Developing a monitoring system for long-distance pipeline leakage incorporating fusion of conflicting evidences

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Abstract: Quite often, during long-distance pipe leakage, critical decisions have to be made quickly and with as much certainty as possible using conflicting and uncertain sources of information. Key to providing quality decision options is an appropriate method of combining, or fusing, these heterogeneous evidence sources. Since the development of belief function theory introduced by Shafer in the 1970s many combination rules have been proposed in the literature because in highly conflicting situations the emblematic Dempster's rule generates counter-intuitive and unacceptable results in practical applications. In this work the Dezert-Smarandache theory (DSmT) will be explored, in particular the PCR5 and PCR6 rules of proportional conflict redistribution, and a simple long-distance pipe leakage monitoring method is developed to help in the development of a method for more complicated situations.

Keywords: pipeline leakage, evidence fusion, conflicting evidence.

1 Introduction

When transporting petroleum products, the use of long-distance pipelines have become commonplace, due to their inherent efficiency and economy (Adair et al., 2014). However, such pipe systems often leak due to corrosion, natural destruction, and, natural aging and weathering, causing harm to the environment, fire etc. The ability to detect leakage with certainty and quickly is therefore very important. The chances of leakage from pipes are considered high (Costa, 2001) and leaks often cause dire consequences to the environment. Many methods have been developed to monitor the movement of petroleum products (ADEC, 1999, Ellul, 1989a, Stouffs and Giot, 1993, da Silva et al., 2005). Generally these methods fall into two categories: hardware based (direct) and software based (inferential), including model based state/parameter estimation based methods (Emara-Shabaik et al., 2004, Emara-Shabaik et al., 2002). Direct methods, such as acoustic methods and infrared thermography, as well as technologies like hydrocarbon sensing via optical fiber or dielectric cables, detect leakage by monitoring some particular physical measurand fields alongside pipelines. Inferential methods infer the oil leakage by monitoring pipeline internal parameters (i.e. pressure, flow, temperature, etc.).

The methods just given however have limitations, with maybe the most important being the inclusion of simple rules to trigger unwanted alarms. This often happens when measured values change too rapidly or exceed stated thresholds, leading quite often to false alarms. To help alleviate this problem and moderate measured value rapid change and outliers, a data fusion algorithm can be used to combine real-time signals from pressure meters and flow meters with information concerning the historical maintenance records of the pipe, the geological condition and the pipe wall "health" condition. This should help increase reliability of alarms.

Various approaches have been used in different industries to deal with data which can be incomplete, uncertain and conflicting. Such approaches, including Bayesian theory, Dempster-Shafer theory (DS) (Shafer, 1976) and Dezert-Smarandache theory (DSmT) (Smarandache and Dezert, 2004-2009), have been used, for example, to fuse uncertain and unreliable information in areas involving sensor information (Basir and Yuan, 2007) and target identification (Leung and Wu, 2000) where systems are required to deal with imprecise information and conflicts which may arise among sensors. Bayesian methods and evidence theories, most notably Dempster-Shafer, have been commonly used to handle uncertainty. Dempster-Shafer can represent ignorance, and can aggregate beliefs as new evidence becomes available, unlike the Bayesian methods. The Dezert-Smarandache theory is a generalization of the Dempster-Shafer and has the advantage of taking into account paradoxical (conflicting) information (Dezert et al., 2010). In this paper the Dezert-Smarandache theory is used for fusion of pieces of evidence for decision making, in particular the Proportional Conflict Redistribution principles, PCR5 and PCR6 (Smarandache and Dezert, 2013).

A simple long-distance pipeline leakage monitoring system is developed with the fusion of various evidences at its centre using both the PCR5 and PCR6 proportional conflict redistribution principles.

