Rough sets in Fuzzy Neutrosophic approximation space

C. Antony Crispin Sweety* & I. Arockiarani*

* Department of Mathematics

Nirmala College for Women, Coimbatore-18.

Email: riosweety@gmail.com

Abstract

A rough set is a formal approximation of a crisp set which gives lower and upper approximation of original set to deal with uncertainties. The concept of neutrosophic set is a mathematical tool for handling imprecise, indeterministic and inconsistent data. In this paper, we introduce the concepts of Rough Fuzzy Neutrosophic Sets and Fuzzy Neutrosophic Rough Sets and investigate some of their properties. Further as the characterisation of fuzzy neutrosophic rough approximation operators, we introduce various notion of cut sets of neutrosophic fuzzy sets.

1 Introduction

Rough set theory is a [9], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. Moreover, it is a mathematical tool for machine learning, information sciences and expert systems and successfully applied in data analysis and data mining. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of rough set. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set . In classical rough set theory partition or equilence relation is the basic concept. Now fuzzy sets are combined with rough sets in a fruitful way and defined by rough fuzzy sets and fuzzy rough sets [5,6]. Also fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vague sets are introduced. The theory of rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe. One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache. Neutrosophic sets described by three functions: Truth function indeterminacy function and false function that are independently related. The theories of neutrosophic set have achieved great success in various areas such as medical diagnosis, database, topology, image processing, and decision making problem. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness. Neutrosophic sets and rough sets are two different topics, none conflicts the other. Recently many researchers applied the notion of neutrosophic sets to relations, group theory, ring theory, Soft set theory and so on. In this paper we combine the mathematical tools fuzzy sets, rough sets and neutrosophic sets and introduce a new class of set called fuzzy neutrosophic rough sets. Here we give rough approximation of a fuzzy neutrosophic set and introduce fuzzy neutrosophic rough sets

2 Preliminaries

Definition 2.1[1]

A Neutrosophic set A on the universe of discourse X is defined as
A $= \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \ / \ x \in X \}$ where
$T, I, F : X \longrightarrow \ ] \, 0^-, 1^+ \, [$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2[1]

A neutrosophice set A is contained in another neutrosophic set B (ie)
$A \subseteq B \Leftrightarrow T_A(x) \leq T_B(x), \ I_A(x) \leq I_B(x), \ F_A(x) \geq F_B(x)$.

Definition 2.3[1]

A fuzzy Neutrosophic set A on the universe of discourse X is defined as
$A = \{\langle \ x, T_A(x), I_A(x), F_A(x) \rangle, x \in X\}$ where $T, F, I : X \longrightarrow [0, 1]$ and
$0 \leq T_A(x) + I_A(x) + F_A(x) \leq B$.

Definition 2.4[1]

If $A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle \ /x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x) \rangle \ /x \in X\}$ are any two fuzzy neutrosophic sets of X then

(i) $A \subseteq B \ \leftrightarrow T_A(x) \leq T_B(x); \ I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$

(ii) $A = B \ \leftrightarrow T_A(x) = T_B(x); \ I_A(x) = I_B(x)$ and $F_A(x) = F_B(x) \ \forall x \in X$

(iii) $\bar{A} = \{\langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle \ /x \in X\}$

(iv) $A \cap B = \{\langle x, T_{(A \cap B)}(x), \ \ I_{(A \cap B)}(x), F_{(A \cap B)}(x) \rangle / x \in X\}$ where
$T_{A \cap B}(x) = min\{T_A(x), T_B(x)\} \ \ I_{A \cap B}(x) = min\{I_A(x), I_B(x)\} \ \ F_{A \cap B}(x) = max\{F_A(x), F_B(x)\}$

(v) $A \cup B = \{\langle x, T_{(A \cup B)}(x), \ \ I_{(A \cup B)}(x), F_{(A \cup B)}(x) \rangle / x \in X\}$ where
$T_{A \cup B}(x) = max\{T_A(x), T_B(x)\} \ \ I_{A \cup B}(x) = max\{I_A(x), I_B(x)\} \ \ F_{A \cup B}(x) = min\{F_A(x), F_B(x)\}$.

Definition 2.5[5]

Let $R \subseteq U \times U$ be a crisp binary relation on U. R is referred to as reflexive if $(x, x) \in R$ for all $x \in U$.R is referred to as symmetric if for all (x,y) $\in$ U, (x,y) $\in$ R implies (y,x) $\in$ R and R is referred to as transitive if for all x,y,z $\in$ U, (x,y) $\in$ R and (y,z) $\in$ R imply (x,z) $\in$ R.

Definition 2.6[5]

Let U be a non empty universe of discourse and $R \subseteq U \times U$, an arbitrary crisp relation on U. Denote $xR = y \in U/(x, y) \in R \ \ x \in U$

xR is called the R-after set of x (Bandler and kohout 1980) or successor neighbourhood of x with respect to R (Yao 1998 b). The pair (U,R) is called a crisp approximation space. For any $A \subseteq U$ the upper and lower approximation of A with respect to (U,R) denoted by $\bar{R}$ and $\underline{R}$ are respectively defined as follows

$\bar{R} = \{x \in U/xR \cap A \neq \varphi\}$

$\underline{R} = \{x \in U/xR \subseteq A\}$

The pair $(\underline{R}(A), \bar{R}(A))$ is referred to as crisp rough set of A with respect to (U,R) and $\bar{R}, \underline{R} : \rho(U) \longrightarrow \rho(U)$ are referred to upper and lower crisp approximation operator respectively.

