Smarandache Soft Groupoids

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Abstract: In this paper, Smarandache soft groupoids shortly (SS-groupoids) are introduced as a generalization of Smarandache Soft semigroups (SS-semigroups) . A Smarandache Soft groupoid (F, A) is an approximated collection of Smarandache subgroupoids of a groupoid G . Further, we introduced parameterized Smarandache groupoid and strong soft semigroup over a groupoid G. Smarandache soft ideals are presented in this paper. We also discussed some of their core and fundamental properties and other notions with sufficient amount of examples. At the end, we introduced Smarandache soft groupoid homomorphism.

Key words: Smarndache groupoid, soft set, soft groupoid, Smarandache soft groupoid.

1. Introduction

In 1998, Raul [27] introduced the notions of Smarandache semigroups in the article "Smaradache Algebraic Structures". Smarandache semigroups are analogous to the notions smarandache groups. Smarandache in [33] first studied the theory of Smarandache algebraic structures in "Special Algebraic Structures". The Smarandache groupoid [18] are introduced by Kandassamy which exhibits the characteristics and features of both semigroups and groupoidss simultaneously. The Smarandache groupoidss are a class of compleltly new and conceptually a new study of associative and non-associative strucutres in nature. Smarandache algebraic structures almost show their existance in all algebraic structure in some sense such as Smarandache semigroups [17], Smarandache rings [21], Smarandache semirings [19], semifields [19], semivector spaces [19], Smarandache loops [20] etc.

Molodtsov [26] introduced the innovative and novel concept of soft sets in 1995. Soft set theory is a kind of mathematical framework that is free from the inadequacy of parameterization, syndrome of fuzzy sets, rough sets, probability etc. Soft set theory has found their applications in several areas such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability theory etc. Recently soft set gain much potential in the research since its beggining. Severalmalgebraic structures and their properties studied in the context of soft sets. Aktas and Cagmann [1] introduced soft groups. Other soft algebraic structures are soft semigroups [16], soft semirings, soft rings etc. Some more work on soft sets can be seen in [13,14]. Further, some other properties and related algebra may be found in [15]. Some other related concepts and notions together with fuzzy sets and rough sets were discussed in [24,25]. Some useful study on soft neutrosophic algebraic structures can be seen in [3,4,5,6,7,8,9,10,12,29,30,32]. Recently, mumtaz studied Smarandache soft semigroups in [11].

Rest of the paper is organized as follows. In first section 2, we studied some basic concepts and notions of smarandache groupoids, soft sets, and soft groupoid. In the next section 3, the notions of Smarandache soft groupoids shortly SS-groupoids are introduced. In this section some related properties and characterization is also discussed with illustrative examples. In the further section 4, Smarandache soft ideals and Smarandache soft groupoid homomorphism is studied with some of their basic properties. Conclusion is given in section 5.

2. Basic Concepts

In this section, fundamental concepts about Smarandache semigroups, soft sets, and soft semigroups is presented with some of their basic properties.

2.1 Smarandache Groupoids

Definition 2.1.1[18]: A Smarandache groupoid is define to be a groupoid G which has a proper subset S such that S is a semigroup with respect to the same induced operation of G. A Smarandache groupoid G is denoted by SG.

Definition 2.1.2 [18]: Let G be a Smarandache groupoid. If atleast one proper subset A in G which is a semigroup is commutative, then G is said to be a Smarandache commutative groupoid.

Definition 2.1.4 [18]: Let G be a Smarandache groupoid. A proper subset A of G is called a Smarandache subgroupoid if A itself is a Smarandach groupoid under the binary operation of G.

Definition 2.1.5 [18]: A Smarandache left ideal A of the Smarandache groupoid G satisfies the following conditions.

1. A is a Smarandache subgroupoid
2. $x \in G$ and $a \in A$, then $xa \in A$.

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 2.2.1 [24,26]: A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$.
In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Definition 2.2.2 [24]: For two soft sets (F,A) and (H,B) over U, (F,A) is called a soft subset of (H,B) if

1. $A \subseteq B$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F,A) \subset (H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A) \supset (H,B)$.

