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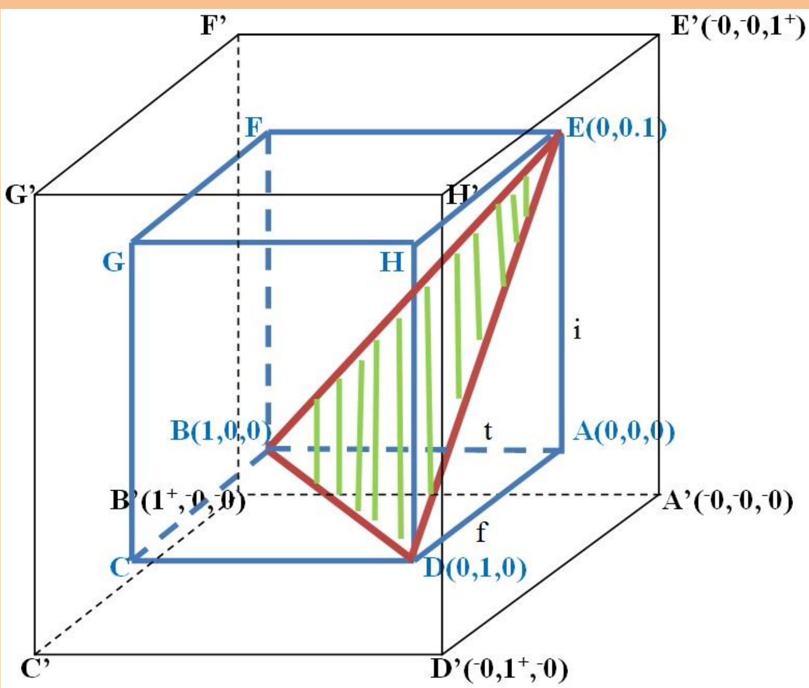

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Neutrosophic Sets and Systems

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The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a degree of indeter
minacy (*I*), and a degree of falsity (*F*), where *T*, *I*, *F* are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Neutrosophic Closed Set and Neutrosophic Continuous Functions

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Abstract

In this paper, we introduce and study the concept of "neutrosophic closed set "and "neutrosophic continuous function". Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Closed Set, Neutrosophic Set; Neutrosophic Topology; Neutrosophic Continuous Function.

1 INTRODUCTION

The idea of "neutrosophic set" was first given by Smarandache [11, 12]. Neutrosophic operations have been investigated by Salama at el. [1-10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts [9, 13]. Here we shall present the neutrosophic crisp version of these concepts. In this paper, we introduce and study the concept of "neutrosophic closed set "and "neutrosophic continuous function".

2 TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular the work of Smarandache in [11, 12], and Salama at el. [1-10].

2.1 Definition [5]

A neutrosophic topology (NT for short) an a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms

(NT_1) $O_N, 1_N \in \tau$,

(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(NT_3) $\cup G_i \in \tau \quad \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is known as neuterosophic open set (NOS for short) in X . The elements of τ are called open neutrosophic sets, A neutrosophic set F is closed if and only if it C (F) is neutrosophic open.

2.1 Definition [5]

The complement of (C (A) for short) of is called a neutrosophic closed set (for short) in A . NOSA NCS X**.**

3 Neutrosophic Closed Set .

3.1 Definition

Let (X, τ) be a neutrosophic topological space. A neutrosophic set A in (X, τ) is said to be neutrosophic closed (in shortly N-closed).

If Ncl (A) $\subseteq$ G whenever A $\subseteq$ G and G is neutrosophic open; the complement of neutrosophic closed set is Neutrosophic open.

3.1 Proposition

If A and B are neutrosophic closed sets then A$\cup$B is Neutrosophic closed set.

3.1 Remark

The intersection of two neutrosophic closed (N-closed for short) sets need not be neutrosophic closed set.

3.1 Example

Let X = {a, b, c} and

A = <(0.5,0.5,0.5) , (0.4,0.5,0.5) , (0.4,0.5,0.5)>
B = <(0.3,0.4,0.4) , (0.7,0.5,0.5) , (0.3,0.4,0.4)>
Then T = { 0_N ,1_N, A, B} is a neutrosophic topology on X. Define the two neutrosophic sets A_1 and A_2 as follows,
A_1 = <(0.5,0.5,0.5),(0.6,0.5,0.5),(0.6,0.5,0.5)>
A_2 = <(0.7,0.6,0.6)(0.3,0.5,0.5),(0.7,0.6,0.6)>
A_1 and A_2 are neutrosophic closed set but $A_1 \cap A_2$ is not a neutrosophic closed set.

3.2 Proposition

Let (X,τ) be a neutrosophic topological space. If B is neutrosophic closed set and B ⊆ A ⊆ Ncl (B), then A is N-closed.

3.4 Proposition

In a neutrosophic topological space (X,T), T=ℑ (the family of all neutrosophic closed sets) iff every neutrosophic subset of (X,T) is a neutrosophic closed set.

Proof.
suppose that every neutrosophic set A of (X,T) is N-closed. Let A∈T, since A ⊆ A and A is N-closed, Ncl (A) ⊆ A. But A ⊆ Ncl (A). Hence, Ncl (A) =A. thus, A ∈ ℑ. Therefore, T ⊆ ℑ. If B ∈ ℑ then 1-B ∈T ⊆ ℑ. and hence B∈T, That is, ℑ ⊆ T. Therefore T=ℑ conversely, suppose that A be a neutrosophic set in (X,T). Let B be a neutrosophic open set in (X,T). such that A ⊆ B. By hypothesis, B is neutrosophic N-closed. By definition of neutrosophic closure, Ncl (A) ⊆ B. Therefore A is N-closed.

3.5 Proposition

Let (X,T) be a neutrosophic topological space. A neutrosophic set A is neutrosophic open iff B ⊆ NInt (A), whenever B is neutrosophic closed and B ⊆ A.

Proof

Let A a neutrosophic open set and B be a N-closed, such that B ⊆ A. Now, B ⊆ A⇒ 1−A ⇒1−B and 1−A is a neutrosophic closed set ⇒ Ncl (1−A) ⊆ 1−B. That is, B=1−(1−B) ⊆ 1−Ncl (1−A). But 1−Ncl (1−A) = Nint (A). Thus, B ⊆ Nint (A). Conversely, suppose that A be a neutrosophic set, such that B ⊆ Nint (A) whenever B is neutrosophic closed and B ⊆ A. Let 1−A ⊆ B ⇒ 1−B ⊆ A. Hence by assumption 1−B ⊆ Nint (A). that is, 1−Nint (A) ⊆ B. But 1−Nint (A) =Ncl (1−A). Hence Ncl(1−A) ⊆ B. That is 1−A is neutrosophic closed set. Therefore, A is neutrosophic open set

3.6 Proposition

If Nint (A) ⊆ B ⊆ A and if A is neutrosophic open set then B is also neutrosophic open set.

4 Neutrosophic Continuous Functions

4.1 Definition

i) If $B=\langle \mu_B,\sigma_B,\nu_B\rangle$ is a NS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NS in X defined by $f^{-1}(B)=\left\langle f^{-1}(\mu_B), f^{-1}(\sigma_B), f^{-1}(\nu_\nu)\right\rangle$.

ii) If $A=\langle \mu_A,\sigma_A,\nu_A\rangle$ is a NS in X, then the image of A under f, denoted by $f(A)$, is the a NS in Y defined by $f(A)=\left\langle f(\mu_A), f(\sigma_A), f(\nu_A)^c\right\rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following sections .

4.1 Corollary

Let A, $\{A_i : i \in J\}$, be NSs in X, and B, $\{B_j : j \in K\}$ NS in Y, and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,
$B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$.

(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B$, .

(d) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$,

(e) $f(\cup A_i) = \cup f(A_i)$; $f(\cap A_i) \subseteq \cap f(A_i)$; and if f is injective, then $f(\cap A_i) = \cap f(A_i)$;

(f) $f^{-1}(!_N) = 1_N$ $f^{-1}(0_N) = 0_N$.

(g) $f(0_N) = 0_N$, $f(1_N) = 1_N$ if f is subjective.

Proof

Obvious.

4.2 Definition

Let (X,Γ_1) and (Y,Γ_2) be two NTSs, and let $f : X \to Y$ be a function. Then f is said to be continuous iff the preimage of each NCS in Γ_2 is a NS in Γ_1.

4.3 Definition

Let (X,Γ_1) and (Y,Γ_2) be two NTSs and let $f : X \to Y$ be a function. Then f is said to be open iff the image of each NS in Γ_1 is a NS in Γ_2.

4.1 Example

Let (X,Γ_o) and (Y,ψ_o) be two NTSs

(a) If $f : X \to Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of Definition 5.1 too. Here we consider the NTs on X and Y, respectively, as follows : $\Gamma_1 = \left\{\left\langle \mu_G, 0, \mu_G^c\right\rangle : G \in \Gamma_o\right\}$ and

$\Gamma_2 = \left\{ \left\langle \mu_H, 0, \mu_H^c \right\rangle : H \in \Psi_o \right\}$,
In this case we have, for each $\left\langle \mu_H, 0, \mu_H^c \right\rangle \in \Gamma_2$, $H \in \Psi_o$,

$f^{-1}\left\langle \mu_H, 0, \mu_H^c \right\rangle = \left\langle f^{-1}(\mu_H), f^{-1}(0), f^{-1}(\mu_H^c) \right\rangle$
$= \left\langle f^{-1}\mu_H, f(0), (f(\mu)^c \right\rangle \in \Gamma_1$.

(b) If $f : X \to Y$ is neutrosophic open in the usual sense, then in this case, f is neutrosophic open in the sense of Definition 3.2.

Now we obtain some characterizations of neutrosophic continuity:

4.1 Proposition

Let $f : (X, \Gamma_1) \to (Y, \Gamma_2)$.

f is neutrosop continuous iff the preimage of each NS (neutrosophic closed set) in Γ_2 is a NS in Γ_2 **.**

4.2 Proposition

The following are equivalent to each other:

(a) $f : (X, \Gamma_1) \to (Y, \Gamma_2)$ is neutrosophic continuous .

(b) $f^{-1}(NInt(B) \subseteq NInt(f^{-1}(B))$ for each CNS B in Y.

(c) $NCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for each NCB in Y.

4.2 Example

Let (Y, Γ_2) be a NTS and $f : X \to Y$ be a function. In this case $\Gamma_1 = \left\{ f^{-1}(H) : H \in \Gamma_2 \right\}$is a NT on X. Indeed, it is the coarsest NT on X which makes the function $f : X \to Y$ continuous. One may call it the initial neutrosophic crisp topology with respect to f.

4.4 Definition

Let (X,T) and (Y,S) be two neutrosophic topological space, then

(a) A map f : (X,T) → (Y,S) is called N-continuous (in short N-continuous) if the inverse image of every closed set in (Y,S) is Neutrosophic closedin (X,T).

(b) A map f:(X,T)→ (Y,S) is called neutrosophic-gc irresolute if the inverse image of every Neutrosophic closedset in (Y,S) is Neutrosophic closedin (X,T). Equivalently if the inverse image of every Neutrosophic open set in (Y,S) is Neutrosophic open in (X,T).

(c) A map f:(X,T)→ (Y,S) is said to be strongly neutrosophic continuous if f^{-1}(A) is both neutrosophic open and neutrosophic closed in (X,T) for each neutrosophic set A in (Y,S).

(d) A map f : (X,T) → (Y,S) is said to be perfectly neutrosophic continuous if f^{-1} (A) is both neutrosophic open and neutrosophic closed in (X,T) for each neutrosophic open set A in (Y,S).

(e) A map f:(X,T)→(Y,S) is said to be strongly N-continuous if the inverse image of every Neutrosophic open set in (Y,S) is neutrosophic open in (X,T).

(F) A map f:(X,T)→(Y,S) is said to be perfectly N-continuous if the inverse image of every Neutrosophic open set in (Y,S) is both neutrosophic open and neutrosophic closed in (X,T).

4.3 Proposition

Let (X,T) and (Y,S) be any two neutrosophic topological spaces. Let f : (X,T) → (Y,S) be generalized neutrosophic continuous. Then for every neutrosophic set A in X, f(Ncl(A)) ⊆ Ncl(f(A)).

4.4 Proposition

Let (X,T) and (Y,S) be any two neutrosophic topological spaces. Let f : (X,T) → (Y,S) be generalized neutrosophic continuous. Then for every neutrosophic set A in Y, Ncl(f^{-1}(A)) ⊆ f^{-1}(Ncl(A)).

4.5 Proposition

Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If A is a Neutrosophic closedset in (X,T) and if f : (X,T) → (Y,S) is neutrosophic continuous and neutrosophic-closed then f(A) is Neutrosophic closedin (Y,S).

Proof.

Let G be a neutrosophic-open in (Y,S). If f(A) ⊆ G, then A ⊆ f^{-1}(G) in (X,T). Since A is neutrosophic closedand f^{-1}(G) is neutrosophic open in (X,T), Ncl(A) ⊆ f^{-1}(G), (i.e) f(Ncl(A)⊆G. Now by assumption, f(Ncl(A)) is neutrosophic closed and Ncl(f(A)) ⊆ Ncl(f(Ncl(A))) = f(Ncl(A)) ⊆ G. Hence, f(A) is N-closed.

4.5 Proposition

Let (X,T) and (Y,S) be any two neutrosophic topological spaces, If f : (X,T) → (Y,S) is neutrosophic continuous then it is N-continuous.

The converse of proposition 4.5 need not be true. See Example 4.3.

4.3 Example

Let X ={a,b,c} and Y ={a,b,c}. Define neutrosophic sets A and B as follows A = $\langle (0.4,0.4,0.5), (0.2,0.4,0.3), (0.4,0.4,0.5) \rangle$

B = $\langle (0.4,0.5,0.6), (0.3,0.2,0.3), (0.4,0.5,0.6) \rangle$

Then the family T = {0_N, 1_N, A} is a neutrosophic topology on X and S = {0_N, 1_N, B} is a neutrosophic topology on Y. Thus (X,T) and (Y,S) are neutrosophic topological spaces. Define f : (X,T) → (Y,S) as f(a) = b , f(b) = a, f(c) = c. Clearly f is N-continuous. Now f is not neutrosophic continuous, since f^{-1}(B) ∉ T for B ∈ S.

4.4 Example

Let X = {a,b,c}. Define the neutrosophic sets A and B as follows.

A = $\langle (0.4,0.5,0.4), (0.5,0.5,0.5), (0.4,0.5,0.4) \rangle$

B = $\langle (0.7,0.6,0.5), (0.3,0.4,0.5), (0.3,0.4,0.5) \rangle$
and C = $\langle (0.5,0.5,0.5), (0.4,0.5,0.5), (0.5,0.5,0.5) \rangle$
T = $\{0_N, 1_N, A, B\}$
and S = $\{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus (X,T) and (X,S) are neutrosophic topological spaces. Define f: (X,T) $\to$ (X,S) as follows f(a) = b, f(b) = b, f(c) = c. Clearly f is N-continuous. Since
D = $\langle (0.6,0.6,0.7), (0.4,0.4,0.3), (0.6,0.6,0.7) \rangle$
is neutrosophic open in (X,S), f^{-1}(D) is not neutrosophic open in (X,T).

4.6 Proposition
Let (X,T) and (Y,S) be any two neutrosophic topological space. If f : (X,T) $\to$ (Y,S) is strongly N-continuous then f is neutrosophic continuous.
The converse of Proposition 3.19 is not true. See Example 3.3

4.5 Example
Let X ={a,b,c}. Define the neutrosophic sets A and B as follows.
A = $\langle (0.9,0.9,0.9), (0.1,0.1,0.1), (0.9,0.9,0.9) \rangle$
B = $\langle (0.9,0.9,0.9), (0.1,0.1,0), (0.9,0.1,0.8) \rangle$
and C = $\langle (0.9,0.9,0.9), (0.1,0,0.1), (0.9,0.9,0.9) \rangle$
T = $\{0_N, 1_N, A, B\}$ and S = $\{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus (X,T) and (X,S) are neutrosophic topological spaces. Also define f :(X,T)$\to$ (X,S) as follows f(a) = a, f(b) = c, f(c) = b. Clearly f is neutrosophic continuous. But f is not strongly N-continuous. Since
D = $\langle (0.9,0.9,0.99), (0.05,0,0.01), (0.9,0.9,0.99) \rangle$
Is an Neutrosophic open set in (X,S), f^{-1}(D) is not neutrosophic open in (X,T).

4.7 Proposition
Let (X,T) and (Y, S) be any two neutrosophic topological spaces. If f: (X,T) $\to$ (Y,S) is perfectly N-continuous then f is strongly N-continuous.
The converse of Proposition 4.7 is not true. See Example 4.6

4.6 Example
Let X = {a,b,c}. Define the neutrosophic sets A and B as follows.
A = $\langle (0.9,0.9,0.9), (0.1,0.1,0.1), (0.9,0.9,0.9) \rangle$
B = $\langle (0.99,0.99,0.99), (0.01,0,0), (0.99,0.99,0.99) \rangle$
And C = $\langle (0.9,0.9,0.9), (0.1,0.1,0.05), (0.9,0.9,0.9) \rangle$
T = $\{0_N, 1_N, A, B\}$ and S = $\{0_N, 1_N, C\}$ are neutrosophic topologies space on X. Thus (X,T) and (X,S) are neutrosophic topological spaces. Also define f : (X,T) $\to$ (X,S) as follows f(a) = a, f(b) = f(c) = b. Clearly f is strongly N-continuous. But f is not perfectly N continuous. Since D = $\langle (0.9,0.9,0.9), (0.1,0.1,0), (0.9,0.9,0.9) \rangle$
Is an Neutrosophic open set in (X,S), f^{-1}(D) is neutrosophic open and not neutrosophic closed in (X,T).

4.8 Proposition
Let (X,T) and (Y,S) be any neutrosophic topological spaces. If f: (X,T) $\to$ (Y,S) is strongly neutrosophic continuous then f is strongly N-continuous.
The converse of proposition 3.23 is not true. See Example 4.7

4.7 Example
Let X = {a,b,c} and Define the neutrosophic sets A and B as follows.
A = $\langle (0.9,0.9,0.9), (0.1,0.1,0.1), (0.9,0.9,0.9) \rangle$
B = $\langle (0.99,0.99,0.99), (0.01,0,0), (0.99,0.99,0.99) \rangle$
and C = $\langle (0.9,0.9,0.9), (0.1,0.1,0.05), (0.9,0.9,0.9) \rangle$
T = $\{0_N, 1_N, A, B\}$ and S = $\{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus (X,T) and (X,S) are neutrosophic topological spaces. Also define f : (X,T) $\to$ (X,S) as follows: f(a) = a, f(b) = f(c) = b. Clearly f is strongly N-continuous. But f is not strongly neutrosophic continuous. Since
D = $\langle (0.9,0.9,0.9), (0.1,0.1,0), (0.9,0.9,0.9) \rangle$
be a neutrosophic set in (X,S), f^{-1}(D) is neutrosophic open and not neutrosophic closed in (X,T).

4.9 Proposition
Let (X,T),(Y,S) and (Z,R) be any three neutrosophic topological spaces. Suppose f : (X,T) $\to$ (Y,S), g : (Y,S) $\to$ (Z,R) be maps. Assume f is neutrosophic gc-irresolute and g is N-continuous then g o f is N-continuous.

4.10 Proposition
Let (X,T), (Y,S) and (Z,R) be any three neutrosophic topological spaces. Let f : (X,T) $\to$ (Y,S), g : (Y,S) $\to$ (Z,R) be map, such that f is strongly N-continuous and g is N-continuous. Then the composition g o f is neutrosophic continuous.

4.5 Definition
A neutrosophic topological space (X,T) is said to be neutrosophic $T_{1/2}$ if every Neutrosophic closed set in (X,T) is neutrosophic closed in (X,T).

4.11 Proposition
Let (X,T),(Y,S) and (Z,R) be any neutrosophic topological spaces. Let f : (X,T) $\to$ (Y,S) and g : (Y,S) $\to$ (Z,R) be mapping and (Y,S) be neutrosophic $T_{1/2}$ if f and g are N-continuous then the composition g o f is N-continuous.
The proposition 4.11 is not valid if (Y,S) is not neutrosophic $T_{1/2}$.

4.8 Example
Let X = {a,b,c}. Define the neutrosophic sets A,B and C as follows.
A = $\langle (0.4,0.4,0.6), (0.4,0.4,0.3) \rangle$
B = $\langle (0.4,0.5,0.6), (0.3,0.4,0.3) \rangle$
and C = $\langle (0.4,0.6,0.5), (0.5,0.3,0.4) \rangle$

Then the family T = $\{0_N, 1_N, A\}$, S = $\{0_N, 1_N, B\}$ and R = $\{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus (X,T),(X,S) and (X,R) are neutrosophic topological spaces. Also define f : (X,T) $\rightarrow$ (X,S) as f(a) = b, f(b) = a, f(c) = c and g : (X,S) $\rightarrow$ (X,R) as g(a) = b, g(b) = c, g(c) = b. Clearly f and g are N-continuous function. But g o f is not N-continuous. For 1 − C is neutrosophic closed in (X,R). $f^{-1}(g^{-1}(1-C))$ is not N closed in (X,T). g o f is not N-continuous.

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Neutrosophic Vector Spaces

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Abstract. The objective of this paper is to study *neutrosophic* vector spaces. Some basic definitions and properties of the classical vector spaces are generalized. It is shown that every *neutrosophic* vector space over a *neutrosophic* field (resp. a field) is a vector space. Also, it is shown that an element of a *neutrosophic* vector space over a *neutrosophic* field can be infinitely expressed as a linear combination of some elements of the *neutrosophic* vector space. *Neutrosophic* quotient spaces and *neutrosophic* vector space homomorphism are also studied.

Keywords: Weak *neutrosophic* vector space, strong *neutrosophic* vector space, field, *neutrosophic* field.

1 Introduction and Preliminaries

The theory of fuzzy set introduced by L. Zadeh [9] is mainly concerned with the measurement of the degree of membership and non-membership of a given abstract situation. Despite its wide range of real life applications, fuzzy set theory cannot be applied to model an abstract situation where indeterminancy is involved. In his quest to modelling situations involving indeterminates, F. Smarandache introduced the theory of *neutrosophy* in 1995. *Neutrosophic* logic is an extension of the fuzzy logic in which indeterminancy is included. In the *neutrosophic* logic, each proposition is characterized by the degree of truth in the set (T), the degree of falsehood in the set (F) and the degree of indeterminancy in the set (I) where T,F,I are subsets of] -,0,1,+ [. *Neutrosophic* logic has wide applications in science, engineering, IT, law, politics, economics, finance, etc. The concept of *neutrosophic* algebraic structures was introduced by F. Smarandache and W.B. Vasantha Kandasamy in 2006. However, for details about *neutrosophy* and *neutrosophic* algebraic structures, the reader should see [1, 2, 3, 4, 5, 6, 7, 8].

Definition 1.1. Let U be a universe of discourse and let M be a subset of U. M is called a neutrosophic set if an element $x = x(T, I, F) \in U$ belongs to M in the following way:

(1) x is t% true in M,
(2) x is i% indeterminate in M, and
(3) x is f% false in M,

where $t \in T$, $i \in I$ and $f \in F$.

It is possible to have t+i+f=1 as in the case of classical and fuzzy logics and probability. Also, it is possible to have t+i+f<1 as in the case of intuitionistic logic and as in the case of paraconsistent logic, it is possible to have t+i+f>1.

Remark 1. Statically, T,I,F are subsets of]-,0,1,+[but dynamically, they are functions/operators depending on many known or unknown parameters.

Example 1. The probability of that a student will pass his final year examination in Mathematics is 60% true according to his Mathematics Teacher from Year 1, 25 or 30-35% false according to his present poor performance, and 15 or 20% indeterminate due to sickness during his final year examination.

Definition 1.2. Let $(G, *)$ be any group and let $G(I) = \langle G \cup I \rangle$. The couple $(G(I), *)$ is called a *neutrosophic* group generated by G and I under the binary operation $*$. The indeterminancy factor I is such that $I * I = I$. If $*$ is ordinary multiplication, then $IastI * ... * I = I^n = I$ and if $*$ is ordinary addition, then $I * I * I * ... * I = nI$ for $n \in \mathbb{N}$.

If $a * b = b * a$ for all $a, b \in G(I)$, we say that G(I) is commutative. Otherwise, G(I) is called a non-commutative *neutrosophic* group.

