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Neutrosophic soft matrices and NSM-decision making

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Abstract. In this paper, we have firstly redefined the notion of neutrosophic soft set and its operations in a new way to handle the indeterminate information and inconsistent information which exists commonly in belief systems. Then, we defined neutrosophic soft matrix and their operators which are more functional to make theoretical studies and application in the neutrosophic soft set theory. The matrix is useful for storing a neutrosophic soft set in computer memory which are very useful and applicable. We finally construct a decision making method, called NSM-decision making, based on the neutrosophic soft sets.

Keywords: Soft sets, soft matrices, neutrosophic sets, neutrosophic soft sets, neutrosophic soft matrix, NSM-decision making

1. Introduction

In recent years, a number of theories have been proposed to deal with problems that contain uncertainties. Some theories such as probability set theory, fuzzy set theory [25], intuitionistic fuzzy set theory [24], interval valued intuitionistic fuzzy set theory [23], vague set theory [52], rough set theory [55] are consistently being utilized as efficient tools for dealing with diverse types of uncertainties. However, each of these theories have their inherent difficulties as pointed out by Molodtsov [7]. Later on, many interesting results of soft set theory have been obtained by embedding the idea of fuzzy set, intuitionistic fuzzy set, rough set and so on. For example, fuzzy soft sets [39], intuitionistic fuzzy soft set [31, 36], rough soft sets [9, 10] and interval valued intuitionistic fuzzy soft sets [49, 51, 54]. The theories have been developed in many directions and applied to wide variety of fields such as the soft decision makings [27, 50], the fuzzy soft decision makings [2, 32, 33, 56], the relation of fuzzy soft sets [6, 47] and the relation of intuitionistic fuzzy soft sets [5].

At present, researchers published several papers on fuzzy soft matrices and intuitionistic fuzzy soft matrices which have been applied in many fields, for instance [1, 17, 34]. Recently, Çağman et al. [28] introduced soft matrices and applied them in decision making problem. They also introduced fuzzy soft matrices [30]. Further, Saikia et al. [4] defined generalized fuzzy soft matrices with four different products of generalized intuitionstic fuzzy soft matrices and presented an application in medical diagnosis. Next, Broumi et al. [43] studied fuzzy soft matrix based on reference function and defined some new operations such fuzzy soft complement matrix on reference function. Also, Mondal et al. [18–20] introduced fuzzy and intuitionstic fuzzy soft matrices with multi criteria decision making based on three basic t-norm operators. The matrices have differently developed in many directions and applied to wide variety of fields in [3, 26, 40, 48].

The concept of neutrosophic set proposed by Smarandache [11] handles indeterminate data whereas fuzzy theory and intuitionistic fuzzy set theory failed when the relations are indeterminate. A neutrosophic

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set defined on universe of discourse, associates each element in the universe with three membership function: truth membership function, indeterminacy membership function and falsity membership function. In soft set theory, there is no limited condition to the description of objects; so researchers can choose the form of parameters they need, which greatly simplifies the decision making process more efficient in the absence of partial information.

The soft set is a mapping from parameter to the crisp subset of universe. The soft set theory is expanded by Maji [37] to a neutrosophic one in which the neutrosophic character of parameters in real world is taken into consideration. The concept of neutrosophic soft set is a parameterized family of all neutrosophic set of a universe and describes a collection of approximation of an object. Also, the neutrosophic soft sets are a generalization of fuzzy soft sets and intuitionistic fuzzy soft sets. The neutrosophic set theory has been developed in many directions and applied to wide variety of fields such as the eutrosophic soft sets [15, 38], the generalized neutrosophic soft sets [41], the intuitionistic neutrosophic soft sets [42], the interval valued neutrosophic set [12], the interval valued neutrosophic soft sets [13], the neutrosophic decision making problems [14, 16, 21, 22, 35, 44–46] and so on.

In this paper, our objective is to introduce the concept of neutrosophic soft matrices and their applications in decision making problem. The remaining part of this paper is organized as follows. Section 2 contains basic definitions and notations that are used in the remaining parts of the paper. In Section 3, we redefine neutrosophic soft set and some operations by taking inspiration from [29, 53] and compared our definitions of neutrosophic soft set with the definitions given by Maji [37]. In Section 4, we introduce the concept of neutrosophic matrices and present some of theirs basic properties. In Section 5, we present two special products of neutrosophic soft matrices. In Section 6, we present a soft decision making method, called neutrosophic soft matrix decision making (NSM-decision making) method, based on and-product of neutrosophic soft matrices. Finally, a conclusion is made in Section 7.