2 Basics of Dezert-Smarandache Theory (DSmT)

In DSmT the frame of discernment denoted by $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ contains a finite set of n elements $\theta_i, i = 1, \dots, n$ assumed to be exhaustive but not necessarily exclusive due to the intrinsic nature of its elements and in order to deal with vague, fuzzy and relative concepts (Smarandache and Dezert, 2004-2009, Vol. 2). This is in contrast to Shafter's model which consists of a discrete finite set of exclusive and exhaustive hypotheses (Shafer, 1976). The rules of the DSmT framework work for any model of the frame, ranging from no exclusive constraint between θ_i to when θ_i are all exclusive, and, including hybrid models consisting of some exclusive θ_i. The set of subsets for DSmT are denoted by the hyper power-set D^Θ (Dedekind's lattice) (Smarandache and Dezert, 2004-2009, Vol. 1), which is created with $\cup$ and $\cap$ operators. When Shafer's model holds, that is, the constraints on all the elements are exclusive, D^Θ reduces to the power set 2^Θ. It is convenient to adopt the generic notation G^Θ to represent the fusion space depending on the application and underlying model chosen for the frame Θ, i.e. G^Θ would be either 2^Θ or D^Θ for Shafer's model or DSmT respectively, depending on the nature of the problem.

A quantitative basic belief assignment (bba) expressing the belief assigned to the elements of G^Θ provided by an evidential source or body of evidence is a mapping function $m(\cdot): G^\Theta \to [0,1]$ such that: $m(\emptyset) = 0$ and $\sum_{A \in G^\Theta} m(A) = 1$. Elements $A \in G^\Theta$ having $m(A) > 0$ are called focal elements of the bba $m(\cdot)$. The general belief and plausibility functions are defined respectively in almost the same manner as Shafer, 1976, i.e.,

$$Bel(A) = \sum_{B \in G^\Theta, B \subseteq A} m(B) \qquad \text{and} \qquad Pl(A) = \sum_{B \in G^\Theta, B \cap A \neq \emptyset} m(B) \tag{1}$$

The method of combining evidence differs between the Dempster-Shafer and Dezert-Smarandache theories.

For Dempster's rule of combination, symbolized by the operator $\oplus$, is used to combine two distinct sources of evidence over the same frame Θ. If Bel_1 and Bel_2 represent two belief functions over the same frame Θ and $m_1(\cdot)$ and $m_2(\cdot)$ are their respective basic belief assignments (bbas), then the combined belief function $Bel = Bel_1 \oplus Bel_2$ is obtained by the combination of $m_1(\cdot)$ and $m_2(\cdot)$ as: $m(\emptyset) = 0$ and $\forall C \neq \emptyset \subseteq \Theta$

$$m_{12}(C) = [m_1 \oplus m_2](C) = \frac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \emptyset} m_1(A) m_2(B)} \tag{2}$$

Dempster's rule of combination is associative $([m_1 \oplus m_2] \oplus m_3 = m_1 \oplus [m_2 \oplus m_3])$ and commutative $(m_1 \oplus m_2 = m_2 \oplus m_1)$.

In the Dezert-Smarandache theory the Proportional Conflict Redistribution Rule no. 5 (PCR5) has been proposed as a serious alternative to Dempster's rule for dealing with conflicting belief functions. When using PCR5 with two sources, $m_1(\cdot)$ and $m_2(\cdot)$ are considered to be independent bbas, and the rule of combination for two sources of evidence is defined as (Smarandache and Dezert, 2004-2009): $m_{PCR5}(\emptyset) = 0$ and $\forall (X \neq \emptyset) \in G^\Theta$

$$m_{PCR5}(A) = \sum_{\substack{X_1, X_2 \in G^\Theta \\ X_1 \cap X_2 = A}} m_1(X_1) m_2(X_2) + \sum_{\substack{X \in G^\Theta \\ X \cap A = \emptyset}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)} \right] \tag{3}$$

All fractions in equation (3), which have zero denominators are discarded and all propositions/sets are considered in a canonical form. The disjunctive normal form, which is a disjunction of conjunctions and which is unique in Boolean algebra is used. For example, $X = A \cap B \cap (A \cup B \cup C)$ is not in a canonical form, but when simplified to $X = A \cap B$ it is in a canonical form. PCR5 is not associative and the optimal fusion result is obtained by combining the sources altogether at the same time when possible. Some PCR5 properties are found in (Dezert and Smarandache, 2010) and it allows non-Bayesian reasoning.

