Improved Weighted Correlation Coefficient based on Integrated Weight for Interval Neutrosophic Sets and its Application in Multi-criteria Decision Making Problems

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Abstract: The paper presents a new correlation coefficient measures satisfying the requirement that the correlation coefficient measure equals one if and only if two interval neutrosophic sets (INSs) are same and the weighted correlation coefficient measure of INSs based on the extension of the correlation coefficient measures of intuitionstic fuzzy sets (IFSs). Firstly, an improved correlation coefficient measure, the weighted correlation coefficient measure and their properties for INSs are developed. Secondly, the notion that the weight in the weighted correlation coefficient should be the integrated weight which is the integration of subjective weight and objective weight is put forward. Thirdly, an objective weight of INSs based on entropy is presented to unearth and utilize the deeper uncertainty information. Then, a decision making method is proposed by the use of the weighted correlation coefficient measure of INSs. Specially, this method takes the influence of the evaluations' uncertainty into consideration and takes both the objective weight and the subjective weight into consideration. Finally, an illustrative example demonstrates the concrete application of the proposed method and the comparison analysis is given.

Keywords: interval neutrosophic sets; objective weight; integrated weight; correlation coefficient; multi-criteria decision making

1. Introduction

The remarkable theory of fuzzy sets (FS) proposed by Zadeh in 1965 [1] is regarded as an important tool for solving multi-criteria decision making (MCDM) problems [2, 3]. Since then, many new extensions encountering imprecise, incomplete and uncertain information have been presented [4]. For example,

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Turksen [5] introduced interval-valued fuzzy set (IVFS) using an interval data instead of one specific value to define the membership degree. In order to depict the fuzzy information comprehensively, Atanassov and Gargov [6, 7] defined IFSs and interval-valued intuitionistic fuzzy sets (IVIFSs) which can handle incomplete and inconsistent information. To deal with the situations that people are hesitant in expressing their preference regarding objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra and Narukawa [8]. And all these extensions of FSs have been developed by many researchers in various fields [9-11]. Some other extensions are still going on [12-15]. Particularly, Florentin Smarandache [16, 17] introduced neutrosophic logic and neutrosophic sets (NS) in 1995, in which a NS is characterized by the function of truth, indeterminacy and falsity. And the three functions' values lie in $]0^-, 1^+[$, the non-standard unit interval [17], which is the extension to the standard interval $[0, 1]$ of IFS. And the uncertainty shown here, i.e. indeterminacy factor, is immune to truth and falsity values while the incorporated uncertainty depends on the degree of belongingness and degree of non-belongingness in IFS [18].

Obviously, NS is difficult to apply in realistic problems. Thus, single-valued neutrosophic set (SVNS) was put forward and manifold MCDM methods have been proposed under the single-valued neutrosophic environment [19-22]. In consideration of the fact that sometimes using exact numbers to describe the degree of truth, falsity and indeterminacy about a particular statement is infeasible in the real situations, Wang et al. [23] proposed the concept of INSs and present the set-theoretic operators of INS. And the operations of INS were discussed in [24]. To correct deficiencies in [23], Zhang et al. [25] refined the INS operations and proposed the comparison approach between interval neutrosophic numbers (INNs) and the aggregation operators for INSs. Additionally, kinds of MCDM methods of INSs were put forward, by using aggregation operators [25], fuzzy cross-entropy [26] and similarity measures [27], etc.

Correlation coefficient is an important tool to judge the relation between two objects. And under fuzzy circumstances, correlation coefficient is a principal vehicle to calculate fuzziness of information in fuzzy set theory and has been widely developed. For example, Chiang and Lin [28] introduced the correlation of fuzzy sets and Gerstenkorn and Manko [29] defined the correlation of IFSs in 1991. However, Hong and Hwang [30] pointed out that the correlation coefficient in [29] did not satisfy the property that $K(A,B)=1$ if and only if $A=B$, where $K(A,B)$ denotes the correlation coefficient between two fuzzy sets A and B. They also generalized the correlation coefficient of IFSs in a probability space [30] and proved the method they proposed conquered the shortcoming mentioned above in the case of finite spaces. Furthermore, Hung and Wu [31] defined the correlation coefficient of IFSs utilizing the concept of centroid and introduced the concept of positively and negatively correlated. Based on [31], Hanafy et al. [32] defined the correlation coefficient of generalized intuitionistic fuzzy sets whose degree of membership and non-membership lie between 0 and 0.5. Moreover, Bustince and Burillo [33] discussed the correlation coefficient under the interval-valued intuitionistic fuzzy environment and demonstrated their properties. Moreover, in the environment with IVIFS, correlation coefficient can also be an effective vehicle. Based on the correlation coefficient method of IVIFSs proposed in [33], Ye [34] developed a weighted correlation coefficient measure to solve the MCDM problems with incompletely known criterion weight information and the weight is determined by the entropy measure. Besides, the correlation coefficient has been diffusely applied in a diversity of scientific fields such as decision making [35-37], pattern recognition [38], machine learning [39] and market prediction [40].

Correlation coefficient measure is also effective under the neutrosophic circumstance. Hanafy et al. [41] defined the correlation and correlation coefficient of Neutrosophic sets and Ye [19] presented the correlation and correlation coefficient of SVNSs and utilized the measure to solve the MCDM problems. Following the correlation coefficient in [41], Broumi and Smarandache [42] proposed the correlation coefficient measure and the weighted correlation coefficient measure of INSs. Nevertheless, some drawbacks are existing in some situations in the correlation coefficient measure defined in [19]. In order to overcome these disadvantages, Ye [43] developed an improved correlation coefficient measure of SVNSs and extended it to INSs.

As to MCDM problems, alternatives are evaluated under various criteria. Therefore, weights of criteria reflect the relative importance in ranking alternatives from a set of available alternatives. With respect to multiple weights, they can be divided into two categories: subjective weights and objective weights [45]. The subjective weights are related to the preference or judgments of decision makers while the objective weights usually mean the relative importance of various criteria without any consideration of the decision maker's preferences. Both the subjective weight measure and the objective weight measure have been extensively studied.

For the subjective weight measure, Saatty [47, 48] put forward an eigenvector method using pairwise weight ratios to obtain the weights of belongings of each member to the set. Then, Keeney and Raiffa [46] discussed the direct assessing methods to determine the subjective weight. Based on [48], Cogger and Yu [49] presented a new eigenweight vector whose computation is easier than Saatty's method. Moreover, Chu [50] proposed a weighted least-square method and several example were shown to compare with the eigenvector method. In order to deal with the mixed multiplicative and fuzzy preference relations, Wang et al. [51] presented a chi-square method.

Matter of the objective weight, based on the notion of contrast intensity and the conflicting character of the evaluation criteria, Diakoulaki [52] proposed the Criteria Importance Through Intercriteria Correlation (CITIC) method to obtain the objective weight. Besides, another objective weight approach called the maximizing deviation method was proposed by Wang [53]. Wu [54] made use of the maximizing deviation method and constructed a non-liner programming model to obtain the objective weight. Furthermore, Zou [55] put forward a new weight evaluation process utilizing entropy measure and applied it in water quality assessment.

In general, the subjective method reflects the preference of the decision maker while the objective method makes use of mathematical models to unearth the objective information. However, the subjective method may be influenced by the level of the decision maker's knowledge and the objective method neglects the decision maker's preference. In order to overcome this shortage and benefit from not only decision makers' expertise but also the relative importance of evaluation information, the most common way is to integrate the subjective weight and objective weight to explore a decision making process approaching as near as possible to the actual one. Ma et al. [56] set up a two-objective programming model by integrated the subjective approach and the objective approach to solve the MADM problems, and this two-objective programming problem can be solved making use of the linear weighted summation method. Similarly, Wang and Parkan [57] utilized linear programming technique to integrating the subjective fuzzy preference relation and the objective decision matrix information together in three different ways. Different from the linear programming method, Chen and Wang [58] proposed a new approach to obtain the integrated weight in which the integrated weight is determined by the normalization of the product with both subjective weight and objective weight.

As noted above, many objective weight measures have been proposed and the entropy weight is one of the widely used approaches to solve MCDM problems [55, 58-61].