2. Preliminary

In this section, we give the basic definitions and results of neutrosophic set theory [11], soft set theory [7], soft matrix theory [28] and neutrosophic soft set theory [37] that are useful for subsequent discussions.

Definition 1. [11] Let E be a universe. A neutrosophic sets (NS) K in E is characterized by a truth-membership function T_K, an indeterminacy-membership function I_K and a falsity-membership function F_K. $T_K(x)$; $I_K(x)$ and $F_K(x)$ are real standard or non-standard elements of $]0^-, 1^+[$.

It can be written as

$$K = \{ < x, (T_K(x), I_K(x), F_K(x)) >: x \in E,\\ T_K(x), I_K(x), F_K(x) \in]^-0, 1[^+\}$$

There is no restriction on the sum of $T_K(x)$, $I_K(x)$ and $F_K(x)$, so $0^- \leq T_K(x) + I_K(x) + F_K(x) \leq 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard elements of $]\,0^-, 1^+[$. For application in real scientific and engineering areas, Wang et al. [53] gave the concept of an single valued neutrosophic set (SVNS), which is an instance of neutrosophic set. In the following, we propose the definition of SVNS.

Definition 2. [53] Let E be a universe. A single valued neutrosophic sets (SVNS) A, which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0, 1]. It can be written as

$$A = \{ < x, (T_A(x), I_A(x), F_A(x)) >: x \in E,\\ T_A(x), I_A(x), F_A(x) \in [0, 1]\}$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

As an illustration, let us consider the following example.

Example 1. Assume that the universe of discourse $U = \{x_1, x_2, x_3\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. They are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set (NS) of U, such that,

$$A = \{ < x_1, (0.3, 0.5, 0.4) >, < x_2, (0.1, 0.3, 0.6) >,\\ < x_3, (0.2, 0.4, 0.4) >\}$$

where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 3. [7] Let U be a universe, E be a set of parameters that describe the elements of U, and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping $f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \in E - A$ where f_A is called approximate function of the soft set F_A. In other words, the soft set is a parameterized family of subsets of the set U, and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \emptyset \text{ if } x \in E - A\}$$

The subscript A in the f_A indicates that f_A is the approximate function of F_A. The value $f_A(x)$ is a set called *x-element* of the soft set for every $x \in E$.

Definition 4. [8] t-norms are associative, monotonic and commutative two valued functions t that map from $[0, 1] \times [0, 1]$ into $[0, 1]$. These properties are formulated with the following conditions: $\forall a, b, c, d \in [0, 1]$,

1. $t(0, 0) = 0$ and $t(a, 1) = t(1, a) = a$,
2. If $a \leq c$ and $b \leq d$, then $t(a, b) \leq t(c, d)$
3. $t(a, b) = t(b, a)$
4. $t(a, t(b, c)) = t(t(a, b), c))$

For example; $t(a, b) = min\{a, b\}$

Definition 5. [8] t-conorms (s-norm) are associative, monotonic and commutative two placed functions s which map from $[0, 1] \times [0, 1]$ into $[0, 1]$. These properties are formulated with the following conditions: $\forall a, b, c, d \in [0, 1]$,

1. $s(1, 1) = 1$ *and* $s(a, 0) = s(0, a) = a$,
2. if $a \leq c$ and $b \leq d$, then $s(a, b) \leq s(c, d)$
3. $s(a, b) = s(b, a)$
4. $s(a, s(b, c)) = s(s(a, b, c)$

For example; $s(a, b) = max\{a, b\}$

3. On neutrosophic soft sets

The notion of the neutrosophic soft set theory is first given by Maji [37]. In this section, we have modified the definition of neutrosophic soft sets and operations as follows. Some of it is quoted from [5, 11, 29, 37].

Definition 6. Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, E be a set of parameters that are describing the elements of U Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping

$$f_N : E \rightarrow N(U)$$

where f_N is called an approximate function of the neutrosophic soft set N. For $x \in E$, the set $f_N(x)$ is called x-approximation of the neutrosophic soft set N which may be arbitrary, some of them may be empty and some may have a nonempty intersection. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $N(U)$, and therefore it can be written a set of ordered pairs,

$$N = \{ (x, \{< u, T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) > : x \in U\} : x \in E\}$$

where $T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) \in [0, 1]$

Definition 7. Let N_1 and N_2 be two neutrosophic soft sets. Then, the complement of a neutrosophic soft set N_1 denoted by N_1^c and is defined by

$$N_1{}^c = \{ (x, \{< u, F_{f_{N_1(x)}}(u), 1 - I_{f_{N_1(x)}}(u), T_{f_{N_1(x)}}(u) >: x \in U\} : x \in E\}$$