Theorem 1.3. *[5] Let G(I) be a neutrosophic group. Then,*

(1) G(I) in general is not a group;
(2) G(I) always contain a group.

Example 2. [3] Let G(I)={e, a, b, c, I, aI, bI, cI} be a set, where $a^2=b^2=c^2=e$, bc=cb=a, ac=ca=b, ab=ba=c. Then (G(I),.) is a commutative *neutrosophic* group.

Definition 1.4. Let (K,+,.) be any field and let $K(I) = \langle K \cup I \rangle$ be a *neutrosophic* set generated by K and I. The triple (K(I),+,.) is called a *neutrosophic* field. The zero element $0 \in K$ is represented by 0+0I in K(I) and $1 \in K$ is represented by 1+0I in K(I).

Definition 1.5. Let K(I) be a neutrosophic field and let F(I) be a nonempty subset of K(I). F(I) is called a neutrosophic subfield of K(I) if F(I) is itself a neutrosophic field. It is essential that F(I) contains a proper subset which is a field.

Example 3. $\mathbb{Q}(I)$, $\mathbb{R}(I)$ and $\mathbb{C}(I)$ are respectively neutrosophic fields of rational, real and complex numbers. $\mathbb{Q}(I)$ is a neutrosophic subfield of $\mathbb{R}(I)$ and $\mathbb{R}(I)$ is a neutrosophic subfield of $\mathbb{C}(I)$.

2 Neutrosophic Vector Spaces

Definition 2.1. Let (V,+,.) be any vector space over a field K and let $V(I) = < V \cup I >$ be a *neutrosophic* set generated by V and I. The triple (V(I),+,.) is called a weak *neutrosophic* vector space over a field K. If V(I) is a *neutrosophic* vector space over a *neutrosophic* field K(I), then V(I) is called a strong *neutrosophic* vector space. The elements of V(I) are called *neutrosophic* vectors and the elements of K(I) are called *neutrosophic* scalars.

If $u = a + bI, v = c + dI \in V(I)$ where a,b,c and d are vectors in V and $\alpha = k + mI \in K(I)$ where k and m are scalars in K, we define:

$$u + v = (a + bI) + (c + dI)$$
$$= (a + c) + (b + d)I, \text{ and}$$
$$\alpha u = (k + mI).(a + bI)$$
$$= k.a + (k.b + m.a + m.b)I.$$

Example 4. (1) $\mathbb{R}(I)$ is a weak *neutrosophic* vector space over a field $\mathbb{Q}$ and it is a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{Q}(I)$.

(2) $\mathbb{R}^n(I)$ is a weak *neutrosophic* vector space over a field $\mathbb{R}$ and it is a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{R}(I)$.

(3) $M_{m\times n}(I) = \{[a_{ij}] : a_{ij} \in \mathbb{Q}(I)\}$ is a weak *neutrosophic* vector space over a field $\mathbb{Q}$ and it is a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{Q}(I)$.

Theorem 2.2. Every strong *neutrosophic* vector space is a weak *neutrosophic* vector space.

Proof. Suppose that V(I) is a strong *neutrosophic* space over a *neutrosophic* field K(I). Since $K \subseteq K(I)$ for every field K, it follows that V(I) is a weak *neutrosophic* vector space.

Theorem 2.3. *Every weak (strong) neutrosophic vector space is a vector space.*

Proof. Suppose that V(I) is a strong *neutrosophic* space over a *neutrosophic* field K(I). Obviously, (V(I),+,.) is an abelian group. Let $u = a + bI, v = c + dI \in V(I)$, $\alpha = k + mI, \beta = p + nI \in K(I)$ where $a, b, c, d \in V$ and $k, m, p, n \in K$. Then

(1) $\alpha(u + v) = (k + mI)(a + bI + c + dI)$
$= ka + kc + [kb + kd + ma + mb + mc + md]I$
$= (k + mI)(a + bI) + (k + mI)(c + dI)$
$= \alpha u + \alpha v$.

(2) $(\alpha + \beta)u = (k + mI + p + nI)(a + bI)$
$= ka + pa + [kb + pb + ma + na + mb + nb]I$
$= (k + mI)(a + bI) + (p + nI)(a + bI)$
$= \alpha u + \beta u$

(3) $(\alpha\beta)u = ((k + mI)(p + nI))(a + bI)$
$= kpa + [kpb + kna + mpa + mna + knb + mpb + mnb]I$
$= (k + mI)((p + nI)(a + bI))$
$= \alpha(\beta u)$

(4) For $1 + 1 + 0I \in K(I)$, we have
$1u = (1 + 0I)(a + bI)$
$= a(b + 0 + 0)I$
$= a + bI$.

Accordingly, V(I) is a vector space.

Lemma 2.4. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let* u=a+bI,v=c+dI, $w = e + fI \in V(I), \alpha = k + mI \in K(I)$. *Then:*

(1) u+w=v+w implies u=v.
(2) α0=0.
(3) 0u=0.
(4) (-α)u=α(-u)=-(αu)

Definition 2.5. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I) and let W(I) be a nonempty subset of V(I). W(I) is called a strong *neutrosophic* subspace of V(I) if W(I) is itself a strong *neutrosophic* vector space over K(I). It is essential that W(I) contains a proper subset which is a vector space.

Definition 2.6. Let V(I) be a weak *neutrosophic* vector space over a field K and let W(I) be a nonempty subset of V (I). W(I) is called a weak *neutrosophic* subspace of V(I) if W(I) is itself a weak *neutrosophic* vector space over K. It is essential that W(I) contains a proper subset which is a vector space.

Theorem 2.7. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let W(I) be a non-empty subset of V(I). W(I) is a strong neutrosophic sub-space of V(I) if and only if the following conditions hold*:

(1) $u, v \in W(I)$ *implies* $u + v \in W(I)$.

(2) $u \in W(I)$ *implies* $\alpha u \in W(I)$ for all $\alpha \in K(I)$.

(3) W(I) contains a proper subset which is a vector space.

Corollary 2.8. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I) and let W(I) be a nonempty subset of V(I). W(I) is a strong *neutrosophic* subspace of V(I) if and only if the following conditions hold:

(1) $u, v \in W(I)$ *implies* $\alpha u + \beta v \in W(I)$ for all $\alpha, \beta \in K(I)$.

(2) W(I) contains a proper subset which is a vector space.

Example 5. Let V(I) be a weak (strong) *neutrosophic* vector space. V(I) is a weak (strong) *neutrosophic* subspace called a trivial weak (strong) *neutrosophic* subspace.

Example 6. Let $V(I) = \mathbb{R}^3(I)$ be a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{R}(I)$ and let

$$W(I) = \{(u = a + bI, v = c + dI, 0 = 0 + 0I) \in V(I) : a, b, c, d \in V\}.$$

Then W(I) is a strong *neutrosophic* subspace of V(I).

Example 7. Let $V(I) = M_{m\times n}(I) = \{[a_{ij}] : a_{ij} \in \mathbb{R}(I)\}$ be a strong *neutrosophic* vector space over $\mathbb{R}(I)$ and let

$$W(I) = A_{m\times n}(I) = \{[b_{ij}] : b_{ij} \in \mathbb{R}(I) \text{ and } trace(A) = 0\}.$$

Then W(I) is a strong *neutrosophic* subspace of V(I).

Theorem 2.9. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let* $\{W_n(I)\}_{n\in\Lambda}$ *be a family of strong neutrosophic subspaces of V(I). Then* $\bigcap W_n(I)$ *is a strong neutrosophic subspace of V(I).*

Remark 2. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I) and let $W_1(I)$ and $W_2(I)$ be two distinct strong *neutrosophic* subspaces of V(I). Generally, $W_1(I) \cup W_1(I)$ is not a strong *neutrosophic* subspace of V(I). However, if $W_1(I) \subseteq W_2(I)$ or $W_2(I) \subseteq W_1(I)$, then $W_1(I) \cup W_2(I)$ is a strong *neutrosophic* subspace of V(I).

Definition 2.10. Let U(I) and W(I) be any two strong *neutrosophic* subspaces of a strong *neutrosophic* vector space V(I) over a *neutrosophic* field K(I).

(1) The sum of U(I) and W(I) denoted by U(I)+W(I) is defined by the set

$$\{u + w : u \in U(I), w \in W(I)\}.$$

(2) V(I) is said to be the direct sum of U(I) and W(I) written $V(I) = U(I) \oplus W(I)$ if every element $v \in V(I)$ can be written uniquely as v=u+w where $u \in U(I)$ and $w \in W(I)$.

Example 8. Let $V(I) = \mathbb{R}^3(I)$ be a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{R}(I)$ and let

$U(I) = \{(u, v, 0) : u, v \in \mathbb{R}(I)\}$ and

$W(I) = \{(0, 0, w) : w \in \mathbb{R}(I)\}$.

Then $V(I) = U(I) \oplus W(I)$.

Lemma 2.11. *Let W(I) be a strong neutrosophic subspace of a strong neutrosophic vector space V(I) over a neutrosophic field K(I). Then:*

(1) W(I)+W(I)=W(I).

(2) *w+W(I)=W(I) for all* $w \in W(I)$.

Theorem 2.12. Let U(I) and W(I) be any two strong *neutrosophic* subspaces of a strong *neutrosophic* vector space V(I) over a *neutrosophic* field K(I). Then:

(1) U(I)+W(I) is a strong *neutrosophic* subspace of V(I).

(2) U(I) and W(I) are contained in U(I)+W(I).

Proof. (1) Obviously, U+W is a subspace contained in U(I)+W(I). Let $u, w \in U(I) + W(I)$ and let

$\alpha, \beta \in K(I)$. Then $u = (u_1 + u_2 I) + (w_1 + w_2 I)$, $w = (u_3 + u_4 I) + (w_3 + w_4 I)$ where $u_i \in U, w_i \in W$, i=1,2,3,4, $\alpha = k + mI, \beta = p + nI$ where $k, m, p, n \in K$. Now,

$$\alpha u + \beta w = [(ku_1 + pu_3) + [ku_2 + mu_1 + pu_4 + nu_3 + nu_4]I] + [(kw_1 + pw_3) + [kw_2 + mw_1 + pw_4 + nw_3 + nw_4]I] \in U(I) + W(I).$$

Accordingly, U(I)+W(I) is a strong *neutrosophic* subspace of V(I).

(2) Obvious.

Theorem 2.13. *Let U(I) and W(I) be strong neutrosophic subspaces of a strong neutrosophic vector space V(I) over a neutrosophic field K(I).* $V(I) = U(I) \oplus W(I)$ *if and only if the following conditions hold:*

(1) V(I)=U(I)+W(I) and

(2) $U(I) \cap W(I) = \{0\}$.

Theorem 2.14. *Let $V_1(I)$ and $V_2(I)$ be strong neutrosophic vector spaces over a neutrosophic field K(I). Then*

$$V_1(I) \times V_2(I) = \{(u_1, u_2) : u_1 \in V_1(I), u_2 \in V_2(I)\}$$

is a strong neutrosophic vector space over K(I) where addition and multiplication are defined by

$$(u_1 + u_2) + (v_1 + v_2) = (u_1 + v_1, u_2 + v_2),$$

$$\alpha(u_1, u_2) = (\alpha u_1, \alpha u_2).$$

Definition 2.15. Let W(I) be a strong *neutrosophic* subspace of a strong *neutrosophic* vector space V(I) over a *neutrosophic* field K(I). The quotient V(I)/W(I) is defined by the set

$$\{v + W(I) : v \in V(I)\}.$$

V(I)/W(I) can be made a strong *neutrosophic* vector space over a *neutrosophic* field K(I) if addition and multiplication are defined for all $u + W(I)), (v + W(I) \in V(I)/W(I)$ and $\alpha \in K(I)$ as follows:

$$(u + W(I)) + (v + W(I)) = (u + v) + W(I),$$

$$\alpha(u + W(I)) = \alpha u + W(I).$$

The strong *neutrosophic* vector space (V(I)/W(I),+,.) over a *neutrosophic* field K(I) is called a strong *neutrosophic* quotient space.

Example 9. Let V(I) be any strong *neutrosophic* vector space over a *neutrosophic* field K(I). Then V(I)/V(I) is strong *neutrosophic* zero space.

Definition 2.16. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I) and let $v_1, v_2, ..., v_3 \in V(I)$.

(1) An element $v \in V(I)$ is said to be a linear combination of the $v_i s$ if

$$v = \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n, \text{ where } \alpha_i \in K(I).$$

(2) $v_i s$ are said to be linearly independent if

$$\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n = 0$$

implies that $\alpha_1 = \alpha_2 = ... = \alpha_n = 0$. In this case, the set $\{v_1, v_2, ..., v_n\}$ is called a linearly independent set.

(3) $v_i s$ are said to be linearly dependent if

$$\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n = 0$$

implies that not all α_i are equal to zero. In this case, the set $\{v_1, v_2, ..., v_n\}$ is called a linearly dependent set.

Definition 2.17. Let V(I) be a weak *neutrosophic* vector space over a field K(I) and let $v_1, v_2, ..., v_3 \in V(I)$.

(1) An element $v \in V(I)$ is said to be a linear combination of the $v_i s$ if

$$v = k_1 v_1 + k_2 v_2 + ... + k_n v_n, \text{ where } k_i \in K(I).$$

(2) $v_i s$ are said to be linearly independent if

$$k_1 v_1 + k_2 v_2 + ... + k_n v_n = 0$$

implies that $k_1 = k_2 = ... = k_n = 0$. In this case, the set $\{v_1, v_2, ..., v_n\}$ is called a linearly independent set.

(3) $v_i s$ are said to be linearly dependent if

$$k_1 v_1 + k_2 v_2 + ... + k_n v_n = 0$$

implies that not all α_i are equal to zero. In this case, the set $\{v_1, v_2, ..., v_n\}$ is called a linearly dependent set.

Theorem 2.18. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let U[I] and W[I] be subsets of V(I) such that* $U[I] \subseteq W[I]$. *If U[I] is linearly dependent, then then W[I] is linearly dependent.*

Corrolary 2.19. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I). Every subset of a linearly dependent set in V(I) is linearly dependent.

Theorem 2.20. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let U[I] and W[I] be subsets of V(I) such that* $U[I] \subseteq W[I]$. *If U[I] is linearly independent, then then W[I] is linearly independent.*

Example 10. Let $V(I) = \mathbb{R}(I)$ be a weak *neutrosophic* vector space over a field $K(I) = \mathbb{R}(I)$. An element $v = 7 + 24I \in V(I)$ is a linear combination of the elements $v_1 = 1 + 2I, v_2 = 2 + 3I \in V(I)$ since 7+24I= 27(1+2I)-10(2+3I).

Example 11. Let $V(I) = \mathbb{R}(I)$ be a strong *neutrosophic* vector space over a field $K(I) = \mathbb{R}(I)$. An element $v = 7 + 24I \in V(I)$ is a linear combination of the elements $v_1 = 1 + 2I, v_2 = 2 + 3I \in V(I)$ since

$$\begin{aligned} 7 + 24I &= (1+I)(1+2I) + (3+2I)(2+3I), \\ &\quad \text{where } 1+I, 3+2I \in K(I) \\ &= (5 + (16/3)I)(1+2I) + (1-I)(2+3I), \\ &\quad \text{where } 5+(16/3)I, 1-I \in K(I) \\ &= (9 + (4/3)I)(1+2I) + (-1+I)(2+3I), \\ &\quad \text{where } 9+(4/3)I, -1+I \in K(I) \\ &= (13-I)(1+2I) + (-3+2I)(2+3I), \\ &\quad \text{where } 13-I, -3+2I \in K(I) \end{aligned}$$

This example shows that the element v=7+24I can be infinitely expressed as a linear combination of the elements $v_1 = 1 + 2I, v_2 = 2 + 3I \in V(I)$. This observation is recorded in the next Theorem.

Theorem 2.21. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I) and let $v_1, v_2, ..., v_3 \in V(I)$. An element $v \in V(I)$ can be infinitely expressed as a linear combination of the $v_i s$.

Proof. Suppose that $v = \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n$, where $v = a + bI, v_1 = a_1 + b_1 I, v_2 = a_2 + b_2 I, ...,$ $v_n = a_n + b_n I$ and $\alpha_1 = k_1 + m_1 I, \alpha_2 = k_2 + m_2 I, ...,$ $\alpha_n = k_n + m_n I \in K(I)$. Then

$$\begin{aligned} a + bI = &(k_1 + m_1 I)(a_1 + b_1 I) + (k_2 + m_2 I)(a_2 + b_2 I) \\ &+ ... + (k_n + m_n I)(a_n + b_n I) \end{aligned}$$

from which we obtain

$$a_1 k_1 + a_2 k_2 + ... + a_n k_n = a,$$
$$b_1 k_1 + a_1 m_1 + b_1 m_1 + b_2 k_2 + a_2 m_2 + ... + b_n k_n + a_n m_n + b_n m_n = b$$

This is a linear system in unknowns $k_i, m_i, i=1,2,3,...,n$. Since the system is consistent and have infinitely many solutions, it follows that the $v_i s$ can be infinitely combined to produce v.

Remark 3. In a strong *neutrosophic* vector space V(I) over a *neutrosophic* field K(I), it is possible to have $0 \neq v \in V(I), 0 \neq \alpha \in K(I)$ and yet $\alpha v = 0$. For instance, if v=k-kI and $\alpha = mI$ where $0 \neq k, m \in K$, we have $\alpha v = mI(k - kI) = mkI - mkI = 0$.

Theorem 2.22. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let* $v_1 = k_1 - k_1 I, v_2 = k_2 - k_2 I, ..., v_n = k_n - k_n I$ *be elements of V(I) where* $0 \neq k_i \in K$. *Then* $\{v_1, v_2, ..., v_n\}$ *is a linearly dependent set.*

Proof. Let $\alpha_1 = p_1 + q_1 I, \alpha_2 = p_2 + q_2 I, ...,$ $\alpha_n = p_n + q_n I$ be elements of K(I). Then

$$\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n = 0$$

which implies that

$$\begin{aligned} &(p_1 + q_1 I)(k_1 - k_1 I) + (p_2 + q_2 I)(k_2 - k_2 I) + ... \\ &+ (p_n + q_n I)(k_n - k_n I) = 0 \end{aligned}$$

from which we obtain

$$k_1 p_1 + k_2 p_2 + ... + k_n p_n = 0.$$

This is a homogeneous linear system in unknowns $p_i, i = 1, 2, ..., n$. It is clear that the system has infinitely many nontrivial solutions. Hence are not all zero and therefore, $\{v_1, v_2, ..., v_n\}$ is a linearly dependent set.

Example 12. (1) Let $V(I) = \mathbb{R}^n(I)$ be a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{R}(I)$. The set

$$\{v_1 = (1,0,0,...,0), v_2 = (0,1,0,...,0),...,$$
$$v_n = (0,0,0,...,1)\}$$

is a linearly independent set in V(I).

(2) Let $V(I) = \mathbb{R}^n(I)$ be a weak *neutrosophic* vector space over a *neutrosophic* field $\mathbb{R}$. The set

$$\{v_1 = (1,0,0,...,0), v_2 = (0,1,0,...,0),...,$$
$$v_k = (0,0,0,...,1), v_{k+1} = (I,0,0,...,0),$$
$$v_{k+2} = (0,I,0,...,0),..., v_n = (0,0,0,...,I)\}$$

is a linearly independent set in V(I).

Theorem 2.23. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let X[I] be a nonempty subset of V(I). If L(X[I]) is the set of all linear combinations of elements of X[I], then:*

(1) *L(X[I]) is a strong neutrosophic subspace of V(I) containing X[I].*
(2) *If W(I) is any strong neutrosophic subspace of V(I) containing X[I], then* $L(X[I]) \subset W(I)$.

Proof. (1) Obviously, L(X[I]) is nonempty since X[I] is nonempty. Suppose that $v = a + bI \in X[I]$ is arbitrary, then for $\alpha = 1 + 0I \in K(I)$, we have $\alpha v = (1+0I)$ $(a+bI) = a + bI \in L(X[I])$. Therefore, X[I] is contained in L(X[I]). Lastly, let $v, w \in L(X[I])$. Then

$$v = \alpha_1 v_1 + \alpha_2 v_2 + ..., \alpha_n v_n,$$
$$w = \beta_1 w_1 + \beta_2 w_2 + ..., \beta_n w_n,$$

Where $v_i, w_j \in X(I), \alpha_i, \beta_j \in K(I)$. For $\alpha, \beta \in K(I)$, it can be shown that $\alpha v + \beta w \in L(X[I])$. Since L(X) is a proper subset of L(X[I]) which is a subspace of V containing X, it follows that L(X[I]) is a strong *neutrosophic* subspace of V(I) containing X[I].

(2) Same as the classical case and omitted.

Definition 2.24. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I).

(1) The strong neutrosophic subspace L(X[I]) of Theorem 2.23 is called the span of X[I] and it is denoted by *span*(X[I]).
(2) X[I] is said to span V(I) if V(I)=*span*(X[I]).
(3) A linearly independent subset $B[I] = \{v_1, v_2, ..., v_n\}$ of V(I) is called a basis for V(I) if $B[I]$ spans V(I).

Example 13. (1) Let $V(I) = \mathbb{R}^n(I)$ be a strong *neutrosophic* vector space over a *neutrosophic* field $\mathbb{R}(I)$. The set

$$B[I] = \{v_1 = (1,0,0,...,0), v_2 = (0,1,0,...,0),...,$$
$$v_n = (0,0,0,...,1)\}$$

is a basis for V(I).

(2) Let $V(I) = \mathbb{R}^n(I)$ be a weak *neutrosophic* vector space over $\mathbb{R}$. The set

$$B = \{v_1 = (1,0,0,...,0), v_2 = (0,1,0,...,0),...,$$
$$v_k = (0,0,0,...,1), v_{k+1} = (I,0,0,...,0),$$
$$v_{k+2} = (0,I,0,...,0),..., v_n = (0,0,0,...,I)\}$$

is a basis for V(I).

Theorem 2.25. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I). The bases of V(I) are the same as the bases of V over a field K.*

Proof. Suppose that $B = \{v_1, v_2, ..., v_n\}$ is an arbitrary basis for V over the field K. Let v=a+bI be an arbitrary element of V(I) and let $\alpha_1 = k_1 + m_1 I,\ \alpha_2 = k_2 + m_2 I,$ $..., \alpha_n = k_n + m_n I$ be elements of K(I). Then from $\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n = 0$, we obtain

$$k_1 v_1 + k_2 v_2 + ... + k_n v_n = 0,$$
$$m_1 v_1 + m_2 v_2 + ... + m_n v_n = 0.$$

Since $v_i s$ are linearly independent, we have k_i=0 and m_j=0 where i,j=1,2,...,n. Hence $\alpha_i = 0, i = 1,2,...,n$. This shows that B is also a linearly independent set in V(I). To show that B spans V(I), let v=a+bI $= \alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n$, .Then we have

$$a = k_1 v_1 + k_2 v_2 + ... + k_n v_n,$$
$$b = m_1 v_1 + m_2 v_2 + ... + m_n v_n.$$

Since $a, b \in V$, it follows that v=a+bI can be written uniquely as a linear combination of $v_i s$. Hence, B is a basis for V(I). Since B is arbitrary, the required result follows.

Theorem 2.26. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I). Then the bases of V(I) over K(I) are contained in the bases of the weak neutrosophic vector space V(I) over a field K(I).*

Definition 2.27. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I). The number of elements in the basis for V(I) is called the dimension of V(I) and it is denoted by $\dim_s(V(I))$. If the number of elements in the basis for V(I) is finite, V(I) is called a finite dimensional strong *neutrosophic* vector space. Otherwise, V(I) is called an infinite dimensional strong *neutrosophic* vector space.

Definition 2.28. Let V(I) be a weak *neutrosophic* vector space over a field K(I). The number of elements in the basis for V(I) is called the dimension of V(I) and it is denoted by $\dim_s(V(I))$. If the number of elements in the basis for V(I) is finite, V(I) is called a finite dimensional strong *neutrosophic* vector space. Otherwise, V(I) is called an infinite dimensional weak *neutrosophic* vector space.

Example 14. (1) The strong *neutrosophic* vector space of Example 12(1) is finite dimensional and $\dim_s(V(I))$ =n.

(2) The weak *neutrosophic* vector space of Example 12(2) is finite dimensional and $\dim_w(V(I)) = n$.