Definition 2.2.3 [24]: Two soft sets (F,A) and (H,B) over U are called soft equal if (F,A) is a soft subset of (H,B) and (H,B) is a soft subset of (F,A).

Definition 2.2.4 [24]: Let (F,A) and (K,B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F,A) \cap_R (K,B) = (H,C)$ where (H,C) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in C = A \cap B$.

Definition 2.2.5 [24]: The extended intersection of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B, \\ G(c) & \text{if c} \in B - A, \\ F(c) \cap G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F,A) \cap_\varepsilon (K,B) = (H,C)$.

Definition 2.2.6 [24]: The restricted union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = F(c) \cup G(c)$ for all $c \in C$. We write it as $(F,A) \cup_R (K,B) = (H,C)$.

Definition 2.2.7 [24]: The extended union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B, \\ G(c) & \text{if c} \in B - A, \\ F(c) \cup G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F,A) \cup_\varepsilon (K,B) = (H,C)$.

Definition 2.2.8: A soft set (F,A) over G is called a soft groupoid over G if and only if $\phi \neq F(a)$ is a subgroupoid of G for all $a \in A$.

3. Smarandache Soft Groupoids

Definition 3.1: Let G be a groupoid and (F, A) be a soft groupoid over G. Then (F, A) is said to be a smarandache soft groupoid over G if a proper soft subset (K, B) of (F, A) is a soft semigroup with respect to the operation of G. We denote a smarandache soft groupoid by SS-groupoid.

In other words a smarandache soft groupoid is a parameterized collection of smarandache subgroupoids of G which has a parameterized family of subsemigroups of G.

We now give an examples to illustrate the notion of SS-groupoids.

Example 3.2: Let $G = \{0,1,2,3,4,5\}$ be a groupoid under the binary operation $*$ modulo 6 with the following table.

*	0	1	2	3	4	5
0	0	3	0	3	0	3
1	1	4	1	4	1	4
2	2	5	2	5	2	5
3	3	0	3	0	3	0
4	4	1	4	1	4	1
5	5	2	5	2	5	2

Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Let (F, A) be a soft groupoid over G, where

$$F(a_1) = \{0,1,3,4\}, \mathrm{F}(a_2) = \{0,2,3,5\},$$

$$F(a_3) = \{1,2,4,5\}, \; F(a_4) = \{0,1,2,3,4,5\}.$$

Let $B = \{a_1, a_2, a_4\} \subset A$. Then (G, B) is a soft subsemigroup of (F, A) over G, where

$$\mathrm{K}(a_1) = \{0,3\}, \mathrm{K}(a_2) = \{2,5\},$$

$$\mathrm{K}(a_4) = \{1,4\}.$$

This shows clearly that (F, A) is a smarandache soft groupoid over G.

Proposition 3.3: If G is a smarandache groupoid, then (F, A) over G is also a smarandache soft groupoid.

Proof: It is trivial.

Proposition 3.4: The extended union of two SS-groupoids (F, A) and (K, B) over G is a SS-groupoid over G.

Proposition 3.5: The extended intersection of two SS-groupoids (F, A) and (K, B) over G is again a SS-groupoid.

Proposition 3.6: The restricted union of two SS-groupoids (F, A) and (K, B) over G is a SS-groupoid over G.

Proposition 3.7: The restricted intersection of two SS-groupoids (F, A) and (K, B) over G is a SS-groupoid over G.

Proposition 3.8: The AND operation of two SS-groupoids (F, A) and (K, B) over G is a SS-groupoid over G.

Proposition 3.9: The OR operation of two SS-groupoids (F, A) and (K, B) over G is a SS-groupoid over G.

Definition 3.10: Let (F, A) be a SS-groupoid over a groupoid G. Then (F, A) is called a commutative SS-groupoid if atleast one proper soft subset (K, B) of (F, A) is a commutative semigroup.

Example 3.11: In Example 3.2, (F, A) is a commutative SS-groupoid over G.

Proposition 3.12: If G is a commutative S-groupoid, then (F, A) over G is also a commutative SS-groupoid.