The crisp approximation operator satisfies the following properties for all A, B $\in \rho(U)$

$(L_1)\ \underline{R}(A) = \bar{R}'(A')$	$(U_1)\bar{R} = \ \underline{R}(\ A)$
$(L_2)\underline{R}(U) = U$	$(U_2)\bar{R}\ \ \varphi = \varphi$
$(L_3)\ \underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$	$(U_3)\bar{R}(A \cap B) = \bar{R}(A) \cup \bar{R}(B)$
$(L_4)\ A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$	$(U_4)A \subseteq B = \bar{R}(A) \subseteq \bar{R}(B)$
$(L_5)\ \underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$	$(U_5)\bar{R}(A \cap B) \subseteq \bar{R}(A) \cap \bar{R}(B)$

Properties $(L_1)$and(U_1) show that the approximation operators $\underline{R}$and$\bar{R}$ are dual to each other. Properties with the same number may be considered as a dual properties. If R is equivalence relation in U then the pair (U,R) is called a Pawlak approximation space and $(\underline{R}(A), \bar{R}(A))$ is a Pawlak rough set, in such a case the approximation operators have additional properties.

3 Fuzzy Neutrosophic rough sets

In this section, we introduce fuzzy neutrosophic approximation operators induced from the same. Further we define a new type of set called fuzzy neutrosophic rough set and investigate some of its properties.

Definition 3.1:

A constant fuzzy Neutrosophic set $\overline{\overline{(\alpha, \beta, \gamma)}} = \{\ \langle x, \alpha, \beta, \gamma\ \rangle /\ x \in U\}$

where $0 \leq \alpha, \beta, \gamma \leq 1$ and $\alpha + \beta + \gamma \leq 3$.

We introduce a special Fuzzy Neutrosophic set ly for y $\in$ U as follows

$$T_{1_y}(x) = \begin{cases} 1, & if \quad x = y \\ 0, & if \quad x \neq y \end{cases}$$

$$T_{1_{u-\{y\}}}(x) = \begin{cases} 0, & if \quad x = y \\ 1, & if \quad x \neq y \end{cases}$$

$$I_{1_y} = \begin{cases} 1, & if \quad x = y \\ 0, & if \quad x \neq y \end{cases}$$

$$I_{1_{u-\{y\}}}(x) = \begin{cases} 0, & if \quad x = y \\ 1, & if \quad x \neq y \end{cases}$$

$$F_{1_y}(x) = \begin{cases} 0, & if \quad x = y \\ 1, & if \quad x \neq y \end{cases}$$

$$F_{1_{u-\{y\}}}(x) = \begin{cases} 1, & if \quad x = y \\ 0, & if \quad x \neq y. \end{cases}$$

Definition 3.2:

A Fuzzy Neutrosophic relation on U is a Fuzzy Neutrosophic subset
$R = \{\langle x, y\rangle, T_R(x, y), I_R(x, y), F_R(x, y)/x, y \in U\}$
$T_R : U \times U \longrightarrow [0, 1]$; $I_R : U \times U \longrightarrow [0, 1]$; $F_R : U \times U \longrightarrow [0, 1]$ satisfies
$0 \leq T_R(x, y) + I_R(x, y) + F_R(x, y) \leq 3$ for all $(x, y) \in U \times U$. We denote the family of all Fuzzy Neutrosophic relation on U by FNR(U $\times$ U).

Definition 3.3:

Let U be a non empty universe of discourse. For an arbitrary fuzzy neutrosophic relation R over U $\times$ U the pair (U,R) is called fuzzy neutrosophic approximation space. For any A $\in$ FN(U), we define the upper and lower approoximations with respect to (U, R), denoted by $\underline{R}(A)$ and $\bar{R}(A)$ respectively.

$$\bar{R}(A) = \{\langle x, T_{\bar{R}(A)}(x), I_{\bar{R}(A)}(x), F_{\bar{R}(A)}(x)\rangle/x \in U\}$$

$$\underline{R}(A) = \{\langle x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x)\rangle/x \in U\}$$

where,
$T_{\bar{R}(A)}(x) = \bigvee_{y \in U} [\ T_R(x, y) \wedge T_A(y)\ ]$

$$I_{\bar{R}(A)}(x) = \bigvee_{y \in U} [\ I_R(x,y) \wedge I_A(y)\ ]$$

$$F_{\bar{R}(A)}(x) = \bigwedge_{y \in U} [\ F_R(x,y) \vee F_A(y)\ ]$$

$$T_{\underline{R}(A)}(x) = \bigwedge_{y \in U} [\ F_R(x,y) \vee T_A(y)\ ]$$

$$I_{\underline{R}(A)}(x) = \bigwedge_{y \in U} [\ 1 - I_R(x,y) \vee I_A(y)\ ]$$

$$F_{\underline{R}(A)}(x) = \bigvee_{y \in U} [\ T_R(x,y) \wedge F_A(y)\ ]$$

The pair $(\underline{R}(A), \bar{R}(A))$ is called Fuzzy Neutrosophic Rough set of A with respect to (U,R) and $\underline{R}, \bar{R} : FN(U) \longrightarrow FN(U)$ are referred to as upper and lower Fuzzy Neutrosophic rough approximation operators respectively.

Remark 3.4:

If R is an intuitionistic fuzzy relation on U then (U,R) is a intuitionistic Fuzzy approximation space, Fuzzy neutrosophic rough operators are induced from a intuitionistic fuzzy approximation space that is

$\bar{R}(A) = \{\langle x, T_{\bar{R}(A)}(x), I_{\bar{R}(A)}(x), F_{\bar{R}(A)}(x)\rangle / x \in U\}\ A \in FN(U)$