Definition 8. Let N_1 and N_2 be two neutrosophic soft sets. Then, the union of N_1 and N_2 is denoted by $N_3 = N_1 \tilde{\cup} N_2$ and is defined by

$$N_3 = \{ (x, \{< u, T_{f_{N_3(x)}}(u), I_{f_{N_3(x)}}(u), F_{f_{N_3(x)}}(u) >: x \in U\} : x \in E\}$$

where

$$T_{f_{N_3(x)}}(u) = s(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)),$$

$$I_{f_{N_3(x)}}(u) = t(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$$

and

$$F_{f_{N_3(x)}}(u) = t(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$$

Definition 9. Let N_1 and N_2 be two neutrosophic soft sets. Then, the intersection of N_1 and N_2 is denoted by $N_4 = N_1 \tilde{\cap} N_2$ and is defined by

$$N_4 = \{ (x, \{< u, T_{f_{N_4(x)}}(u), I_{f_{N_4(x)}}(u), F_{f_{N_4(x)}}(u) >: x \in U\} : x \in E\}$$

where

$$T_{f_{N_4(x)}}(u) = t(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)),$$

$$I_{f_{N_4(x)}}(u) = s(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$$

and

$$F_{f_{N_4(x)}}(u) = s(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$$

Proposition 1. *Let N_1, N_2 and N_3 be any three neutrosophic soft sets. Then,*

1. $N_1 \widetilde{\cup} N_2 = N_2 \widetilde{\cup} N_1$
2. $N_1 \widetilde{\cap} N_2 = N_2 \widetilde{\cap} N_1$
3. $N_1 \widetilde{\cup} (N_2 \widetilde{\cup} N_3) = (N_1 \widetilde{\cup} N_2) \widetilde{\cup} N_3$
4. $N_1 \widetilde{\cap} (N_2 \widetilde{\cap} N_3) = (N_1 \widetilde{\cap} N_2) \widetilde{\cap} N_3$

Proof. The proofs can be easily obtained since the t-norm and s-norm functions are commutative and associative.

3.1. Comparision of the Definitions

In this subsection, we compared our definitions of neutrosophic soft sets with the definitions given by Maji [37] by inspiring from [29].

Let us compare our definitions of neutrosophic soft sets with the definitions given Maji [37] in Table 1.

Let us compare our complement definitions of neutrosophic soft sets with the definition given by Maji [37] in Table 2.

Let us compare our union definitions of neutrosophic soft sets with the definition given by Maji [37] in Table 3.

Let us compare our intersection definitions of neutrosophic soft set with the definition given by Maji [37] in Table 4.

4. Neutrosophic soft matrices

In this section, we presented neutrosophic soft matrices (NS-matrices) which are representative of the neutrosophic soft sets. The matrix is useful for storing a neutrosophic soft set in computer memory which are very useful and applicable. Some of it is quoted from [28, 30, 48]. This section is an attempt to extend the concept of soft matrices matrices [28], fuzzy soft matrices [30] and intuitionistic fuzzy soft matrices [48].

Table 1
Definition of the neutrosophic soft sets

Our approach	Maji's approach
$N = \{(x, f_N(x)) : x \in E\}$	$N = \{(x, f_N(x)) : x \in A\}$
where	
E parameter set and	$A \subseteq E$
$f_N : E \to N(U)$	$f_N : A \to N(U)\}$

Table 2
Complement of the neutrosophic soft sets

Our approach	Maji's approach
N_1^c	N_1°
$f^c_{N_1} : E \to N(U)$	$f^\circ_{N_1} : \neg E \to N(U)$
$T_{f_{N_1^c(x)}}(u) = F_{f_{N_1(x)}}(u)$	$T_{f_{N_1^\circ(x)}}(u) = F_{f_{N_1(x)}}(u)$
$I_{f_{N_1^c(x)}}(u) = 1 - I_{f_{N_1(x)}}(u)$	$I_{f_{N_1^\circ(x)}}(u) = I_{f_{N_1(x)}}(u)$
$F_{f_{N_1^c(x)}}(u) = T_{f_{N_1(x)}}(u)$	$F_{f_{N_1^\circ(x)}}(u) = T_{f_{N_1(x)}}(u)$

Definition 10. Let N be a neutrosophic soft set over $N(U)$. Then a subset of $N(U) \times E$ is uniquely defined by

$R_N = \{(f_N(x), x) : x \in E,\ f_N(x) \in N(U)\}$ which is called a relation form of (N, E). The characteristic function of R_N is written by

$$\Theta_{R_N} : N(U) \times E \to [0, 1] \times [0, 1] \times [0, 1],$$

$$\Theta_{R_N}(u, x) = (T_{f_N(x)}(u), I_{f_N(x)}(u), F_{f_N(x)}(u))$$

where $T_{f_N(x)}(u)$, $I_{f_N(x)}(u)$ and $F_{f_N(x)}(u)$ are the truth-membership, indeterminacy-membership and falsity-membership of $u \in U$, respectively.