As mentioned in [34], entropy is also an important concept in the fuzzy sets. The fuzzy entropy was first introduced by Zadeh [1, 4] to measure uncertain information. In 1972, Luca and Termini [62] introduced the axiomatic definition of entropy of FSs and defined entropy using no probabilistic concept. Trillas and Riera [63] proposed general expressions for the entropy. Besides, Yager [64] defined the fuzziness degree of a fuzzy set in terms of a lack of distinction between the fuzzy set and its complement in 1982. Fan and Xie [65] proposed the fuzzy entropy measure induced by distance. Similarly, entropy has been widely developed in intuitionistic fuzzy environment. Bustince and Burillo [66] gave an axiom definition of intuitionistic fuzzy entropy. Based on the axiomatic definition of entropy of Luca et al. in [62], Szmidt et al. [67] extended it into IFSs and proposed an entropy measure for IFSs as a result of a geometric interpretation of IFSs using a ratio of distances between them, and what's more, they also proposed some new entropy measures based on similarity measures in [68]. As for the neutrosophic circumstance, Majumdar et al. [69] introduced the entropy of SVNSs by giving an axiomatic definition based on entropy's definition of fuzzy set proposed by Luca et al. [62] and proposed a new entropy measure based on the notion that uncertainty of a SVNS is due to the belongingness and non-belongingness part and the indeterminacy part. Moreover, the relationships among similarity measures, distance measures and entropy measures of FSs, IVFSs, IFSs and NSs have also been investigated [69-73]. Entropy is also effective in dealing with practical problems. For example, as mentioned above, entropy can be used to obtain the objective weight in MCDM problems [34, 55, 61, 74].

However, most contributions on the measure of correlation coefficient and entropy concentrate on the extensions of fuzzy set and little effort as to it has been made on INSs and this will restrict its real scientific and engineering applications. Besides, the current researches about correlation coefficient mostly utilize objective measure only under the environment where the information about criterion weight for alternatives is completely unknown or incompletely unknown [34, 74]. However, the influence caused by the uncertainty of evaluation still exists while the information about criterion weight is known and the objective weight can avert the non-determinacy and arbitrariness caused by the subjective weight [58]. Thus, lots of studies on this issue are needed to be done. Therefore, the correlation coefficient measure, the weighted correlation coefficient measure and the entropy measure for INSs are extended in this paper and an objective weight measure based on entropy for INSs is also proposed. Besides, the notion that the weighted correlation coefficient measure should take use of the integrated weight is put forward. Furthermore, a MCDM procedure is established based on the weighted correlation coefficient measure which considers both the subjective weight and the objective weight, and an illustrative example is given to demonstrate the application of the proposed measures.

The rest of this paper is organized as follows. Section 2 briefly introduces IVIFSs, NSs, SVNSs and INSs as well as some operations for INSs such as intersection, union. And the correlation coefficient measure, the weighted correlation coefficient measure, entropy measure, and their properties for INSs are developed in Section 3. In addition, an objective weight measure making use of entropy for INSs is explored. In Section 4, decision making procedure based on the weighted correlation coefficient measures using the integrated weight for MCDM problems are given. In Section 5, an illustrative example is presented to illustrate the proposed methods and the comparative analysis and discussion are given. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, some basic concepts and definitions related to INSs are introduced, which will be used in the rest of the paper.

Definition 1 [7]. Let X be a space of points (objects), with a generic element in X denoted by x. An IFS A in X is characterized by a membership function $\mu_A(x)$, and a non-membership function $\nu_A(x)$. For each point x in X, we have $\mu_A(x), \nu_A(x) \subseteq [0,1]$, $x \in X$. Thus, the IFS A can be denoted by

$$A=\{<x,\mu_A(x),\nu_A(x)>|x\in X\}$$

Definition 2 [75]. Let A and B be two IFSs in the universe of discourse $X=\{x_1,x_2,\cdots,x_n\}$ and $A=\{<x_i,\mu_A(x_i),\nu_A(x_i)>|x_i\in X\}$, $B=\{<x_i,\mu_B(x_i),\nu_B(x_i)>|x_i\in X\}$, then the correlation coefficient of A and B is defined by

$$C(A,B)=\frac{\sum_{i=1}^{n}(\mu_A(x_i)\cdot\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)+\pi_A(x_i)\pi_B(x_i))}{\max(\sum_{i=1}^{n}(\mu_A^2(x_i)+\nu_A^2(x_i)+\pi_A^2(x_i)),\sum_{i=1}^{n}(\mu_B^2(x_i)+\nu_B^2(x_i)+\pi_B^2(x_i)))}$$

where $\pi_A(x_i)=1-\mu_A(x_i)-\nu_A(x_i)$ and $\pi_B(x_i)=1-\mu_B(x_i)-\nu_B(x_i)$ are called the degree of uncertainty (or hesitation).

Definition 3 [76]. Let X be a space of points (objects), with a generic element in X denoted by x. A NS A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$, that is, $T_A(x):X\to]0^-,1^+[$, $I_A(x):X\to]0^-,1^+[$, and $F_A(x):X\to]0^-,1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^-\le\sup T_A(x)+\sup I_A(x)+\sup F_A(x)\le 3^+$.

Definition 4 [76]. A NS A is contained in the other NS B, denoted as $A\subseteq B$, if and only if $\inf T_A(x)\le\inf T_B(x)$, $\sup T_A(x)\le\sup T_B(x)$, $\inf I_A(x)\le\inf I_B(x)$, $\sup I_A(x)\le\sup I_B(x)$, $\inf F_A(x)\le\inf F_B(x)$ and $\sup F_A(x)\le\sup F_B(x)$ for $x\in X$.

Since it is difficult to apply NSs to practical problems, Ye [20] reduced NSs of nonstandard intervals into a kind of SVNSs of standard intervals that will preserve the operations of the NSs.

Definition 5 [20]. Let X be a space of points (objects), with a generic element in X denoted by x. A NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x):X\to[0,1]$, $I_A(x):X\to[0,1]$, and $F_A(x):X\to[0,1]$. Then, a simplification of A is denoted by

$$A=\{<x,T_A(x),I_A(x),F_A(x)>|\,x\in X\}$$

which is called a SVNS. It is a subclass of NSs.

Definition 6 [76]. Let X be a space of points (objects) with generic elements in X denoted by X. An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)=[\inf T_A(x),\sup T_A(x)]$, $I_A(x)=[\inf I_A(x),\sup I_A(x)]$, $F_A(x)=[\inf F_A(x),\sup F_A(x)]\subseteq[0,1]$, and $0\le \sup T_A(x)+\sup I_A(x)+\sup F_A(x)\le 3$, $x\in X$. We only consider the subunitary interval of [0, 1]. It is the subclass of a NS. Therefore, all INSs are clearly NSs.

For any fuzzy set A, its complement A^c is defined by $m_{A^c}(x)=1-m_A(x)$, for all x in X. The complement of an INS A is also denoted by A^c.

Definition 7 [76, 77]. Let A and B be two INSs, then

(1) $A=B$, if and only if $A\subseteq B$ and $A\supseteq B$;

(2) $A^c=\{<x,[\inf F_A(x),\sup F_A(x)],[1-\sup I_A(x),1-\inf I_A(x)],[\inf T_A(x),\sup T_A(x)]>\}$;

(3) $A\subseteq B$ if and only if $\inf T_A(x)\le \inf T_B(x)$, $\sup T_A(x)\le \sup T_B(x)$, $\inf I_A(x)\ge \inf I_B(x)$, $\sup I_A(x)\ge \sup I_B(x)$, $\inf F_A(x)\ge \inf F_B(x)$ and $\sup F_A(x)\ge \sup F_B(x)$, for any $x\in X$.

A distance function or metric is a generalization of the concept of physical distance. In fuzzy set theory, it describes how far one element is away from another. Ye [78] defined the Hamming distance measure between two INSs.