The idea behind PCR5 is to transfer the conflicting mass only to the elements involved in the conflict and proportionally to their individual masses, so that the specificity of the information is entirely preserved through this fusion process. For example, as described in (Smarandache et al., 2010), consider three bba's, $m_1(\cdot)$ and $m_2(\cdot)$,$m_3(\cdot)$ $A \cap B = \emptyset$ for the model of Θ. With PCR5 the partial conflicting mass, $m_1(A)m_2(B)m_3(B)$ is redistributed back to A and B only with respect to the following proportions,

$$\frac{x_A^{PCR5}}{m_1(A)} = \frac{x_B^{PCR5}}{m_2(B)m_3(B)} = \frac{m_1(A)m_2(B)m_3(B)}{m_1(A) + m_2(B)m_3(B)} \tag{4}$$

The extension and a variant of PCR5, called PCR6 has been proposed (Martin and Oswald, 2006, Martin et al., 2008) for combining $s > 2$ sources. For two sources PCR6 coincides with PCR5, with the difference being in the way the proportional conflict redistribution is done as three or more sources are involved in the fusion.

With the PCR6 fusion rule, the partial conflicting mass $m_1(A)m_2(B)m_3(C)$ is redistributed back to A and B only with respect to the following proportions,

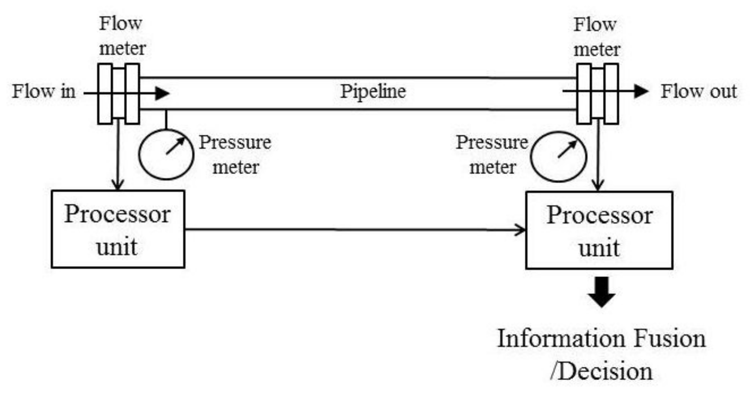
$$\frac{x_A^{PCR6}}{m_1(A)} = \frac{x_B^{PCR6}}{m_2(B)} = \frac{x_C^{PCR6}}{m_3(C)} = \frac{m_1(A)m_2(B)m_3(C)}{m_1(A) + m_2(B) + m_3(C)} \tag{5}$$

From an implementation point of view, PCR6 is more simple than PCR5. The actual form of the equation used in this work for PCR6 is given in the Appendix.