Theorem 2.29. *Let V(I) be a finite dimensional strong neutrosophic vector space over a field K(I). Then every basis of V(I) has the same number of elements.*

Theorem 2.30. *Let V(I) be a finite dimensional weak (strong) neutrosophic vector space over a field K(resp. over a neutrosophic field K(I)). If* $\dim_s(V(I)) = n$ *, then* $\dim_w(V(I)) = 2n$.

Theorem 2.31. Let W(I) be a strong neutrosophic subspace of a finite dimensional strong neutrosophic vector space V(I) over a neutrosophic field K(I). Then W(I) is finite dimensional and $\dim_s(W(I)) \leq \dim_s(V(I))$ .If $\dim_s(W(I)) = \dim_s(V(I))$, then W(I)=V(I).

Theorem 2.32. *Let U(I) and W(I) be a finite dimensional strong neutrosophic subspaces of a strong neutrosophic vector space V(I) over a neutrosophic field K(I). Then U(I)+W(I) is a finite dimensional strong neutrosophic subspace of V(I) and*

$$\dim_s(U(I)+W(I)) = \dim_s(U(I)) + \dim_s(W(I)) - \dim_s(U(I)\cap W(I)).$$

If $V(I) = U(I) \oplus W(I)$, then

$$\dim_s(U(I)+W(I)) = \dim_s(U(I)) + \dim_s(W(I))$$

Definition 2.33. Let V(I) and W(I) be strong *neutrosophic* vector spaces over a *neutrosophic* field K(I) and let $\phi: V(I) \to W(I)$ be a mapping of V(I) into W(I). ϕ is called a *neutrosophic* vector space homomorphism if the following conditions hold:

(1) ϕ is a vector space homomorphism.

(2) $\phi(I) = I$.

If ϕ is a bijective *neutrosophic* vector space homomorphism, then ϕ is called a *neutrosophic* vector space isomorphism and we write $V(I) \cong W(I)$.

Definition 2.34. Let V (I) and W(I) be strong *neutrosophic* vector spaces over a *neutrosophic* field K(I) and let $\phi: V(I) \to W(I)$ be a *neutrosophic* vector space homomorphism.

(1) The kernel of ϕ denoted by $Ker\phi$ is defined by the set $\{v \in V(I) : \phi(v) = 0\}$.

(2) The image of ϕ denoted by $\operatorname{Im}\phi$ is defined by the set $\{w \in W(I) : \phi(v) = w$ for some $v \in V(I)\}$.

Example 15. Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I).

(1) The mapping $\phi: V(I) \to V(I)$ defined by $\phi(v) = v$ for all $v \in V(I)$ is *neutrosophic* vector space homomorphism and $Ker\phi$=0.

(2) The mapping $\phi: V(I) \to V(I)$ defined by $\phi(v) = 0$ for all $v \in V(I)$ is *neutrosophic* vector space homomorphism since $I \in V(I)$ but

$\phi(I) \neq 0$.

Definition 2.35. *Let V (I) and W(I) be strong neutrosophic vector spaces over a neutrosophic field K(I) and let* $\phi: V(I) \rightarrow W(I)$ *be a neutrosophic vector space homomorphism. Then:*

(1) $Ker\phi$ *is not a strong neutrosophic subspace of V(I) but a subspace of V.*

(2) $\operatorname{Im}\phi$ *is a strong neutrosophic subspace of W(I).*

Proof. (1) Obviously, $I \in V(I)$ but $\phi(I) \neq 0$. That $Ker\phi$ is a subspace of V is clear. (2) Clear.

Theorem 2.36. *Let V (I) and W(I) be strong neutrosophic vector spaces over a neutrosophic field K(I) and let* $\phi: V(I) \rightarrow W(I)$ *be a neutrosophic vector space homomorphism. If* $B = \{v_1, v_2, ..., v_n\}$ *is a basis for V(I), then* $\phi(B) = \{\phi(v_1), \phi(v_2), ..., \phi(v_n)\}$ *is a basis for W(I).*

Theorem 2.37. *Let W(I) be a strong neutrosophic subspace of a strong neutrosophic vector space V(I) over a neutrosophic field K(I). Let* $\phi: V(I) \rightarrow V(I)/W(I)$ *be a mapping defined by* $\phi(v) = v + W(I)$ *for all* $v \in V(I)$. *Then* ϕ *is not a neutrosophic vector space homomorphism.*

Proof. Obvious since $\phi(I) = I + W(I) = W(I) \neq I$.

Theorem 2.38. *Let W(I) be a strong neutrosophic subspace of a strong neutrosophic vector space V(I) over a neutrosophic field K(I) and let* $\phi: V(I) \rightarrow U(I)$ *be a neutrosophic vector space homomorphism from V(I) into a strong neutrosophic vector space U(I) over K(I). If* $\phi_{W(I)}: W(I) \rightarrow U(I)$ *is the restriction of* ϕ *to W(I) is defined by* $\phi_{W(I)}(w) = \phi(w)$ *for all, then:*

(1) $\phi_{W(I)}$ is a neutrosophic vector space homomorphism.

(2) $Ker\phi_{W(I)} = Ker\phi \cap W(I)$.

(3) $\operatorname{Im}\phi_{W(I)} = \phi(W(I))$.

Remark 4. If V(I) and W(I) are strong *neutrosophic* vector spaces over a *neutrosophic* field K(I) and $\phi, \psi: V(I) \rightarrow W(I)$ are *neutrosophic* vector space homomorphisms, then $(\phi + \psi)$ and $(\alpha\phi)$ are not *neutrosophic* vector space homomorphisms since $(\phi + \psi)(I) = \phi(I) + \psi(I) = I + I = 2I \neq I$ and $(\alpha\phi)(I) = \alpha\phi(I) = \alpha I \neq I$ for all $\alpha \in K(I)$. Hence, if $Hom(V(I), W(I))$ is the collection of all *neutrosophic* vector space homomorphisms from V(I) into W(I), then $Hom(V(I), W(I))$ is not a *neutrosophic* vector space over K(I). This is different from what is obtainable in the classical vector spaces.

Definition 2.39. Let U(I), V(I) and W(I) be strong *neutrosophic* vector spaces over a *neutrosophic* field K(I) and let $\phi: U(I) \rightarrow V(I), \psi: V(I) \rightarrow W(I)$ be *neutrosophic* vector space homomorphisms. The composition $\psi\phi: U(I) \rightarrow W(I)$ is defined by $\psi\phi(u) = \psi(\phi(u))$ for all $u \in U(I)$.

Theorem 2.40. *Let U(I), V(I) and W(I) be strong neutrosophic vector spaces over a neutrosophic field K(I) and let* $\phi: U(I) \rightarrow V(I), \psi: V(I) \rightarrow W(I)$ *be neutrosophic vector space homomorphisms. Then the composition* $\psi\phi: U(I) \rightarrow W(I)$ *is a neutrosophic vector space homomorphism.*

Proof. Clearly, $\psi\phi$ is a vector space homomorphism. For $u = I \in U(I)$, we have:

$$\begin{aligned} \psi\phi(I) &= \psi(\phi(I)) \\ &= \psi(I) \\ &= I. \end{aligned}$$

Hence $\psi\phi$ is a *neutrosophic* vector space homomorphism.

Corrolary 2.41. Let L(V(I)) be the collection of all *neutrosophic* vector space homomorphisms from V(I) onto V(I). Then $\phi(\psi\lambda) = (\phi\psi)\lambda$ for all $\phi, \psi, \lambda \in L(V(I))$.

Theorem 2.42. *Let U(I), V(I) and W(I) be strong neutrosophic vector spaces over a neutrosophic field K(I) and let* $\phi: U(I) \rightarrow V(I), \psi: V(I) \rightarrow W(I)$ *be neutroso-*

phic vector space homomorphisms. Then

(1) If $\psi\phi$ is injective, then ϕ is injective.

(2) If $\psi\phi$ is surjective, then ψ is surjective.

(3) If ψ and ϕ are injective, then $\psi\phi$ is injective.

Let V(I) be a strong *neutrosophic* vector space over a *neutrosophic* field K(I) and let $\phi : V(I) \to V(I)$ be a *neutrosophic* vector space homomorphism. If $B = \{v_1, v_2, ..., v_n\}$ is a basis for V(I), then each $\phi(v_i) \in V(I)$ and thus for $\alpha_{ij} \in K(I)$, we can write

$$\phi(v_1) = \alpha_{11}v_1 + \alpha_{12}v_2 + ... + \alpha_{1n}v_n$$
$$\phi(v_1) = \alpha_{21}v_1 + \alpha_{22}v_2 + ... + \alpha_{2n}v_n$$
$$...$$
$$\phi(v_n) = \alpha_{n1}v_1 + \alpha_{n2}v_2 + ... + \alpha_{nn}v_n$$

Let

$$[\phi]_B = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix}$$

$[\phi]_B$ is called the matrix representation of ϕ relative to the basis B.

Theorem 2.43. *Let V(I) be a strong neutrosophic vector space over a neutrosophic field K(I) and let* $\phi : V(I) \to V(I)$ *be a neutrosophic vector space homomorphism. If B is a basis for V(I) and v is any element of V(I), then*

$$[\phi]_B[v]_B = [\phi(v)]_B.$$

Example 16. Let $V(I) = \mathbb{R}^3(I)$ be a strong *neutrosophic* vector space over a *neutrosophic* field K(I)= $\mathbb{R}(I)$ and let $v = (1+2I, I, 3-2I) \in V(I)$. If $\phi : V(I) \to V(I)$ is a *neutrosophic* vector space homomorphism defined by $\phi(v) = v$ for all $v \in V(I)$, then relative to the basis $B = (v_1 = (1,0,0), v_2 = (0,1,0), v_3 = (0,0,1)\}$ for V(I), the matrix of ϕ is obtained as

$$[\phi]_B = \begin{bmatrix} 1+0I & 0+0I & 0+0I \\ 0+0I & 1+0I & 0+0I \\ 0+0I & 0+0I & 1+0I \end{bmatrix}$$

For $v = (1+2I, I, 3-2I) \in V(I)$, we have

$$\phi(v) = v = (1+2I)v_1 + Iv_2 + (3-2I)v_3$$

So that

$$[v]_B = \begin{bmatrix} 1+2I \\ I \\ 3-2I \end{bmatrix} = [\phi(v)]_B$$

And we have

$$[\phi]_B[v]_B = [\phi(v)]_B$$

Example 17. Let $V(I) = \mathbb{R}^3(I)$ be a weak *neutrosophic* vector space over a *neutrosophic* field K= $\mathbb{R}$ and let $v = (1-2I, 3-4I) \in V(I)$. If $\phi : V(I) \to V(I)$ is a *neutrosophic* vector space homomorphism defined by $\phi(v) = v$ for all $v \in V(I)$, then relative to the basis $B = (v_1 = (1,0), v_2 = (0,1), v_3 = (I,0), v_4 = (0,I)\}$ for V(I), the matrix of ϕ is obtained as

$$[\phi]_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For $v = (1-2I, 3-4I) \in V(I)$, we have

$$\phi(v) = v = v_1 + 3v_2 - 2v_3 - 4v_4.$$

Therefore,

$$[v]_B = \begin{bmatrix} 1 \\ 3 \\ -2 \\ -4 \end{bmatrix} = [\phi(v)]_B$$

And thus

$$[\phi]_B[v]_B = [\phi(v)]_B$$

3 Conclusion

In this paper, we have studied *neutrosophic* vector spaces. Basic definitions and properties of the classical vector spaces were generalized. It was shown that every weak (strong) *neutrosophic* vector space is a vector space.

Also, it was shown that an element of a strong *neutrosophic* vector space can be infinitely expressed as a linear combination of some elements of the *neutrosophic* vector space. *Neutrosophic* quotient spaces and *neutrosophic* vector space homomorphisms were also studied. Matrix representations of *neutrosophic* vector space homomorphisms were presented.

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Neutrosophic Bi-LA-Semigroup and Neutrosophic N-LA-Semigroup

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Abstract. In this paper we define neutrosophic bi-LA-semigroup and neutrosophic N-LA-semigroup. Infact this paper is an extension of our previous paper neutrosophic left almost semigroup shortly neutrosophic LA-semigroup. We also extend the neutrosophic ideal to neutrosophic biideal and neutrosophic N-ideal. We also find some new type of neutrosophic ideal which is related to the strong or pure part of neutrosophy. We have given sufficient amount of examples to illustrate the theory of neutrosophic bi-LA-semigroup, neutrosophic N-LA-semigroup and display many properties of them this paper.

Keywords: Neutrosophic LA-semigroup, neutrosophic ideal, neutrosophic bi-LA-semigroup, neutrosophic biideal, neutrosophic N-LA-semigroup, neutrosophic N-ideal.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in $[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

A left almost semigroup abbreviated as LA-semigroup is an algebraic structure which was introduced by M .A. Kazim and M. Naseeruddin $[3]$ in 1972. This structure is basically a midway structure between a groupoid and a commutative semigroup. This structure is also termed as Able-Grassmann's groupoid abbreviated as AG-groupoid $[6]$. This is a non associative and non commutative algebraic structure which closely resemble to commutative semigroup. The generalization of semigroup theory is an LA-semigroup and this structure has wide applications in collaboration with semigroup.

We have tried to develop the ideal theory of LA-semigroups in a logical manner. Firstly, preliminaries and basic concepts are given for neutrosophic LA-semigroup. Then we presented the newly defined notions and results in neutrosophic bi-LA-semigroups and neutrosophic N-LA-semigroups. Various types of neutrosophic biideals and neutrosophic N-ideal are defined and elaborated with the help of examples.

2 Preliminaries

Definition 1. Let $(S,*)$ be an LA-semigroup and let $\langle S\cup I\rangle=\{a+bI : a,b\in S\}$. The neutrosophic LA-semigroup is generated by S and I under $*$ denoted as $N(S)=\{\langle S\cup I\rangle,*\}$, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and

$0.I = 0.I^{-1}$, the inverse of I is not defined and hence does not exist.
Similarly we can define neutrosophic RA-semigroup on the same lines.

Definition 2. Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Definition 3. A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$ are neutrosophic elements.

Definition 4. Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if $N(S)N(K) \subseteq N(K)$
{ $N(K)N(S) \subseteq N(K)$ }.
If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Definition 5. A neutrosophic ideal $N(K)$ is called strong neutrosophic ideal or pure neutrosophic ideal if all of its elements are neutrosophic elements.

3 Neutrosophic Bi-LA-Semigroup

Definition 6. Let $(BN(S), *, \circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(S), *, \circ)$ is said to be a neutrosophic bi-LA-semigroup if $BN(S) = P_1 \cup P_2$ where atleast one of $(P_1, *)$ or $(P_2, \circ)$ is a neutrosophic LA-semigroup and other is just an LA- semigroup. P_1 and P_2 are proper subsets of $BN(S)$.
Similarly we can define neutrosophic bi-RA-semigroup on the same lines.

Theorem 1. All neutrosophic bi-LA-semigroups contains the corresponding bi-LA-semigroups.

Example 1. Let $BN(S) = \{\langle S_1 \cup I\rangle \cup \langle S_2 \cup I\rangle\}$ be a neutrosophic bi-LA-semigroup where

$\langle S_1 \cup I\rangle = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$\langle S_2 \cup I\rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Definition 7. Let $(BN(S) = P_1 \cup P;: *, \circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T, \circ, *)$ is said to be a neutrosophic sub bi-LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic LA-semigroup.

Example 2: $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 1. Then $P = \{1, 1I\} \cup \{3, 3I\}$ and $Q = \{2, 2I\} \cup \{1, 1I\}$ are neutrosophic sub bi-LA-semigroups of $BN(S)$.

Theorem 2. Let $BN(S)$ be a neutrosophic bi-LA-semigroup and $N(H)$ be a proper subset of $BN(S)$. Then $N(H)$ is a neutrosophic sub bi-LA-semigroup of $BN(S)$ if $N(H).N(H) \subseteq N(H)$.

Definition 8. Let $(BN(S) = P_1 \cup P, *, \circ)$ be any neutrosophic bi-LA-semigroup. Let J be a proper subset of $BN(S)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(S)$.

Example 3. Let $BN(S) = \{\langle S_1 \cup I\rangle \cup \langle S_2 \cup I\rangle\}$ be a neutrosophic bi-LA-semigroup, where

$\langle S_1 \cup I\rangle = \{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $\langle S_2 \cup I\rangle = \{1,2,3,I,2I,3I\}$ be another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Then $P = \{1,1\mathrm{I},3,3I\} \cup \{2,2I\}$, $Q = \{1,3,1I,3I\} \cup \{2,3,2\mathrm{I},3\mathrm{I}\}$ are neutrosophic biideals of $BN(S)$.

Proposition 1. Every neutrosophic biideal of a neutrosophic bi-LA-semigroup is trivially a Neutrosophic sub bi-LA-semigroup but the conver is not true in general.
One can easily see the converse by the help of example.

3 Neutrosophic Strong Bi-LA-Semigroup

Definition 9: If both $(P_1, *)$ and $(P_2, \circ)$ in the Definition 6. are neutrosophic strong LA-semigroups then we call $(BN(S), *, \circ)$ is a neutrosophic strong bi-LA-semigroup.

Definition 10. Let $(BN(S) = P_1 \cup P, *, \circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T, \circ, *)$ is said to be a neutrosophic strong sub bi-LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. $(T_1, \circ)$ and $(T_2, *)$ are neutrosophic strong LA-semigroups.

Example 4. Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 3.
Then $P = \{1\mathrm{I}, 3I\} \cup \{2I\}$, and $Q = \{1I, 3I\} \cup \{2\mathrm{I}, 3\mathrm{I}\}$ are neutrosophic strong sub bi-LA-semigroup of $BN(S)$.

Theorem 4: Every neutrosophic strong sub bi-LA-semigroup is a neutrosophic sub bi-LA-semigroup.

Definition 11. Let $(BN(S), *, \circ)$ be a strong neutrosophic bi-LA-semigroup where $BN(S) = P_1 \cup P_2$ with $(P_1, *)$ and $(P_2, \circ)$ be any two neutrosophic LA-semigroups. Let J be a proper subset of $BN(S)$ where $I = I_1 \cup I_2$ with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic LA-semigroups P_1 and P_2 respectively. Then I is called or defined as the neutrosophic strong biideal of $BN(S)$.

Theorem 5: Every neutrosophic strong biideal is trivially a neutrosophic sub bi-LA-semigroup.

Theorem 6: Every neutrosophic strong biideal is a neutrosophic strong sub bi-LA-semigroup.

Theorem 7: Every neutrosophic strong biideal is a neutrosophic biideal.

Example 5. Let $BN(S)$ be a neutrosophic bi-LA semigroup in Example (*) .Then $P=\{1I,3I\}\cup\{2I\}$, and $Q=\{1I,3I\}\cup\{2I,3I\}$ are neutrosophic strong biideal of $BN(S)$.

4 Neutrosophic N-LA-Semigroup

Definition 12. Let $\{S(N),*_1,...,*_2\}$ be a non-empty set with N -binary operations defined on it. We call $S(N)$ a neutrosophic N -LA-semigroup (N a positive integer) if the following conditions are satisfied.

1) $S(N)=S_1\cup...S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i\subset S_j$ or $S_j\subset S_i$ if $i\neq j$.
2) $(S_i,*_i)$ is either a neutrosophic LA-semigroup or an LA-semigroup for $i=1,2,3,...,N$.

Example 6. Let $S(N)=\{S_1\cup S_2\cup S_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup where $S_1=\{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$S_2=\{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $S_3=\{1,2,3,I,2I,3I\}$ is another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Theorem 8 All neutrosophic N-LA-semigroups contains the corresponding N-LA-semigroups.

Definition 13. Let $S(N)=\{S_1\cup S_2\cup....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P=\{P_1\cup P_2\cup....P_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic sub N -LA-semigroup if $P_i=P\cap S_i, i=1,2,...,N$ are sub LA-semigroups of S_i in which atleast some of the sub LA-semigroups are neutrosophic sub LA-semigroups.

Example 7: Let $S(N)=\{S_1\cup S_2\cup S_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup in above Example 6. Then clearly $P=\{1,1I\}\cup\{2,3,3I\}\cup\{2,2I\}$, $Q=\{2,2I\}\cup\{1,3,1I,3I\}\cup\{2,3,2I,3I\}$, and

$R=\{4,4I\}\cup\{1I,3I\}\cup\{2\mathrm{I},3\mathrm{I}\}$ are neutrosophic sub 3-LA-semigroups of $\mathrm{S(N)}$.

Theorem 19. Let $N(S)$ be a neutrosophic N-LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is a neutrosophic sub N-LA-semigroup of $N(S)$ if $N(H).N(H)\subseteq N(H)$.

Definition 14. Let $S(N)=\{S_1\cup S_2\cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-LA-semigroup. A proper subset $P=\{\mathrm{P}_1\cup P_2\cup ....\cup P_N, *_1, *_2, ..., *_N\}$ of $S(N)$ is said to be a neutrosophic N-ideal, if the following conditions are true,

1. P is a neutrosophic sub N-LA-semigroup of $S(N)$.
2. Each $P_i=S\cap P_i, i=1,2,...,N$ is an ideal of S_i.

Example 8. Consider Example 6.
Then $I_1=\{1,1I\}\cup\{3,3I\}\cup\{2,2I\}$, and $I_2=\{2,2I\}\cup\{1I,3I\}\cup\{2,3,3\mathrm{I}\}$ are neutrosophic 3-ideals of $S(N)$.

Theorem 10: Every neutrosophic N-ideal is trivially a neutrosophic sub N-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

5 Neutrosophic Strong N-LA-Semigroup

Definition 15: If all the N-LA-semigroups $(S_i, *_i)$ in Definition ( ) are neutrosophic strong LA-semigroups (i.e. for $i=1,2,3,...,N$) then we call $S(N)$ to be a neutrosophic strong N-LA-semigroup.

Definition 16. Let $S(N)=\{S_1\cup S_2\cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-LA-semigroup. A proper subset $T=\{\mathrm{T}_1\cup T_2\cup ....\cup \mathrm{T}_N, *_1, *_2, ..., *_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N-LA-semigroup if each $(T_i, *_i)$ is a neutrosophic strong sub LA-semigroup of $(S_i, *_i)$ for $i=1,2,...,N$ where $T_i=S_i\cap T$.

Theorem 11: Every neutrosophic strong sub N-LA-semigroup is a neutrosophic sub N-LA-semigroup.

Definition 17. Let $S(N)=\{S_1\cup S_2\cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-LA-semigroup. A proper subset $J=\{\mathrm{J}_1\cup J_2\cup ....\mathrm{J}_N, *_1, *_2, ..., *_N\}$ where $J_t=J\cap S_t$ for $t=1,2,...,N$ is said to be a neutrosophic strong N-ideal of $S(N)$ if the following conditions are satisfied.

1) Each it is a neutrosophic sub LA-semigroup of $S_t, t=1,2,...,N$ i.e. It is a neutrosophic strong N-sub LA-semigroup of $S(N)$.
2) Each it is a two sided ideal of S_t for $t=1,2,...,N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $S(N)$.
A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

Theorem 12: Every neutrosophic strong Nideal is trivially a neutrosophic sub N-LA-semigroup.

Theorem 13: Every neutrosophic strong N-ideal is a neutrosophic strong sub N-LA-semigroup.

Theorem 14: Every neutrosophic strong N-ideal is a N-ideal.

Conclusion

In this paper we extend neutrosophic LA-semigroup to neutrosophic bi-LA-semigroup and neutrosophic N-LA-semigroup. The neutrosophic ideal theory of neutrosophic LA-semigroup is extend to neutrosophic biideal and neutrosophic N-ideal. Some new type of neutrosophic ideals are discovered which is strongly neutrosophic or purely neutrosophic. Related examples are given to illustrate neutrosophic bi-LA-semigroup, neutrosophic N-LA-semigroup and many theorems and properties are discussed.

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Neutrosophic Parameterized Soft Relations and Their Applications

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Abstract. The aim of this paper is to introduce the concept of relation on neutrosophic parameterized soft set (NP- soft sets) theory. We have studied some related properties and also put forward some propositions on neutrosophic parameterized soft relation with proofs and examples. Finally the notions of symmetric, transitive, reflexive, and equivalence neutrosophic parameterized soft set relations have been established in our work. Finally a decision making method on NP-soft sets is presented.

Keywords: Soft set, neutrosophic parameterized soft set, NP-soft relations.

1. Introduction

Neutrosophic set theory was introduced in 1995 with the study of Smarandache [21] as mathematical tool for handling problem involving imprecise, indeterminacy and inconsistent data. The concept of neutrosophic set generalizes the concept of fuzzy sets [22], intuitionistic fuzzy sets [1] and so on. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and falsity-membership are independent. Neutrosophic set theory has successfully used in logic, economics, computer science, decision making process and so on.