Definition 11. Let $U = \{u_1, u_2, \ldots, u_m\}$, $E = \{x_1, x_2, \ldots, x_n\}$ and N be a neutrosophic soft set over $N(U)$. Then

R_N	$f_N(x_1)$	$f_N(x_2)$	$\cdots$	$f_N(x_n)$
u_1	$\Theta_{R_N}(u_1, x_1)$	$\Theta_{R_N}(u_1, x_2)$	$\cdots$	$\Theta_{R_N}(u_1, x_n)$
u_2	$\Theta_{R_N}(u_2, x_1)$	$\Theta_{R_N}(u_2, x_2)$	$\cdots$	$\Theta_{R_N}(u_2, x_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\Theta_{R_N}(u_m, x_1)$	$\Theta_{R_N}(u_m, x_2)$	$\cdots$	$\Theta_{R_N}(u_m, x_n)$

If $a_{ij} = \Theta_{R_N}(u_i, x_j)$, we can define a matrix

$$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

such that

$$a_{ij} = (T_{f_N(x_j)}(u_i), I_{f_N(x_j)}(u_i), F_{f_N(x_j)}(u_i)) = (T^a_{ij}, I^a_{ij}, F^a_{ij})$$

which is called an $m \times n$ neutrosophic soft matrix (or namely NS-matrix) of the neutrosophic soft set N over $N(U)$.

According to this definition, a neutrosophic soft set N is uniquely characterized by matrix $[a_{ij}]_{m \times n}$. Therefore, we shall identify any neutrosophic soft set with its soft NS-matrix and use these two concepts as interchangeable. The set of all $m \times n$ NS-matrix over $N(U)$ will be denoted by $\widetilde{N}_{m \times n}$. From now on we shall delete th subscripts $m \times n$ of $[a_{ij}]_{m \times n}$, we use $[a_{ij}]$

Table 3
Union of the neutrosophic soft sets

Our approach	Maji's approach
$N_3 = N_1 \tilde{\cup} N_2$ $f_{N_3} : E \to N(U)$ *where*	$N_3 = N_1 \acute{\cup} N_2$ $f_{N_3(x)} : A \to N(U)$
$T_{f_{N_3(x)}}(u) = s(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u))$	$T_{f_{N_3(x)}}(u) = \begin{cases} T_{f_{N_1(x)}}(u), & x \in A - B \\ T_{f_{N_2(x)}}(u), & x \in B - A \\ max\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}, & x \in A \cap B \end{cases}$
$I_{f_{N_3(x)}}(u) = t(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$	$I_{f_{N_3(x)}}(u) = \begin{cases} I_{f_{N_1(x)}}(u), & x \in A - B \\ I_{f_{N_2(x)}}(u), & x \in B - A \\ \frac{(I_{f_{N_1(x)}}(u) + I_{f_{N_2(x)}}(u))}{2}, & x \in A \cap B \end{cases}$
$F_{f_{N_3(x)}}(u) = t(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$	$F_{f_{N_3(x)}}(u) = \begin{cases} F_{f_{N_1(x)}}(u), & x \in A - B \\ F_{f_{N_2(x)}}(u), & x \in B - A \\ min\{I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)\}, & x \in A \cap B \end{cases}$

Table 4
Intersection of the neutrosophic soft sets

Our approach	Maji's approach
$N_3 = N_1 \tilde{\cap} N_2$	$N_3 = N_1 \acute{\cap} N_2$
$f_{N_3} : E \to N(U)$	$f_{N_3(x)} : A \to N(U)$
where	
$T_{f_{N_3(x)}}(u) = t(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u))$	$T_{f_{N_3(x)}}(u) = min\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}$
$I_{f_{N_3(x)}}(u) = s(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$	$I_{f_{N_3(x)}}(u) = \frac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}$
$F_{f_{N_3(x)}}(u) = s(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$	$F_{f_{N_3(x)}}(u) = max\{F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)\}$

instead of $[a_{ij}]_{m \times n}$, since $[a_{ij}] \in \widetilde{N}_{m \times n}$ means that $[a_{ij}]$ is an $m \times n$ NS-matrix for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Example 2. Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$. N_1 be a neutrosophic soft sets over neutrosophic as

$$N = \Big\{ (x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >\}), (x_2, \{< u_1, (0.5, 0.7, 0.8) >, < u_2, (0.5, 0.9, 0.3) >, < u_3, (0.5, 0.6, 0., 8) >\}), (x_3, \{< u_1, (0.8, 0.6, 0.9) >, < u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.4) >\}) \Big\}$$