3 Hydraulic Monitoring Method

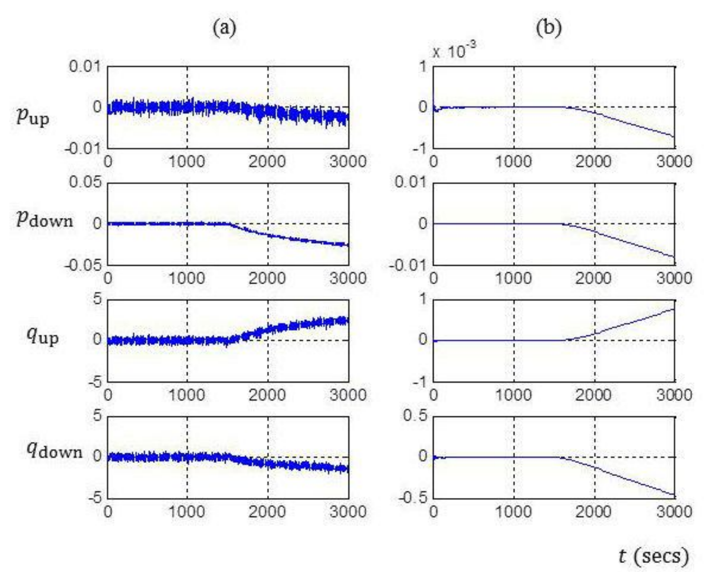
A hydraulic model is used to determine a leakage by simulating its effects (i.e., for example, an increase in flow and drop in pressure) and compare them with values obtained from pressure and flow sensors upstream and downstream as shown on Figure 1.

Figure 1 Pipeline monitoring system



Each sensor records the waveform of a particular parameter and the corresponding processor unit classifies the waveform definite, i.e. true or false or uncertain. Before processing using the Dezert-Smarandache theory, a probability $m(\cdot)$ for belief in the classification is given.

Figure 2 Typical pressure and flow characteristics when a leak occurs (a) original signal. (b) with Kalman filtering



On Figure 2 the characteristics of pressure and flow are shown when a leak occurs at 1500 s. An extended Kalman filtering (Brown and Hwang, 1992) is implemented with the corresponding noisy measurement residuals and their averages generated and shown on the figure. Pressure and flow sensors upstream and downstream of the leak were used to capture the characteristics and the figure clearly shows the effect of the leak. The measured pressure residual averages are negative as pressure is reduced due to the leak. On the other hand flow residual averages are positive for the upstream location of the leak and negative downstream.

Expert decisions have to be employed to help with the provision of bbas. Each expert can use the following points to help guide the assignment of these bbas.

With the definitions,

$$\delta p = p(t) - p(t-1), \qquad \delta q = q(t) = q(t-1) \tag{6}$$

1. Relative to normal conditions, i.e., steady state conditions or normal line pack conditions, $\delta p < 0 \Rightarrow$ leak condition.
2. If several pressure sensors are used the higher the drop in pressure, the nearer to a leak i.e., $\max|\delta p| \Longrightarrow$ nearest to a leak.
3. Usually flow rate increases/decreases upstream/downstream of a leak (but note as will be shown in Table 2 there are several permutations covering this observation) i.e., $\delta q > 0$ upstream of a leak, $\delta q < 0$ downstream of a leak.
4. As the new steady state with a leak is established,

$$q_{up} \to q_{limit,up}, \qquad q_{down} \to q_{limit,down}, \quad q_{limit,up} > q_{limit,down}$$

5. Magnitude of leak, $|q_L| = q_{limit,up} - q_{limit,down}$

Petroleum products are usually transferred through a pipeline at elevated pressure relative to atmosphere, and when a leak occurs a negative pressure wave originates and then propagates along the pipeline upstream and downstream. The negative pressure wave method based on pressure sensors is the most widely used leak location technology (Ellul, 1989b). Assuming, in reference to Figure 1, that $p_i(t)$ and $p_o(t)$ are the normalized waveforms of negative pressure at the inlet and outlet respectively.

As the pressure waves originate from the same source they can be correlated with the probability of a leak evaluated using,

$$\rho_{p_i p_o}(t') = \int_{-\infty}^{\infty} p_i(t) \cdot p_o(t + t')\mathrm{d}t, \qquad t' \in [o, \frac{L}{v_p - v_{pp}}] \tag{7}$$

where L is the length of the pipeline, v_p is the propagation speed of the negative pressure wave, and v_{pp} is the transmission speed of the petroleum products inside the pipeline. Assuming that $\rho_{p_i p_o}(t')$ reaches its maximum value when $t' = t'_{max}$, the leakage probability, denoted as $m_p(a_1)$, is given according to,

$$m_p(a_1) = \rho_{p_i p_o}(t'_{max}) \quad \text{and} \quad m_p(\Theta) = 1 - r(t'_{max}) \tag{8}$$

where, a_1 is the primitive hypothesis of leak occurrence.