$\underline{R}(A) = \{\langle x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x)\rangle / x \in U\}\ A \in FN(U)$

where,

$$T_{\bar{R}(A)}(x) = \bigvee_{y \in U} [\ \mu_R(x,y) \wedge T_A(y)\ ]$$

$$I_{\bar{R}(A)}(x) = \bigvee_{y \in U} [\ 1 - (\mu_R(x,y) + \gamma_R(x,y))\ \wedge I_R(y)]$$

$$F_{\bar{R}(A)}(x) = \bigwedge_{y \in U} [\ \gamma_R(x,y) \vee F_A(y)\ ]$$

$$T_{\underline{R}(A)}(x) = \bigwedge_{y \in U} [\ \gamma_R(x,y) \vee T_A(y)\ ]$$

$$I_{\underline{R}(A)}(x) = \bigwedge_{y \in U} [\ (\mu_R(x,y) + \gamma_R(x,y)) \vee I_A(y)\ ]$$

$$F_{\underline{R}(A)}(x) = \bigvee_{y \in U} [\ \mu_R(x,y) \wedge F_A(y)\ ].$$

Remark 3.5:

If R is a crisp binary relation on U then (U,R) is a crisp approximation space, the Fuzzy neutrosophic rough approximation operators are induced from a crisp approximation space, that $\forall A \in FN(U)$

$\bar{R}(A) = \{\langle x, T_{\bar{R}(A)}(x), I_{\bar{R}(A)}(x), F_{\bar{R}(A)}(x)\rangle / x \in U\}$

$\underline{R}(A) = \{\langle x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x)\rangle / U \in U\}$
where,

$$T_{\bar{R}(A)}(x) = \bigvee_{y\in[x]_R} T_A(y) \quad I_{\bar{R}(A)}(x) = \bigvee_{y\in[x]_R} I_A(y) \quad F_{\bar{R}(A)}(x) = \bigwedge_{y\in[x]_R} F_A(y)$$

$$T_{\underline{R}(A)}(x) = \bigwedge_{y\in[x]_R} T_A(y) \quad I_{\underline{R}(A)}(x) = \bigwedge_{y\in[x]_R} I_A(y) \quad F_{\underline{R}(A)}(x) = \bigvee_{y\in[x]_R} F_A(y)$$

Theorem 3.6:

Let (U,R) be a Fuzzy Neutrosophic approximation space. Then the upper and lower fuzzy Neutrosophic rough approximation operators induced from (U,R) satisfy the following properties. $\forall\ A, B \in FN(U)$, $\forall\ \alpha, \beta, \gamma \in [0, 1]$ with $\alpha + \beta + \gamma \ \leq\ 3$

$(FNL1)\underline{R}(A) = \bar{R}'(A'),$ $\qquad (FNU1)\bar{R}(A) = \underline{R}'(A)'$
$(FNL2)\underline{R}(A \cup \overline{\overline{\alpha, \beta, \gamma}}) = \underline{R}(A) \cup (\overline{\overline{\alpha, \beta, \gamma}}),$ $\qquad (FNU2)\bar{R}(A \cap \overline{\overline{\alpha, \beta, \gamma}}) = \bar{R}(A) \cap (\overline{\overline{\alpha, \beta, \gamma}})$
$(FNL3)\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B),$ $\qquad (FNU3)\bar{R}(A \cup B) = \bar{R}(A) \cup \bar{R}(B)$
$(FNL4)A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$ $\qquad (FNU4)A \subseteq B \Rightarrow \bar{R}(A) \subseteq \bar{R}(B)$
$(FNL5)\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$ $\qquad (FNU5)\bar{R}(A \cap B) \subseteq \bar{R}(A) \cap \bar{R}(B)$
$(FNL6)R_1 \subseteq R_2 \Rightarrow \underline{R_1}(A) \supseteq \underline{R_2}(A)$ $\qquad (FNU6)R_1 \subseteq R_2 \Rightarrow \bar{R}_1(A) \subseteq \bar{R}_2(A)$

proof:

It follows directly from Definition 3.1.
Properties (FNL1) and (FNU1) show that FN rough approximation operators $\underline{R}$ and $\bar{R}$ are dual to each other.

Remark 3.7: The properties (FNL2) and (FNU2) imply, following properties $(FNL2)'$ and $(FNU2)'$
$(FNL2)'\ \ \underline{R}(U) = U \qquad (FNU2)' = \bar{R}(\varphi) = \varphi$

Example 3.8:

Let (U,R) be a FN approximation space where U $= \{x_1, x_2, x_3\}$ and $R \in FNR(U \times U)$ is defined as
$R = \{\langle (x_1, x_1)0.8, 0.7, 0.1\rangle\ \ \langle (x_1, x_2), 0.2, 0.5, 0.4\rangle\ \ \langle (x_1, x_3)0.6, 0.5, 0.7\rangle\ \ \langle (x_2, x_1)0.4, 0.6, 0.3\rangle$
$\langle (x_2, x_2)0.7, 0.8, 0.1\rangle\ \ \langle (x_2, x_3)0.5, 0.3, 0.1\rangle\ \ \langle (x_3, x_1)0.6, 0.2, 0.1\rangle\ \ \langle (x_3, x_2)0.7, 0.8, 0.1\rangle\ \ \langle (x_3, x_3)1, 0.9, 0.1\rangle\}$
If a Fuzzy Neutrosophic set
$A = \{\langle x_1, 0.8, 0.9, 0.1\rangle\ \ \langle x_2, 0.5, 0.4, 0.3\rangle\ \ \langle x_3, 0.5, 0.4, 0.7\rangle\}$
then $T_{\overline{R}(A)}(x_1) = \bigvee_{y\in Y} [T_R(x_1, y) \wedge T_A(y)] = 0.8$
$I_{\overline{R}(A)}(x_1) = \bigvee_{y\in Y} [I_R(x_1, y) \wedge I_A(y)] = 0.7$

$F_{\overline{R}(A)}(x_1) = \bigwedge_{y \in Y} [F_R(x_1, y) \vee F_A(y)] = 0.1$

Similarly we have $T_{\overline{R}(A)}(x_2) = 0.7$, $I_{\overline{R}(A)}(x_2) = 0.6$, $F_{\bar{R}(A)}(x_2) = 0.3$ and

$T_{\overline{R}(A)}(x_3) = 0.6$, $I_{\bar{R}(A)}(x_3) = 0.4$, $F_{\overline{R}(A)}(x_3) = 0.1$.