Then, the NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.5, 0.7, 0.8) & (0.8, 0.6, 0.9) \\ (0.4, 0.2, 0.8) & (0.5, 0.9, 0.3) & (0.5, 0.9, 0.9) \\ (0.9, 0.1, 0.5) & (0.5, 0.6, 0.8) & (0.7, 0.5, 0.4) \end{bmatrix}$$

Definition 12. Let $[a_{ij}] \in \widetilde{N}_{m \times n}$. Then $[a_{ij}]$ is called

1. A zero NS-matrix, denoted by $[\tilde{0}]$, if $a_{ij} = (0, 1, 1)$ for all i and j.
2. A universal NS-matrix, denoted by $[\tilde{1}]$, if $a_{ij} = (1, 0, 0)$ for all i and j.

Example 3. Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$. Then, a zero NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \\ (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \\ (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \end{bmatrix}.$$

and a universal NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (1, 0, 0) & (1, 0, 0) & (1, 0, 0) \\ (1, 0, 0) & (1, 0, 0) & (1, 0, 0) \\ (1, 0, 0) & (1, 0, 0) & (1, 0, 0) \end{bmatrix}.$$

Definition 13. Let $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m \times n}$. Then

1. $[a_{ij}]$ is an NS-submatrix of $[b_{ij}]$, denoted, $[a_{ij}] \tilde{\subseteq} [b_{ij}]$, if $T^b_{ij} \geq T^a_{ij}$, $I^a_{ij} \geq I^b_{ij}$ and $F^a_{ij} \geq F^b_{ij}$, for all i and j.
2. $[a_{ij}]$ is a proper NS-submatrix of $[b_{ij}]$, denoted, $[a_{ij}] \tilde{\subset} [b_{ij}]$, if $T^a_{ij} \geq T^b_{ij}$, $I^a_{ij} \leq I^b_{ij}$ and $F^a_{ij} \leq F^b_{ij}$ for at least $T^a_{ij} > T^b_{ij}$ and $I^a_{ij} < I^b_{ij}$ and $F^a_{ij} < F^b_{ij}$ for all i and j.
3. $[a_{ij}]$ and $[b_{ij}]$ are IFS equal matrices, denoted by $[a_{ij}] = [b_{ij}]$, if $a_{ij} = b_{ij}$ for all i and j.

Definition 14. Let $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. Then

1. Union of $[a_{ij}]$ and $[b_{ij}]$, denoted, $[a_{ij}]\tilde{\cup}[b_{ij}]$, if $c_{ij} = (T^c_{ij}, I^c_{ij}, F^c_{ij})$, where $T^c_{ij} = \max\{T^a_{ij}, T^b_{ij}\}$, $I^c_{ij} = \min\{I^a_{ij}, I^b_{ij}\}$ and $F^c_{ij} = \min\{F^a_{ij}, F^b_{ij}\}$ for all i and j.
2. Intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted, $[a_{ij}]\tilde{\cap}[b_{ij}]$, if $c_{ij} = (T^c_{ij}, I^c_{ij}, F^c_{ij})$, where $T^c_{ij} = \min\{T^a_{ij}, T^b_{ij}\}$, $I^c_{ij} = \max\{I^a_{ij}, I^b_{ij}\}$ and $F^c_{ij} = \max\{F^a_{ij}, F^b_{ij}\}$ for all i and j.
3. Complement of $[a_{ij}]$, denoted by $[a_{ij}]^c$, if $c_{ij} = (F^a_{ij}, 1 - I^a_{ij}, T^a_{ij})$ for all i and j.

Definition 15. Let $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. Then $[a_{ij}]$ and $[b_{ij}]$ are disjoint, if $[a_{ij}]\tilde{\cap}[b_{ij}] = [\tilde{0}]$ for all i and j.

Proposition 2. *Let* $[a_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

1. $\left([a_{ij}]^c\right)^c = [a_{ij}]$
2. $[\tilde{0}]^c = [\tilde{1}]$.