The concept of soft set theory is another mathematical theory dealing with uncertainty and vagueness, developed by Russian researcher [20]. The soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Many interesting results of soft set theory have been studied by embedding the ideas of fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets and so on. For example; fuzzy soft sets [3,9,17], on intuitionistic fuzzy soft set theory [10,18], on possibility intuitionistic fuzzy soft set [2], on neutrosophic soft set [19], on intuitionistic neutrosophic soft set [4,7], on generalized neutrosophic soft set [5], on interval-valued neutrosophic soft set [6], on fuzzy parameterized soft set theory [14,15,16], on intuitionistic fuzzy parameterized soft set theory [12], on IFP−fuzzy soft set theory [13], on fuzzy parameterized fuzzy soft set theory [11].

Later on, Broumi et al. [8] defined the neutrosophic parameterized soft sets (NP-soft sets) which is a generalization of fuzzy parameterized soft sets (FP-soft sets) and intuitionistic fuzzy parameterized soft sets (IFP-soft sets).

In this paper our main objective is to extend the concept relations on FP-soft sets[14] to the case of NP-soft sets. The paper is structured as follows. In Section 2, some basic definition and preliminary results are given which will be used in the rest of the paper. In Section 3, we define relations on NP-soft sets and some of its algebraic properties are studied. In section 4, we present decision making method on NP-soft relations. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of neutrosophic set, soft set and neutrosophic parameterized soft set. For more details, the reader could refer to [8,20,21].

Definition 2.1. [21] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\},$$

where the functions $\mu, \nu, \omega : U \to]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \le \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \le 3^+ \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$$A_{NS} = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x)\rangle | x \in X\}$$

And

$$B_{NS} = \{\langle x, \mu_B(x), \nu_B(x), \omega_B(x)\rangle | x \in X\}$$

Then,

1. $A_{NS} \subseteq B_{NS}$ if and only if $\mu_A(x) \le \mu_B(x), \nu_A(x) \ge \nu_B(x)$ and $\omega_A(x) \ge \omega_B(x)$.
2. $A_{NS} = B_{NS}$ if and only if, $\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x)$ for any $x \in X$.
3. The complement of A_{NS} is denoted by A°_{NS} and is defined by
$$A^{\circ}_{NS} = \{\langle x, \omega_A(x), 1-\nu_A(x), \mu_A(x)\rangle | x \in X\}$$
4. $$A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_A(x)\}, \max\{\omega_A(x), \omega_B(x)\}\rangle : x \in X\}$$
5. $$A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_A(x)\}, \max\{\omega_A(x), \omega_B(x)\}\rangle : x \in X\}$$

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse $U=\{x_1, x_2, x_3\}$. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] Then, A is a neutrosophic set (NS) of U, such that,

$$A = \{\langle x_1, 0.7, 0.5, 0.2\rangle, \langle x_2, 0.4, 0.5, 0.5\rangle, \langle x_3, 0.4, 0.5, 0.6\rangle\}$$

Definition 2.3.[8] Let *U* be an initial universe, *P (U)* be the power set of *U*, *E* be a set of all parameters and *K* be a ne trosophic set over *E*. Then a neutrosophic parameterized soft sets

$$\psi_K = \{(\langle x, \mu_K(x), \nu_K(x), \omega_K(x)\rangle, f_K(x)) : x \in E\}$$

where $\mu_K : E \to [0, 1]$, $\nu_K : E \to [0, 1]$, $\omega_K : E \to [0, 1]$ and $f_K : E \to P(U)$ such that $f_K(x) = \Phi$ if $\mu_K(x) = 0$, $\nu_K(x) = 1$ and $\omega_K(x) = 1$.

Here, the function μ_K, ν_K and ω_K called membership function, indeterminacy function and non-membership function of neutrosophic parameterized soft set (NP-soft set), respectively.

Example 2.4. Assume that $U = \{u_1, u_2, u_3\}$ is a universal set and $E = \{x_1, x_2\}$ is a set of parameters. If

$$K = \{\langle x_1, 0.7, 0.3, 0.4\rangle, \langle x_2, 0.7, 0.5, 0.4\rangle\}$$

and

$$f_K(x_1) = \{u_2, u_5\},\ f_K(x_2) = U.$$

Then a neutrosophic parameterized soft set Ψ_k is written by

$$\psi_K = \{(\langle x_1, 0.7, 0.3, 0.4\rangle, \{u_2, u_5\}), (\langle x_2, 0.7, 0.5, 0.4\rangle, U\}$$

3. Relations on the NP-Soft Sets

In this section, after given the cartesian products of two NP- soft sets, we define a relations on NP- soft sets and study their desired properties.

Definition 3.1. Let $\psi_K, \Omega_L \in NPS(U)$. Then, a Cartesian product of ψ_K and Ω_L, denoted by $\psi_K \hat{\times} \Omega_L$, is defined as;

$$\psi_K \hat{\times} \Omega_L = \{\langle (x, y), \mu_{K\hat{\times}L}(x, y), \nu_{K\hat{\times}L}(x, y), \omega_{K\hat{\times}L}(x, y)\rangle, f_{K\hat{\times}L}(x, y) : (x, y) \in E \hat{\times} E\}$$

Where

$$f_{K\hat{\times}L}(x, y) = f_K(x) \cap f_L(y)$$

and

$$\mu_{K\hat{X}L}(x, y) = \min\{\mu_K(x), \mu_L(y)\}$$
$$\nu_{K\hat{X}L}(x, y) = \max\{\nu_K(x), \nu_L(y)\}$$
$$\omega_{K\hat{X}L}(x, y) = \max\{\omega_K(x), \omega_L(y)\}$$

Here $\mu_{K\hat{X}L}(x, y), \nu_{K\hat{X}L}(x, y), \omega_{K\hat{X}L}(x, y)$ is a t-norm.

Example 3.2. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}\}$,

$E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$,

$$K = \{\langle x_2, 0.5, 0.6, 0.3\rangle, \langle x_3, 0.3, 0.2, 0.9\rangle, \langle x_5, 0.6, 0.7, 0.3\rangle, \langle x_6, 0.1, 0.4, 0.6\rangle, \langle x_7, 0.7, 0.5, 0.3\rangle\}$$

and

$$L = \{\langle x_1, 0.5, 0.6, 0.3\rangle, \langle x_2, 0.5, 0.6, 0.3\rangle, \langle x_4, 0.9, 0.8, 0.1\rangle, \langle x_8, 0.3, 0.2, 0.9\rangle\}$$

be to neutrosophic sets of *E*. Suppose that

$$\begin{aligned}\psi_K = \{&(\langle x_2, 0.5, 0.6, 0.3\rangle, \{u_1, u_2, u_4, u_5, u_7, u_8, u_{10}, u_{12}, u_{14}, u_{15}\}),\\ &(\langle x_3, 0.3, 0.2, 0.9\rangle, \{u_2, u_5, u_8, u_{11}, u_{15}\}),\\ &(\langle x_5, 0.6, 0.7, 0.3\rangle, \{u_2, u_3, u_4, u_7, u_8, u_{11}, u_{12}, u_{15}\}),\\ &(\langle x_6, 0.1, 0.4, 0.6\rangle, \{u_2, u_4, u_6, u_7, u_{10}, u_{12}\}),\\ &(\langle x_7, 0.7, 0.5, 0.3\rangle, \{u_2, u_5, u_6, u_8, u_9, u_{13}, u_{15}\})\}\end{aligned}$$

and

$$\begin{aligned}\Omega_L = \{&(\langle x_1, 0.7, 0.4, 0.6\rangle, \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}),\\ &(\langle x_2, 0.5, 0.6, 0.3\rangle, \{u_1, u_2, u_4, u_5, u_7, u_8, u_{10}, u_{12}, u_{14}, u_{15}\}),\\ &(\langle x_4, 0.9, 0.8, 0.1\rangle, \{u_2, u_5, u_9, u_{10}, u_{11}, u_{14}\}),\\ &(\langle x_8, 0.4, 0.7, 0.2\rangle, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\})\}\end{aligned}$$

Then, the Cartesian product of ψ_K and Ω_L is obtained as follows;

$$\begin{aligned}\psi_K \hat{\text{x}} \Omega_L = \{ & (\langle (x_2, x_1), 0.5, 0.6, 0.6 \rangle, \{u_1, u_5, u_{10}\}), \\ & (\langle (x_2, x_2), 0.5, 0.6, 0.3 \rangle, \\ & \{u_1, u_2, u_4, u_5, u_7, u_8, u_{10}, u_{12}, u_{14}, u_{15}\}) \\ & (\langle (x_2, x_4), 0.5, 0.8, 0.3 \rangle, \{u_2, u_5, u_{10}, u_{14}\}), \\ & (\langle (x_2, x_8), 0.4, 0.7, 0.3 \rangle, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\}), \\ & (\langle (x_3, x_1), 0.3, 0.4, 0.9 \rangle, \{u_5\}), \\ & (\langle (x_3, x_2), 0.3, 0.6, 0.9 \rangle, \{u_2, u_5, u_8, u_{15}\}), \\ & (\langle (x_3, x_4), 0.3, 0.8, 0.9 \rangle, \{u_2, u_5, u_{11}\}), \\ & (\langle (x_3, x_8), 0.3, 0.7, 0.9 \rangle, \{u_2, u_5, u_8\}), \\ & (\langle (x_5, x_1), 0.6, 0.7, 0.6 \rangle, \varnothing), \\ & (\langle (x_5, x_2) 0.5, 0.7, 0.3 \rangle, \{u_2, u_4, u_7, u_8, u_{12}, u_{15}\}), \\ & (\langle (x_5, x_4), 0.6, 0.8, 0.3 \rangle, \{u_2, u_{11}\}), \\ & (\langle (x_5, x_8), 0.4, 0.7, 0.3 \rangle, \{u_2, u_8, u_{12}\}), \\ & (\langle (x_6, x_1), 0.1, 0.4, 0.6 \rangle, \{u_6, u_{10}\}), \\ & (\langle (x_6, x_2), 0.1, 0.6, 0.6 \rangle, \{u_2, u_4, u_7, u_{10}, u_{12}\}), \\ & (\langle (x_6, x_4), 0.1, 0.8, 0.6 \rangle, \{u_2, u_{10}\}), \\ & (\langle (x_6, x_8), 0.1, 0.7, 0.6 \rangle, \{u_2, u_{10}, u_{12}\}), \\ & (\langle (x_7, x_1), 0.7, 0.5, 0.6 \rangle, \{u_5, u_6, u_9, u_{13}\}), \\ & (\langle (x_7, x_2), 0.5, 0.6, 0.3 \rangle, \{u_2, u_5, u_8, u_{15}\}), \\ & (\langle (x_7, x_4), 0.7, 0.8, 0.3 \rangle, \{u_2, u_5, u_9\}), \\ & (\langle (x_7, x_8), 0.4, 0.7, 0.3 \rangle, \{u_2, u_5, u_8\})\}\end{aligned}$$

Definition 3.3. Let $\psi_K, \Omega_L \in NPS(U)$. Then, a *NP*-soft relation from ψ_K to Ω_L, denoted by R_N, is a *NP*-soft subset of $\psi_K \hat{\text{x}} \Omega_L$. Any *NP*-soft subset of $\psi_K \hat{\text{x}} \Omega_L$ is called a *NP*-soft relation on ψ_K.

Note that if $\alpha = (\langle x, \mu_K(x), \nu_K(x), \omega_K(x) \rangle, f_K(x)) \in \psi_K$ and $\beta = (\langle y, \mu_L(y), \nu_L(y), \omega_L(y) \rangle, f_L(y)) \in \Omega_L$, then

$$\begin{aligned}\alpha R_N \beta \Leftrightarrow (& \langle (x, y), \mu_{K\hat{\text{X}}L}(x, y), \nu_{K\hat{\text{X}}L}(x, y), \\ & \omega_{K\hat{\text{X}}L}(x, y) \rangle, f_{K\hat{\text{X}}L}(x, y) \in R_N)\end{aligned}$$

Where $f_{K\hat{\text{x}}L}(x, y) = f_K(x) \cap f_L(y)$.

Example 3.4. Let us consider the Example 3.2. Then, we define a *NP*-soft relation, from ψ_K to Ω_L, as follows

$$\begin{aligned}\alpha R_N \beta \Leftrightarrow (& \langle (x_i, y_j), \mu_{K\hat{\text{X}}L}(x_i, y_j), \nu_{K\hat{\text{X}}L}(x_i, y_j), \\ & \omega_{K\hat{\text{X}}L}(x_i, y_j) \rangle, f_{K\hat{\text{X}}L}(x_i, y_j)) (1 \le i, j \le 8)\end{aligned}$$

Such that

$$\mu_{K\hat{\text{X}}L}(x_i, y_j) \ge 0.3$$
$$\nu_{K\hat{\text{X}}L}(x_i, y_j) \le 0.5$$
$$\omega_{K\hat{\text{X}}L}(x_i, y_j) \le 0.7$$

Then

$$\begin{aligned}R_N = \{ & (\langle (x_2, x_1), 0.5, 0.6, 0.6 \rangle, \{u_1, u_5, u_{10}\}), \\ & (\langle (x_2, x_2), 0.5, 0.6, 0.3 \rangle, \\ & \{u_1, u_2, u_4, u_5, u_7, u_8, u_{10}, u_{12}, u_{14}, u_{15}\}) \\ & (\langle (x_2, x_4), 0.5, 0.8, 0.3 \rangle, \{u_2, u_5, u_{10}, u_{14}\}), \\ & (\langle (x_2, x_8), 0.4, 0.7, 0.3 \rangle, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\}), \\ & (\langle (x_5, x_2), 0.5, 0.7, 0.3 \rangle, \{u_2, u_4, u_7, u_8, u_{12}, u_{15}\}), \\ & (\langle (x_5, x_4), 0.6, 0.8, 0.3 \rangle, \{u_2, u_{11}\}), \\ & (\langle (x_5, x_8), 0.4, 0.7, 0.3 \rangle, \{u_2, u_8, u_{12}\}), \\ & (\langle (x_7, x_1), 0.7, 0.5, 0.6 \rangle, \{u_5, u_6, u_9, u_{13}\}), \\ & (\langle (x_7, x_2), 0.5, 0.6, 0.3 \rangle, \{u_2, u_5, u_8, u_{15}\}), \\ & (\langle (x_7, x_4), 0.7, 0.8, 0.3 \rangle, \{u_2, u_5, u_9\}), \\ & (\langle (x_7, x_8), 0.4, 0.7, 0.3 \rangle, \{u_2, u_5, u_8\})\}.\end{aligned}$$

Definition 3.5. Let ψ_K, $\Omega_L \in NPS(U)$ and R_N be *NP*-soft relation from ψ_K to Ω_L. Then domain and range of R_N respectively is defined as;

$$D(R_N)=\{\alpha\in\psi_K : \alpha R_N\beta\}$$
$$R(R_N)=\{\beta\in\Omega_L : \alpha R_N\beta\}.$$

Example 3.6. Let us consider the Example 3.4

$$\begin{aligned}D(R_N)=\{&(\langle x_2,0.5,0.6,0.3\rangle,\\ &\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\}),\\ &(\langle x_3,0.3,0.2,0.9\rangle,\{u_2,u_5,u_8,u_{11},u_{15}\}),\\ &(\langle x_5,0.6,0.7,0.3\rangle,\{u_2,u_3,u_4,u_7,u_8,u_{11},u_{12},u_{15}\}),\\ &(\langle x_6,0.1,0.4,0.6\rangle,\{u_2,u_4,u_6,u_7,u_{10},u_{12}\}),\\ &(\langle x_7,0.7,0.5,0.3\rangle,\{u_2,u_5,u_6,u_8,u_9,u_{13},u_{15}\})\}\end{aligned}$$

$$\begin{aligned}R(R_N)=\{&(\langle x_1,0.7,0.4,0.6\rangle,\{u_1,u_5,u_6,u_9,u_{10},u_{13}\}),\\ &(\langle x_2,0.5,0.6,0.3\rangle,\\ &\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\}),\\ &(\langle x_4,0.9,0.8,0.1\rangle,\{u_2,u_5,u_9,u_{10},u_{11},u_{14}\}),\\ &(\langle x_8,0.4,0.7,0.2\rangle,\{u_2,u_5,u_8,u_{10},u_{12},u_{14}\})\}\end{aligned}$$

Definition 3.7. Let R_N be a *NP*-soft relation from ψ_K to Ω_L. Then, inverse of R_N, R_N^{-1} from ψ_K to Ω_L is a *NP*-soft relation defined as;

$$\alpha R_N^{-1}\beta=\beta R_N\alpha$$

Example 3.8. Let us consider the Example 4.4. Then R_N^{-1} is from ψ_K to Ω_L is obtained by

$$\begin{aligned}R_N^{-1}=\{&(\langle(x_1,x_2),0.5,0.6,0.6\rangle,\{u_1,u_5,u_{10}\}),\\ &(\langle(x_2,x_2),0.5,0.6,0.3\rangle\\ &,\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\})\\ &(\langle(x_4,x_2),0.5,0.8,0.3\rangle,\{u_2,u_5,u_{10},u_{14}\}),\\ &(\langle(x_8,x_2),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8,u_{10},u_{12},u_{14}\}),\\ &(\langle(x_2,x_5),0.5,0.7,0.3\rangle,\{u_2,u_4,u_7,u_8,u_{12},u_{15}\}),\\ &(\langle(x_4,x_5),0.6,0.8,0.3\rangle,\{u_2,u_{11}\}),\\ &(\langle(x_8,x_5),0.4,0.7,0.3\rangle,\{u_2,u_8,u_{12}\}),\\ &(\langle(x_1,x_7),0.7,0.5,0.6\rangle,\{u_5,u_6,u_9,u_{13}\}),\\ &(\langle(x_2,x_7),0.5,0.6,0.3\rangle,\{u_2,u_5,u_8,u_{15}\}),\\ &(\langle(x_4,x_7),0.7,0.8,0.3\rangle,\{u_2,u_5,u_9\}),\\ &(\langle(x_8,x_7),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8\})\}.\end{aligned}$$

Proposition 3.9. Let R_{N_1} and R_{N_2} be two NP-soft relation. Then

1. $\left(R_{N_1}^{-1}\right)^{-1}=R_{N_1}$
2. $R_{N_1}\subseteq R_{N_2}\Rightarrow R_{N_1}^{-1}\subseteq R_{N_2}^{-1}$

Proof:

1. $\alpha\left(R_{N_1}^{-1}\right)^{-1}\beta=\beta R_{N_1}^{-1}\alpha=\alpha R_{N_1}\beta$
2. $\alpha R_{N_1}\beta\subseteq\alpha R_{N_2}\beta\Rightarrow\beta R_{N_1}^{-1}\alpha\subseteq\beta R_{N_2}^{-1}\alpha$
   $\Rightarrow R_{N_1}^{-1}\subseteq R_{N_2}^{-1}$

Definition 3.10. If R_{N_1} **and** R_{N_2} are two NP- soft relation from ψ_K to Ω_L, then a compostion of two *NP*-soft relations R_{N_1} and R_{N_2} is defined by

$$\alpha\left(R_{N_1}oR_{N_2}\right)\gamma=\left(aR_{N_1}\beta\right)\wedge\left(\beta R_{N_2}\gamma\right)$$

Proposition 3.11. Let R_{N_1} and R_{N_2} be two *NP*-soft relation from ψ_K to Ω_L.

Then, $\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}{}^{-1} o R_{N_1}{}^{-1}$

Proof:

$$\begin{aligned}\alpha\left(R_{N_1} o R_{N_2}\right)^{-1}\gamma = \gamma\left(R_{N_1} o R_{N_2}\right)\alpha \\ &= \left(\gamma R_{N_1}\beta\right)\wedge\left(\beta R_{N_2}\alpha\right) \\ &= \left(\beta R_{N_2}\alpha\right)\wedge\left(\gamma R_{N_1}\beta\right) \\ &= \left(\alpha R_{N_2}{}^{-1}\beta\right)\wedge\left(\beta R_{N_1}{}^{-1}\gamma\right) \\ &= \alpha\left(R_{N_2}{}^{-1} o R_{N_1}{}^{-1}\right)\gamma\end{aligned}$$

Therefore we obtain

$$\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}{}^{-1} o R_{N_1}{}^{-1}$$

Definition 3.12. A $NP-soft$ relation R_N on ψ_K is said to be a NP-soft symmetric relation

If $\alpha R_{N_1}\beta \Rightarrow \beta R_{N_1}\alpha,\ \forall\alpha,\beta\in\psi_K$.

Definition 3.13. A $NP-soft$ relation R_N on ψ_K is said to be a *NP*-soft transitive relation

if $R_N o R_N \subseteq R_N$,

that is,

$\alpha R_N\beta$ and $\beta R_N\gamma \Rightarrow \alpha R_N\gamma, \forall\alpha,\beta,\gamma\in\psi_K$.

Definition 3.14. A $NP-soft$ relation R_N on ψ_K is said to be a *NP*-soft reflexive relation

If $\alpha R_N\alpha,\ \forall\alpha\in\psi_K$.

Definition 3.15. A *NP*-soft relation R_N on ψ_K is said to be a *NP*-soft equivalence relation if it is symmetric, transitive and reflexive.

Example 3.16. Let

$U=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$,

$E=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$ and

$X=\{\langle x_1,0.5,0.4,0.7\rangle,\langle x_2,0.6,0.8,0.4\rangle,$
$\langle x_3 0.2,0.5,0.1\rangle\}$.

Suppose that

$$\begin{aligned}\psi_K = \{&(\langle x_1,0.5,0.4,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\ &(\langle x_2 0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\ &(\langle x_3,0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

Then, a cartesian product on ψ_K is obtained as follows.

$$\begin{aligned}\psi_K\hat{\mathrm{x}}\psi_K = \{&(\langle(x_1,x_1),0.5,0.4,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\ &(\langle(x_1,x_2),0.5,0.8,0.7\rangle,\{u_2,u_6,u_8\}),\\ &(\langle(x_1,x_3),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\ &(\langle(x_2,x_1),0.5,0.8,0.7\rangle,\{u_2,u_6,u_8\}),\\ &(\langle(x_2,x_2),0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\ &(\langle(x_2,x_3),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\ &(\langle(x_3,x_1),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\ &(\langle(x_3,x_2),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\ &(\langle(x_3,x_3),0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

Then, we get a neutrosophic parameterized soft relation R_N on ψ_K as follows

$$\alpha R_N \beta \Leftrightarrow \Big(\Big\langle (x_i, y_j), \mu_{K\hat{X}L}(x_i, y_j), \nu_{K\hat{X}L}(x_i, y_j), \omega_{K\hat{X}L}(x_i, y_j)\Big\rangle, f_{K\hat{X}L}(x_i, y_j)\Big)(1 \le i, j \le 8)$$

Where

$\mu_{K\hat{X}L}(x_i, y_j) \ge 0.3$

$\nu_{K\hat{X}L}(x_i, y_j) \le 0.5$

$\omega_{K\hat{X}L}(x_i, y_j) \le 0.7$

Then

$$R_N = \{(\langle (x_1, x_1), 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\}), (\langle (x_1, x_2), 0.5, 0.8, 0.7\rangle, \{u_2, u_6, u_8\}), (\langle (x_2, x_1), 0.5, 0.8, 0.7\rangle, \{u_2, u_6, u_8\}), (\langle (x_2, x_2), 0.6, 0.8, 0.4\rangle, \{u_2, u_6, u_8\})\}.$$

R_N on ψ_K is an NP-soft equivalence relation because it is symmetric, transitive and reflexive.

Proposition 3.17. If R_N is symmetric, if and only if R_N^{-1} is so.

Proof: If R_N is symmetric, then

$aR_N^{-1}\beta = \beta R_N \alpha = \alpha R_N \beta = \beta R_N^{-1}\alpha.$ So, R_N^{-1} is symmetric.

Conversely, if R_N^{-1} is symmetric,

then

$\alpha R_N^{-1}\beta = \alpha(R_N^{-1})^{-1}\beta = \beta(R_N^{-1})\alpha = \alpha(R_N^{-1})\beta = \beta R_N \alpha$

So, R_N is symmetric.

Proposition 3.18. R_N is symmetric if and only if $R_N^{-1} = R_N$.

Proof: If R_N is symmetric, then

$\alpha R_N^{-1}\beta = \beta R_N \alpha = \alpha R_N \beta.$ So, $R_N^{-1} = R_N$.