Hence $\overline{R}(A) = \{\langle x_1, 0.8, 0.7, 0.1\rangle \; \langle x_2, 0.7, 0.6, 0.3\rangle , \langle x_3, 0.6, 0.4, 0.1\rangle \; \}$.

Likewise we can calculate

$\underline{R}(A) = \{\langle x_1, 0.5, 0.5, 0.4\rangle \; \langle x_2, 0.5, 0.4, 0.3\rangle , \langle x_3, 0.5, 0.4, 0.7\rangle \; \}$

Definition 3.9:

Let $A \in FN(U)$ and $\alpha, \beta, \gamma \in [0,1]$ with $\alpha+\beta+\gamma \leq 3$ with (α, β, γ) level set of A denoted by $A^{(\alpha\beta\gamma)}$ is defined as

$A^{(\alpha\beta\gamma)} = \{x \in U / T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}$

We define

$A_\alpha = \{x \in U / T_A(x) \geq \alpha\}$

$A_{\alpha+} = \{x \in U / T_A(x) > \alpha\}$

the α level cut and strong α level cut of truth function generated by A.

$A_\beta = \{x \in U / I_A(x) \geq \beta\}$

$A_{\beta+} = \{x \in U / I_A(x) > \beta\}$

the β level cut and strong β level cut of indeterminacy function generated by A.

and

$A^\gamma = \{x \in U / F_A(x) \leq \gamma\}$

$A^{\gamma+} = \{x \in U / F_A(x) < \gamma\}$

the γ level cut and strong γ level cut of false value function generated by A.

Similarly, We can define other types level cuts

$A^{(\alpha+,\beta+,\gamma+)} = \{x \in U / T_A(x) > \alpha, I_A(x) > \beta, F_A(x) < \gamma\}$ which is $(\alpha + \beta + \gamma+)$ level cut set of A.

$A^{(\alpha+,\beta,\gamma)} = \{x \in U / T_A(x) > \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}$

$A^{(\alpha,\beta+,\gamma)} = \{x \in U / T_A(x) \geq \alpha, I_A(x) > \beta, F_A(x) \leq \gamma\}$

$A^{(\alpha,\beta,\gamma+)} = \{x \in U / T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) < \gamma\}$

Like wise other level cuts can be defined.

Theorem 3.10:

The level cut sets of Fuzzy Neutrosophic sets satisfy the following properties $\forall\ A, B \in FN(U)$,

$\alpha, \beta, \gamma \in [0,1]$ with $\alpha + \beta + \gamma \leq 3$

$\alpha_1, \beta_1, \gamma_1 \in [0,1]$ with $\alpha_1 + \beta_1 + \gamma_1 \leq 3$

$\alpha_2, \beta_2, \gamma_2 \in [0,1]$ with $\alpha_2 + \beta_2 + \gamma_2 \leq 3$

1) $A^{(\alpha,\beta,\gamma)} = A_\alpha \bigcap A\beta \bigcap A^\gamma$

2) $(A^{'})_\alpha = (A^{'})\alpha+ : (A^{'})\beta = (A^{'})(1-\beta+); (A^{'})^\gamma = (A^{'})_{\alpha+}$

3) $\left(\bigcap_{i\in J} A_i\right)_\alpha = \bigcap_{i\in J}(A_i)_\alpha$

$\left(\bigcap_{i\in J} A_i\right)\beta = \bigcap_{i\in J}(A_i)\beta$

$\left(\bigcap_{i\in J} A_i\right)^\gamma = \bigcap_{i\in J}(A_i)^\gamma$

5) $\left(\bigcup_{i\in J} A_i\right)_\alpha = \bigcup_{i\in J}(A_i)_\alpha$

$\left(\bigcup_{i\in J} A_i\right)\beta = \bigcup_{i\in J}(A_i)\beta$

$\left(\bigcup_{i\in J} A_i\right)^\gamma = \bigcup_{i\in J}(A_i)^\gamma$

6) $\left(\bigcup_{i\in J} A_i\right)^{(\alpha,\beta,\gamma)} \supseteq \bigcup_{i\in J}(A_i)^{(\alpha,\beta,\gamma)}$

7) $\left(\bigcap_{i\in J} A_i\right)^{(\alpha,\beta,\gamma)} \supseteq \bigcap_{i\in J}(A_i)^{(\alpha,\beta,\gamma)}$

8) For $\alpha_1 \geq \alpha_2 \quad \beta_1 \geq \beta_2 \quad \gamma_1 \leq \gamma_2$

$A_{\alpha_1} \subseteq A_{\alpha_2}; \quad A\beta_1 \subseteq A\beta_2 \quad A^{\gamma_1} \subseteq A^{\gamma_2}$

$A^{(\alpha_1,\beta_1,\gamma_1)} \subseteq A^{(\alpha_2,\beta_2,\gamma_2)}$

Proof

1) and 3) follow directly from definition 3.9

2) Since $A = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle / x \in U\}$

$(A^{'})_\alpha = \{x \in U / F_A(x) \geq \alpha\}$

By definition,

$A^{\alpha+} = \{x \in U / F_A(x) < \alpha\}$

$A^{\alpha+} = \{x \in U / F_A(x) \geq \alpha\}$

$\Rightarrow (\ A)_\alpha = \ (A^{\alpha+})$

Similarly we can prove,

$(\ A)\beta = \ (A^{1-\beta+})$

$(\ A)^\gamma = \ (A^{\gamma+})$

4) $\bigcap\limits_{i\in J} A_i = \left\{\langle x, \bigwedge\limits_{i\in J} T_{A_i}(x), \bigwedge\limits_{i\in J} I_{A_i}(x), \bigvee\limits_{i\in J} F_{A_i}(x)\rangle / x \in U\right\}$