Proposition 3. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

1. $[a_{ij}] \subseteq [\tilde{1}]$
2. $[\tilde{0}]\tilde{\subseteq}[a_{ij}]$
3. $[a_{ij}]\tilde{\subseteq}[a_{ij}]$
4. $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ and $[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow [a_{ij}]\tilde{\subseteq}[c_{ij}]$

Proposition 4. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

1. $[a_{ij}] = [b_{ij}]$ and $[b_{ij}] = [c_{ij}] \Leftrightarrow [a_{ij}] = [c_{ij}]$
2. $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ and $[b_{ij}]\tilde{\subseteq}[a_{ij}] \Leftrightarrow [a_{ij}] = [b_{ij}]$

Proposition 5. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

1. $[a_{ij}]\tilde{\cup}[a_{ij}] = [a_{ij}]$
2. $[a_{ij}]\tilde{\cup}[\tilde{0}] = [a_{ij}]$
3. $[a_{ij}]\tilde{\cup}[\tilde{1}] = [\tilde{1}]$
4. $[a_{ij}]\tilde{\cup}[b_{ij}] = [b_{ij}]\tilde{\cup}[a_{ij}]$
5. $([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cup}[c_{ij}] = [a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cup}[c_{ij}])$

Proposition 6. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

1. $[a_{ij}]\tilde{\cap}[a_{ij}] = [a_{ij}]$
2. $[a_{ij}]\tilde{\cap}[\tilde{0}] = [\tilde{0}]$
3. $[a_{ij}]\tilde{\cap}[\tilde{1}] = [a_{ij}]$
4. $[a_{ij}]\tilde{\cap}[b_{ij}] = [b_{ij}]\tilde{\cap}[a_{ij}]$
5. $([a_{ij}]\tilde{\cap}[b_{ij}])\tilde{\cap}[c_{ij}] = [a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cap}[c_{ij}])$

Proposition 7. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. *Then De Morgan's laws are valid*

1. $([a_{ij}]\tilde{\cup}[b_{ij}])^c = [a_{ij}]^c\tilde{\cap}[b_{ij}]^c$
2. $([a_{ij}]\tilde{\cap}[b_{ij}])^c = [a_{ij}]^c\tilde{\cup}[b_{ij}]^c$

Proof. *i.*

$$\begin{aligned}([a_{ij}]\tilde{\cup}[b_{ij}])^c &= ([(T^a_{ij}, I^a_{ij}, F^a_{ij})]\tilde{\cup}[(T^b_{ij}, I^b_{ij}, F^b_{ij})])^c\\ &= [(\max\{T^a_{ij}, T^b_{ij}\}, \min\{I^a_{ij}, I^b_{ij}\},\\ &\quad \min\{F^a_{ij}, F^b_{ij}\})]^c\\ &= [(\min\{F^a_{ij}, F^b_{ij}\}, \max\{1 - I^a_{ij},\\ &\quad 1 - I^b_{ij}\}, \max\{T^a_{ij}, T^b_{ij}\}))]\\ &= [(F^a_{ij}, I^a_{ij}, T^a_{ij})]\tilde{\cap}[(F^b_{ij}, I^b_{ij}, T^b_{ij})]\\ &= [a_{ij}]^c\tilde{\cap}[b_{ij}]^c\end{aligned}$$

i.

$$\begin{aligned}([a_{ij}]\tilde{\cap}[b_{ij}])^c &= ([(T^a_{ij}, I^a_{ij}, F^a_{ij})]\tilde{\cap}[(T^b_{ij}, I^b_{ij}, F^b_{ij})])^c\\ &= [(\min\{T^a_{ij}, T^b_{ij}\}, \max\{I^a_{ij}, I^b_{ij}\},\\ &\quad \max\{F^a_{ij}, F^b_{ij}\})]^c\\ &= [(\max\{F^a_{ij}, F^b_{ij}\}, \min\{1 - I^a_{ij},\\ &\quad 1 - I^b_{ij}\}, \min\{T^a_{ij}, T^b_{ij}\}))]\\ &= [(F^a_{ij}, I^a_{ij}, T^a_{ij})]\tilde{\cup}[(F^b_{ij}, I^b_{ij}, T^b_{ij})]\\ &= [a_{ij}]^c\tilde{\cup}[b_{ij}]^c\end{aligned}$$

Proposition 8. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

1. $[a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cup}[c_{ij}]) = ([a_{ij}]\tilde{\cap}([b_{ij}])\tilde{\cup}([a_{ij}]\tilde{\cap}[c_{ij}])$
2. $[a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cap}[c_{ij}]) = ([a_{ij}]\tilde{\cup}([b_{ij}])\tilde{\cap}([a_{ij}]\tilde{\cup}[c_{ij}])$

5. Products of NS-matrices

In this section, we define two special products of NS-matrices to construct soft decision making methods.