Here, t'_{max} is the time difference to when the negative pressure wave arrives at the inlet and outlet. The distance L_o from the point of leakage to the pipeline outlet can be evaluated using,

$$L_o = L - \left\{ \frac{1}{2v_p} \left[L\left(v_p - v_{pp}\right) + \left(v_p^2 - v_{pp}^2\right) \cdot t'_{max} \right] \right\} \tag{9}$$

There is a certain amount of uncertainty in detection of leaks using the negative pressure wave method however and therefore it is a candidate for treatment by the current decision algorithm. The installation of pressure sensors required for this method necessitates localized deconstruction of the pipeline. So pressure sensors are usually installed at input and output points of a pipeline and seldom anywhere in between. This can lead to a large signal attenuation and interference, leading to high rates of false alarms whilst reducing the precision of locating algorithms. In addition the location formula assumes that the propagation velocity of the negative pressure wave and the velocity of the petroleum product in the pipeline are constants. This does not normally match with the actual situation and will result in a positioning error. Therefore to reduce false alarms moderation is needed. The proposal here is to incorporate the use of flow meters and the DSmT theory of evidence fusion.

For the flow meters, during real-time steady state flow, there are a number of permutations to indicate pipeline leakage as shown in Table 1.

Table 1 Flow meter readings permutations

Permutation	Readings	Explanation
A	$q_{in}(const.) > q_{out}(falling)$	Growing leakage
B	$q_{in}(const.) > q_{out}(const.)$	Leakage of fixed magnitude
C	$q_{in}(rising) > q_{out}(falling)$	Growing Leakage, mechanical malfunction
D	$q_{in}(rising) > q_{out}(rising)$	Leakage, mechanical malfunction, normal fulfilling pipe delay
E	No signal	Malfunction
F	$q_{in}(const.) = q_{out}(const.)$	No leakage

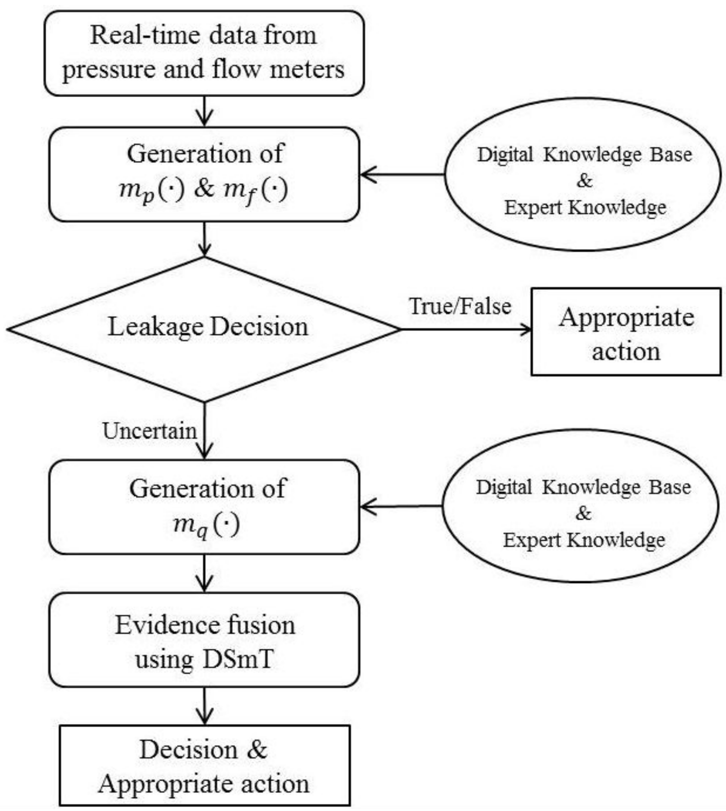
Therefore the frame of discernment of q_{in} vs. q_{out} is then $\Theta' = \{A, B, C, D, E\}$. A mass function m_f can be generated from the comparisons q_{in} vs. q_{out} together with their changing states over a short time period. Cases A, B, C and D contribute to the primitive hypothesis a_1 of leakage-occurrence, and cases $C, D,$ and E to the primitive hypothesis a_2 of either mechanical malfunctions, normal fulfilling pipe delay or no signal. The mass function m_f on Θ' can be transformed into the mass function $m_q(\cdot)$ on Θ as follows (Demotier et al., 2004),

