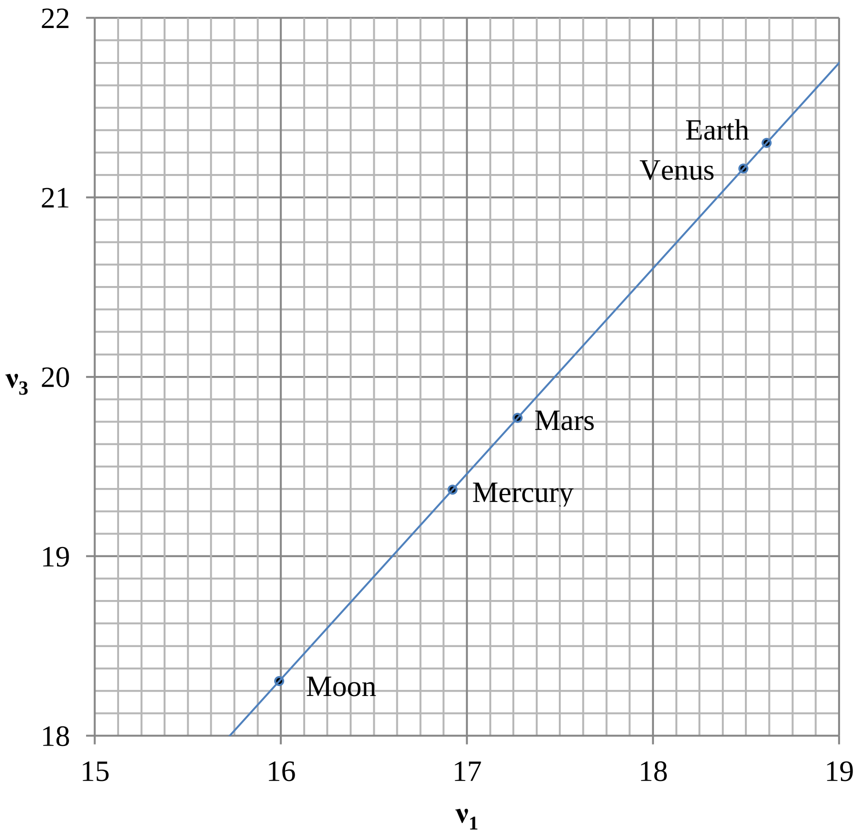


Figure 10: Gravitational binding energies of the rocky planets of the Solar System, and the binding energy of the Moon, on the trans-Planckian levels of Sequences 1 and 3.

Discussion

Our four-dimensional world encompasses both cis-Planckian and trans-Planckian mass/energy scales. Through the 10D/4D correspondence, we have found that black holes and planetary bodies occupy trans-Planckian mass levels of ten-dimensional origin [5, 6]. Planetary bodies, in particular, should therefore be regarded not just as composite objects but as bona fide trans-Planckian bodies. The clue to this realisation lay in the discovery that atomic nuclei occupy mass levels on cis-Planckian scales [12]. Having discovered that atomic and nuclear binding energies coincide with energy levels within the cis-Planckian scheme it was no surprise that gravitational binding energies lay within the trans-Planckian scheme.

References

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