Definition 16. Let $[a_{ij}], [b_{ik}] \in \widetilde{N}_{m\times n}$. Then, *And*-product of $[a_{ij}]$ and $[b_{ij}]$ is defined by

$$\wedge : \widetilde{N}_{m\times n} \times \widetilde{N}_{m\times n} \to \widetilde{N}_{m\times n^2}$$

$$[a_{ij}] \wedge [b_{ik}] = [c_{ip}] = (T^c_{ip}, I^c_{ip}, F^c_{ip})$$

where
$T^c_{ip} = t(T^a_{ij}, T^b_{jk})$, $I^c_{ip} = s(I^a_{ij}, I^b_{jk})$ and
$F^c_{ip} = s(F^a_{ij}, F^b_{jk})$ such that $p = n(j - 1) + k$

Definition 17. Let $[a_{ij}], [b_{ik}] \in \widetilde{N}_{m\times n}$. Then, *And*-product of $[a_{ij}]$ and $[b_{ij}]$ is defined by

$$\vee : \widetilde{N}_{m\times n} \times \widetilde{N}_{m\times n} \to \widetilde{N}_{m\times n^2}$$

$$[a_{ij}] \vee [b_{ik}] = [c_{ip}] = (T^c_{ip}, I^c_{ip}, F^c_{ip})$$

where

$T^c_{ip} = s(T^a_{ij}, T^b_{jk})$, $I^c_{ip} = t(I^a_{ij}, I^b_{jk})$ and

$F^c_{ip} = t(F^a_{ij}, F^b_{jk})$ such that $p = n(j-1)+k$

Example 4. Assume that $[a_{ij}], [b_{ik}] \in \widetilde{N}_{3\times 2}$ are given as follows

$$[a_{ij}] = \begin{bmatrix} (1.0, 0.1, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.2, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

$$[b_{ij}] = \begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.5, 0.1) \end{bmatrix}$$

$[a_{ij}] \wedge [b_{ij}] =$

$$\begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) & (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

$[a_{ij}] \vee [b_{ij}] =$

$$\begin{bmatrix} (1.0, 0.1, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.4, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.2, 0.1) & (1.0, 0.2, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.8, 0.1) & (1.0, 0.5, 0.1) & (1.0, 0.5, 0.1) \end{bmatrix}$$

Proposition 9. *Let $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. Then the De morgan's types of results are true.*

1. $([a_{ij}] \vee [b_{ij}])^c = [a_{ij}]^c \wedge [b_{ij}]^c$
2. $([a_{ij}] \wedge [b_{ij}])^c = [a_{ij}]^c \vee [b_{ij}]^c$

6. NSM-decision making

In this section, we present a soft decision making method, called neutrosophic soft matrix decision making (NSM-decision making) method, based on the and-product of neutrosophic soft matrices. The definitions and application on soft set defined in [28] are extended to the case of neutrosophic soft sets.

Definition 18. Let $[(\mu_{ip}, \nu_{ip}, w_{ip})] \in NSM_{m\times n^2}$, $I_k = \{p : \exists_i, (\mu_{ip}, \nu_{ip}, w_{ip}) \neq (0,0,0), 1 \leq i \leq m, (k-1)n < p \leq kn\}$ for all $k \in I = \{1, 2, ..., n\}$. Then NS-max-min-min decision function, denoted D_{Mmm}, is defined as follows

$$D_{Mmm} : NSM_{m\times n^2} \to NSM_{m\times 1},$$

For $t_{ik} = (\mu_{ik}, \nu_{ik}, w_{ik})$

$$\begin{aligned} D_{Mmm} &= Mmm[(\mu_{ip}, \nu_{ip}, w_{ip})] = [d_{i1}] \\ &= [(\max_k\{\mu_{ik}\}, \min_k\{\nu_{ik}\}, \min_k\{w_{ik}\})] \end{aligned}$$

where

$$\mu_{ik} = \begin{cases} \min_{p\in I_k}\{\mu_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

$$\nu_{ik} = \begin{cases} \max_{p\in I_k}\{\nu_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

$$w_{ik} = \begin{cases} \max_{p\in I_k}\{w_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

The one column NS-matrix $Mmm[(\mu_{ip}, \nu_{ip}, w_{ip})]$ is called max-min-min decision NS-matrix.