Definition 8 [78]. Let A and B be two INSs in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$, then the distance measure between them can be defined:

The Hamming distance:

$$d_H(A,B)=\frac{1}{6}\sum_{i=1}^{n}\big[|\inf T_A(x_i)-\inf T_B(x_i)|+|\sup T_A(x_i)-\sup T_B(x_i)| \\ +|\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)| \\ +|\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|\big] \tag{1}$$

The normalized Hamming distance:

$$d_{nH}(A,B)=\frac{1}{6n}\sum_{i=1}^{n}\big[|\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)| \\ +|\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)| \\ +|\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|\big] \tag{2}$$

Definition 9 [42]. Let A and B be two INSs in the universe of discourse $X=\{x_1,x_2,\cdots,x_n\}$ and $A=\{<x_i,[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]>|x_i\in X\}$, $B=\{<x_i,[\inf T_B(x_i),\sup T_B(x_i)],[\inf I_B(x_i),\sup I_B(x_i)],[\inf F_B(x_i),\sup F_B(x_i)]>|x_i\in X\}$, then the correlation coefficient of A and B is defined by

$$K(A,B)=\frac{C(A,B)}{\sqrt{E(A)\cdot E(B)}}, \tag{3}$$

where the correlation of two INSs A and B is given by

$$C(A,B)=\frac{1}{2}\sum_{i=1}^{n}\big[\inf T_A(x_i)\cdot\inf T_B(x_i)+\sup T_A(x_i)\cdot\sup T_B(x_i)+\inf I_A(x_i)\cdot\inf I_B(x_i) \\ +\sup I_A(x_i)\cdot\sup I_B(x_i)+\inf F_A(x_i)\cdot\inf F_B(x_i)+\sup F_A(x_i)\cdot\sup F_B(x_i)\big],$$

and the informational intuitional energies of two IVIFSs A and B are defined as

$$E(A)=\sum_{i=1}^{n}\frac{1}{2}\Big[(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2\Big],$$
$$E(B)=\sum_{i=1}^{n}\frac{1}{2}\Big[(\inf T_B(x_i))^2+(\sup T_B(x_i))^2+(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+(\inf F_B(x_i))^2+(\sup F_B(x_i))^2\Big].$$

However, as Ye [43] mentioned, this correlation coefficient measure in Definition 9 cannot guarantee that the correlation coefficient of two INSs equals one if and only if two INSs are the same [43]. In some cases, several different kinds of weight may be taken into account at the same time. In order to solve this problem, the integration measure of different kinds of weights is necessary.

Definition 10 [58]. Let $w=(w_1,w_2,\cdots,w_n)$ and $\theta=(\theta_1,\theta_2,\cdots,\theta_n)$ be two different types of weight vector.

The final integrated weight vector $W=(W_1,W_2,\cdots,W_n)$ can be calculated:

$$W_i=\frac{w_i\theta_i}{\sum_{i=1}^{n}w_i\theta_i} \tag{4}$$

3. The weighted correlation coefficient measure for INS

In this section, a new correlation coefficient measure, the weighted correlation coefficient measure for INSs and their properties are developed. Besides, we also explore an objective weight measure for INS utilizing entropy.

3.1. Correlation coefficient measure for INS

In order to overcome the deficiency presented in Definition 9, we propose a novel correlation coefficient measure motivated by the correlation coefficient measure of IFSs proposed by Xu [75].

Definition 11. A mapping *K*: $INS(X)\times INS(X)\rightarrow[0,1]$ is called INSs correlation coefficient measure, if *K* satisfies the following properties:

($KP1$) $0\le K(A,B)\le 1$;

($KP2$) $K(A,B)=K(B,A)$;

($KP3$) $K(A,B)=1$ if and only if $A=B$.

Definition 12. Let two INSs A and B in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$ be $A=\{<x_i,[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]>|x_i\in X\}$ and $B=\{<x_i,[\inf T_B(x_i),\sup T_B(x_i)],[\inf I_B(x_i),\sup I_B(x_i)],[\inf F_B(x_i),\sup F_B(x_i)]>|x_i\in X\}$. Then we define a measure between A and B by the following formula:

$$K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}=\frac{\sum_{i=1}^{n}C(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^{n}T(A(x_i)),\sum_{i=1}^{n}T(B(x_i))\right)} \tag{5}$$

where $C(A,B)$ means the correlation between two INSs A and B and $T(A)$, $T(B)$ refer to the information energies of the two INSs respectively. And they are given by

$$C(A(x_i),B(x_i))=\frac{1}{2}[\inf T_A(x_i)\cdot\inf T_B(x_i)+\sup T_A(x_i)\cdot\sup T_B(x_i)+\inf I_A(x_i)\cdot\inf I_B(x_i) \\ +\sup I_A(x_i)\cdot\sup I_B(x_i)+\inf F_A(x_i)\cdot\inf F_B(x_i)+\sup F_A(x_i)\cdot\sup F_B(x_i)], \quad (6)$$

$$T(A(x_i))=\frac{(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+(\inf I_A(x_i))^2+(\sup I_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2}{2}, \quad (7)$$

$$T(B(x_i))=\frac{(\inf T_B(x_i))^2+(\sup T_B(x_i))^2+(\inf I_B(x_i))^2+(\sup I_B(x_i))^2+(\inf F_B(x_i))^2+(\sup F_B(x_i))^2}{2}. \quad (8)$$

Theorem 1. The proposed measure $K(A,B)$ satisfies all the axioms given in Definition 11.

Proof.

(KP1) According to Definition 6, $[\inf T_A(x_i),\sup T_A(x_i)]$, $[\inf I_A(x_i),\sup I_A(x_i)]$, $[\inf F_A(x_i),\sup F_A(x_i)]$, $[\inf T_B(x_i),\sup T_B(x_i)]$, $[\inf I_B(x_i),\sup I_B(x_i)]$ and $[\inf F_B(x_i),\sup F_B(x_i)]\subseteq[0,1]$ exist for any $i\in\{1,2,\cdots,n\}$. Thus, it holds that $C(A,B)\ge 0$, $T(A)\ge 0$ and $T(B)\ge 0$. Therefore, $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}\ge 0$. According to the Cauchy–Schwarz inequality: $\left(a_1b_1+a_2b_2+\cdots+a_nb_n\right)^2\le\left(a_1^2+a_2^2+\cdots+a_n^2\right)\cdot\left(b_1^2+b_2^2+\cdots+b_n^2\right)$ where $a_i,b_i\in R$, $i=1,2,\cdots,n$, $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}\le 1$. Therefore, $0\le K(A,B)\le 1$ holds.

(KP2) According to Equation (6), we know that $C(A,B)=C(B,A)$, and it's obviously $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}=\frac{C(A,B)}{\max\left(T(B),T(A)\right)}=K(B,A)$.

(KP3) If $A=B$, we have that $\inf T_A(x_i)=\inf T_B(x_i)$, $\sup T_A(x_i)=\sup T_B(x_i)$, $\inf I_A(x_i)=\inf I_B(x_i)$, $\sup I_A(x_i)=\sup I_B(x_i)$, $\inf F_A(x_i)=\inf F_B(x_i)$ and $\sup F_A(x_i)=\sup F_B(x_i)$. So, $C(A,B)=\frac{1}{2}\sum_{i=1}^{n}[(\inf T_A(x))^2+(\sup T_A(x_i))^2+(\inf I_A(x_i))^2+(\sup I_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2]$ and $T(A)=T(B)=\frac{1}{2}\sum_{i=1}^{n}[(\inf T_A(x_i))^2+(\sup T_A(x_i))^2+(\inf I_A(x_i))^2+(\sup I_A(x_i))^2+(\inf F_A(x_i))^2+(\sup F_A(x_i))^2]$, i.e. $C(A,B)=T(A)=T(B)$. Thus, it is clear that $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}=1$.