Conversely, if $R_N^{-1} = R_N$, then

$\alpha R_N \beta = \alpha R_N^{-1}\beta = \beta R_N \alpha.$ So, R_N is symmetric .

Proposition 3.19. If R_{N_1} and R_{N_2} are symmetric relations on ψ_K, then $R_{N_1} o R_{N_2}$ is symmetric on ψ_K if and only if $R_{N_1} o R_{N_2} = R_{N_2} o R_{N_1}$

Proof: If R_{N_1} and R_{N_2} are symmetric, then it implies $R_{N_1}^{-1} = R_{N_1}$ and $R_{N_2}^{-1} = R_{N_2}$. We have

$\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}^{-1} o R_{N_1}^{-1}.$

Then $R_{N_1} o R_{N_2}$ is symmetric. It implies

$R_{N_1} o R_{N_2} = \left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}^{-1} o R_{N_1}^{-1} = R_{N_2} o R_{N_1}.$

Conversely,

$\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}^{-1} o R_{N_1}^{-1} = R_{N_2} o R_{N_1} = R_{N_1} o R_{N_2}.$

So, $R_{N_1} o R_{N_2}$ is symmetric.

Corollary 3.20. If R_N is symmetric, then $R^n{}_N$ is symmetric for all positive integer n, where $R^n{}_N = \underbrace{R_N o R_N o....o R_N}_{n \text{ times}}$.

Proposition 3.21. If R_N is transitive, then R_N^{-1} is also transitive.

Proof:

$$\begin{aligned}
\alpha R_N^{-1}\beta = \beta R_N \alpha &\supseteq \beta\left(R_N o R_N\right)\alpha \\
&= \left(\beta R_N \alpha\right) \wedge \left(\gamma R_N \alpha\right) \\
&= \left(\gamma R_N \alpha\right) \wedge \left(\beta R_N \alpha\right) \\
&= \left(\alpha R_N^{-1}\gamma\right) \wedge \left(\gamma R_N^{-1}\beta\right) \\
&= \alpha\left(R_N^{-1} o R_N^{-1}\right)\beta
\end{aligned}$$

So, $R_N^{-1} o R_N^{-1} \subseteq R_N^{-1}.$ The proof is completed.

Proposition 3.22. If R_N is reflexive, then R_N^{-1} is so.

Proof: $aR_N^{-1}\beta = \beta R_N \alpha \subseteq \alpha R_N \alpha = \alpha R_N^{-1}\alpha$ and

$\beta R_N^{-1}\alpha = aR_N \beta \subseteq \alpha R_N \alpha = \alpha R_N^{-1}\alpha.$

The proof is completed.

Proposition 3.23. If R_N is symmetric and transitive, then R_N is reflexive.

Proof: Its clearly.

Definition 3.24. Let $\psi_K \in NPS(U)$, R_N be an $NP-soft$ equivalence relation on ψ_K and $\alpha \in R_N$. Then, an equivalence class of α, denoted by $[\alpha]_{R_N}$, is defined as

$$[\alpha]_{R_N} = \{\beta : \alpha R_N \beta\}.$$

Example 3.25. Let us consider the Example 3. 16 Then an equivalence class of $(\langle x_1, 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\})$ will be as follows.

$$\left[\left(\langle x_1, 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\}\right)\right]_{R_N} =$$
$$\{(\langle x_1, 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\}),$$
$$(\langle x_2 0.6, 0.8, 0.4\rangle, \{u_2, u_6, u_8\}),$$
$$(\langle x_3, 0.2, 0.5, 0.1\rangle, \{u_1, u_2, u_4, u_5, u_7, u_8\})\}$$

4. Decision Making Method

In this section, we construct a soft neutrosophication operator and a decision making method on $NP-soft$ relations.

Definition 4.1. Let $\psi_K \in NPS(U)$ and R_N be a NP-soft relation on ψ_K. The neutrosophication operator, denoted by sR_N, is defined by

$$sR_N : R_N \to F(U),\ sR_N(X \times X, U) = \{\langle u, \mu_{R_N}(u), \nu_{R_N}(u), \omega_{R_N}(u)\rangle, u \in U\}$$

Where

$$\mu_{R_N}(u) = \frac{1}{|X \times X|}\sum_i \sum_j \mu_{R_N}(x_i, x_j)\chi(u)$$
$$\nu_{R_N}(u) = \frac{1}{|X \times X|}\sum_i \sum_j \nu_{R_N}(x_i, x_j)\chi(u)$$
$$\omega_{R_N}(u) = \frac{1}{|X \times X|}\sum_i \sum_j \omega_{R_N}(x_i, x_j)\chi(u)$$

and where

$$\chi(u) = \begin{cases} 1, & u \in f_{R_N}(x_i, x_j) \\ 0, & u \notin f_{R_N}(x_i, x_j) \end{cases}$$

Note that $|X \times X|$ is the cardinality of $X \times X$.

Definition 4.2. Let $\Psi_K \in$ NP-soft set and sR_N a neutrosophication operator, then a reduced fuzzy set of $\tilde{\psi}_K$ is a *fuzzy* set over U denoted by

$$\tilde{\psi}_K(u) = \left\{ \frac{\mu_{sR_N}(u)}{u} : u \in U \right\}$$

Where $\mu_{sR_N} : U \to [0,1]$ and

$$\mu_{sR_N}(u) = \frac{|\mu_K(u) + \nu_K(u) - \omega_K(u)|}{2}$$

Now; we can construct a decision making method on $NP-soft$ relation by the following algorithem;

1. construct a feasible neutrosophic subset X over E,
2. construct a $NP-soft\ set\ \psi_K$ over U,
3. construct a $NP-soft$ relation R_N over ψ_K according to the requests,
4. calculate the neutrosophication operator sR_N over R_N,
5. calculate the reduced fuzzy set $\tilde{\psi}_K$
6. select the objects from $\tilde{\psi}_K$, which have the largest membership value.

Example 4.3. A customer, Mr. *X*, comes to the auto gallery agent to buy a car which is over middle class. Assume that an auto gallery agent has a set of different types of car $U=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$, which may be characterized by a set of parameters $E=\{x_1,x_2,x_3,x_4\}$. For $i=1,2,3,4$ the parameters x_i stand for ty", "cheap", "modern" and "large", respectively. If Mr. *X* has to consider own set of parameters, then we select a car on the basis of the set of customer parameters by using the algorithm as follows.

1. Mr. *X* constructs a neutrosophic set *X* over *E*,
 X= {<x_1,0.5, 0.4 ,0.7>,<x_2,0.6, 0.8 ,0.4>,<x_3,0.2, 0.5 ,0.1>}
2. Mr. *X* constructs a $NP-soft$ set ψ_K over U,

$$\begin{aligned}\psi_K=\{&(\langle x_1,0.5,0.4,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\&(\langle x_2,0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\&(\langle x_3,0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

3. The neutrosophic parameterized soft relation R_N over ψ_K is calculated according to the Mr X's request (The car must be a over middle class, it means the membership degrees are over 0.5),

$$\begin{aligned}R_N=\{&(\langle(x_1,x_1),0.4,0.6,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\&(\langle(x_1,x_2),0.4,0.6,0.7\rangle,\{u_2,u_6,u_8\}),\\&(\langle(x_1,x_3),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\&(\langle(x_2,x_1),0.4,0.6,0.7\rangle,\{u_2,u_6,u_8\})\\&(\langle(x_2,x_2),0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\&(\langle(x_2,x_3),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\&(\langle(x_3,x_1),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\&(\langle(x_3,x_2),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\&(\langle(x_3,x_3),0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

4. The soft neutrosophication operator s_{R_N} over R_N calculated as follows

$$\begin{aligned}sR_N=\{&\langle u_1,0.022,0.055,0.011\rangle,\\&\langle u_2,0.311,0.633,0.533\rangle,\langle u_3,0.044,0.066,0.077\rangle,\\&\langle u_4,0.022,0.055,0.011\rangle,\langle u_5,0.111,0.233,0.244\rangle,\\&\langle u_6,0.2,0.288,0.277\rangle,\langle u_7,0.133,0.322,0.288\rangle,\\&\langle u_8,0.311,0.633,0.533\rangle\}\end{aligned}$$

5. Reduced fuzzy set $\tilde{\psi}_K$ calculated as follows

$$\tilde{\psi}_K(u)=\left\{\frac{0.033}{u_1},\frac{0.205}{u_2},\frac{0.016}{u_3},\frac{0.033}{u_4},\frac{0.05}{u_5},\frac{0.105}{u_6},\frac{0.083}{u_7},\frac{0.205}{u_8};\ u\in U\right\}$$

6. Now, Mr. *X* select the optimum car u_2 and u_8 which have the biggest membership degree 0.205 among the other cars.

5. Conclusion

In this work, we have defined relation on NP-soft sets and studied some of their properties. We also defined symmetric, transitive, reflexive and equivalence relations on the

NP-soft sets. Finally, we construct a decision making method and gave an application which shows that this method successfully works. In future work, we will extend this concept to interval valued neutrosophic parameterized soft sets.

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A novel method for single-valued neutrosophic multi-criteria decision making with incomplete weight information

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Abstract.
A single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) are two instances of a neutrosophic set, which can efficiently deal with uncertain, imprecise, incomplete, and inconsistent information. In this paper, we develop a novel method for solving single-valued neutrosophic multi-criteria decision making with incomplete weight information, in which the criterion values are given in the form of single-valued neutrosophic sets (SVNSs), and the information about criterion weights is incompletely known or completely unknown. The developed method consists of two stages. The first stage is to use the maximizing deviation method to establish an optimization model, which derives the optimal weights of criteria under single-valued neutrosophic environments. After obtaining the weights of criteria through the above stage, the second stage is to develop a single-valued neutrosophic TOPSIS (SVNTOPSIS) method to determine a solution with the shortest distance to the single-valued neutrosophic positive ideal solution (SVNPIS) and the greatest distance from the single-valued neutrosophic negative ideal solution (SVNNIS). Moreover, a best global supplier selection problem is used to demonstrate the validity and applicability of the developed method. Finally, the extended results in interval neutrosophic situations are pointed out and a comparison analysis with the other methods is given to illustrate the advantages of the developed methods.

Keywords: neutrosophic set, single-valued neutrosophic set (SVNS), interval neutrosophic set (INS), multi-criteria decision making (MCDM), maximizing deviation method; TOPSIS.

1. Introduction

Neutrosophy, originally introduced by Smarandache [12], is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra [12]. As a powerful general formal framework, neutrosophic set [12] generalizes the concept of the classic set, fuzzy set [24], interval-valued fuzzy set [14,25], vague set [4], intuitionistic fuzzy set [1], interval-valued intuitionistic fuzzy set [2], paraconsistent set [12], dialetheist set [12], paradoxist set [12], and tautological set [12]. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and falsity-membership are independent, which is a very important assumption in many applications such as information fusion in which the data are combined from different sensors [12]. Recently, neutrosophic sets have been successfully applied to image processing [3,5,6].

The neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications [16,17]. Therefore, Wang et al. [17] defined a single valued neutrosophic set (SVNS), and then provided the set theoretic operators and various properties of single valued neutrosophic sets (SVNSs). Furthermore, Wang et al. [16] proposed the set-theoretic operators on an instance of neutrosophic set called interval neutrosophic set (INS). A single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) are two instances of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. Single valued neutrosophic sets and interval neutrosophic sets are different from intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. Intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets can only handle incomplete information, but cannot handle the indeterminate information and inconsistent information which exist commonly in real situations. The connectors in the intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set are defined with respect to membership and non-membership only (hence the indeterminacy is what is left from 1), while in the single valued neutrosophic set and interval neutrosophic set, they can be defined with respect to any of them (no restriction). For example [17], when we ask the opinion of an expert about certain statement, he or she may say that the possibility in which the statement is true is 0.6 and the statement is false is 0.5 and the degree in which he or she is not sure is 0.2. This situation can be expressed as a single valued neutrosophic set $\langle 0.6, 0.2, 0.5 \rangle$, which is beyond the scope of the intuitionistic fuzzy set.

For another example [16], suppose that an expert may say that the possibility that the statement is true is between 0.5 and 0.6, and the statement is false is between 0.7 and 0.9, and the degree that he or she is not sure is between 0.1 and 0.3. This situation can be expressed as an interval neutrosophic set $\langle[0.5,0.6],[0.1,0.3],[0.7,0.9]\rangle$, which is beyond the scope of the interval-valued intuitionistic fuzzy set.

Due to their abilities to easily reflect the ambiguous nature of subjective judgments, single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs) are suitable for capturing imprecise, uncertain, and inconsistent information in the multi-criteria decision analysis [20,21,22,23]. Most recently, some methods [20,21,22,23] have been developed for solving the multi-criteria decision making (MCDM) problems with single-valued neutrosophic or interval neutrosophic information. For example, Ye [20] developed a multi-criteria decision making method using the correlation coefficient under single-valued neutrosophic environments. Ye [21] defined the single valued neutrosophic cross entropy, based on which, a multi-criteria decision making method is established in which criteria values for alternatives are single valued neutrosophic sets (SVNSs). Ye [23] proposed a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator, and then utilized two aggregation operators to develop a method for multi-criteria decision making problems under simplified neutrosophic environments. Ye [22] defined the similarity measures between interval neutrosophic sets (INSs), and then utilized the similarity measures between each alternative and the ideal alternative to rank the alternatives and to determine the best one. However, it is noted that the aforementioned methods need the information about criterion weights to be exactly known. When using these methods, the associated weighting vector is more or less determined subjectively and the decision information itself is not taken into consideration sufficiently. In fact, in the process of multi-criteria decision making (MCDM), we often encounter the situations in which the criterion values take the form of single valued neutrosophic sets (SVNSs) or interval neutrosophic sets (INSs), and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain [18]. Considering that the existing methods are inappropriate for dealing with such situations, in this paper, we develop a novel method for single valued neutrosophic or interval neutrosophic MCDM with incomplete weight information, in which the criterion values take the form of single valued neutrosophic sets (SVNSs) or interval neutrosophic sets (INSs), and the information about criterion weights is incompletely known or completely unknown. The developed method is composed of two parts. First, we establish an optimization model based on the maximizing deviation method to objectively determine the optimal criterion weights. Then, we develop an extended TOPSIS method, which we call the single valued neutrosophic or interval neutrosophic TOPSIS, to calculate the relative closeness coefficient of each alternative to the single valued neutrosophic or interval neutrosophic positive ideal solution and to select the optimal one with the maximum relative closeness coefficient. Two illustrative examples and comparison analysis with the existing methods show that the developed methods can not only relieve the influence of subjectivity of the decision maker but also remain the original decision information sufficiently.

To do so, the remainder of this paper is set out as follows. Section 2 briefly recalls some basic concepts of neutrosophic sets, single-valued neutrosophic sets (SVNSs), and interval neutrosophic sets (INSs). Section 3 develops a novel method based on the maximizing deviation method and the single-valued neutrosophic TOPSIS (SVNTOPSIS) for solving the single-valued neutrosophic multi-criteria decision making with incomplete weight information. Section 4 develops a novel method based on the maximizing deviation method and the interval neutrosophic TOPSIS (INTOPSIS) for solving the interval neutrosophic multi-criteria decision making with incomplete weight information. Section 5 provides two practical examples to illustrate the effectiveness and practicality of the developed methods. Section 6 ends the paper with some concluding remarks.

2 Neutrosophic sets and and SVNSs

In this section, we will give a brief overview of neutrosophic sets [12], single-valued neutrosophic set (SVNSs) [17], and interval neutrosophic sets (INSs) [16].

2.1 Neutrosophic sets

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [12], and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view.

Smarandache [12] defined a neutrosophic set as follows:

Definition 2.1 [12]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$. That is $T_A(x): X \to]0^-,1^+[$, $I_A(x): X \to]0^-,1^+[$, and $F_A(x): X \to]0^-,1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Definition 2.2 [12]. The complement of a neutrosophic set A is denoted by A^c and is defined as $T_{A^c}(x) = \{1^+\} \ominus T_A(x)$, $I_{A^c}(x) = \{1^+\} \ominus I_A(x)$, and $F_{A^c}(x) = \{1^+\} \ominus F_A(x)$ for every x in X.

Definition 2.3 [12]. A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$ if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for every x in X.

2.2 Single-valued neutrosophic sets (SVNSs)

A single-valued neutrosophic set (SVNS) is an instance of a neutrosophic set, which has a wide range of applications in real scientific and engineering fields. In the following, we review the definition of a SVNS proposed by Wang et al. [17].

Definition 2.4 [17]. Let X be a space of points (objects) with generic elements in X denoted by x. A single-valued neutrosophic set (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$ for each point x in X.

A SVNS A can be written as

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| x \in X \right\} \quad (1)$$

Let $A = \left\{ \left\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i) \right\rangle \middle| x_i \in X \right\}$ and $B = \left\{ \left\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i) \right\rangle \middle| x_i \in X \right\}$ be two single-valued neutrosophic sets (SVNSs) in $X = \{x_1, x_2, \cdots, x_n\}$. Then we define the following distances for A and B.

(i) The Hamming distance

$$d(A,B) = \frac{1}{3}\sum_{i=1}^{n} \begin{pmatrix} \left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \\ \left|F_A(x_i) - F_B(x_i)\right| \end{pmatrix} \quad (2)$$

(ii) The normalized Hamming distance

$$d(A,B) = \frac{1}{3n}\sum_{i=1}^{n} \begin{pmatrix} \left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \\ \left|F_A(x_i) - F_B(x_i)\right| \end{pmatrix} \quad (3)$$

(iii) The Euclidean distance

$$d(A,B) = \sqrt{\frac{1}{3}\sum_{i=1}^{n} \begin{pmatrix} \left|T_A(x_i) - T_B(x_i)\right|^2 + \left|I_A(x_i) - I_B(x_i)\right|^2 \\ + \left|F_A(x_i) - F_B(x_i)\right|^2 \end{pmatrix}} \quad (4)$$

(iv) The normalized Euclidean distance

$$d(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n} \begin{pmatrix} \left|T_A(x_i) - T_B(x_i)\right|^2 + \left|I_A(x_i) - I_B(x_i)\right|^2 \\ + \left|F_A(x_i) - F_B(x_i)\right|^2 \end{pmatrix}} \quad (5)$$

2.3 Interval neutrosophic sets (INSs)

Definition 2.5 [16]. Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (INS) $\tilde{A}$ in X is characterized by a truth-membership function $\tilde{T}_{\tilde{A}}(x)$, an indeterminacy-membership function $\tilde{I}_{\tilde{A}}(x)$, and a falsity-membership function $\tilde{F}_{\tilde{A}}(x)$. For each point x in X, we have that $\tilde{T}_{\tilde{A}}(x) = \left[\inf \tilde{T}_{\tilde{A}}(x), \sup \tilde{T}_{\tilde{A}}(x)\right] \subseteq [0,1]$, $\tilde{I}_{\tilde{A}}(x) = \left[\inf \tilde{I}_{\tilde{A}}(x), \sup \tilde{I}_{\tilde{A}}(x)\right] \subseteq [0,1]$, $\tilde{F}_{\tilde{A}}(x) = \left[\inf \tilde{F}_{\tilde{A}}(x), \sup \tilde{F}_{\tilde{A}}(x)\right] \subseteq [0,1]$, and $0 \le \sup \tilde{T}_{\tilde{A}}(x) + \sup \tilde{I}_{\tilde{A}}(x) + \sup \tilde{F}_{\tilde{A}}(x) \le 3$.

Let $\tilde{A} = \left\{ \left\langle x_i, \tilde{T}_{\tilde{A}}(x_i), \tilde{I}_{\tilde{A}}(x_i), \tilde{F}_{\tilde{A}}(x_i) \right\rangle \middle| x_i \in X \right\}$ and $\tilde{B} = \left\{ \left\langle x_i, \tilde{T}_{\tilde{B}}(x_i), \tilde{I}_{\tilde{B}}(x_i), \tilde{F}_{\tilde{B}}(x_i) \right\rangle \middle| x_i \in X \right\}$ be two interval neutrosophic sets (INSs) in $X = \{x_1, x_2, \cdots, x_n\}$, where $\tilde{T}_{\tilde{A}}(x_i) = \left[\inf \tilde{T}_{\tilde{A}}(x_i), \sup \tilde{T}_{\tilde{A}}(x_i)\right]$, $\tilde{I}_{\tilde{A}}(x_i) = \left[\inf \tilde{I}_{\tilde{A}}(x_i), \sup \tilde{I}_{\tilde{A}}(x_i)\right]$, $\tilde{F}_{\tilde{A}}(x_i) = \left[\inf \tilde{F}_{\tilde{A}}(x_i), \sup \tilde{F}_{\tilde{A}}(x_i)\right]$, $\tilde{T}_{\tilde{B}}(x_i) = \left[\inf \tilde{T}_{\tilde{B}}(x_i), \sup \tilde{T}_{\tilde{B}}(x_i)\right]$, $\tilde{I}_{\tilde{B}}(x_i) = \left[\inf \tilde{I}_{\tilde{B}}(x_i), \sup \tilde{I}_{\tilde{B}}(x_i)\right]$, and $\tilde{F}_{\tilde{B}}(x_i) = \left[\inf \tilde{F}_{\tilde{B}}(x_i), \sup \tilde{F}_{\tilde{B}}(x_i)\right]$. Then Ye [22] defined the following distances for A and B.

(i) The Hamming distance

$$d(A,B) = \frac{1}{6}\sum_{i=1}^{n} \begin{pmatrix} \left|\inf \tilde{T}_{\tilde{A}}(x_i) - \inf \tilde{T}_{\tilde{B}}(x_i)\right| + \left|\sup \tilde{T}_{\tilde{A}}(x_i) - \sup \tilde{T}_{\tilde{B}}(x_i)\right| + \\ \left|\inf \tilde{I}_{\tilde{A}}(x_i) - \inf \tilde{I}_{\tilde{B}}(x_i)\right| + \left|\sup \tilde{I}_{\tilde{A}}(x_i) - \sup \tilde{I}_{\tilde{B}}(x_i)\right| + \\ \left|\inf \tilde{F}_{\tilde{A}}(x_i) - \inf \tilde{F}_{\tilde{B}}(x_i)\right| + \left|\sup \tilde{F}_{\tilde{A}}(x_i) - \sup \tilde{F}_{\tilde{B}}(x_i)\right| \end{pmatrix} \quad (6)$$

(ii) The normalized Hamming distance

$$d(A,B)=\frac{1}{6n}\sum_{i=1}^{n}\left(\begin{array}{l}\left|\inf\tilde{T}_{\tilde{A}}(x_i)-\inf\tilde{T}_{\tilde{B}}(x_i)\right|+\left|\sup\tilde{T}_{\tilde{A}}(x_i)-\sup\tilde{T}_{\tilde{B}}(x_i)\right|+\\ \left|\inf\tilde{I}_{\tilde{A}}(x_i)-\inf\tilde{I}_{\tilde{B}}(x_i)\right|+\left|\sup\tilde{I}_{\tilde{A}}(x_i)-\sup\tilde{I}_{\tilde{B}}(x_i)\right|+\\ \left|\inf\tilde{F}_{\tilde{A}}(x_i)-\inf\tilde{F}_{\tilde{B}}(x_i)\right|+\left|\sup\tilde{F}_{\tilde{A}}(x_i)-\sup\tilde{F}_{\tilde{B}}(x_i)\right|\end{array}\right) \quad (7)$$

(iii) The Euclidean distance

$$d(A,B)=\sqrt{\frac{1}{6}\sum_{i=1}^{n}\left(\begin{array}{l}\left|\inf\tilde{T}_{\tilde{A}}(x_i)-\inf\tilde{T}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{T}_{\tilde{A}}(x_i)-\sup\tilde{T}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{I}_{\tilde{A}}(x_i)-\inf\tilde{I}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{I}_{\tilde{A}}(x_i)-\sup\tilde{I}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{F}_{\tilde{A}}(x_i)-\inf\tilde{F}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{F}_{\tilde{A}}(x_i)-\sup\tilde{F}_{\tilde{B}}(x_i)\right|^2\end{array}\right)} \quad (8)$$

(iv) The normalized Euclidean distance

$$d(A,B)=\sqrt{\frac{1}{6n}\sum_{i=1}^{n}\left(\begin{array}{l}\left|\inf\tilde{T}_{\tilde{A}}(x_i)-\inf\tilde{T}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{T}_{\tilde{A}}(x_i)-\sup\tilde{T}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{I}_{\tilde{A}}(x_i)-\inf\tilde{I}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{I}_{\tilde{A}}(x_i)-\sup\tilde{I}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{F}_{\tilde{A}}(x_i)-\inf\tilde{F}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{F}_{\tilde{A}}(x_i)-\sup\tilde{F}_{\tilde{B}}(x_i)\right|^2\end{array}\right)} \quad (9)$$

3 A novel method for single-valued neutrosophic multi-criteria decision making with incomplete weight information

3.1 Problem description

The aim of multi-criteria decision making (MCDM) problems is to find the most desirable alternative(s) from a set of feasible alternatives according to a number of criteria or attributes. In general, the multi-criteria decision making problem includes uncertain, imprecise, incomplete, and inconsistent information, which can be represented by SVNSs. In this section, we will present a method for handling the MCDM problem under single-valued neutrosophic environments. First, a MCDM problem with single-valued neutrosophic information can be outlined as: let $A=\{A_1,A_2,\cdots,A_m\}$ be a set of m alternatives and $C=\{c_1,c_2,\ldots,c_n\}$ be a collection of n criteria, whose weight vector is $w=(w_1,w_2,\cdots,w_n)^T$, with $w_j\in[0,1]$, $j=1,2,\cdots,n$, and $\sum_{j=1}^{n}w_j=1$. In this case, the characteristic of the alternative A_i ($i=1,2,\cdots,m$) with respect to all the criteria is represented by the following SVNS:

$$A_i=\left\{\left\langle c_j,T_{A_i}(c_j),I_{A_i}(c_j),F_{A_i}(c_j)\right\rangle\middle|c_j\in C\right\}$$

where $T_{A_i}(c_j),I_{A_i}(c_j),F_{A_i}(c_j)\in[0,1]$, and $0\le T_{A_i}(c_j)+I_{A_i}(c_j)+F_{A_i}(c_j)\le 3$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$).