$$\begin{cases} m_q(a_1) = m_f(A) + m_f(B) + \ m_f(A,B) \\ m_q(a_2) = m_f(E) \\ m_q(a_1,a_2) = m_f(C) + m_f(D) + m_f(C,D) \\ m_q(\Theta) = 1 - m_f(a_1) - m_f(a_2) - m_f(a_1,a_2) \end{cases} \tag{10}$$

4 Combining Decisions based on DSmT

For assessment of leakage from long-distance oil pipelines the evidences provided by the pressure and flow meters reading need to be combined. An outline of the decision algorithm which fuses evidence is shown on Figure 3. Real time data is gathered form pressure and flow meters and in conjunction with a digital knowledge base and expert knowledge. Following, a decision is made as to whether a leak is present, absent or uncertainty exists. If uncertainty exists using information from a digital data-base along with expert knowledge, basic probability assessments can be assigned to the mass functions, $m_p(\cdot)$and $m_f(\cdot)$. Using equation (10), values of the mass functions $m_q(\cdot)$ are then generated, followed by fusion of the evidence using the Dezert-Smarandache theory (PCR5 and PCR6).

Figure 3 Outline of decision algorithm



To demonstrate some of the numerics of the monitoring method, it is assumed that, at a certain time, the respective bbas, $m_p(a_1), m_q(a_1), m_q(a_2), m_q(a_1, a_2)$, derived as discussed above, are as given in Table 2.

If $\lambda_1 = \{a_1\}$ means that there is a leakage, $\lambda_1 = \{a_2\}$ means that there is some sort of malfunction and $\lambda_1 = \{a_1, a_2\}$ means that there may be the possibility of leakage as well as some sort of malfunction, then following distribution of bbas can be generated.

Table 2 Basic belief assignments

	$\lambda_1 = \{a_1\}$	$\lambda_1 = \{a_2\}$	$\lambda_1 = \{a_1, a_2\}$	Θ
$m_p(a_1)$	0.82	0.15	0.0	0.03
$m_q(a_1)$	0.10	0.20	0.60	0.10
$m_q(a_2)$	0.15	0.60	0.15	0.10
$m_q(a_1, a_2)$	0.15	0.30	0.50	0.05

PCR5 and PCR6 are then used to fuse the information, with an assumption that all sources are equal in terms of reliability and priority with the results shown in Table 3. Here $m_{12} \ldots . m_{1234}$ corresponds to the sequential fusion of the sources $m_1 \ldots, m_4$ using PCR5. The values given for PCR6 are obtained by applying this rule globally. It can be seen however that both methods indicate the possibility of oil leakage.

Table 3 Dezert-Smarandache evidence fusion method using PCR5 (sequential) and PCR6 (global)

		PCR5		PCR6
	m_{12}	m_{123}	m_{1234}	$m(\cdot)$
$\lambda_1 = \{a_1\}$	0.7872	0.5914	**0.5490**	**0.4361**
$\lambda_1 = \{a_2\}$	0.1754	0.3832	0.4318	0.3123
$\lambda_1 = \{a_1, a_2\}$	0.0171	0.0055	0.0123	0.2351
Θ	0.0202	0.0198	0.0068	0.0164

It can be seen that the results for combining evidences using PCR5 and PCR6 are different. This perhaps surprising result is remarked on (Smarandache and Dezert, 2013) in that the PCR6 fusion results are valid if and only if the PCR6 rule is applied globally, and not sequentially. If PCR6 is sequentially applied it becomes equivalent to PCR5 sequentially applied and it will generate incorrect results for combining $s > 2$ sources of evidence. Smarandache and Dezert ,2013 strongly recommend taking the PCR6 result in Table 3 rather than the PCR5.