If $K(A,B)=\frac{C(A,B)}{\max\left(T(A),T(B)\right)}=1$, then $C(A,B)=\max(T(A),T(B))$. According to the Cauchy–Schwarz inequality, $C(A,B) \le \sqrt{T(A)\cdot T(B)} \le \max(T(A),T(B))$. Thus, $C(A,B) = \sqrt{T(A)\cdot T(B)} = \max(T(A),T(B))$. If $C(A,B) = \sqrt{T(A)\cdot T(B)}$, there exists a nonzero real number η such that $\inf T_A(x_i) = \eta \inf T_B(x_i)$, $\sup T_A(x_i) = \eta \sup T_B(x_i)$, $\inf I_A(x_i) = \eta \inf I_B(x_i)$, $\sup I_A(x_i) = \eta \sup I_B(x_i)$, $\inf F_A(x_i) = \eta \inf F_B(x_i)$ and $\sup F_A(x_i) = \eta \sup F_B(x_i)$ for any $x_i \in X$. Besides, if $\sqrt{T(A)\cdot T(B)} = \max(T(A),T(B))$, $T(A) = T(B)$. Based on these two conditions, it is obvious that $\eta = 1$ (i.e. $A = B$).

Hence, the theorem 1 is true which means the measure $K(A,B)$ defined in Definition 12 is a correlation coefficient measure.

Property 1. $K(A,A)$ is the supremum of all $K(A,B)$, which in other words, $K(A,A) \ge K(A,B)$, $\forall A,B \in INS$.

Proof.

Property 1 is easy to yield from Property 1. According to Property 1, $0 \le K(A,B) \le 1$ and $K(A,A) = 1$. Thus, Property 1 is true.

Property 1 implies that the correlation coefficient between an INS and itself is always greater than or equal to the correlation coefficient between the INS and any other INS defined in the same universe.

Example 1. Assume $A = \{< x,[0.7,0.8],[0.0,0.1],[0.1,0.2] >\}$, and $B = \{< x,[0.4,0.5],[0.2,0.3],[0.3,0.4] >\}$, then $C(A,B) = 0.41$, $T(A) = 0.595$, $T(B) = 0.395$, thus, $K(A,B) = \frac{C(A,B)}{\max\left(T(A),T(B)\right)} = \frac{0.41}{\max(0.595,0.395)} = 0.689$.

3.2. The weighted correlation coefficient measure for INS

In Section 3.1, we proposed a correlation coefficient measure for INSs. However, this correlation coefficient measure does not take the relative importance of each INN in INSs into consideration. In many situations, different INNs may have different weights, such as MCDM [34, 74]. In the following, based on the correlation coefficient measure between INSs defined in Definition 12, the weighted correlation coefficient between INSs will be introduced.

Definition 13. Let $A=\{<x_i,[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]>\left|x_i \in X\right.\}$ and $B=\{<x_i,[\inf T_B(x_i),\sup T_B(x_i)],[\inf I_B(x_i),\sup I_B(x_i)],[\inf F_B(x_i),\sup F_B(x_i)]>\left|x_i \in X\right.\}$ be two INSs in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$. Let $w=\{w_1,w_2,\cdots,w_n\}$ be the weight vector of the elements $x_i\ (i=1,2,\cdots,n)$. Then we can define a measure between A and B by the following formula:

$$K(A,B)=\frac{\sum_{i=1}^{n} w_i C(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^{n} w_i T(A(x_i)),\sum_{i=1}^{n} w_i T(B(x_i))\right)} \tag{9}$$

where $C(A(x_i),B(x_i))$, $T(A(x_i))$ and $T(B(x_i))$ satisfy Equations (6)-(8).

Theorem 2. The proposed measure $K(A,B)$ in Definition 13 satisfies all the axioms given in Definition 11.

Proof.

$(P1)$ According to Property 1, $C(A(x_i),B(x_i))\geq 0$, $T(A(x_i))\geq 0$ and $T(B(x_i))\geq 0$ $(i=1,2,\cdots,n)$. Besides, $w_i\geq 0$, thus, $K(A,B)=\frac{\sum_{i=1}^{n} w_i C(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^{n} w_i T(A(x_i)),\sum_{i=1}^{n} w_i T(B(x_i))\right)}>0$. According to the Cauchy–Schwarz inequality, $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i))\leq \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))}\leq \max\left(\sum_{i=1}^{n} w_i T(A(x_i)),\sum_{i=1}^{n} w_i T(B(x_i))\right)$. Therefore, $K(A,B)\leq 1$.

$(P2)$ According to Property 1, we know that $C(A(x_i),B(x_i))=C(B(x_i),A(x_i))$ exists for any $i\in\{i,2,\cdots,n\}$. Therefore, it's obvious that $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i))=\sum_{i=1}^{n} w_i C(B(x_i),A(x_i))$. Thus,

$$K(A,B)=\frac{\sum_{i=1}^{n} w_i C(A(x_i),B(x_i))}{\max\left(\sum_{i=1}^{n} w_i T(A(x_i)),\sum_{i=1}^{n} w_i T(B(x_i))\right)}=\frac{\sum_{i=1}^{n} w_i C(B(x_i),A(x_i))}{\max\left(\sum_{i=1}^{n} w_i T(B(x_i)),\sum_{i=1}^{n} w_i T(A(x_i))\right)}=K(B,A).$$

(P3) According to Property 1, $C(A(x_i),B(x_i)) = \max(T(A(x_i)),T(B(x_i)))$ is true for any $i \in \{i,2,\cdots,n\}$ if $A=B$. Therefore, $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i)) = \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$ is proved to be right. Hence, if $A=B$, $K(A,B)=1$.

If $K(A,B)=1$, $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i)) = \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$. According to the Cauchy–Schwarz inequality: $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i)) \le \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))} \le \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$. Thus, $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i)) = \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))} = \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$. If $\sum_{i=1}^{n} w_i C(A(x_i),B(x_i)) = \sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))}$, there exists a nonzero real number η such that $\inf T_A(x_i) = \eta \inf T_B(x_i)$, $\sup T_A(x_i) = \eta \sup T_B(x_i)$, $\inf I_A(x_i) = \eta \inf I_B(x_i)$, $\sup I_A(x_i) = \eta \sup I_B(x_i)$, $\inf F_A(x_i) = \eta \inf F_B(x_i)$ and $\sup F_A(x_i) = \eta \sup F_B(x_i)$ for any $x_i \in X$. Besides, if $\sqrt{\sum_{i=1}^{n} w_i T(A(x_i))}\sqrt{\sum_{i=1}^{n} w_i T(B(x_i))} = \max\left(\sum_{i=1}^{n} w_i T(A(x_i)), \sum_{i=1}^{n} w_i T(B(x_i))\right)$, $T(A)=T(B)$. Based on these two conditions, it is obvious that $\eta = 1$ (i.e. $A=B$).

Thus, the theorem 2 holds which signifies the measure $K(A,B)$ defined by Equation (9) is a correlation coefficient measure. For convenience, we call it a weighted correlation coefficient measure.

Example 2. Assume $A = \{<x_1,[0.7,0.8],[0.0,0.1],[0.1,0.2]>,<x_2,[0.6,0.7],[0.1,0.2],[0.1,0.3]>\}$, and $B = \{<x_1,[0.4,0.5],[0.2,0.3],[0.3,0.4]>,<x_2,[0.4,0.6],[0.1,0.3],[0.2,0.4]>\}$, and $w=\{0.4,0.6\}$. Thus, $C(A(x_1),B(x_1)) = 0.41$, $C(A(x_2),B(x_2)) = 0.435$, $T(A(x_1)) = 0.595$, $T(A(x_2)) = 0.5$, $T(B(x_1)) = 0.395$, $T(B(x_2)) = 0.41$, so, $\sum_{i=1}^{2} w_i C(A(x_i),B(x_i)) = 0.425$, $\sum_{i=1}^{2} w_i T(A(x_i)) = 0.538$, $\sum_{i=1}^{2} w_i T(B(x_i)) = 0.404$. Thus, $K(A,B) = 0.790$.

As noted in Section 1, the integrated weight can benefit from not only decision makers' expertise but also the relative importance of evaluation information. In order to assess the relative importance or weights accurately and comprehensively, it's better to utilize the integrated weight rather than only the subjective weight or objective weight to obtain the weighted correlation coefficient here.

The subjective weight and the objective weight should be calculated in order to compute the integrated weight. The subjective weight mirroring the individual preference can be evaluated by the decision maker while the objective weight reflecting the relative importance contained in the decision matrix should be calculated by mathematical methods. Certainly, many kinds of objective weight measures have been proposed and every measure has its own advantages [79, 80]. Because of the fact that the more equivocal the information is, the less important it will be [81], we utilize the entropy weight measure to obtain the objective weight.