Here, $T_{A_i}(c_j)$ indicates the degree to which the alternative A_i satisfies the criterion c_j, $I_{A_i}(c_j)$ indicates the indeterminacy degree to which the alternative A_i satisfies or does not satisfy the criterion c_j, and $F_{A_i}(c_j)$ indicates the degree to which the alternative A_i does not satisfy the criterion c_j. For the sake of simplicity, a criterion value $\left\langle c_j,T_{A_i}(c_j),I_{A_i}(c_j),F_{A_i}(c_j)\right\rangle$ in A_i is denoted by a single-valued neutrosophic value (SVNV) $a_{ij}=\left\langle T_{ij},I_{ij},F_{ij}\right\rangle$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$), which is usually derived from the evaluation of an alternative A_i with respect to a criterion C_j by means of a score law and data processing in practice [19,22]. All a_{ij} ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$) constitute a single valued neutrosophic decision matrix $A=(a_{ij})_{m\times n}=\left(\left\langle T_{ij},I_{ij},F_{ij}\right\rangle\right)_{m\times n}$ (see Table 1):

Table 1: Single valued neutrosophic decision matrix A.

1	c_1	$\cdots$	c_j	$\cdots$	c_n
A_1	$\langle T_{11},I_{11},F_{11}\rangle$	$\cdots$	$\langle T_{1j},I_{1j},F_{1j}\rangle$	$\cdots$	$\langle T_{1n},I_{1n},F_{1n}\rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_i	$\langle T_{i1},I_{i1},F_{i1}\rangle$	$\cdots$	$\langle T_{ij},I_{ij},F_{ij}\rangle$	$\cdots$	$\langle T_{in},I_{in},F_{in}\rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m	$\langle T_{m1},I_{m1},F_{m1}\rangle$	$\cdots$	$\langle T_{mj},I_{mj},F_{mj}\rangle$	$\cdots$	$\langle T_{mn},I_{mn},F_{mn}\rangle$

3.2 Obtaining the optimal weights of criteria by the maximizing deviation method

Due to the fact that many practical MCDM problems are complex and uncertain and human thinking is inherently subjective, the information about criterion weights is usually incomplete. For convenience, let Δ be a set of the known weight information [8,9,10,11], where Δ can be constructed by the following forms, for $i\ne j$:

Form 1. A weak ranking: $\{w_i\ge w_j\}$;

Form 2. A strict ranking: $\{w_i - w_j \geq \alpha_i\}$ ($\alpha_i > 0$);

Form 3. A ranking of differences: $\{w_i - w_j \geq w_k - w_l\}$, for $j \neq k \neq l$;

Form 4. A ranking with multiples: $\{w_i \geq \alpha_i w_j\}$ ($0 \leq \alpha_i \leq 1$);

Form 5. An interval form: $\{\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i\}$ ($0 \leq \alpha_i \leq \alpha_i + \varepsilon_i \leq 1$).

Wang [15] proposed the maximizing deviation method for estimating the criterion weights in MCDM problems with numerical information. According to Wang [15], if the performance values of all the alternatives have small differences under a criterion, it shows that such a criterion plays a less important role in choosing the best alternative and should be assigned a smaller weight. On the contrary, if a criterion makes the performance values of all the alternatives have obvious differences, then such a criterion plays a much important role in choosing the best alternative and should be assigned a larger weight. Especially, if all available alternatives score about equally with respect to a given criterion, then such a criterion will be judged unimportant by most decision makers and should be assigned a very small weight. Wang [15] suggests that zero weight should be assigned to the criterion of this kind.

Here, based on the maximizing deviation method, we construct an optimization model to determine the optimal relative weights of criteria under single valued neutrosophic environments. For the criterion $c_j \in C$, the deviation of the alternative A_i to all the other alternatives can be defined as below:

$$D_{ij} = \sum_{q=1}^{m} d\left(a_{ij}, a_{qj}\right) = \sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$$

$$i = 1, 2, \cdots, m, \ j = 1, 2, \cdots, n \tag{10}$$

where $d\left(a_{ij}, a_{qj}\right) = \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$ denotes the single valued neutrosophic Euclidean distance between two single-valued neutrosophic values (SVNVs) a_{ij} and a_{qj} defined as in Eq. (4).

Let

$$D_j = \sum_{i=1}^{m} D_{ij} = \sum_{i=1}^{m}\sum_{q=1}^{m} d\left(a_{ij}, a_{qj}\right) = \sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$$

$$j = 1, 2, \cdots, n \tag{11}$$

then D_j represents the deviation value of all alternatives to other alternatives for the criterion $c_j \in C$.

Further, let

$$D(w) = \sum_{j=1}^{n} w_j D_j = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}} \tag{12}$$

then $D(w)$ represents the deviation value of all alternatives to other alternatives for all the criteria.

Based on the above analysis, we can construct a non-linear programming model to select the weight vector w by maximizing $D(w)$, as follows:

$$\max D(w) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$$

$$\text{s.t.} \quad w_j \geq 0, \ j = 1, 2, \cdots, n, \quad \sum_{j=1}^{n} w_j^2 = 1 \tag{M-1}$$

To solve this model, we construct the Lagrange function as follows:

$$L(w, \lambda) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}} + \frac{\lambda}{2}\left(\sum_{j=1}^{n} w_j^2 - 1\right) \tag{13}$$

where λ is the Lagrange multiplier.

Differentiating Eq. (13) with respect to w_j ($j = 1, 2, \cdots, n$) and λ, and setting these partial derivatives equal to zero, then the following set of equations is obtained:

$$\frac{\partial L}{\partial w_j} = \sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}} + \lambda w_j = 0 \tag{14}$$

$$\frac{\partial L}{\partial \lambda} = \frac{1}{2}\left(\sum_{j=1}^{n} w_j^2 - 1\right) = 0 \tag{15}$$

It follows from Eq. (14) that

$$w_j = \frac{-\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}}{\lambda} \tag{16}$$

Putting Eq. (16) into Eq. (15), we get

$$\lambda=-\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}\right)^2} \quad (17)$$

Then, by combining Eqs. (16) and (17), we have

$$w_j=\frac{\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}\right)^2}} \quad (18)$$

By normalizing w_j ($j=1,2,\cdots,n$), we make their sum into a unit, and get

$$w_j^*=\frac{w_j}{\sum_{j=1}^{n}w_j}=\frac{\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}} \quad (19)$$

which can be considered as the optimal weight vector of criteria.

However, it is noted that there are practical situations in which the information about the weight vector is not completely unknown but partially known. For such cases, we establish the following constrained optimization model:

$$\begin{aligned}&\max D(w)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m}w_j\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}\\&\text{s.t.}\quad w\in\Delta,\quad w_j\geq 0,\ j=1,2,\cdots,n,\quad \sum_{j=1}^{n}w_j=1\end{aligned} \quad \text{(M-2)}$$

It is noted that the model (M-2) is a linear programming model that can be solved using the MATLAB mathematics software package. Suppose that the optimal solution to the model (M-2) is $w=\left(w_1,w_2,\cdots,w_n\right)^T$, which can be considered as the weight vector of criteria.

3.3. Extended TOPIS method for the MCDM with single valued neutrosophic information

TOPSIS method, initially introduced by Hwang and Yoon [7], is a widely used method for dealing with MCDM problems, which focuses on choosing the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). After obtaining the criterion weight values on basis of the maximizing deviation method, in the following, we will extend the TOPSIS method to take single-valued neutrosophic information into account and utilize the distance measures of SVNVs to obtain the final ranking of the alternatives.

In general, the criteria can be classified into two types: benefit criteria and cost criteria. The benefit criterion means that a higher value is better while for the cost criterion is valid the opposite. Let C_1 be a collection of benefit criteria and C_2 be a collection of cost criteria, where $C_1\bigcup C_2=C$ and $C_1\bigcap C_2=\varnothing$. Under single-valued neutrosophic environments, the single-valued neutrosophic PIS (SVNPIS), denoted by A^+, can be identified by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives as follows:

$$A^+=\left\{a_1^+,a_2^+,\cdots,a_n^+\right\} \quad (20)$$

where

$$a_j^+=\begin{cases}\left\langle \max_i\left\{T_{ij}\right\},\min_i\left\{I_{ij}\right\},\min_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j\in C_1,\\ \left\langle \min_i\left\{T_{ij}\right\},\max_i\left\{I_{ij}\right\},\max_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j\in C_2.\end{cases} \quad (21)$$

The single-valued neutrosophic NIS (SVNNIS), denoted by A^-, can be identified by using a minimum operator for the benefit criteria and a maximum operator for the cost criteria to determine the worst value of each criterion among all alternatives as follows:

$$A^-=\left\{a_1^-,a_2^-,\cdots,a_n^-\right\} \quad (22)$$

where

$$a_j^-=\begin{cases}\left\langle \min_i\left\{T_{ij}\right\},\max_i\left\{I_{ij}\right\},\max_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j\in C_1,\\ \left\langle \max_i\left\{T_{ij}\right\},\min_i\left\{I_{ij}\right\},\min_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j\in C_2.\end{cases} \quad (23)$$

The separation measures, d_i^+ and d_i^-, of each alternative from the SVNPIS A^+ and the SVNNIS A^-, respectively, are derived from

$$d_i^+ = \sum_{j=1}^{n} w_j d\left(a_{ij}, a_j^+\right)$$

$$= \sum_{j\in C_1} w_j \sqrt{\frac{\left(\left|T_{ij} - \max_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \min_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \min_i\left\{F_{ij}\right\}\right|^2\right)}{3}} +$$

$$\sum_{j\in C_2} w_j \sqrt{\frac{\left(\left|T_{ij} - \min_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \max_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \max_i\left\{F_{ij}\right\}\right|^2\right)}{3}} \quad (24)$$

$$d_i^- = \sum_{j=1}^{n} w_j d\left(a_{ij}, a_j^-\right)$$

$$= \sum_{j\in C_1} w_j \sqrt{\frac{\left(\left|T_{ij} - \min_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \max_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \max_i\left\{F_{ij}\right\}\right|^2\right)}{3}} +$$

$$\sum_{j\in C_2} w_j \sqrt{\frac{\left(\left|T_{ij} - \max_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \min_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \min_i\left\{F_{ij}\right\}\right|^2\right)}{3}} \quad (25)$$

The relative closeness coefficient of an alternative A_i with respect to the single-valued neutrosophic PIS A^+ is defined as the following formula:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (26)$$

where $0 \le C_i \le 1$, $i = 1, 2, \cdots, m$. Obviously, an alternative A_i is closer to the single-valued neutrosophic PIS A^+ and farther from the single-valued neutrosophic NIS A^- as C_i approaches 1. The larger the value of C_i, the more different between A_i and the single-valued neutrosophic NIS A^-, while the more similar between A_i and the single-valued neutrosophic PIS A^+. Therefore, the alternative(s) with the maximum relative closeness coefficient should be chosen as the optimal one(s).

Based on the above analysis, we will develop a practical approach for dealing with MCDM problems, in which the information about criterion weights is incompletely known or completely unknown, and the criterion values take the form of single-valued neutrosophic information.

The flowchart of the proposed approach for MCDM is provided in Fig. 1. The proposed approach is composed of the following steps:

Step 1. For a MCDM problem, the decision maker (DM) constructs the single-valued neutrosophic decision matrix $A = \left(a_{ij}\right)_{m\times n} = \left(\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle\right)_{m\times n}$, where $a_{ij} = \left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle$ is a single-valued neutrosophic value (SVNV), given by the DM, for the alternative A_i with respect to the attribute c_j.

Step 2. If the information about the criterion weights is completely unknown, then we use Eq. (19) to obtain the criterion weights; if the information about the criterion weights is partly known, then we solve the model (M-2) to obtain the criterion weights.

Step 3. Utilize Eqs. (20), (21), (22), and (23) to determine the single-valued neutrosophic positive ideal solution (SVNPIS) A^+ and the single-valued neutrosophic negative ideal solution (SVNNIS) A^-.

Step 4. Utilize Eqs. (24) and (25) to calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the single-valued neutrosophic positive ideal solution (SVNPIS) A^+ and the single-valued neutrosophic negative ideal solution (SVNNIS) A^-, respectively.

Step 5. Utilize Eq. (26) to calculate the relative closeness coefficient C_i of each alternative A_i to the single-valued neutrosophic positive ideal solution (SVNPIS) A^+.

Step 6. Rank the alternatives A_i ($i = 1, 2, \cdots, m$) according to the relative closeness coefficients C_i ($i = 1, 2, \cdots, m$) to the single-valued neutrosophic positive ideal solution (SVNPIS) A^+ and then select the most desirable one(s).

4 A novel method for interval neutrosophic multi-criteria decision making with incomplete weight information

In this section, we will extend the results obtained in Section 3 to interval neutrosophic environments.

4.1. Problem description

Similar to Subsection 3.1, a MCDM problem under interval neutrosophic environments can be summarized as follows: let $A = \left\{A_1, A_2, \cdots, A_m\right\}$ be a set of m alternatives and $C = \left\{c_1, c_2, \ldots, c_n\right\}$ be a collection of n criteria, whose weight vector is $w = \left(w_1, w_2, \cdots, w_n\right)^T$, with $w_j \in [0,1]$, $j = 1, 2, \cdots, n$, and $\sum_{j=1}^{n} w_j = 1$. In this case, the characteristic of the alternative A_i ($i = 1, 2, \cdots, m$) with respect to all the criteria is represented by the following INS:

$$\tilde{A}_i = \left\{\left\langle c_j, \tilde{T}_{\tilde{A}_i}\left(c_j\right), \tilde{I}_{\tilde{A}_i}\left(c_j\right), \tilde{F}_{\tilde{A}_i}\left(c_j\right)\right\rangle \middle| c_j \in C\right\}$$

$$= \left\{\left\langle \begin{array}{l} c_j, \left[\inf \tilde{T}_{\tilde{A}}\left(c_j\right), \sup \tilde{T}_{\tilde{A}}\left(c_j\right)\right], \left[\inf \tilde{I}_{\tilde{A}}\left(c_j\right), \sup \tilde{I}_{\tilde{A}}\left(c_j\right)\right], \\ \left[\inf \tilde{F}_{\tilde{A}}\left(c_j\right), \sup \tilde{F}_{\tilde{A}}\left(c_j\right)\right] \end{array}\right\rangle \middle| c_j \in C\right\}$$

where $\tilde{T}_{\tilde{A}_i}\left(c_j\right) = \left[\inf \tilde{T}_{\tilde{A}}\left(c_j\right), \sup \tilde{T}_{\tilde{A}}\left(c_j\right)\right] \subseteq [0,1]$, $\tilde{I}_{\tilde{A}_i}\left(c_j\right) = \left[\inf \tilde{I}_{\tilde{A}}\left(c_j\right), \sup \tilde{I}_{\tilde{A}}\left(c_j\right)\right] \subseteq [0,1]$,

$\tilde{F}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{F}_{\tilde{A}}(c_j), \sup \tilde{F}_{\tilde{A}}(c_j)\right] \subseteq [0,1]$, and $\sup \tilde{T}_{\tilde{A}}(c_j)+\sup \tilde{I}_{\tilde{A}}(c_j)+\sup \tilde{F}_{\tilde{A}}(c_j) \le 3$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$).

Here, $\tilde{T}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{T}_{\tilde{A}}(c_j), \sup \tilde{T}_{\tilde{A}}(c_j)\right]$ indicates the interval degree to which the alternative A_i satisfies the criterion c_j, $\tilde{I}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{I}_{\tilde{A}}(c_j), \sup \tilde{I}_{\tilde{A}}(c_j)\right]$ indicates the indeterminacy interval degree to which the alternative A_i satisfies or does not satisfy the criterion c_j, and $\tilde{F}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{F}_{\tilde{A}}(c_j), \sup \tilde{F}_{\tilde{A}}(c_j)\right]$ indicates the interval degree to which the alternative A_i does not satisfy the criterion c_j. For the sake of simplicity, a criterion value $\left\langle c_j, \tilde{T}_{\tilde{A}_i}(c_j), \tilde{I}_{\tilde{A}_i}(c_j), \tilde{F}_{\tilde{A}_i}(c_j)\right\rangle$ in $\tilde{A}_i$ is denoted by an interval neutrosophic value (INV) $\tilde{a}_{ij}=\left\langle \tilde{T}_{ij}, \tilde{I}_{ij}, \tilde{F}_{ij}\right\rangle=\left\langle \left[T_{ij}^L, T_{ij}^U\right], \left[I_{ij}^L, I_{ij}^U\right], \left[F_{ij}^L, F_{ij}^U\right]\right\rangle$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$), which is usually derived from the evaluation of an alternative A_i with respect to a criterion c_j by means of a score law and data processing in practice [19,22]. All $\tilde{a}_{ij}$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$) constitute an interval neutrosophic decision matrix $\tilde{A}=\left(\tilde{a}_{ij}\right)_{m\times n}=\left(\left\langle \tilde{T}_{ij}, \tilde{I}_{ij}, \tilde{F}_{ij}\right\rangle\right)_{m\times n}=\left(\left\langle \left[T_{ij}^L, T_{ij}^U\right], \left[I_{ij}^L, I_{ij}^U\right], \left[F_{ij}^L, F_{ij}^U\right]\right\rangle\right)_{m\times n}$ (see Table 2):

Table 2: Interval neutrosophic decision matrix $\tilde{A}$.

2	c_1	$\cdots$	c_j	$\cdots$	c_n
A_1	$\langle \tilde{T}_{11}, \tilde{I}_{11}, \tilde{F}_{11}\rangle$	$\cdots$	$\langle \tilde{T}_{1j}, \tilde{I}_{1j}, \tilde{F}_{1j}\rangle$	$\cdots$	$\langle \tilde{T}_{1n}, \tilde{I}_{1n}, \tilde{F}_{1n}\rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_i	$\langle \tilde{T}_{i1}, \tilde{I}_{i1}, \tilde{F}_{i1}\rangle$	$\cdots$	$\langle \tilde{T}_{ij}, \tilde{I}_{ij}, \tilde{F}_{ij}\rangle$	$\cdots$	$\langle \tilde{T}_{in}, \tilde{I}_{in}, \tilde{F}_{in}\rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m	$\langle \tilde{T}_{m1}, \tilde{I}_{m1}, \tilde{F}_{m1}\rangle$	$\cdots$	$\langle \tilde{T}_{mj}, \tilde{I}_{mj}, \tilde{F}_{mj}\rangle$	$\cdots$	$\langle \tilde{T}_{mn}, \tilde{I}_{mn}, \tilde{F}_{mn}\rangle$

4.2. Obtaining the optimal weights of criteria under interval neutrosophic environments by the maximizing deviation method

In what follows, similar to Subsection 3.2, based on the maximizing deviation method, we construct an optimization model to determine the optimal relative weights of criteria under interval neutrosophic environments. For the attribute $c_j \in C$, the deviation of the alternative A_i to all the other alternatives can be defined as below:

$$D_{ij}=\sum_{q=1}^{m} d\left(\tilde{a}_{ij}, \tilde{a}_{qj}\right)=\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}$$

$$i=1,2,\cdots,m,\ j=1,2,\cdots,n \tag{27}$$

where $d\left(\tilde{a}_{ij}, \tilde{a}_{qj}\right)=\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}$ denotes the interval neutrosophic Euclidean distance between two interval neutrosophic values (INVs) $\tilde{a}_{ij}$ and $\tilde{a}_{qj}$ defined as in Eq. (8).

Let

$$D_j=\sum_{i=1}^{m} D_{ij}=\sum_{i=1}^{m}\sum_{q=1}^{m} d\left(\tilde{a}_{ij}, \tilde{a}_{qj}\right)=\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}$$

$$j=1,2,\cdots,n \tag{28}$$

then D_j represents the deviation value of all alternatives to other alternatives for the criterion $c_j \in C$.

Further, let

$$D(w)=\sum_{j=1}^{n} w_j D_j=\sum_{j=1}^{n} w_j\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}\right) \tag{29}$$

then $D(w)$ represents the deviation value of all alternatives to other alternatives for all the criteria.

From the above analysis, we can construct a non-linear programming model to select the weight vector w by maximizing $D(w)$, as follows:

$$\max D(w)=\sum_{j=1}^{n} w_j \left(\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}} \right)$$

$$\text{s.t.}\quad w_j \ge 0,\ j=1,2,\cdots,n,\quad \sum_{j=1}^{n} w_j^2=1 \tag{M-3}$$

To solve this model, we construct the Lagrange function:

$$L(w,\lambda)=\sum_{j=1}^{n} w_j \left(\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}} \right)+ \frac{\lambda}{2}\left(\sum_{j=1}^{n} w_j^2-1\right) \tag{30}$$

where λ is the Lagrange multiplier.

Differentiating Eq. (30) with respect to w_j ($j=1,2,\cdots,n$) and λ, and setting these partial derivatives equal to zero, then the following set of equations is obtained:

$$\frac{\partial L}{\partial w_j}=\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}+ \lambda w_j=0 \tag{31}$$

$$\frac{\partial L}{\partial \lambda}=\frac{1}{2}\left(\sum_{j=1}^{n} w_j^2-1\right)=0 \tag{32}$$

It follows from Eq. (32) that

$$w_j=\frac{-\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}}{\lambda} \tag{33}$$

Putting Eq. (33) into Eq. (32), we get

$$\lambda=-\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}} \right)^2} \tag{34}$$

Then, by combining Eqs. (33) and (34), we have

$$w_j=\frac{\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}} \right)^2}} \tag{35}$$

By normalizing w_j ($j=1,2,\cdots,n$), we make their sum into a unit, and get

$$w_j^*=\frac{w_j}{\sum_{j=1}^{n} w_j}=\frac{\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}} \tag{36}$$

which can be considered as the optimal weight vector of criteria.

However, it is noted that there are practical situations in which the information about the weight vector is not completely unknown but partially known. For such cases, we establish the following constrained optimization model:

$$\max D(w)=\sum_{j=1}^{n} w_j \left(\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}} \right)$$

$$\text{s.t.}\quad w\in\Delta,\quad w_j \ge 0,\ j=1,2,\cdots,n,\quad \sum_{j=1}^{n} w_j=1 \tag{M-4}$$

It is noted that the model (M-4) is a linear programming model that can be solved using the MATLAB mathematics software package. Suppose that the optimal solution to the model (M-4) is $w=\left(w_1,w_2,\cdots,w_n\right)^T$, which can be considered as the weight vector of criteria.

4.3. Extended TOPIS method for the MCDM with interval neutrosophic information

After obtaining the weights of criteria on basis of the maximizing deviation method, similar to Subsection 3.3, we next extend the TOPSIS method to interval neutrosophic environments and develop an extended TOPSIS method to obtain the final ranking of the alternatives.