We have $\left(\bigcap\limits_{i\in J} A_i\right)_\alpha = \left\{x \in U / \bigwedge\limits_{i\in J} T_{A_i}(x) \geq \alpha\right\} = \{x \in U / T_{A_i}(x) \geq \alpha\} = \bigcap\limits_{i\in J} (A_i)_\alpha$

Similarly,

$\left(\bigcap\limits_{i\in J} A_i\right)\beta = \left\{x \in U / \bigwedge\limits_{i\in J} I_{A_i}(x) \geq \beta\right\} = \{x \in U / I_{A_i}(x) \geq \beta \forall i \in J\} = \bigcap\limits_{i\in J} (A_i)\beta$ and

$\left(\bigcap\limits_{i\in J} A_i\right)^\gamma = \left\{x \in U / \bigvee\limits_{i\in J} F_{A_i}(x) \leq \gamma\right\} = \{x \in U / F_{A_i}(x) \leq \gamma \forall i \in J\} = \bigcap\limits_{i\in J} (A_i)_\beta$

We can conclude

$$\left(\bigcap\limits_{i\in J} A_i\right)^{\alpha,\beta,\gamma} = \left(\bigcap\limits_{i\in J} A_i\right)_\alpha \cap \left(\bigcap\limits_{i\in J} A_i\right)\beta \cap \left(\bigcap\limits_{i\in J} A_i\right)^\gamma = \bigcap\limits_{i\in J} ((A_i)_\alpha \cap (A_i)\beta \cap (A_i)^\gamma) = \bigcap\limits_{i\in J} (A_i)^{(\alpha,\beta,\gamma)}$$

5) We know

$\bigcup\limits_{i\in J} (A_i) = \left\{\langle x, \bigvee\limits_{i\in J} T_{A_i}(x), \bigvee\limits_{i\in J} I_{A_i}(x), \bigwedge\limits_{i\in J} F_{A_i}(x)\rangle / x \in U\right\}$

$\left(\bigcup\limits_{i\in J} A_i\right)_\alpha = \left\{x \in U / \bigvee\limits_{i\in J} T_{A_i}(x) \geq \alpha\right\} = \left\{x \in U / \bigvee\limits_{i\in J} T_{A_i}(x) \geq \alpha,\ \exists i \in J\right\} = \bigcup\limits_{i\in J} (A_i)_\alpha$

$\left(\bigcup\limits_{i\in J} A_i\right)\beta = \left\{x \in U / \bigvee\limits_{i\in J} I_{A_i}(x) \geq \beta\right\} = \{x \in U / I_{A_i}(x) \geq \beta,\ \forall i \in J\} = \bigcup\limits_{i\in J} (A_i)\beta$

$\left(\bigcup\limits_{i\in J} A_i\right)^\gamma = \left\{x \in U / \bigwedge\limits_{i\in J} F_{A_i}(x) \leq \gamma\right\} = \{x \in U / F_{A_i}(x) \leq \gamma,\ \forall i \in J\} = \bigcup\limits_{i\in J} (A_i)^\gamma$

6) For any $x \in A_\alpha$, according to definition(*) we have for $T_A(x) \geq \alpha_1 \geq \alpha_2$, we obtain $A_{\alpha_1} \subseteq A_{\alpha_2}$.

Similarly for $\beta_1 \geq \beta_2$ and $\gamma_1 \leq \gamma_2$ we obtain $A\beta_1 \subseteq A\beta_2$ and $A^{\gamma_1} \subseteq A^{\gamma_2}$.

Hence we have,

$A^{(\alpha_1,\beta_1,\gamma_1)} \subseteq A^{(\alpha_2,\beta_2,\gamma_2)}$.

Corollary 3.11:

Assume that R is a Fuzzy Neutrosophic relation in U,

$R_\alpha = \{(x, y) \in U \times U / T_R(x, y) \geq \alpha\}$

$R_\alpha(x) = \{y \in U / T_R(x, y) \geq \alpha\}$

$R_{\alpha+} = \{(x, y) \in U \times U / T_R(x, y) > \alpha\}$

$R_{\alpha+}(x) = \{y \in U / T_R(x, y) > \alpha\}$

$R\beta = \{(x, y) \in U \times U / I_R(x, y) \geq \beta\}$
$R\beta(x) = \{y \in U / I_R(x, y) \geq \beta\}$
$R\beta+ = \{(x, y) \in U \times U / I_R(x, y) > \beta\}$
$R\beta+(x) = \{y \in U / I_R(x, y) > \beta\}$

$R^\gamma = \{(x, y) \in U \times U / F_R(x, y) \leq \gamma\}$
$R^\gamma(x) = \{y \in U / F_R(x, y) \leq \gamma\}$
$R^{\gamma+} = \{(x, y) \in U \times U / F_R(x, y) < \gamma\}$
$R^{\gamma+}(x) = \{y \in U / F_R(x, y) < \alpha\}$
$R^{(\alpha,\beta,\gamma)} = \{(x, y) \in U \times U / T_R(x, y) \geq \alpha, I_R(x, y) \geq \beta, F_R(x, y) \leq \gamma\}$
$R^{(\alpha,\beta,\gamma)}(x) = \{y \in U / T_R(x, y) \geq \alpha, I_R(x, y) \geq \beta, F_R(x, y) \leq \gamma\}$
Then for all $R_\alpha, R_{\alpha+}, R\beta, R\beta+, R^\gamma, R^{\gamma+}, R^{(\alpha\beta\gamma)}$ are crisp relation in U and
1) If R is reflexive then the above level cuts are reflexive.
2) If R is symmetric then the above level cuts are symmetric.
3) If R is transitive then the above level cuts are transitive.