3.3 The entropy weight measure for INS

In this section, we propose the entropy measure and an objective weight measure based on entropy for INS.

Entropy is an important concept named after Claude Shannon who introduced the concept first. In information theory, entropy is a measure for calculating the uncertainty associated with a random variable. It characterizes the uncertainty about the source of information. Thus entropy is as a measure of uncertainty. Based on the axiomatic definition of entropy measure for SVNSs in [69], the entropy for INSs can be defined as follows.

Definition 14. A real function $E: INS(X) \rightarrow [0,1]$ is called entropy on INS(*X*), if *E* satisfies the following properties,

$(EP1)$ $E(A)=0$ (minimum) if A is a crisp set $(\forall A \in P(X))$;

($EP2$) $E(A)=1$ (maximum) if $T_A(x)=I_A(x)=F_A(x)$ (i.e. $\inf T_A(x)=\inf I_A(x)=\inf F_A(x)$ and $\sup T_A(x)=\sup I_A(x)=\sup F_A(x)$) for any $x\in X$;

($EP3$) $E(A)\le E(B)$ if A is less fuzzy than B or B is more uncertain than A, i.e. (1) $\inf T_A(x)-\inf F_A(x)\le \inf I_B(x)-\inf F_B(x)$ and $\sup T_A(x)-\sup F_A(x)\le \sup I_B(x)-\sup F_B(x)$ for $\inf T_A(x)\ge \inf F_A(x)$ and $\sup T_A(x)\le \sup F_A(x)$ or $\inf T_A(x)-\inf F_A(x)\ge \inf I_B(x)-\inf F_B(x)$ and $\sup T_A(x)-\sup F_A(x)\ge \sup I_B(x)-\sup F_B(x)$ for $\inf T_A(x)\le \inf F_A(x)$ and $\sup T_A(x)\le \sup F_A(x)$ and $\sup T_A(x)\ge \sup T_B(x)$; and (2) $\inf I_A(x)\le \inf I_B(x)$ and $\sup I_A(x)\le \sup I_B(x)$ for $\inf I_B(x)+\sup I_B(x)\le 1$ or $\inf I_A(x)\ge \inf I_B(x)$ and $\sup I_A(x)\ge \sup I_B(x)$ for $\inf I_B(x)+\sup I_B(x)\ge 1$;

($EP4$) $E(A)=E(A^c)$.

Great quantities of researches have demonstrated the connection among the distance measure, the similarity measure and the entropy measure of fuzzy sets [69-73]. According to these studies, we propose the entropy measure of INSs based on the distance measure defined in Definition 8.

Definition 15. Let A be an INS in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$, and assume that $E(A):N(X)\to[0,1]$. $E(A)$ is a measure such that

$$E(A)=1-d(A,A^c). \tag{10}$$

where $d(A,A^c)$ refers to the distance measure between INS A and its complementary set A^c utilizing Equation (2).

Theorem 3. The proposed measure $E(A)$ satisfies all the axioms given in Definition 14.

Proof. Let $A=\{<x_i,[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]>|x_i\in X\}$ and $B=\{<x_i,[\inf T_B(x_i),\sup T_B(x_i)],[\inf I_B(x_i),\sup I_B(x_i)],[\inf F_B(x_i),\sup F_B(x_i)]>|x_i\in X\}$ be two INSs.

($EP1$) If an INS A is a crisp set, i.e. $\inf T_A(x_i)=\sup T_A(x_i)=1$, $\inf I_A(x_i)=\sup I_A(x_i)=0$, $\inf F_A(x_i)=\sup F_A(x_i)=0$ or $\inf T_A(x_i)=\sup T_A(x_i)=0$, $\inf I_A(x_i)=\sup I_A(x_i)=1$,

$\inf F_A(x_i)=\sup F_A(x_i)=1$. According to Definition 7, we can calculate the complementary set of A, i.e. $\inf T_{A^c}(x_i)=\sup T_{A^c}(x_i)=0$, $\inf I_{A^c}(x_i)=\sup I_{A^c}(x_i)=1$, $\inf F_{A^c}(x_i)=\sup F_{A^c}(x_i)=1$ or $\inf T_{A^c}(x_i)=\sup T_{A^c}(x_i)=1$, $\inf I_{A^c}(x_i)=\sup I_{A^c}(x_i)=0$, $\inf F_{A^c}(x_i)=\sup F_{A^c}(x_i)=0$ respectively. Therefore, it's obvious that $E(A)=0$.

($EP2$) If $T_A(x_i)=I_A(x_i)=F_A(x_i)$ and $\inf T_A(x_i)+\sup T_A(x_i)=1$, according to Equation (10), the entropy can be calculated: $E(A)=1-\frac{1}{6n}\sum_{i=1}^{n}[|\inf T_A(x_i)-\inf F_A(x_i)|+|\sup T_A(x_i)-\sup F_A(x_i)|$ $+|\inf I_A(x_i)+\sup I_A(x_i)-1|+|\sup I_A(x_i)+\inf I_A(x_i)-1|+|\inf F_A(x_i)-\inf T_A(x_i)|+|\sup F_A(x_i)-\sup T_A(x_i)|]$ $=1$.

($EP3$) $\inf T_A(x_i)-\inf F_A(x_i)\le\inf I_B(x_i)-\inf F_B(x_i)$ and $\sup T_A(x_i)-\sup F_A(x_i)\le\sup I_B(x_i)-\sup F_B(x_i)$ for $\inf T_A(x_i)\ge\inf F_A(x_i)$ and $\sup T_A(x_i)\le\sup F_A(x_i)$ or $\inf T_A(x_i)-\inf F_A(x_i)\ge\inf I_B(x_i)-\inf F_B(x_i)$ and $\sup T_A(x_i)-\sup F_A(x_i)\ge\sup I_B(x_i)-\sup F_B(x_i)$ for $\inf T_A(x_i)\le\inf F_A(x_i)$ and $\sup T_A(x_i)\le\sup F_A(x_i)$ and $\sup T_A(x_i)\ge\sup T_B(x_i)$. Thus, it is obviously that $|\inf T_A(x)-\inf F_A(x)|+|\sup T_A(x_i)-\sup F_A(x_i)|\ge|\inf T_B(x_i)-\inf F_B(x_i)|+|\sup T_B(x_i)-\sup F_B(x_i)|$. $\inf I_A(x_i)\le\inf I_B(x_i)$ and $\sup I_A(x_i)\le\sup I_B(x_i)$ for $\inf I_B(x_i)+\sup I_B(x_i)\le 1$ or $\inf I_A(x_i)\ge\inf I_B(x_i)$ and $\sup I_A(x_i)\ge\sup I_B(x_i)$ for $\inf I_B(x_i)+\sup I_B(x_i)\ge 1$, so $\left|(\inf F_A(x_i)+\sup F_A(x_i))-1\right|\ge\left|(\inf F_B(x_i)+\sup F_B(x_i))-1\right|$. Therefore, $2\left[\left|\inf T_A(x_i)-\inf F_A(x_i)\right|+\left|\sup T_A(x_i)-\sup F_A(x_i)\right|+\left|\inf I_A(x_i)+\sup I_A(x_i)-1\right|\right]\ge 2\left[|\inf T_B(x_i)-\inf F_B(x_i)|\right.$ $\left.+\left|\sup T_B(x_i)-\sup F_B(x_i)\right|+\left|\inf I_B(x_i)+\sup I_B(x_i)-1\right|\right]$ $(i=1,2,\cdots,n)$. Thus, $E(A)\le E(B)$.

($EP4$) According to Equation (10), we can calculate $E(A)$ and $E(A^c)$ respectively:

$$E(A)=1-\frac{1}{6n}\times 2\times\sum_{i=1}^{n}[\left|\inf T_A(x_i)-\inf F_A(x_i)\right|+\left|\sup T_A(x_i)-\sup F_A(x_i)\right|+\left|\inf I_A(x_i)+\sup I_A(x_i)-1\right|].$$

$$E(A^c)=1-\frac{1}{6n}\times 2\times\sum_{i=1}^{n}[\left|\inf F_A(x_i)-\inf T_A(x_i)\right|+\left|\sup F_A(x_i)-\sup T_A(x_i)\right|+\left|1-(\inf I_A(x_i)+\sup I_A(x_i))\right|].$$

Therefore, it is clear that $E(A)=E(A^c)$.