Under interval neutrosophic environments, the interval neutrosophic PIS (INPIS), denoted by $\tilde{A}^+$, and the interval neutrosophic NIS (INNIS), denoted by $\tilde{A}^-$, can be defined as follows:

$$\tilde{A}^+=\left\{\tilde{a}_1^+,\tilde{a}_2^+,\cdots,\tilde{a}_n^+\right\} \tag{37}$$

$$\tilde{A}^-=\left\{\tilde{a}_1^-,\tilde{a}_2^-,\cdots,\tilde{a}_n^-\right\} \tag{38}$$

where

$$\tilde{a}_j^+=\begin{cases}\left\langle \max\limits_i\left\{\tilde{T}_{ij}\right\},\min\limits_i\left\{\tilde{I}_{ij}\right\},\min\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle = \\ \left\langle \begin{array}{l}\left[\max\limits_i T_{ij}^L,\max\limits_i T_{ij}^U\right],\left[\min\limits_i I_{ij}^L,\min\limits_i I_{ij}^U\right], \\ \left[\min\limits_i F_{ij}^L,\min\limits_i F_{ij}^U\right]\end{array}\right\rangle, & \text{if } j\in C_1, \\ \left\langle \min\limits_i\left\{\tilde{T}_{ij}\right\},\max\limits_i\left\{\tilde{I}_{ij}\right\},\max\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle \\ =\left\langle \begin{array}{l}\left[\min\limits_i T_{ij}^L,\min\limits_i T_{ij}^U\right],\left[\max\limits_i I_{ij}^L,\max\limits_i I_{ij}^U\right], \\ \left[\max\limits_i F_{ij}^L,\max\limits_i F_{ij}^U\right]\end{array}\right\rangle, & \text{if } j\in C_2.\end{cases} \tag{39}$$

$$\tilde{a}_j^-=\begin{cases}\left\langle \min\limits_i\left\{\tilde{T}_{ij}\right\},\max\limits_i\left\{\tilde{I}_{ij}\right\},\max\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle = \\ \left\langle \begin{array}{l}\left[\min\limits_i T_{ij}^L,\min\limits_i T_{ij}^U\right],\left[\max\limits_i I_{ij}^L,\max\limits_i I_{ij}^U\right], \\ \left[\max\limits_i F_{ij}^L,\max\limits_i F_{ij}^U\right]\end{array}\right\rangle, & \text{if } j\in C_1, \\ \left\langle \max\limits_i\left\{\tilde{T}_{ij}\right\},\min\limits_i\left\{\tilde{I}_{ij}\right\},\min\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle = \\ \left\langle \begin{array}{l}\left[\max\limits_i T_{ij}^L,\max\limits_i T_{ij}^U\right],\left[\min\limits_i I_{ij}^L,\min\limits_i I_{ij}^U\right], \\ \left[\min\limits_i F_{ij}^L,\min\limits_i F_{ij}^U\right]\end{array}\right\rangle, & \text{if } j\in C_2.\end{cases} \tag{40}$$

The separation measures, $\tilde{d}_i^+$ and $\tilde{d}_i^-$, of each alternative A_i from the INPIS $\tilde{A}^+$ and the INNIS $\tilde{A}^-$, respectively, are derived from

$$\begin{aligned}\tilde{d}_i^+ &= \sum_{j=1}^n w_j d\left(\tilde{a}_{ij},\tilde{a}_j^+\right) \\ &= \sum_{j\in C_1} w_j\sqrt{\frac{\left(\begin{array}{l}\left|T_{ij}^L-\max\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\max\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\min\limits_i\left\{I_{ij}^L\right\}\right|^2+ \\ \left|I_{ij}^U-\min\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\min\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\min\limits_i\left\{F_{ij}^U\right\}\right|^2\end{array}\right)}{6}}+ \\ &\quad \sum_{j\in C_2} w_j\sqrt{\frac{\left(\begin{array}{l}\left|T_{ij}^L-\min\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\min\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\max\limits_i\left\{I_{ij}^L\right\}\right|^2+ \\ \left|I_{ij}^U-\max\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\max\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\max\limits_i\left\{F_{ij}^U\right\}\right|^2\end{array}\right)}{6}}\end{aligned} \tag{41}$$

$$\begin{aligned}\tilde{d}_i^- &= \sum_{j=1}^n w_j d\left(\tilde{a}_{ij},\tilde{a}_j^-\right) \\ &= \sum_{j\in C_1} w_j\sqrt{\frac{\left(\begin{array}{l}\left|T_{ij}^L-\min\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\min\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\max\limits_i\left\{I_{ij}^L\right\}\right|^2+ \\ \left|I_{ij}^U-\max\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\max\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\max\limits_i\left\{F_{ij}^U\right\}\right|^2\end{array}\right)}{6}}+ \\ &\quad \sum_{j\in C_2} w_j\sqrt{\frac{\left(\begin{array}{l}\left|T_{ij}^L-\max\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\max\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\min\limits_i\left\{I_{ij}^L\right\}\right|^2+ \\ \left|I_{ij}^U-\min\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\min\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\min\limits_i\left\{F_{ij}^U\right\}\right|^2\end{array}\right)}{6}}\end{aligned} \tag{42}$$

The relative closeness coefficient of an alternative A_i with respect to the interval neutrosophic PIS $\tilde{A}^+$ is defined as the following formula:

$$\tilde{C}_i=\frac{\tilde{d}_i^-}{\tilde{d}_i^+ + \tilde{d}_i^-} \tag{43}$$

where $0\le\tilde{C}_i\le1$, $i=1,2,\cdots,m$. Obviously, an alternative A_i is closer to the interval neutrosophic PIS $\tilde{A}^+$ and farther from the interval neutrosophic NIS $\tilde{A}^-$ as $\tilde{C}_i$ approaches 1. The larger the value of $\tilde{C}_i$, the more different between A_i and the interval neutrosophic NIS $\tilde{A}^-$, while the more similar between A_i and the interval neutrosophic PIS $\tilde{A}^+$. Therefore, the alternative(s) with the maximum relative closeness coefficient should be chosen as the optimal one(s).

Based on the above analysis, analogous to Subsection 3.3, we will develop a practical approach for dealing with MCDM problems, in which the information about criterion weights is incompletely known or completely unknown, and the criterion values take the form of interval neutrosophic information.

The flowchart of the proposed approach for MCDM is provided in Fig. 1. The proposed approach is composed of the following steps:

Step 1. For a MCDM problem, the decision maker constructs the interval neutrosophic decision matrix $\tilde{A}=\left(\tilde{a}_{ij}\right)_{m\times n}=\left(\left\langle \tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\right\rangle\right)_{m\times n}$, where $\tilde{a}_{ij}=\left\langle \tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\right\rangle$ is an interval neutrosophic value (INV), given by the DM, for the alternative A_i with respect to the criterion c_j.

Step 2. If the information about the criterion weights is completely unknown, then we use Eq. (36) to obtain the criterion weights; if the information about the criterion weights is partly known, then we solve the model (M-4) to obtain the criterion weights.

Step 3. Utilize Eqs. (37), (38), (39), and (40) to determine the interval neutrosophic positive ideal solution (INPIS) $\tilde{A}^+$ and the interval neutrosophic negative ideal solution (INNIS) $\tilde{A}^-$.

Step 4. Utilize Eqs. (41) and (42) to calculate the separation measures $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative A_i from the interval neutrosophic positive ideal solution (INPIS) $\tilde{A}^+$ and the interval neutrosophic negative ideal solution (INNIS) $\tilde{A}^-$, respectively.

Step 5. Utilize Eq. (43) to calculate the relative closeness coefficient $\tilde{C}_i$ of each alternative A_i to the interval neutrosophic positive ideal solution (INPIS) $\tilde{A}^+$.

Step 6. Rank the alternatives A_i ($i=1,2,\cdots,m$) according to the relative closeness coefficients $\tilde{C}_i$ ($i=1,2,\cdots,m$) to the interval neutrosophic positive ideal solution (INPIS) A^+ and then select the most desirable one(s).

5 Illustrative examples

5.1. A practical example under single-valued neutrosophic environments

Example 5.1 [13]. In order to demonstrate the application of the proposed approach, a multi-criteria decision making problem adapted from Tan and Chen [13] is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A=\{A_1,A_2,A_3,A_4,A_5\}$ whose core competencies are evaluated by means of the following four criteria:

(1) the level of technology innovation (c_1),

(2) the control ability of flow (c_2),

(3) the ability of management (c_3),

(4) the level of service (c_4).

It is noted that all the criteria c_j ($j=1,2,3,4$) are the benefit type attributes. The selection of the best global supplier can be modeled as a hierarchical structure, as shown in Fig. 2. According to [21], we can obtain the evaluation of an alternative A_i ($i=1,2,3,4,5$) with respect to a criterion c_j ($j=1,2,3,4$) from the questionnaire of a domain expert. Take a_{11} as an example. When we ask the opinion of an expert about an alternative A_1 with respect to a criterion c_1, he or she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.1. In this case, the evaluation of the alternative A_1 with respect to the criterion c_1 is expressed as a single-valued neutrosophic value $a_{11}=\langle 0.5,0.1,0.3\rangle$. Through the similar method from the expert, we can obtain all the evaluations of all the alternatives A_i ($i=1,2,3,4,5$) with respect to all the criteria c_j ($j=1,2,3,4$), which are listed in the following single valued neutrosophic decision matrix $A=\left(a_{ij}\right)_{m\times n}=\left(\left\langle T_{ij},I_{ij},F_{ij}\right\rangle\right)_{m\times n}$ (see Table 3).

Table 3: Single valued neutrosophic decision matrix A.

3	c_1	c_2	c_3	c_4
A_1	$\langle 0.5,0.1,0.3\rangle$	$\langle 0.5,0.1,0.4\rangle$	$\langle 0.3,0.2,0.3\rangle$	$\langle 0.7,0.2,0.1\rangle$
A_2	$\langle 0.6,0.1,0.2\rangle$	$\langle 0.5,0.2,0.2\rangle$	$\langle 0.5,0.4,0.1\rangle$	$\langle 0.4,0.2,0.3\rangle$
A_3	$\langle 0.9,0.0,0.1\rangle$	$\langle 0.3,0.2,0.3\rangle$	$\langle 0.2,0.2,0.5\rangle$	$\langle 0.4,0.3,0.2\rangle$
A_4	$\langle 0.8,0.1,0.1\rangle$	$\langle 0.5,0.0,0.4\rangle$	$\langle 0.6,0.2,0.1\rangle$	$\langle 0.2,0.3,0.4\rangle$
A_5	$\langle 0.7,0.2,0.1\rangle$	$\langle 0.4,0.3,0.2\rangle$	$\langle 0.6,0.1,0.3\rangle$	$\langle 0.5,0.4,0.1\rangle$

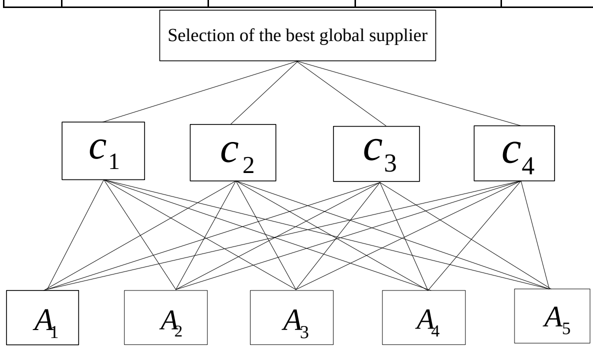

Fig. 2: Hierarchical structure.

In what follows, we utilize the developed method to find the best alternative(s). We now discuss two different cases.

Case 1: Assume that the information about the criterion weights is completely unknown; in this case, we use the following steps to get the most desirable alternative(s).

Step 1. Considering that the information about the criterion weights is completely unknown, we utilize Eq. (19) to get the optimal weight vector of attributes:

$$w=\left(0.2184,0.2021,0.3105,0.2689\right)^T$$

Step 2. Utilize Eqs. (20), (21), (22), and (23) to determine the single valued neutrosophic PIS A^+ and the single valued neutrosophic NIS A^-, respectively:

$$A^+=\left\{\langle 0.9,0.0,0.1\rangle,\langle 0.5,0.0,0.2\rangle,\langle 0.6,0.1,0.1\rangle,\langle 0.7,0.2,0.1\rangle\right\}$$

$$A^-=\left\{\langle 0.5,0.2,0.3\rangle,\langle 0.3,0.3,0.4\rangle,\langle 0.2,0.4,0.5\rangle,\langle 0.2,0.4,0.4\rangle\right\}$$

Step 3: Utilize Eqs. (24) and (25) to calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the single valued neutrosophic PIS A^+ and the single valued neutrosophic NIS A^-, respectively:

$d_1^+=0.1510$, $d_1^-=0.1951$, $d_2^+=0.1778$, $d_2^-=0.1931$, $d_3^+=0.1895$, $d_3^-=0.1607$, $d_4^+=0.1510$, $d_4^-=0.2123$, $d_5^+=0.1523$, $d_5^-=0.2242$.

Step 4: Utilize Eq. (26) to calculate the relative closeness coefficient C_i of each alternative A_i to the single valued neutrosophic PIS A^+:

$C_1=0.5638$, $C_2=0.5205$, $C_3=0.4589$, $C_4=0.5845$, $C_5=0.5954$.

Step 5: Rank the alternatives A_i ($i=1,2,3,4,5$) according to the relative closeness coefficient C_i ($i=1,2,3,4,5$). Clearly, $A_5 \succ A_4 \succ A_1 \succ A_2 \succ A_3$, and thus the best alternative is A_5.

Case 2: The information about the criterion weights is partly known and the known weight information is given as follows:

$$\Delta=\left\{\begin{array}{l}0.15\le w_1\le 0.25,\ 0.25\le w_2\le 0.3,\ 0.3\le w_3\le 0.4,\\ 0.35\le w_4\le 0.5,\ w_j\ge 0,\ j=1,2,3,4,\ \sum_{j=1}^{4}w_j=1\end{array}\right\}$$

Step 1: Utilize the model (M-2) to construct the single-objective model as follows:

$$\begin{cases}\max D(w)=2.9496w_1+2.7295w_2+4.1923w_3+3.6315w_4\\ \text{s.t.}\quad w\in\Delta\end{cases}$$

By solving this model, we get the optimal weight vector of criteria $w=\left(0.15,0.25,0.3,0.35\right)^T$.

Step 2. Utilize Eqs. (20), (21), (22), and (23) to determine the single valued neutrosophic PIS A^+ and the single valued neutrosophic NIS A^-, respectively:

$$A^+=\left\{\langle 0.9,0.0,0.1\rangle,\langle 0.5,0.0,0.2\rangle,\langle 0.6,0.1,0.1\rangle,\langle 0.7,0.2,0.1\rangle\right\}$$

$$A^-=\left\{\langle 0.5,0.2,0.3\rangle,\langle 0.3,0.3,0.4\rangle,\langle 0.2,0.4,0.5\rangle,\langle 0.2,0.4,0.4\rangle\right\}$$

Step 3: Utilize Eqs. (24) and (25) to calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the single valued neutrosophic PIS A^+ and the single valued neutrosophic NIS A^-, respectively:

$d_1^+=0.1368$, $d_1^-=0.2260$, $d_2^+=0.1852$, $d_2^-=0.2055$, $d_3^+=0.2098$, $d_3^-=0.1581$, $d_4^+=0.1780$, $d_4^-=0.2086$, $d_5^+=0.1619$, $d_5^-=0.2358$.

Step 4: Utilize Eq. (26) to calculate the relative closeness coefficient C_i of each alternative A_i to the single valued neutrosophic PIS A^+:

$C_1=0.6230$, $C_2=0.5260$, $C_3=0.4297$, $C_4=0.5396$, $C_5=0.5928$.

Step 5: Rank the alternatives A_i ($i=1,2,3,4,5$) according to the relative closeness coefficient C_i ($i=1,2,3,4,5$). Clearly, $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$, and thus the best alternative is A_1.

5.2. The analysis process under interval neutrosophic environments

Example 5.2. Let's revisit Example 5.1. Suppose that the five possible alternatives are to be evaluated under the above four criteria by the form of INVs, as shown in the following interval neutrosophic decision matrix $\tilde{A}$ (see Table 4).

Table 4: Interval neutrosophic decision matrix $\tilde{A}$.

4	c_1	c_2	c_3	c_4
A_1	<[0.7, 0.9], [0.1, 0.2], [0.5, 0.6]>	<[0.3, 0.4], [0.2, 0.3], [0.4, 0.5]>	<[0.3, 0.5], [0.2, 0.3], [0.6, 0.7]>	<[0.7, 0.9], [0.3, 0.4], [0.5, 0.6]>
A_2	<[0.5, 0.6], [0.2,0.3], [0.2, 0.4]>	<[0.2, 0.3], [0.1, 0.3], [0.7, 0.8]>	< [0.5, 0.7], [0.2, 0.3], [0.7, 0.8]>	<[0.8, 0.9], [0.1, 0.2], [0.5, 0.7]>
A_3	< [0.4, 0.5], [0.2, 0.3], [0.4, 0.6]>	<[0.3, 0.4], [0.1, 0.2], [0.7, 0.9]>	<[0.3, 0.5], [0.2, 0.3], [0.6, 0.7]>	<[0.7, 0.9], [0.2, 0.3], [0.5, 0.6]>
A_4	< [0.2,0.3], [0.1, 0.2], [0.4, 0.5]>	<[0.4, 0.5], [0.3, 0.5], [0.2, 0.3]>	<[0.8, 0.9], [0.1, 0.3], [0.3, 0.4]>	< [0.2,0.3], [0.3, 0.5], [0.6, 0.8]>
A_5	<[0.7, 0.8], [0.3,0.4], [0.6,0.7] >	< [0.6, 0.7], [0.1, 0.2], [0.7, 0.9]>	<[0.2, 0.3], [0.1, 0.2], [0.7, 0.8]>	<[0.6, 0.7], [0.3, 0.4], [0.4, 0.5]>

In what follows, we proceed to utilize the developed method to find the most optimal alternative(s), which consists of the following two cases:

Case 1: Assume that the information about the criterion weights is completely unknown; in this case, we use the following steps to get the most desirable alternative(s).

Step 1. Considering that the information about the criterion weights is completely unknown, we utilize Eq. (36) to get the optimal weight vector of attributes:

$$w = (0.2490, 0.2774, 0.2380, 0.2356)^T$$

Step 2. Utilize Eqs. (37), (38), (39), and (40) to determine the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$$\tilde{A}^+ = \begin{Bmatrix} \langle 0.7, 0.9, 0.1, 0.2, 0.2, 0.4 \rangle, \langle 0.6, 0.7, 0.1, 0.2, 0.2, 0.3 \rangle, \\ \langle 0.8, 0.9, 0.1, 0.2, 0.3, 0.4 \rangle, \langle 0.8, 0.9, 0.1, 0.2, 0.4, 0.5 \rangle \end{Bmatrix}$$

$$\tilde{A}^- = \begin{Bmatrix} \{0.2, 0.3, 0.3, 0.4, 0.6, 0.7\}, \{0.2, 0.3, 0.3, 0.5, 0.7, 0.9\}, \\ \{0.2, 0.3, 0.2, 0.3, 0.7, 0.8\}, \{0.2, 0.3, 0.3, 0.5, 0.6, 0.8\} \end{Bmatrix}$$

Step 3: Utilize Eqs. (41) and (42) to calculate the separation measures $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative A_i from the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$\tilde{d}_1^+ = 0.2044$, $\tilde{d}_1^- = 0.2541$, $\tilde{d}_2^+ = 0.2307$, $\tilde{d}_2^- = 0.2405$, $\tilde{d}_3^+ = 0.2582$, $\tilde{d}_3^- = 0.1900$, $\tilde{d}_4^+ = 0.2394$, $\tilde{d}_4^- = 0.2343$, $\tilde{d}_5^+ = 0.2853$, $\tilde{d}_5^- = 0.2268$.

Step 4: Utilize Eq. (43) to calculate the relative closeness coefficient $\tilde{C}_i$ of each alternative A_i to the interval neutrosophic PIS $\tilde{A}^+$:

$\tilde{C}_1 = 0.5543$, $\tilde{C}_2 = 0.5104$, $\tilde{C}_3 = 0.4239$, $\tilde{C}_4 = 0.4946$, $\tilde{C}_5 = 0.4429$.

Step 5: Rank the alternatives A_i ($i = 1, 2, 3, 4, 5$) according to the relative closeness coefficient $\tilde{C}_i$ ($i = 1, 2, 3, 4, 5$). Clearly, $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$, and thus the best alternative is A_1.

Case 2: The information about the attribute weights is partly known and the known weight information is given as follows:

$$\Delta = \begin{Bmatrix} 0.25 \le w_1 \le 0.3,\ 0.25 \le w_2 \le 0.35,\ 0.35 \le w_3 \le 0.4, \\ 0.4 \le w_4 \le 0.45,\ w_j \ge 0,\ j = 1, 2, 3, 4,\ \sum_{j=1}^{4} w_j = 1 \end{Bmatrix}$$

Step 1: Utilize the model (M-4) to construct the single-objective model as follows:

$$\begin{cases} \max D(w) = 4.2748 w_1 + 4.7627 w_2 + 4.0859 w_3 + 4.0438 w_4 \\ \text{s.t.} \quad w \in \Delta \end{cases}$$

By solving this model, we get the optimal weight vector of criteria $w = (0.25, 0.25, 0.35, 0.4)^T$.

Step 2. Utilize Eqs. (37), (38), (39), and (40) to determine the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$$\tilde{A}^+ = \begin{Bmatrix} \langle 0.7, 0.9, 0.1, 0.2, 0.2, 0.4 \rangle, \langle 0.6, 0.7, 0.1, 0.2, 0.2, 0.3 \rangle, \\ \langle 0.8, 0.9, 0.1, 0.2, 0.3, 0.4 \rangle, \langle 0.8, 0.9, 0.1, 0.2, 0.4, 0.5 \rangle \end{Bmatrix}$$

$$\tilde{A}^- = \begin{Bmatrix} \{0.2, 0.3, 0.3, 0.4, 0.6, 0.7\}, \{0.2, 0.3, 0.3, 0.5, 0.7, 0.9\}, \\ \{0.2, 0.3, 0.2, 0.3, 0.7, 0.8\}, \{0.2, 0.3, 0.3, 0.5, 0.6, 0.8\} \end{Bmatrix}$$

Step 3: Utilize Eqs. (41) and (42) to calculate the separation measures $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative A_i from the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$\tilde{d}_1^+ = 0.2566$, $\tilde{d}_1^- = 0.3152$, $\tilde{d}_2^+ = 0.2670$, $\tilde{d}_2^- = 0.3228$, $\tilde{d}_3^+ = 0.2992$, $\tilde{d}_3^- = 0.2545$, $\tilde{d}_4^+ = 0.3056$, $\tilde{d}_4^- = 0.2720$, $\tilde{d}_5^+ = 0.3503$, $\tilde{d}_5^- = 0.2716$.

Step 4: Utilize Eq. (43) to calculate the relative closeness coefficient $\tilde{C}_i$ of each alternative A_i to the interval neutrosophic PIS $\tilde{A}^+$:

$\tilde{C}_1 = 0.5513$, $\tilde{C}_2 = 0.5474$, $\tilde{C}_3 = 0.4596$, $\tilde{C}_4 = 0.4709$, $\tilde{C}_5 = 0.4368$.

Step 5: Rank the alternatives A_i ($i = 1, 2, 3, 4, 5$) according to the relative closeness coefficient $\tilde{C}_i$ ($i = 1, 2, 3, 4, 5$). Clearly, $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$, and thus the best alternative is A_1.

5.3. Comparison analysis with the existing single-valued neutrosophic or interval neutrosophic multi-criteria decision making methods

Recently, some methods [20,21,22,23] have been developed for solving the MCDM problems with single-valued neutrosophic or interval neutrosophic information. In this section, we will perform a comparison analysis between our new methods and these existing methods, and then highlight the advantages of the new methods over these existing methods.

It is noted that these existing methods have some inherent drawbacks, which are shown as follows:

(1) The existing methods [20,21,22,23] need the decision maker to provide the weights of criteria in advance, which is subjective and sometime cannot yield the persuasive results. In contrast, our methods utilize the maximizing deviation method to determine the weight values of criteria, which is more objective and reasonable than the other existing methods [20,21,22,23].