Definition 19. Let $U = \{u_1, u_2, u_3, u_m\}$ be the universe and $D_{Mmm}(\mu_{ip}, \nu_{ip}, w_{ip}) = [d_{i1}]$. Then the set defined by

$$opt^m_{[d_{i1}]}(U) = \{u_i/d_i : u_i \in U, d_i = max\{s_i\}\},$$

where $s_i = \frac{1}{3}(2 + \mu_{ij} - \nu_{ij} - w_{ij})$ (denotes the score function proposed by Ye. J in [21]) which is called an optimum fuzzy set on U.

The algorithm for the solution is given below;

Algorithm

Step 1: Choose feasible subset of the set of parameters.

Step 2: Construct the neutrosophic soft matrices for each parameter.

Step 3: Choose a product of the neutrosophic soft matrices.

Step 4: Find the method max-min-min decision NS-matrices.

Step 5: Find an optimum fuzzy set on U.

Remark 1. We can also define NS-matrices min-max-min decision making methods. One of them may be more useful than the others according to the type of problem.

Case study: Assume that a car dealer stores three different types of cars $U = \{u_1, u_2, u_3\}$ which may be characterize by the set of parameters $E = \{e_1, e_2\}$ where e_1 stands for costly, e_2 stands for fuel efficiency. Then we consider the following example. Suppose a couple Mr. X and Mrs. X come to the dealer to buy a car. If partners have to consider his/her set of parameters, then we select the car on the basis of partner's parameters by using NS-matrices max-min-min decision making as follow.

Step 1: First Mr. X and Mrs. X have to chose the sets of their parameter $E = \{e_1, e_2\}$.

Step 2: Then, we construct the NS-matrices $[a_{ij}]$ and $[b_{ij}]$ according to their set of parameter E as follow:

$$[a_{ij}] = \begin{bmatrix} (1.0, 0.1, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.2, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

and

$$[b_{ij}] = \begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.5, 0.1) \end{bmatrix}$$

Step 3: Now, we can find the And-product of the NS-matrices $[a_{ij}]$ and $[b_{ij}]$ as follow:

$[a_{ij}] \wedge [b_{ij}] =$

$$\begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) & (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

Step 4: Now, to calculate $[d_{i1}]$ we have to d_{i1} for all $i \in \{1, 2, 3\}$. To demonstrate, let us find d_{21}. Since $i = 2$ and $k \in \{1, 2\}$, $d_{21} = (\mu_{21}, \nu_{21}, w_{21})$

Let $t_{2k} = \{t_{21}, t_{22}\}$, where $t_{2k} = (\mu_{2p}, \nu_{2p}, w_{2p})$ then, we have to find t_{2k} for all $k \in \{1, 2\}$. First to find t_{21}, $I_1 = \{p : 0 < p \leq 2\}$ for $k = 1$ and $n = 2$ we have $t_{21} = (min\{\mu_{2p}\}, max\{\nu_{2p}\}, max\{w_{2p}\})$

In here for $p \in \{1, 2\}$ we have

$$\begin{aligned} &(min\{\mu_{21}, \mu_{22}\}, max\{\nu_{21}, \nu_{22}\}, max\{w_{21}, w_{22}\}) \\ &= (min\{1, 1\}, max\{0.5, 0.2\}, max\{0.1, 0.1\}) \\ &= (1, 0.5, 0.1) \end{aligned}$$

Similarly we can find as $t_{22} == (1, 0.5, 0.1)$

Similarly, we can find $d_{11} = (1, 0.7, 0.1)$, $d_{31} = (1, 0.8, 0.1)$,

$$[d_{i1}] = \begin{bmatrix} (1, 0.7, 0.1) \\ (1, 0.5, 0.1) \\ (1, 0.8, 0.1) \end{bmatrix}$$

$$max[s_i] = \begin{bmatrix} 0.73 \\ 0.80 \\ 0.70 \end{bmatrix}$$

where $s_i = \frac{1}{3}(2 + \mu_{ij} - \nu_{ij} - w_{ij})$ denotes the score function proposed by Ye. J in [21]

Step 5: Finally , we can find an optimum fuzzy set on U as:

$$opt^2_{[d_{i1}]}(U) = \{u_1/0.73, u_2/0.80, u_3/0.70\}$$

Thus u_2 has the maximum value. Therefore the couple may decide to buy the car u_2.

7. Conclusion

In this paper, we redefined the operations of neutrosophic soft sets and neutrosophic soft matrices. We also construct NSM-decision making method based on the neutrosophic soft sets with an example.

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