Thus, Theorem 3 holds which indicates the measure put forward in Definition 15 is an entropy measure.

Example 3. Assume $A=\{<x,[0.7,0.8],[0.0,0.1],[0.1,0.2]>\}$, then $A^c=\{<x,[0.1,0.2],[0.9,1.0],[0.7,0.8]>\}$, and $E(A)=1-\frac{1}{6}\times 2\times[|0.7-0.1|+|0.8-0.2|+|0+0.1-1|]=0.3$.

In the following, based on the above entropy measure, we put forward an objective weight measure for INS called the entropy weight measure.

Entropy can be regarded as a measure of the uncertainty degree involved in a fuzzy set [42], and it reflects the objective information contained in the decision values. Thus, utilizing entropy as a vehicle to obtain the objective weight is reasonable. According to the entropy theory [19, 74], if a fuzzy set provide less uncertainty than other ones, it should be paid more attention. Therefore, the bigger weight should be assigned to the less uncertain fuzzy information in MCDM problems. Otherwise, the fuzzy information will be thought unimportant, which means its weight will be a smaller one.

According to these theories, an entropy weight measure is established to determine the objective weight under interval-valued neutrosophic environment:

$$H_j(A)=\frac{1-E\left(A\left(x_j\right)\right)}{n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)} \tag{11}$$

where A is an INS in the universe discourse $X=\{x_1,x_2,\cdots,x_n\}$, $A\left(x_j\right)=\left\{\left\langle[\inf T_A(x_i),\sup T_A(x_i)],[\inf I_A(x_i),\sup I_A(x_i)],[\inf F_A(x_i),\sup F_A(x_i)]\right\rangle\right\}$ and $E\left(A\left(x_j\right)\right)$ is calculated by Equation (10).

Property 2. The proposed weight measure satisfies the following properties:

($W1$) $H_j\left(A\right)\in[0,1]$;

($W2$) $\sum_{j=1}^{n}H_j\left(A\right)=1$.

Proof.

($W1$) Let $H=\left(H_1\left(A\right),H_2\left(A\right),\cdots,H_n\left(A\right)\right)$ be an entropy weight vector calculated according to Equation

(11). According to Theorem 3, we know that entropy value of INSs lies between 0 and 1, i.e. $E\left(A\left(x_j\right)\right)\in[0,1]$, thus, it's obvious that $1-E\left(A\left(x_j\right)\right)\in[0,1]$ and $n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)\in[0,1]$. Besides, $\left(1-E\left(A\left(x_j\right)\right)\right)+\left(n-1+\sum_{i=1,i\neq j}^{n}E\left(A\left(x_j\right)\right)\right)=n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)\geq 0$ and $\left(n-1+\sum_{i=1,i\neq j}^{n}E\left(A\left(x_j\right)\right)\right)\geq 0$ hold which means $n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)\geq\left(1-E\left(A\left(x_j\right)\right)\right)$ is true. Based on these conclusions, we can obtain that $H_j\left(A\right)=\dfrac{1-E\left(A\left(x_j\right)\right)}{n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)}\in[0,1]$.

(W2) It is clear that $\sum_{j=1}^{n}H_j\left(A\right)=\sum_{j=1}^{n}\dfrac{1-E\left(A\left(x_j\right)\right)}{n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)}=\dfrac{\sum_{j=1}^{n}1-E\left(A\left(x_j\right)\right)}{n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)}=\dfrac{n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)}{n-\sum_{j=1}^{n}E\left(A\left(x_j\right)\right)}=1$.

Therefore, Property 2 holds.

Example 4. Assume $A=\{<x_1,[0.7,0.8],[0.0,0.1],[0.1,0.2]>,<x_2,[0.4,0.5],[0.2,0.3],[0.3,0.4]>, <x_3,[0.6,0.7],[0.1,0.2],[0.1,0.3]>\}$. According to Equation (10), we can figure out that $E\left(A\left(x_1\right)\right)=0.3$, $E\left(A\left(x_2\right)\right)=0.767$ and $E\left(A\left(x_3\right)\right)=0.467$. And according to Equation (11), $H_1\left(A\right)=0.477$, $H_1\left(B\right)=0.159$ and $H_1\left(C\right)=0.364$.

Example 5. Assume that there are three INSs $A=\{<x_1,[0.4,0.5],[0.0,0.1],[0.3,0.4]>,<x_2,[0.6,0.7],[0.4,0.5],[0.1,0.3]>\}$, $B=\{<x_1,[0.7,0.8],[0.0,0.1],[0.1,0.2]>,<x_2,[0.2,0.4],[0.5,0.6],[0.2,0.4]>\}$ and $C=\{<x_1,[1,1],[0,0],[0,0]>,<x_2,[1,1],[0,0],[0,0]>\}$, and $w=(0.5,0.5)$ is the subjective weight vector. According to Equation (9), the weighted correlation coefficient based on the subjective weight can be calculated: $K(A,C)=0.55$ and $K(B,C)=0.525$. Therefore, $K(A,C)>K(B,C)$ is true which means that the relative similarity degree between A and C is more than that between B and C. What's more, according to Equation (11), the objective weight matrix can be obtained: $H_A=(0.52,0.48)$, $H_B=(0.95,0.05)$ and according to Equation (4), the integrated weight matrix is $W=\begin{bmatrix}0.52 & 0.48\\ 0.95 & 0.05\end{bmatrix}$.

According to Equation (9), the weighted correlation coefficient based on the subjective weight can be calculated: $K(A,C)=0.546$ and $K(B,C)=0.728$. Thus, $K(A,C)<K(B,C)$ is true which means that the relative similarity degree between A and C is less than that between B and C.

The above example shows that the relative similarity degree may be different when using two different kinds of weight. The cause lies in the fact that the subjective weight reflects only the preference of decision maker and ignores the objective information included in the decision matrix; by contrast, the integrated weight can benefit from not only decision makers' expertise but also the relative importance of evaluation information.

4. The weighted correlation coefficient application to multi-criteria decision making problems

In this section, we present a model for MCDM problems applying the weighted correlation coefficient measures for INSs and taking the integration of objective weight and subjective weight into account.

Assume there are m alternatives $A=\{A_1, A_2, \cdots, A_m\}$ and n criteria $C=\{C_1, C_2, \cdots, C_n\}$, whose subjective weight vector provided by the decision maker is $w=(w_1,w_2,\cdots,w_n)$, where $w_j \ge 0$ ($j=1,2,\cdots,n$), $\sum_{j=1}^{n} w_j = 1$. Let $R=(a_{ij})_{m\times n}$ be the interval neutrosophic decision matrix, where $a_{ij}=\langle T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}}\rangle$ is an evaluation value, denoted by INN, where $T_{a_{ij}}=[\inf T_{a_{ij}}, \sup T_{a_{ij}}]$ indicates the truth-membership function that the alternative A_i satisfies the criterion C_j, $I_{a_{ij}}=[\inf I_{a_{ij}}, \sup I_{a_{ij}}]$ indicates the indeterminacy-membership function that the alternative A_i satisfies the criterion C_j and $F_{a_{ij}}=[\inf F_{a_{ij}}, \sup F_{a_{ij}}]$ indicates the falsity-membership function that the alternative A_i satisfies the criterion C_j.

In MCDM environments, the concept of ideal point has been used to help identify the best alternative in the decision set [34]. An ideal alternative can be identified by using a maximum operator to determine the best value of each criterion among all alternatives [78]. Thus, we defined an ideal INN in the ideal alternative A^* as

$$\begin{aligned}\alpha_j^* &= \left\langle [a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*] \right\rangle \\ &= \left\langle [\max_i(a_{ij}), \max_i(b_{ij})], [\min_i(a_{ij}), \min_i(b_{ij})], [\min_i(a_{ij}), \min_i(b_{ij})] \right\rangle,\end{aligned} \tag{12}$$

where $i \in \{1, 2, \cdots, m\}$ and $j = 1, 2, \cdots, n$.