The final part of this monitoring scheme is to make a decision as to whether a leak is occurring or not. This can be done by a threshold method (Yannian and Zhuanghe, 2005).

Assuming that $\lambda_1, \lambda_2 \in \Theta, m(\lambda_1) = \max\{m(\lambda_i), \lambda_i \subset \Theta\}$, and $m(\lambda_2) = \max\{m(\lambda_i), \lambda_i \neq \lambda_1 \cap \lambda_i \subset \Theta\}$. If λ_1 and λ_2 satisfy the following relations, that is,

$$\begin{cases} m(\lambda_1) - m(\lambda_2) > \varepsilon_1 \\ m(\Theta) < \varepsilon_2 \\ m(\lambda_1) > m(\Theta) \end{cases} \tag{11}$$

the result of the decision is λ_1 where ε_1 and ε_2 are the given thresholds. If a leakage is thought to occur the location of the leakage can be found using equation (9), depending on the quality of the signals.

5 Conclusions

A decision algorithm which fuses evidence from pressure and flow meters has been developed for the detecting of long-distance oil pipeline leakage. The method of evidence fusion chosen here is the Dezert-Smarandache (DSmT) theory and in particular the Proportional Conflict Redistribution Rule nos. 5 and 6 (PCR5 and PCR6). The raw data to determine leakage was obtained from a pair of pressure meters located close to the pipe's inlet and outlet and flow meters located to measure inlet flow and outlet flow. Each type of sensors generates real-time functions to classify leakage as being true, false or uncertain. On using PCR5 and PCR6 the evidence is fused to make a decision which may be true, false, or uncertain concerning leakage. A difference was found after fusing the evidences, with PCR5 used sequentially and PCR6 used globally. It has been recommended for $s > 2$ that PCR6 global fusion is used. The improved method of fusion of evidence should increase reliability and robustness of warnings and reduce false alarms and miss-alarms.

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Appendix

A general formula for PCR6 for $s \geq 2$ sources:

$$m_{PRC6}(A) = m_{12\ldots s} + \sum_{\substack{X_1,X_2,\ldots,X_{s-1}\in G^{\Theta} \\ X_i \neq A, i\in\{1,2,\ldots,s-1\} \\ (\cap_{i=1}^{s-1} X_i)\cap A=\emptyset}} \sum_{k=1}^{s-1} \sum_{(i_1,i_2,\ldots,i_s)\in P(1,2,\ldots,s)} \left[m_{i_1}(A) + m_{i_2}(A) + \cdots + m_{i_k}(A)\right] \cdot$$

$$\cdot \frac{m_{i_1}(A)m_{i_2}(A)\ldots m_{i_k}(A)m_{i_{k+1}}(X_1)\ldots m_{i_s}(X_{s-k})}{m_{i_1}(A) + m_{i_2}(A) + \cdots + m_{i_k}(A) + m_{i_{k+1}}(X_1) + \cdots m_{i_s}(X_{s-k})}$$

where $P(1,2,\ldots,s)$ is the set of all permutations of the elements $\{1,2,\ldots,s\}$. It should be observed that $X_1, X_2, \ldots, X_{s-k}$ may be different from each other, or some of them equal and others different, etc.

The general formula for PCR6 above is written in the style of PCR5 which is different from Martin and Oswald, 2006, but actually does the same thing. In order not to complicate the formula of PCR6, no summations or products are used after the third Sigma.