Definition 3.13: Let (F, A) be a SS-groupoid over a groupoid G. Then (F, A) is called a cyclic SS-groupoid if each proper soft subset (K, B) of (F, A) is a cyclic subsemigroup.

Proposition 3.14: If G is a cyclic S-groupoid, then (F, A) over G is also a cyclic SS-groupoid.

Proposition 3.15: If G is a cyclic S-groupoid, then (F, A) over G is a commutative SS-groupoid.

Definition 3.16: Let G be a groupoid and (F, A) be a SS-groupoid. A proper soft subset (K, B) of (F, A) is said to be a Smarandache soft subgroupoid if (K, B) is itself a Smarandache soft groupoid over G.

Example 3.17: Let $G = \{0,1,2,3,4,5\}$ be a groupoid under the binary operation $*$ modulo 6 with the following table.

*	0	1	2	3	4	5
0	0	5	4	3	2	1
1	4	3	2	1	0	5
2	2	1	0	5	4	3
3	0	5	4	3	2	1
4	4	3	2	1	0	5
5	2	1	0	5	4	3

Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Let (F, A) be a soft groupoid over G, where

$$F(a_1) = \{0,3\}, \mathrm{F}\left(a_2\right) = \{0,2,4\},$$

$$F(a_3) = \{1,3,5\},\ F(a_4) = \{0,1,2,3,4,5\}.$$

Let $B = \{a_2, a_3, a_4\} \subset A$. Then (K, B) is a soft subgroupoid of (F, A) over G, where

$$\mathrm{K}(a_2) = \{0,2,4\}, \mathrm{K}(a_3) = \{1,3,5\},$$

$$\mathrm{K}(a_4) = \{1,3,5\}.$$

Let $C = \{a_3, a_4\} \subset B$. Then (H,C) is aSsmarndache soft subsemigroup of (K, B) over G, where

$$\mathrm{K}(a_3) = \{1,3,5\}, \mathrm{K}(a_4) = \{1,3,5\}.$$

Thus clearly (K,B) is a Smarandache soft subgroupoid of (F, A) over G.

Remark: Every soft subgroupoid of a Smarandache soft groupoid need not be a Smarandache soft subgroupoid in general.

One can easily verify it by the help of examples.

Theorem: If a soft groupoid (F, A) contain a Smarandcahe soft subgroupoid, then (F, A) is SS -groupoid.

Proof: Let (F, A) be a soft groupoid and $(\mathrm{K}, \mathrm{B}) \subset (F, A)$ is a Smarandache soft subgroupoid. Therefore, (K, B) has a proper soft subgroupoid (H, C) which is a soft semigroup and this implies $(\mathrm{H}, \mathrm{C}) \subset (F, A)$ which completes the proof.

Definition 3.18: Let G be a groupoid and (F, A) be a soft set over G. Then G is called a parameterized Smarandache groupoid if $F(a)$ is a semigroup under the operation of G for all $a \in A$.

In this case (F, A) is termed a strong soft semigroup.

A strong soft semigroup (F, A) is a parameterized collection of the subsemigroups of the groupoid G.

Example 3.19: Let $G = \{0,1,2,3,4,5\}$ be a groupoid under the binary operation $*$ modulo 6 with the following table.

*	0	1	2	3	4	5
0	0	3	0	3	0	3
1	1	4	1	4	1	4
2	2	5	2	5	2	5
3	3	0	3	0	3	0
4	4	1	4	1	4	1
5	5	2	5	2	5	2

Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Let (F, A) be a soft groupoid over G, where

$$F(a_1) = \{0,3\}, \mathrm{F}\left(a_2\right) = \{2,5\},$$

$$F(a_3) = \{1,4\}.$$

This clearly shows that (F, A) is a strong soft semigroup over G, and G is a parameterized groupoid.