Proof

Since R is a crisp reflexive $\forall\ x \in U, \quad \alpha, \beta, \gamma \in [0, 1]$
Take, $T_R(x, x) = 1 \quad I_R(x, x) = 1 \quad F_R(x, x) = 0 \quad \forall \ x \in U$
Now, we have R_α is a crisp binary relation in U and $x \in U$, $(x, x) \in R_\alpha$. $\therefore R_\alpha$ is reflexive.

If R is symmetric then $\forall \ x, y \in U$, we have $(x, y) \in R_\alpha \Rightarrow (y, x) \in R_\alpha$. $\therefore R_\alpha$ is symmetry.
Similarly we can prove $R\beta$ and R^γ are symmetric.

If R is transitive then $\forall x, y, z \in U$ and $\alpha, \beta, \gamma \in [0, 1]$
$T_R(x, z) \geq T_R(x, y) \wedge T_R(y, z)$
$I_R(x, z) \geq I_R(x, y) \wedge I_R(y, z)$
$F_R(x, z) \leq F_R(x, y) \vee F_R(y, z)$
for any $(x, y) \in R_\alpha \qquad (y, z) \in R_\alpha$
$(x, y) \in R\beta \qquad (y', z') \in R\gamma$
$(x'', y'') \in R^\gamma \qquad (y'', z'') \in R^\gamma$
(ie) $T_R(x, y) \geq \alpha, \quad T_R(y, z) \geq \alpha \quad \Rightarrow T_R(x, z) \geq \alpha$
$I_R(x', y') \geq \beta, \quad I_R(y', z') \geq \beta \quad \Rightarrow I_R(x', z') \geq \beta$
$F_R(x'', y'') \leq \gamma, \quad F_R(y'', z'') \leq \gamma \quad \Rightarrow F_R(x'', z'') \leq \gamma$
Therefore $R_\alpha, R\beta, R^\gamma$ are transitive.
Hence $R^{(\alpha,\beta,\gamma)}$ is transitive.
Similarly we can prove other level cuts sets are transitive.

Theorem 3.12:

Let (U,R) be a fuzzy neutrosophic approximation space and A $\in$ FN(U), then the upper FN approximation operator can be represented as follows $\forall\ x \in U$.

1) $T_{\bar{R}(A)}(x) = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_\alpha(A_\alpha)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_\alpha(A_{\alpha+})(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_{\alpha+}(A_\alpha)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_{\alpha+}(A_{\alpha+})(x)\right]$

2) $I_{\bar{R}(A)}(x) = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha(A\alpha)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha(A\alpha+)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha+(A\alpha)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha+(A\alpha+)(x)\right]$

3)$F_{\bar{R}(A)}(x) = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^\alpha(A^\alpha)(x)\right] = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^\alpha(A^{\alpha+})(x)\right] = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^{\alpha+}(A^\alpha)(x)\right] = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^{\alpha+}(A^{\alpha+})(x)\right]$

and more over for any $\alpha \in [0,1]$

4)$\left[\bar{R}(A)\right]_{\alpha+} \subseteq \bar{R}_{\alpha+}(A_{\alpha+}) \subseteq \bar{R}_\alpha(A_\alpha) \subseteq \left[\bar{R}(A)\right]_\alpha$

5)$\left[\bar{R}(A)\right]\alpha+ \subseteq \bar{R}\alpha+(A\alpha+) \subseteq \bar{R}\alpha(A\alpha) \subseteq \left[\bar{R}(A)\right]\alpha$

6) $\left[\bar{R}(A)\right]^{\alpha+} \subseteq R^{\bar{\alpha}+}(A^{\alpha+}) \subseteq \bar{R}^\alpha(A^\alpha) \subseteq \left[\bar{R}(A)\right]^\alpha$

7) $\left[\bar{R}(A)\right]_{\alpha+} \subseteq \bar{R}_{\alpha+}(A_{\alpha+}) \subseteq \bar{R}_\alpha(A_\alpha) \subseteq \left[\bar{R}(A)\right]_\alpha$

8)$\left[\bar{R}(A)\right]\alpha+ \subseteq \bar{R}\alpha+(A\alpha+) \subseteq \bar{R}\alpha(A\alpha) \subseteq \left[\bar{R}(A)\right]\alpha$

9) $\left[\bar{R}(A)\right]^{\alpha+} \subseteq R^{\bar{\alpha}+}(A^{\alpha+}) \subseteq \bar{R}^\alpha(A^\alpha) \subseteq \left[\bar{R}(A)\right]^\alpha$

Proof

1) For $x \in U$, we have

$\bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_\alpha(A_\alpha)(x)\right] = Sup\left\{\alpha \in [0,1]/x \in \bar{R}_\alpha(A_\alpha)\right\}$
$= Sup\left\{\alpha \in [0,1]/R_\alpha(x) \cap A_\alpha \neq \varphi\right\}$
$= Sup\left\{\alpha \in [0,1]/\exists y \in U(y \in R_\alpha(x), y \in A_\alpha)\right\}$
$= Sup\left\{\alpha \in [0,1]/\exists y \in U[T_R(x,y) \geq \alpha, T_A(y) \geq \alpha]\right\}$
$= \bigvee_{y\in U} [T_R(x,y) \wedge T_A(y)] = T_{\bar{R}(A)}(x)$

2) $\bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha(A\alpha)(x)\right] = Sup\left\{\alpha \in [0,1]/x \in \bar{R}\alpha(A\alpha)\right\}$
$= Sup\left\{\alpha \in [0,1]/R\alpha(x) \cap A\alpha \neq \varphi\right\}$