Based on Equation (10) and the integrated weight matrix $W = \begin{pmatrix} W_{11} & W_{12} & \cdots & W_{1n} \\ W_{21} & W_{22} & \cdots & W_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1} & W_{m2} & \cdots & W_{mn} \end{pmatrix}$ where W_{ij} is the integrated weight of alternative A_i under criterion C_j, we can denote the weighted correlation coefficient measure between the alternative A_i and the ideal alternative A^* as

$$K(A_i, A^*) = \frac{\sum_{j=1}^{n} W_{ij} [a_j^* \cdot (\inf T_{a_{ij}}) + b_j^* \cdot (\sup T_{a_{ij}}) + c_j^* \cdot (\inf I_{a_{ij}}) + d_j^* \cdot (\sup I_{a_{ij}}) + e_j^* \cdot (\inf F_{a_{ij}}) + f_j^* \cdot (\sup F_{a_{ij}})]}{\max\left(\sum_{j=1}^{n} W_{ij} T\left(A_i\left(x_j\right)\right), \sum_{j=1}^{n} W_{ij} [(a_j^*)^2 + (b_j^*)^2 + (c_j^*)^2 + (d_j^*)^2 + (e_j^*)^2 + (f_j^*)^2]\right)} \tag{13}$$

where $T\left(A_i\left(x_j\right)\right)$ can be obtained based on Equation (7).

The larger the value of the weighted correlation coefficient $K(A_i, A^*)$ is, the better the alternative A_i is, as the closer the alternative A_i is to the ideal alternative A^*. Therefore, all the alternatives can be ranked according to the value of the weighted correlation coefficients so that the best alternative can be selected. In the following, a procedure considering the integrated weight to rank and select the most desirable alternative(s) is proposed based upon the weighted correlation coefficient measure.

Step 1. Calculate the distance between the set $A_{ij} = \{a_{ij}\}$ formed by the rating value a_{ij} and its complementary set A_{ij}^c.

Utilizing Equation (2), the distance matrix $D=\begin{bmatrix} d_{nh}(A_{11},A_{11}^c) & d_{nh}(A_{12},A_{12}^c) & \cdots & d_{nh}(A_{1n},A_{1n}^c) \\ d_{nh}(A_{21},A_{21}^c) & d_{nh}(A_{22},A_{22}^c) & \cdots & d_{nh}(A_{2n},A_{2n}^c) \\ \vdots & \vdots & \ddots & \vdots \\ d_{nh}(A_{m1},A_{m1}^c) & d_{nh}(A_{m2},A_{m2}^c) & \cdots & d_{nh}(A_{mn},A_{mn}^c) \end{bmatrix}$ can be obtained.

Step 2. Calculate the entropy value of the set $A_{ij}=\{a_{ij}\}$.

According to Equation (10) and the distance matrix D, the entropy value matrix

$$E=\begin{bmatrix} E(A_{11}) & E(A_{12}) & \cdots & E(A_{1n}) \\ E(A_{21}) & E(A_{22}) & \cdots & E(A_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ E(A_{m1}) & E(A_{m2}) & \cdots & E(A_{mn}) \end{bmatrix}=\begin{bmatrix} 1-d_{nh}(A_{11},A_{11}^c) & 1-d_{nh}(A_{12},A_{12}^c) & \cdots & 1-d_{nh}(A_{1n},A_{1n}^c) \\ 1-d_{nh}(A_{21},A_{21}^c) & 1-d_{nh}(A_{22},A_{22}^c) & \cdots & 1-d_{nh}(A_{2n},A_{2n}^c) \\ \vdots & \vdots & \ddots & \vdots \\ 1-d_{nh}(A_{m1},A_{m1}^c) & 1-d_{nh}(A_{m2},A_{m2}^c) & \cdots & 1-d_{nh}(A_{mn},A_{mn}^c) \end{bmatrix}$$

can be calculated.

Step 3. Calculate the objective weight matrix H.

According to Equation (11) and the entropy value matrix E, it's easy to calculate the objective weight matrix

$$H=\begin{bmatrix} H(A_{11}) & H(A_{12}) & \cdots & H(A_{1n}) \\ H(A_{21}) & H(A_{22}) & \cdots & H(A_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ H(A_{m1}) & H(A_{m2}) & \cdots & H(A_{mn}) \end{bmatrix}=\begin{bmatrix} \frac{1-E(A_{11})}{\sum_{j=1}^{n}(1-E(A_{1j}))} & \frac{1-E(A_{12})}{\sum_{j=1}^{n}(1-E(A_{1j}))} & \cdots & \frac{1-E(A_{1n})}{\sum_{j=1}^{n}(1-E(A_{1j}))} \\ \frac{1-E(A_{21})}{\sum_{j=1}^{n}(1-E(A_{2j}))} & \frac{1-E(A_{22})}{\sum_{j=1}^{n}(1-E(A_{2j}))} & \cdots & \frac{1-E(A_{2n})}{\sum_{j=1}^{n}(1-E(A_{2j}))} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1-E(A_{m1})}{\sum_{j=1}^{n}(1-E(A_{mj}))} & \frac{1-E(A_{m2})}{\sum_{j=1}^{n}(1-E(A_{mj}))} & \cdots & \frac{1-E(A_{mn})}{\sum_{j=1}^{n}(1-E(A_{mj}))} \end{bmatrix}.$$

Step 4. Calculate the integrated weight matrix W.

According to Equation (4), the subjective weight $w=(w_1,w_2,\cdots,w_n)$ provided by the decision maker and the objective weight can be integrated and the integrated weight matrix is

$$W=\begin{bmatrix} W(A_{11}) & W(A_{12}) & \cdots & W(A_{1n}) \\ W(A_{21}) & W(A_{22}) & \cdots & W(A_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ W(A_{m1}) & W(A_{m2}) & \cdots & W(A_{mn}) \end{bmatrix}=\begin{bmatrix} \frac{w_1H_{11}}{\sum_{j=1}^{n}w_jH_{1j}} & \frac{w_2H_{12}}{\sum_{j=1}^{n}w_jH_{1j}} & \cdots & \frac{w_nH_{1n}}{\sum_{j=1}^{n}w_jH_{1j}} \\ \frac{w_1H_{21}}{\sum_{j=1}^{n}w_jH_{2j}} & \frac{w_2H_{22}}{\sum_{j=1}^{n}w_jH_{2j}} & \cdots & \frac{w_nH_{2n}}{\sum_{j=1}^{n}w_jH_{2j}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_1H_{m1}}{\sum_{j=1}^{n}w_jH_{mj}} & \frac{w_2H_{m2}}{\sum_{j=1}^{n}w_jH_{mj}} & \cdots & \frac{w_nH_{mn}}{\sum_{j=1}^{n}w_jH_{mj}} \end{bmatrix}.$$

Step 5. Calculate the ideal alternative A^*.

According to Equation (12), the ideal alternative A^* can be calculated.

Step 6. Calculate the weighted correlation coefficient between the alternative A_i and the ideal alternative A^*.

According to Equation (13) and the integrated weight matrix, the weighted correlation coefficient value between A_i and A^* can be obtained.

Step 7. Rank the alternatives depending on the weighted correlation coefficient value.

4. Illustrative example

4.1 Example of the weighted correlation coefficient measure for MCDM with INSs

In this section, an example for the multi-criteria decision making problem of alternatives is used as the demonstration of the application of the proposed decision making method, as well as the effectiveness of the proposed method.

Example 6. Let us consider the decision making problem adapted from [82]. There is a panel with four possible alternatives: A_1, A_2, A_3, A_4. The decision must be taken according to the following three criteria: C_1, C_2 and C_3. The weight vector of the criteria is given by $w=(0.35,0.25,0.4)$. The four possible alternatives are evaluated by a decision maker under the above three criteria. In order to reflect the reality more accurately and obtain more uncertainty information, we transform the evaluation values into INNs, as shown in the following interval neutrosophic decision matrix *D*:

$$D=\begin{bmatrix}\langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle\\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle\\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle\\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle\end{bmatrix}$$

Let the ideal alternative is $A^{*}=<[1,1],[0,0],[0,0]>$ Procedure of decision making based on INSs is as follow.