Proposition 3.20: Let (F, A) and (H, B) be two strong soft semigroups over a groupoid G. Then

1. $(F,A)\bigcap_R (H,B)$ is a strong soft semigroup over G.
2. $(F,A)\bigcap_E (H,B)$ is a strong soft semigroup over G.
3. $(F,A)\bigcup_E (H,B)$ is a strong soft semigroup over G.
4. $(F,A)\bigcup_R (H,B)$ is a strong soft semigroup over G.

Proof: The proof of these are straightforward.

Proposition 3.21: Let G be a groupoid and (F,A) be a soft set over G. Then G is a parameterized Smarandache groupoid if (F,A) is a soft semigroup over G.

Proof: Suppose that (F,A) is a soft semigroup over G. This implies that each $F(a)$ is a subsemigroup of the groupoid G for al $a \in A$, and thus G is a parameterized smarandache groupoid.

Example 3.22: Let $\mathbb{Z}_{12} = \{0,1,2,3,...,11\}$ be the groupoid with respect to multiplication modulo 12 and let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Let (F,A) be a soft semigroup over $\mathbb{Z}_{12}$, where

$$F(a_1) = \{3,9\}, \mathrm{F}\left(a_2\right) = \{1,7\},$$

$$F(a_3) = \{1,5\}.$$

Then $\mathbb{Z}_{12}$ is a parameterized Smarandache groupoid.

4. Smarandache Soft Ideal over a Groupoid and Smarandache Soft Ideal of a Smarandache Soft Groupoid

Definition 4.1: Let (F,A) be a SS-groupoid over G. Then (F,A) is called a Smarandache soft ideal over G if and only if $F(a)$ is a Smarandache ideal of G for all a in A.

Definition: Let (F,A) be a SS-groupoid and (K,B) be a soft subset of (F,A). Then (K,B) is called a Smarandache soft left ideal if the following conditions are hold.

1. (K,B) is a Smarandache soft subgroupoid of (F,A), and
2. $x \in G$ and $k \in K(b)$ implies $xk \in K(b)$ for all $b \in B$.

Similarly we can define a Smarandachesoft right ideal. A Smarandache soft ideal is one which both Smarandache soft left and right ideal.

Theorem: Let (F, A) be a SS -groupoid over G . If (K, B) is a Smarandache soft ideal of (F, A), then (K, B) is a soft ideal of (F, A) over G .

Proposition 3.20: Let (F, A) and (H, B) be two Smarandache soft ideals over a groupoid G . Then

1. $(F, A) \bigcap_R (H, B)$ is a Smarandache soft ideal over G .
2. $(F, A) \bigcap_E (H, B)$ is a Smarandache soft ideal over G .
3. $(F, A) \bigcup_E (H, B)$ is a Smarandache soft ideal over G .
4. $(F, A) \bigcup_R (H, B)$ is a Smarandache soft ideal over G .

Proof: The proof of these are straightforward.

Definition: Let (F, A) and (K, B) be two SS -groupoid over G . Then (K, B) is called a Smarandache soft seminormal groupoid if

1. $B \subset A$, and
2. $K(a)$ is a Smaradache seminormal subgroupoid $F(a)$ for all $a \in A$.

Smarandache Soft Homomorphism

Definition: Let (F, A) and (K, B) be two SS -groupoid over $(G, *)$ and $(G, \circ)$ respectively. A map $\phi : (F, A) \rightarrow (\mathrm{K}, \mathrm{B})$ is said to be a Smaradache soft groupoid homomorphism if $\phi : (F^{'}, A^{'}) \rightarrow (\mathrm{K}^{'}, B^{'})$ is a soft semigroup homomorphism where $(F^{'}, A^{'}) \subset (F, A)$ and $(\mathrm{K}^{'}, B^{'}) \subset (\mathrm{K}, B)$ are soft semigroups respectively.

A Smarandache soft groupoid homomorphism is called Smarandache soft groupoid isomorphism if ϕ is a soft semigroup isomorphism.

5. Conclusion

In this paper Smaradache soft groupoids are introduced. Their related properties and results are discussed with illustrative examples to grasp the idea of these new notions. The theory of Smarandache soft groupoids open a new way for researchers to define these type of soft algebraic structures in other areas of soft algebraic structures in the future.

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