$= Sup\{\alpha \in [0,1]/\exists y \in U(y \in R\alpha(x), y \in A\alpha)\}$
$= Sup\{\alpha \in [0,1]/\exists y \in U[I_R(x,y) \geq \alpha, I_A(y) \geq \alpha]\}$
$= \bigvee_{y\in U} [I_R(x,y) \wedge I_A(y)] = I_{\bar{R}(A)}(x)$

3) $\bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}^{\alpha}(A^{\alpha})(x)\right] = inf\{\alpha \in [0,1]/R^{\alpha}(x) \cap A^{\alpha} \neq \varphi\}$
$= inf\{\alpha \in [0,1]/R^{\alpha}(x) \cap A^{\alpha} \neq \varphi\}$
$= inf\{\alpha \in [0,1]/\exists y \in U(y \in R^{\alpha}(x), y \in A^{\alpha})\}$
$= inf\{\alpha \in [0,1]/\exists y \in U[F_R(x,y) \leq \alpha, F_A(y) \leq \alpha]\}$
$= \bigwedge_{y\in U} [F_R(x,y) \vee F_A(y)] = F_{\bar{R}(A)}(x)$
Like wise we can conclude
$T_{\bar{R}(A)}(x) = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_{\alpha}(A_{\alpha+})(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_{\alpha+}(A_{\alpha})(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}_{\alpha+}(A_{\alpha+})(x)\right]$
$I_{\bar{R}(A)}(x) = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha(A\alpha+)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha+(A\alpha)(x)\right] = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge \bar{R}\alpha+(A\alpha+)(x)\right]$
$F_{\bar{R}(A)}(x) = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^{\alpha}(A^{\alpha+})(x)\right] = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^{\alpha+}(A^{\alpha})(x)\right] = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee \bar{R}^{\alpha+}(A^{\alpha+})(x)\right]$

4) Since $\bar{R}_{\alpha+}(A_{\alpha+}) \subseteq \bar{R}_{\alpha+}(A_{\alpha}) \subseteq \bar{R}_{\alpha}(A_{\alpha})$
We prove only $[\bar{R}(A)]_{\alpha+} \subseteq \bar{R}_{\alpha+}(A_{\alpha+})$ and $\bar{R}_{\alpha}(A_{\alpha}) \subseteq [R(A)]_{\alpha}$
For any $x \in [\bar{R}(A)]_{\alpha+}$
$T_{\bar{R}(A)} > \alpha$
$\Rightarrow \bigvee_{y\in U} [T_R(x,y) \wedge T_A(y)] > \alpha\ \exists y' \in U \ni T_R(x,y') \wedge T_R(y") > \alpha$
$\Rightarrow y' \in R_{\alpha+}(x)$ and $y' \in A_{\alpha+}$
$\Rightarrow R_{\alpha+}(x) \cap A_{\alpha+} \neq \varphi$
From the definition of upper crisp approximation operator we have $x \in \bar{R}_{\alpha+}(A_{\alpha+})$
Hence $[\bar{R}(A)]_{\alpha+} \subseteq \bar{R}_{\alpha+}(A_{\alpha+})$
Next, to prove $\bar{R}_{\alpha}(A_{\alpha}) \subseteq [R(A)]_{\alpha}$
For any $x \in \bar{R}_{\alpha}(A_{\alpha}), R_{\alpha}(A_{\alpha})(x) = 1$
If $\exists\ \beta$, then
$T_{\bar{R}(A)}(x) = \bigvee_{\beta\in[0,1]} \left[\beta \wedge \bar{R}_{\beta}(A_{\beta})(x)\right] \geq \alpha \wedge \bar{R}_{\alpha}(A_{\alpha})(x) = \alpha$
We obtained $x \in [\bar{R}(A)]_{\alpha}$
$\bar{R}_{\alpha}(A_{\alpha}) \subseteq [\bar{R}(A)]_{\alpha}$

5) Similar to (4) It is easy to prove
$R\bar{\alpha}+(A\alpha+) \subseteq R\bar{\alpha}+(A\alpha) \subseteq \bar{R}\alpha(A\alpha)$
Hence we prove

$i)[\bar{R}(A)]\alpha+ \subseteq R\bar{\alpha}+(A\alpha+)$

$ii)\bar{R\alpha}(A\alpha) \subseteq [\bar{R}(A)]\alpha$

i) For $x \in [\bar{R}(A)]\alpha+, I_{\bar{R}(A)}(x) > \alpha$

$\Rightarrow \bigvee_{y\in U} [I_R(x,y) \wedge I_A(y)] > \alpha$

$\exists \;\; y^{'} \in U \ni I_R(x,y^{'}) \wedge I_A(y^{'}) > \alpha$

(ie) $I_R(x,y^{'})\alpha$ and $I_A(y^{'})\alpha$

$\Rightarrow y^{'} \in R\alpha + (x)$ and $y^{'} \in A\alpha+$

$y^{'} \in R(x) \cap A\alpha+ \Rightarrow R\alpha + (x) \cap A\alpha+ \neq \varphi$

By the definition of crisp approximation operator we have

$x \in \overline{R\alpha+}(A\alpha+)$

Therefore $[\bar{R}(A)]\alpha+ \subseteq \overline{R\alpha+}(A\alpha+)$

Next for any $x \in \overline{R\alpha}(A\alpha)$, $\bar{R}\alpha(A\alpha)(x) = 1$

If $\exists \;\; \beta$ then

$T_{\bar{R}(A)}(x) = \bigvee_{\beta\in[0,1]} \left[\beta \wedge \bar{R}_\beta(A_\beta)(x)\right] \geq \alpha \wedge \bar{R}\alpha(A\alpha)(x) = \alpha$

We obtain $x \in [\bar{R}(A)]\alpha$ Therefore $\overline{R\alpha}(A\alpha) \subseteq [\bar{R}(A)]\alpha$