Step 1. Calculate the distance between the set $A_{ij}=\{a_{ij}\}$ formed by the rating value a_{ij} and its complementary set A_{ij}^{c}.

Utilizing Equation (2), the distance matrix is $D=\begin{bmatrix}0.23 & 0.33 & 0.40\\ 0.50 & 0.50 & 0.33\\ 0.23 & 0.30 & 0.37\\ 0.70 & 0.53 & 0.23\end{bmatrix}$.

Step 2. Calculate the entropy value of the set $A_{ij}=\{a_{ij}\}$.

According to Equation (10) and the distance matrix D, the entropy value matrix is $E=\begin{bmatrix}0.77 & 0.67 & 0.60\\ 0.50 & 0.50 & 0.67\\ 0.77 & 0.70 & 0.63\\ 0.30 & 0.47 & 0.77\end{bmatrix}$.

Step 3. Calculate the objective weight matrix H.

According to Equation (11) and the entropy value matrix E, it's easy to calculate the objective weight matrix $H=\begin{bmatrix}0.24 & 0.34 & 0.42\\ 0.376 & 0.376 & 0.248\\ 0.26 & 0.33 & 0.41\\ 0.48 & 0.36 & 0.16\end{bmatrix}$.

Step 4. Calculate the integrated weight matrix W.

According to Equation (4), the subjective weight $w=(0.35,0.25,0.4)$ provided by the decision maker and the objective weight can be integrated and the integrated weight matrix is $W=\begin{bmatrix}0.25 & 0.25 & 0.50\\ 0.405 & 0.29 & 0.305\\ 0.27 & 0.24 & 0.49\\ 0.52 & 0.28 & 0.20\end{bmatrix}$.

Step 5. Calculate the ideal alternative A^*.

According to Equation (12), we can obtain the following ideal alternative:

$A^*=\{\langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle,\langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle,\langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle\}$.

Step 6.Calculate the weighted correlation coefficient between the alternative A_i and the ideal alternative A^*.

According to Equation (12) and the integrated weight matrix, the weighted correlation coefficient value between A_i and A^* can be obtained, and $K(A_1,A^*)=0.9148$, $K(A_2,A^*)=0.899$, $K(A_3,A^*)=0.8517$, $K(A_4,A^*)=0.9219$.

Step 6. Rank the alternatives depending on the weighted correlation coefficient value.

Based on the steps above, the final order $A_4 \succ A_1 \succ A_2 \succ A_3$ is obtained. Obviously, A_4 is the best alternative in this example.

4.2. Comparison analysis and discussion

In order to validate the feasibility of the proposed decision-making method, a comparative study was conducted with other methods. The comparison analysis includes two cases. One is compared to the existing methods that were outlined in [82] and [27] using interval value neutrosophic information. In the other, the proposed method is compared to the methods using single valued neutrosophic information introduced in [19], [83] and [43].

Case 1. The proposed approach is compared with some methods using interval neutrosophic information.

With regard to the method in [27], the similarity measures were calculated and used to determine the final ranking order of all the alternatives first, and then two aggregation operators were developed in order to aggregate the interval neutrosophic information [82]. To solve the MCDM problem in Example 6, the results with different methods are shown in Table 1.

Table 1. The results of different methods with INSs.

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method 1 [27]	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1
Method 2 [27]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 3 [82]	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4	A_3
Method 4 [82]	$A_1 \succ A_4 \succ A_2 \succ A_3$	A_1	A_3
The proposed method	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4	A_3

From the results presented in Table 1, the best alternatives in [27] are A_4 and A_2 respectively. And the worst one is A_1. By contrast, the best ones are A_4 and A_1 respectively and the worst one is A_3 by using the methods in [82]. And regard to the proposed method in this paper, the best one is A_4 and A_3 is the worst one. There are a number of reasons why differences exist in the final rankings of all the compared methods and the proposed approach. Firstly, these different measures and aggregation operators also lead to different rankings and it is very difficult for decision-makers to confirm their judgments when using operators and measures that have similar characteristics. Secondly, the proposed method in this paper pays more attention to the impact that uncertainty has on the decision, and takes integrated weight into consideration. Furthermore, different aggregation operators lead to different rankings because the operators emphasize decision-makers' judgments differently. Method 3 in [82] is using interval neutrosophic number weighted averaging (INNWA) operator and method 4 in [82] is using interval neutrosophic number weighted geometric (INNWG) operator. The INNWA operator is based on arithmetic average and emphasizes group's major points while the INNWG operator emphasizes personal major points. That is the reason why results by method 3 and method 4 in [82] are different. By comparison, the proposed method in this paper focuses on weighted correlation coefficient measure which takes both the subjective weight and the subjective weight into consideration. Even so, the rank by the proposed method is as same as that of INNWG operator, which emphasizes personal major points. Therefore, the proposed method is effective.

Case 2. The proposed approach is compared with some methods using simplified neutrosophic information. The comparison results can be found in Table 2.

Table 2. The results of different methods with SVNSs.

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Method 5 [83]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 6 [19]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Method 7 [43]	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
The proposed method	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1

From the results presented in Table 2, the worst alternatives of [83], [19], [43] and the proposed method are same, i.e. A_1. The best alternatives of [83], [19] and [43] are both A_2, but the worst one of the proposed method is A_4. The reason why differences exist in the final rankings of the three compared methods and the proposed approach is given in the following part. As mentioned in Case 1, the proposed weighted correlation coefficient method not only considers the subjective weight which reflects the decision maker's subjective preference but also refers the objective weight which mirrors the objective information in the decision matrix.

It shows that the proposed method can also be used to MCDM problems with single valued neutrosophic set information.

From the comparison analysis presented above, it can be concluded that the proposed approach is more flexible and reliable in handling MCDM problems than the compared methods in the interval neutrosophic environment, which means that the approach developed in this paper has some advantages. Firstly, it can also be used to solve problems with the preference information that is expressed by INSs as well as SVNSs. Secondly, it unearths the deeper uncertainty information and utilizes them to make precise decision-making program. Besides, it is also capable for handling the multi-criteria decision making problems with completely unknown weight for criteria. In addition, it is utilitarian for the amount of computation can be evidently reduced with the assistance of programming software, such as MATLAB.

5. Conclusion

NS has been applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. Correlation coefficient measure is important in NS theory and entropy measure and it captures the uncertainty of NSs. Motivated by the correlation coefficient of IFSs, a new correlation coefficient measure for INSs satisfying the property that the value equals one if and only if two INSs are the same was proposed. Besides, the weighted correlation coefficient measure was extended and its property was developed. Moreover, the entropy measure of INSs was defined based on relationship among distance, similarity and entropy. As well, in order to obtain the integrated weight, we discussed an objective weight measure utilizing entropy for INSs and established the decision making procedure for MCDM problems. Furthermore, an illustrative example demonstrated the application of the proposed decision making method and comparative discussions showed that the proposed methods are appropriate and effective for dealing with MCDM problems.

The advantage of the proposed method is that it is simple and convenient in computing and it contributes to decreasing the loss of evaluation information. The feasibility and validity of the proposed approach have been verified through the illustrative example and comparison analysis. Therefore, this approach has much application potential in dealing with issues with the interval neutrosophic information in the environment of cluster analysis, artificial intelligence and other areas. What's more, the new correlation coefficient measure overcome the shortcoming that the correlation coefficient measure in [42] does not satisfy the property that the value equals one if and only if two INSs are the same. Moreover, this paper elaborates and demonstrates the standpoint that the uncertainty of evaluation has something to do with its importance and through combining the subjective weight and the objective weight can avoid the non-determinacy and arbitrariness resulted from subjective opinions. And based on these viewpoints, this paper takes further use of the uncertainty information and proposes a weighted correlation coefficient decision making method taking both the subjective weight and the objective weight into account which can be helpful to make wiser decision(s).

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