6) The proof of (6) is similar to (4) and (5) we need to prove only $[\bar{R}(A)]^{\alpha+} \subseteq \overline{R^{\alpha+}}(A^{\alpha+})$ and $\overline{R^{\alpha}}(A^{\alpha}) \subseteq [\bar{R}(A)]^{\alpha}$

For any $x \in [\bar{R}(A)]^{\alpha+}, F_{\bar{R}(A)}(x) < \alpha$ (ie) $\bigwedge_{y\in U} [F_R(x,y) \vee F_A(y)] < \alpha$

$\exists \;\; y^{'} \in U \ni F_R(x,Y^{'}) \vee F_A(y^{'}) < \alpha$

Hence $F_R(x,Y^{'}) < \alpha, T_A(y^{'}) < \alpha$

(ie) $y^{'} \in R^{\alpha+}(x)$ and $y^{'} \in A^{\alpha+}$

$R^{\alpha+}(x) \cap A^{\alpha+} \neq \phi$

Therefore $x \in \overline{R^{\alpha+}}(A^{\alpha+})$

$[\bar{R}(A)]^{\alpha+} \subseteq \overline{R^{\alpha+}}(A^{\alpha+})$

Next for any $x \in \overline{R^{\alpha}}(A^{\alpha})$ note $\overline{R^{\alpha}}(A^{\alpha})(x) = 1$ then we have

$F_{\bar{R}(A)}(x) = \bigwedge_{\beta\in[0,1]} \left[\beta \vee \overline{R^{\beta}}(A^{\beta})(x)\right] \leq \alpha \vee \overline{R^{\alpha}}(A^{\alpha})(x) = \alpha$

Thus $x \in [\bar{R}(A)]^{\alpha}$. Hence $\overline{R^{\alpha}}(A^{\alpha}) \subseteq [\bar{R}(A)]^{\alpha}$

The proof of (7), (8), (9) can be obtained similar to (4), (5), (6).

Theorem 3.13:

Let (U,R) be FN approximation space and $A \in FN(U)$ then $\forall x \in U$

$$1)T_{\underline{R}(A)}(x) = \bigwedge_{\alpha\in[0,1]} [\alpha \vee (1 - \underline{R^{\alpha}}(A_{\alpha+})(x))] = \bigwedge_{\alpha\in[0,1]} [\alpha \vee (1 - R^{\alpha}(A_{\alpha})(x))]$$

$\bigwedge_{\alpha\in[0,1]} [\alpha \vee (1 - \underline{R^{\alpha+}}(A_{\alpha+})(x))] = \bigwedge_{\alpha\in[0,1]} [\alpha \vee (1 - R^{\alpha+}(A_{\alpha})(x))]$

2) $I_{\underline{R}(A)}(x) = \bigwedge_{\alpha\in[0,1]} \left[\alpha \vee (1 - \underline{R(1-\alpha)}(A\alpha+)(x))\right] = \bigwedge_{\alpha\in[0,1]} [\alpha \vee (1 - R(1-\alpha)(A\alpha)(x))]$

$\bigwedge_{\alpha\in[0,1]} \left[\alpha \vee (1 - \underline{R(1-\alpha+)}(A\alpha+)(x))\right] = \bigwedge_{\alpha\in[0,1]} [\alpha \vee (1 - R(1-\alpha+)(A\alpha)(x))]$

3) $F_{\underline{R}(A)}(x) = \bigvee_{\alpha\in[0,1]} \left[\alpha \wedge (1 - \underline{R_{\alpha}}(A^{\alpha+})(x))\right] = \bigvee_{\alpha\in[0,1]} [\alpha \wedge (1 - R_{\alpha}(A^{\alpha})(x))]$

$\bigvee_{\alpha\in[0,1]} \left[\alpha \wedge (1 - \underline{R_{\alpha+}}(A^{\alpha+})(x))\right] = \bigvee_{\alpha\in[0,1]} [\alpha \wedge (1 - R_{\alpha+}(A^{\alpha})(x))]$

and for $\alpha \in [0,1]$

4)$[\underline{R}(A)]_{\alpha+} \subseteq \underline{R}^{\alpha}(A_{\alpha+}) \subseteq \underline{R^{\alpha+}}(A_{\alpha+}) \subseteq \underline{R^{\alpha+}}(A_{\alpha}) \subseteq [\underline{R}(A)]_{\alpha}$

5)$[\underline{R}(A)]\,\alpha+ \subseteq R1-\alpha(A\alpha+) \subseteq R1-\alpha+(A\alpha+) \subseteq R1-\alpha+(A\alpha) \subseteq [\underline{R}(A)]\alpha$

6)$[\underline{R}(A)]^{\alpha+} \subseteq \underline{R}_{\alpha}(A^{\alpha+}) \subseteq \underline{R_{\alpha+}}(A^{\alpha+}) \subseteq \underline{R_{\alpha+}}(A^{\alpha}) \subseteq [\underline{R}(A)]^{\alpha}$

7)$[\underline{R}(A)]_{\alpha+} \subseteq \underline{R}^{\alpha}(A_{\alpha+}) \subseteq \underline{R^{\alpha+}}(A_{\alpha}) \subseteq \underline{R^{\alpha+}}(A_{\alpha}) \subseteq [\underline{R}(A)]_{\alpha}$

8)$[\underline{R}(A)]\,\alpha+ \subseteq R1-\alpha+(A\alpha+) \subseteq R\alpha(A\alpha+) \subseteq \underline{R(1-\alpha+)}(A\alpha) \subseteq [\underline{R}(A)]^{\alpha}$

9)$[\underline{R}(A)]^{\alpha+} \subseteq \underline{R}_{\alpha}(A^{\alpha+}) \subseteq \underline{R_{\alpha}}(A^{\alpha}) \subseteq \underline{R_{\alpha+}}(A^{\alpha}) \subseteq [\underline{R}(A)]^{\alpha}$

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