(2) In Ref. [23], Ye proposed a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator, and then utilized two aggregation operators to develop a method for multi-criteria decision making problems under simplified neutrosophic environments. However, it is noted that these

operators and method need to perform an aggregation on the input simplified neutrosophic arguments, which may increase the computational complexity and therefore lead to the loss of information. In contrast, our methods do not need to perform such an aggregation but directly deal with the input simplified neutrosophic arguments, thereby can retain the original decision information as much as possible.

(3) In Ref. [22], Ye defined the Hamming and Euclidean distances between interval neutrosophic sets (INSs) and proposed the similarity measures between INSs on the basis of the relationship between similarity measures and distances. Moreover, Ye [22] utilized the similarity measures between each alternative and the ideal alternative to rank the alternatives and to determine the best one. In order to clearly demonstrate the comparison results, we use the method proposed in [22] to revisit Example 5.2, which is shown as follows:

First, we identify an ideal alternative by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives as:

$$\tilde{A}^{+}=\begin{Bmatrix}\langle 0.7,0.9,0.1,0.2,0.2,0.4\rangle,\langle 0.6,0.7,0.1,0.2,0.2,0.3\rangle,\\ \langle 0.8,0.9,0.1,0.2,0.3,0.4\rangle,\langle 0.8,0.9,0.1,0.2,0.4,0.5\rangle\end{Bmatrix}$$

In order to be consistent with Example 5.2, the same distance measure and the same weights for criteria are adopted here. Then, we apply Eq. (8) to calculate the similarity measure between an alternative A_i ($i=1,2,3,4,5$) and the ideal alternative $\tilde{A}^{+}$ as follows:

$$s\left(\tilde{A}^{+},A_1\right)=1-d\left(\tilde{A}^{+},A_1\right)=1-0.2044=0.7956$$

$$s\left(\tilde{A}^{+},A_2\right)=1-d\left(\tilde{A}^{+},A_2\right)=1-0.2307=0.7693$$

$$s\left(\tilde{A}^{+},A_3\right)=1-d\left(\tilde{A}^{+},A_3\right)=1-0.2582=0.7418$$

$$s\left(\tilde{A}^{+},A_4\right)=1-d\left(\tilde{A}^{+},A_4\right)=1-0.2394=0.7606$$

$$s\left(\tilde{A}^{+},A_5\right)=1-d\left(\tilde{A}^{+},A_5\right)=1-0.2853=0.7147$$

Finally, through the similarity measure $s\left(\tilde{A}^{+},A_i\right)$ ($i=1,2,3,4,5$) between each alternative and the ideal alternative, the ranking order of all alternatives can be determined as: $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$. Thus, the optimal alternative is A_1.

It is easy to see that the optimal alternative obtained by the Ye' method [22] is the same as our method, which shows the effectiveness, preciseness, and reasonableness of our method. However, it is noticed that the ranking order of the alternatives obtained by our method is $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$, which is different from the ranking order obtained by the Ye' method [22]. Concretely, the ranking order between A_3 and A_5 obtained by two methods are just converse, i.e., $A_5 \succ A_3$ for our method while $A_3 \succ A_5$ for the Ye' method [22]. The main reason is that the Ye' method determines a solution which is the closest to the positive ideal solution (PIS), while our method determines a solution with the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). Therefore, the Ye' method is suitable for those situations in which the decision maker wants to have maximum profit and the risk of the decisions is less important for him, while our method is suitable for cautious (risk avoider) decision maker, because the decision maker might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible.

Conclusions

Considering that some multi-criteria decision making problems contain uncertain, imprecise, incomplete, and inconsistent information, and the information about criterion weights is usually incomplete, this paper has developed a novel method for single-valued neutrosophic or interval neutrosophic multi-criteria decision making with incomplete weight information. First, motivated by the idea that a larger weight should be assigned to the criterion with a larger deviation value among alternatives, a maximizing deviation method has been presented to determine the optimal criterion weights under single-valued neutrosophic or interval neutrosophic environments, which can eliminate the influence of subjectivity of criterion weights provided by the decision maker in advance. Then, a single-valued neutrosophic or interval neutrosophic TOPSIS is proposed to calculate the relative closeness coefficient of each alternative to the single-valued neutrosophic or interval neutrosophic positive ideal solution, based on which the considered alternatives are ranked and then the most desirable one is selected. The prominent advantages of the developed methods are that they can not only relieve the influence of subjectivity of the decision maker but also remain the original decision information sufficiently. Finally, the effectiveness and practicality of the developed methods have been illustrated with a best global supplier selection example, and the advantages of the developed methods have been demonstrated with a comparison with the other existing methods.

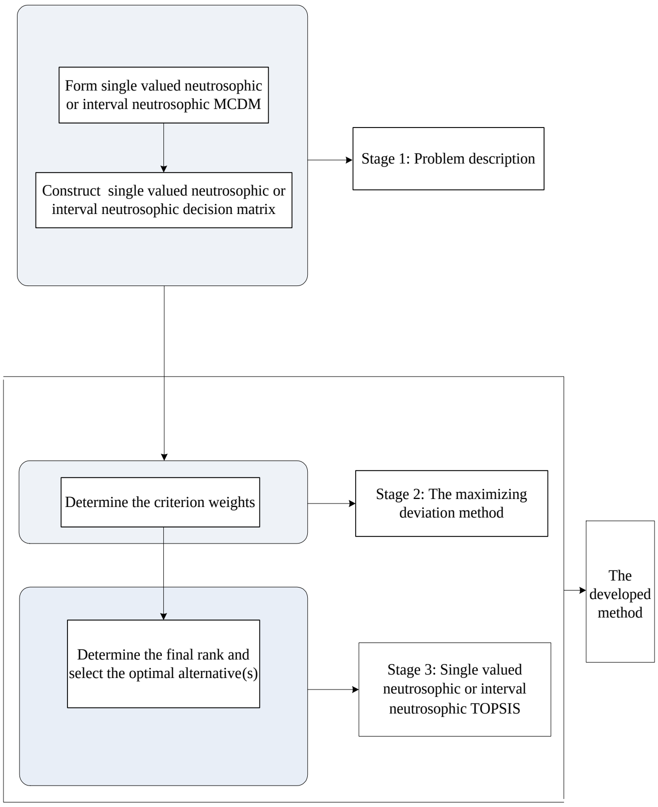

Fig. 1: The flowchart of the developed methods

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New Neutrosophic Crisp Topological Concepts

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Abstract
In this paper, we introduce the concept of ""neutrosophic crisp neighborhoods system for the neutrosophic crisp point ". Added to, we introduce and study the concept of neutrosophic crisp local function, and construct a new type of neutrosophic crisp topological space via neutrosophic crisp ideals. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Crisp Point, Neutrosophic Crisp Ideal; Neutrosophic Crisp Topology; Neutrosophic Crisp Neighborhoods

1 INTRODUCTION

The idea of "neutrosophic set" was first given by Smarandache [14, 15]. In 2012 neutrosophic operations have been investigated by Salama et al. [4-13]. The fuzzy set was introduced by Zadeh [17]. The intuitionstic fuzzy set was introduced by Atanassov [1, 2, 3]. Salama et al. [11]defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [16]. Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts. Here we shall present the neutrosophic crisp version of these concepts. In this paper, we introduce the concept of "neutrosophic crisp points "and "neutrosophic crisp neigbourhoods systems". Added to we define the new concept of neutrosophic crisp local function, and construct new type of neutrosophic crisp topological space via neutrosophic crisp ideals.

2 TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular the work of Smarandache in [14, 15], and Salama et al. [4 -13].

2.1 Definition [13]
Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) A is an object having the form $A=\langle A_1, A_2, A_3\rangle$ where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$.

2.2 Definition [13].
Let X be a nonempty set and $p \in X$ Then the neutrosophic crisp point p_N defined by $p_N = \langle \{p\}, \phi, \{p\}^c \rangle$ is called a neutrosophic crisp point (NCP for short) in X, where NCP is a triple ({only one element in X}, the empty set,{the complement of the same element in X}).

2.3 Definition [13]
Let X be a nonempty set, and $p \in X$ a fixed element in X. Then the neutrosophic crisp set $p_{N_N} = \langle \phi, \{p\}, \{p\}^c \rangle$ is called "vanishing neutrosophic crisp point" (VNCP for short) in X, where VNCP is a triple (the empty set,{only one element in X},{the complement of the same element in X}).

2.4 Definition [13]
Let $p_N = \langle \{p\}, \phi, \{p\}^c \rangle$ be a NCP in X and $A=\langle A_1, A_2, A_3\rangle$ a neutrosophic crisp set in X.

(a) p_N is said to be contained in A ($p_N \in A$ for short) iff $p \in A_1$.

(b) Let p_{N_N} be a VNCP in X, and $A=\langle A_1, A_2, A_3\rangle$ a neutrosophic crisp set in X. Then p_{N_N} is said to be

(b) Let p_{N_N} be a VNCP in X, and $A=\langle A_1, A_2, A_3\rangle$ a neutrosophic crisp set in X. Then p_{N_N} is said to be contained in A ($p_{N_N} \in A$ for short) iff $p \notin A_3$.

2.5 Definition [13].

Let X be non-empty set, and L a non–empty family of NCSs. We call L a neutrosophic crisp ideal (NCL for short) on X if

i. $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

ii. $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic crisp ideal L is called a σ - neutrosophic crisp ideal if $\{M_j\}_{j\in N} \le L$, implies $\underset{j\in J}{\cup} M_{j\in L}$ (countable additivity).

The smallest and largest neutrosophic crisp ideals on a non-empty set X are $\{\phi_N\}$ and the NSs on X. Also, NCL_f, NCL_c are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NCS : B \subseteq A\}$ is an NCL on X. This is called the principal NCL of all NCSs, denoted by NCL$\langle A\rangle$.

2.1 Proposition [13]

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic crisp ideals on a set X. Then $\bigcap_{j\in J} L_j$ and $\bigcup_{j\in J} L_j$ are neutrosophic crisp ideals on X, where

$$\underset{j\in J}{\cap} L_j = \left\langle \underset{j\in J}{\cap} A_{j1}, \underset{j\in J}{\cap} A_{j2}, \underset{j\in J}{\cup} A_{j3} \right\rangle \text{ or}$$

$$\underset{j\in J}{\cap} L_j = \left\langle \underset{j\in J}{\cap} A_{j1}, \underset{j\in J}{\cup} A_{j2}, \underset{j\in J}{\cup} A_{j3} \right\rangle \text{ and}$$

$$\underset{j\in J}{\cup} L_j = \left\langle \underset{j\in J}{\cup} A_{j1}, \underset{j\in J}{\cup} A_{j2}, \underset{j\in J}{\cap} A_{j3} \right\rangle \text{ or}$$

$$\underset{j\in J}{\cup} L_j = \left\langle \underset{j\in J}{\cup} A_{j1}, \underset{j\in J}{\cap} A_{j2}, \underset{j\in J}{\cap} A_{j3} \right\rangle.$$

2,2 Remark [13]

The neutrosophic crisp ideal defined by the single neutrosophic set ϕ_N is the smallest element of the ordered set of all neutrosophic crisp ideals on X.

2.1 Proposition [13]

A neutrosophic crisp set $A=\langle A_1, A_2, A_3\rangle$ in the neutrosophic crisp ideal L on X is a base of L iff every member of L is contained in A.

3. Neutrosophic Crisp Neigborhoods System

3.1 Definition.

Let $A=\langle A_1, A_2, A_3\rangle$, be a neutrosophic crisp set on a set X, then $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, $p_1 \ne p_2 \ne p_3 \in X$ is called a neutrosophic crisp point

An NCP $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, is said to be belong to a neutrosophic crisp set $A=\langle A_1, A_2, A_3\rangle$, of X, denoted by $p \in A$, if may be defined by two types

i) **Type 1:** $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$

ii) **Type 2:** $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$

3.1 Theorem

Let $A=\langle\langle A_1, A_2, A_3\rangle\rangle$, and $B=\langle\langle B_1, B_2, B_3\rangle\rangle$, be neutrosophic crisp subsets of X. Then $A \subseteq B$ iff $p \in A$ implies $p \in B$ for any neutrosophic crisp point p in X.

Proof

Let $A \subseteq B$ and $p \in A$. Then two types

Type 1: $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$ or

Type 2: $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$. Thus $p \in B$.

Conversely, take any x in X. Let $p_1 \in A_1$ and $p_2 \in A_2$ and $p_3 \in A_3$. Then p is a neutrosophic crisp point in X. and $p \in A$. By the hypothesis $p \in B$. Thus $p_1 \in B_1$, or Type 1: $\{p_1\} \subseteq B_1, \{p_2\} \subseteq B_2$ and $\{p_3\} \subseteq B_3$ or

Type 2: $\{p_1\} \subseteq B_1, \{p_2\} \supseteq B_2$ and $\{p_3\} \subseteq B_3$. Hence. $A \subseteq B$.

3.2 Theorem

Let $A=\langle A_1, A_2, A_3\rangle$, be a neutrosophic crisp subset of X. Then $A = \cup\{p : p \in A\}$.

Proof

Since $\cup\{p : p \in A\}$. may be two types

Type 1:
$\langle \cup\{p_1 : p_1 \in A_1\}, \cup\{p_2 : p_2 \in A_2\}, \cap\{p_3 : p_3 \in A_3\}\rangle$ or
Type 2:
$\langle \cup\{p_1 : p_1 \in A_1\}, \cap\{p_2 : p_2 \in A_2\}, \cap\{p_3 : p_3 \in A_3\}\rangle$. Hence $A = \langle A_1, A_2, A_3\rangle$

3.1 Proposition

Let $\{A_j : j \in J\}$ is a family of NCSs in X. Then
(a_1) $p = \langle\{p_1\},\{p_2\},\{p_3\}\rangle \in \bigcap_{j\in J} A_j$ iff $p \in A_j$ for each $j \in J$.
(a_2) $p \in \bigcup_{j\in J} A_j$ iff $\exists j \in J$ such that $p \in A_j$.
.

3.2 Proposition

Let $A = \langle A_1, A_2, A_3\rangle$ and $B = \langle B_1, B_2, B_3\rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p we have $p \in A \Leftrightarrow p \in B$ and for each p we have $p \in A \Rightarrow p \in B$.

b) $A = B$ iff for each p we have $p \in A \Rightarrow p \in B$ and for each p we have $p \in A \Leftrightarrow p \in B$.

3.3 Proposition

Let $A = \langle A_1, A_2, A_3\rangle$ be a neutrosophic crisp set in X. Then
$A = \cup < \{p_1 : p_1 \in A_1\}, \{p_2 : p_2 \in A_2\}, \{p_3 : p_3 \in A_3\}$.

3.2 Definition

Let $f : X \to Y$ be a function and p be a nutrosophic crisp point in X. Then the image of p under f, denoted by $f(p)$, is defined by
$f(p) = \langle\{q_1\},\{q_2\},\{q_3\}\rangle$, where $q_1 = f(p_1), q_2 = f(p_2)$ .and $q_3 = f(p_3)$.

It is easy to see that $f(p)$ is indeed a NCP in Y, namely $f(p) = q$, where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under the function f.

4 4. Neutrosophic Crisp Local functions

4.1 Definition
Let p be a neutrosophic crisp point of a neutrosophic crisp topological space (X,τ). A neutrosophic crisp neighbourhood (NCNBD for short) of a neutrosophic crisp point p if there is a neutrosophic crisp open set(NCOS for short) B in X such that $p \in B \subseteq A$.

4.1 Theorem
Let (X,τ) be a neutrosophic crisp topological space (NCTS for short) of X. Then the neutrosophic crisp set A of X is NCOS iff A is a NCNBD of p for every neutrosophic crisp set $p \in A$.

Proof
Let A be NCOS of X . Clearly A is a NCBD of any $p \in A$. Conversely, let $p \in A$. Since A is a NCBD of p, there is a NCOS B in X such that $p \in B \subseteq A$. So we have $A = \cup\{p : p \in A\} \subseteq \cup\{B : p \in A\} \subseteq A$ and hence $A = \cup\{B : p \in A\}$. Since each B is NCOS.

4.2 Definition
Let (X,τ) be a neutrosophic crisp topological spaces (NCTS for short) and L be neutrsophic crisp ideal (NCL, for short) on X. Let A be any NCS of X. Then the neutrosophic crisp local function $NCA^*(L,\tau)$ of A is the union of all neutrosophic crisp points(NCP, for short) $P = \langle\{p_1\},\{p_2\},\{p_3\}\rangle$, such that if $U \in N((p))$ and $NA^*(L,\tau) = \cup\{p \in X : A \wedge U \not\in L \text{ for every U nbd of N(P)}\}$, $NCA^*(L,\tau)$ is called a neutrosophic crisp local function of A with respect to τ and L which it will be denoted by $NCA^*(L,\tau)$, or simply $NCA^*(\mathrm{L})$.

4.1 Example
One may easily verify that.
If L=$\{\phi_N\}$, then $NCA^*(L,\tau) = NCcl(A)$, for any neutrosophic crisp set $A \in NCSs$ on X.
If $\mathrm{L} = \{\text{all NCSs on X}\}$ then $NCA^*(L,\tau) = \phi_N$, for any $A \in NCSs$ on X .

4.2 Theorem
Let (X,τ) be a NCTS and L_1, L_2 be two topological neutrosophic crisp ideals on X. Then for any neutrosophic crisp sets A, B of X. then the following statements are verified

i) $A \subseteq B \Rightarrow NCA^*(L,\tau) \subseteq NCB^*(L,\tau)$,
ii) $L_1 \subseteq L_2 \Rightarrow NCA^*(L_2,\tau) \subseteq NCA^*(L_1,\tau)$.
iii) $NCA^* = NCcl(A^*) \subseteq NCcl(A)$.

iv) $NCA^{**} \subseteq NCA^{*}$.

v) $NC(A \cup B)^{*} = NCA^{*} \cup NCB^{*}$.,

vi) $NC(A \cap B)^{*}(L) \subseteq NCA^{*}(L) \cap NCB^{*}(L)$.

vii) $\ell \in L \Rightarrow NC(A \cup \ell)^{*} = NCA^{*}$.

viii) $NCA^{*}(L,\tau)$ is neutrosophic crisp closed set .

Proof

i) Since $A \subseteq B$, let $p = \langle \{p_1\},\{p_2\},\{p_3\} \rangle \in NCA^{*}(L_1)$ then $A \cap U \notin L$ for every $U \in N(p)$. By hypothesis we get $B \cap U \notin L$, then $p = \langle \{p_1\},\{p_2\},\{p_3\} \rangle \in NB^{*}(L_1)$.

ii) Clearly. $L_1 \subseteq L_2$ implies $NCA^{*}(L_2,\tau) \subseteq NCA^{*}(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for GIFP $p = \langle \{p_1\},\{p_2\},\{p_3\} \rangle \in NCA^{*}(L_1)$ but P may not be contained in $NCA^{*}(L_2)$.

iii) Since $\{\phi_N\} \subseteq L$ for any NCL on X, therefore by (ii) and Example 3.1, $NCA^{*}(L) \subseteq NCA^{*}(\{O_N\}) = NCcl(A)$ for any NCS A on X. Suppose $P_1 = \langle \{p_1\},\{p_2\},\{p_3\} \rangle \in NCcl(A^{*}(L_1))$. So for every $U \in NC(P_1)$, $NC(A^{*}) \cap U \neq \phi_N$, there exists $P_2 = \langle \{q_1\},\{q_2\},\{q_3\} \rangle \in NCA^{*}(L_1) \cap U$ such that for every V NCNBD of $P_2 \in N(P_2)$, $A \cap U \notin L$. Since $U \wedge V \in N(p_2)$ then $A \cap (U \cap V) \notin L$ which leads to $A \wedge U \notin L$, for every $U \in N(P_1)$ therefore $P_1 \in NC(A^{*}(L))$ and so $NCcl(NA^{*}) \subseteq NCA^{*}$ While, the other inclusion follows directly. Hence $NCA^{*} = NCcl(NCA^{*})$ .But the inequality $NCA^{*} \subseteq Ncl(NCA^{*})$.

iv) The inclusion $NCA^{*} \cup NCB^{*} \subseteq NC(A \cup B)^{*}$ follows directly by (i). To show the other implication, let $p \in NC(A \cup B)^{*}$ then for every $U \in NC(p)$, $(A \cup B) \cap U \notin L$, *i.e*, $(A \cap U) \cup (B \cap U) \notin L$. then, we have two cases $A \cap U \notin L$ and $B \cap U \in L$ or the converse, this means that exist $U_1, U_2 \in N(P)$ such that $A \cap U_1 \notin L$, $B \cap U_1 \notin L$, $A \cap U_2 \notin L$ and $B \cap U_2 \notin L$. Then $A \cap (U_1 \cap U_2) \in L$ and $B \cap (U_1 \cap U_2) \in L$ this gives $(A \cup B) \cap (U_1 \cap U_2) \in L$, $U_1 \cap U_2 \in N(C(P))$ which contradicts the hypothesis. Hence the equality holds in various cases.

vi) By (iii), we have

$$NCA^{*^{*}} = NCcl(NCA^{*})^{*} \subseteq NCcl(NCA^{*}) = NCA^{*}$$

Let (X,τ) be a NCTS and L be NCL on X . Let us define the neutrosophic crisp closure operator $NCcl^{*}(A) = A \cup NC(A^{*})$ for any NCS A of X. Clearly, let $NCcl^{*}(A)$ is a neutrosophic crisp operator. Let $NC\tau^{*}(L)$ be NCT generated by $NCcl^{*}$ .i.e $NC\tau^{*}(L) = \{A : NCcl^{*}(A^{c}) = A^{c}\}$ now $L = \{\phi_N\} \Rightarrow NCcl^{*}(A) = A \cup NCA^{*} = A \cup NCcl(A)$ for every neutrosophic crisp set A. So, $N\tau^{*}(\{\phi_N\}) = \tau$. Again $L = \{all \text{ NCSs on X}\} \Longrightarrow NCcl^{*}(A) = A$, because $NCA^{*} = \phi_N$, for every neutrosophic crisp set A so $NC\tau^{*}(L)$ is the neutrosophic crisp discrete topology on X. So we can conclude by Theorem 4.1.(ii). $NC\tau^{*}(\{\phi_N\}) = NC\tau^{*}(L)$ i.e. $NC\tau \subseteq NC\tau^{*}$, for any neutrosophic ideal L_1 on X. In particular, we have for two topological neutrosophic ideals L_1, and L_2 on X, $L_1 \subseteq L_2 \Rightarrow NC\tau^{*}(L_1) \subseteq NC\tau^{*}(L_2)$.

4.3 Theorem

Let τ_1, τ_2 be two neutrosophic crisp topologies on X. Then for any topological neutrosophic crisp ideal L on X, $\tau_1 \leq \tau_2$ implies $NA^{*}(L,\tau_2) \subseteq NA^{*}(L,\tau_1)$, for every A $\in L$ then $NC\tau^{*}{}_1 \subseteq NC\tau^{*}{}_2$

Proof

Clear.

A basis $NC\beta(L,\tau)$ for $NC\tau^{*}(L)$ can be described as follows:

$NC\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$. Then we have the following theorem

4.4 Theorem

$NC\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Forms a basis for the generated NT of the NCT (X,τ) with topological neutrosophic crisp ideal L on X.

Proof

Straight forward.

The relationship between $NC\tau$ and $NC\tau^{*}(L)$ established throughout the following result which have an immediately proof .

4.5 Theorem

Let τ_1, τ_2 be two neutrosophic crisp topologies on X. Then for any topological neutrosophic ideal L on X, $\tau_1 \subseteq \tau_2$ implies $NC\tau^{*}{}_1 \subseteq NC\tau^{*}{}_2$.

4.6 Theorem

Let (X,τ) be a NCTS and L_1, L_2 be two neutrosophic crisp ideals on X . Then for any neutrosophic crisp set A in X, we have

i) $NCA^*(L_1 \cup L_2,\tau) = NCA^*\left(L_1, NC\tau^*(L_1)\right) \wedge NCA^*\left(L_2, NC\tau^*(L_2)\right)$ ii) $NC\tau^*(L_1 \cup L_2) = \left(NC\tau^*(L_1)\right)^*(L_2) \wedge \left(NC\tau^*(L_2)\right)^*(L_1)$

Proof

Let $p \notin (L_1 \cup L_2,\tau)$, this means that there exists $U \in NC(P)$ such that $A \cap U_p \in (L_1 \cup L_2)$ i.e. There exists $\ell_1 \in L_1$ and $\ell_2 \in L_2$ such that $A \cap U \in (\ell_1 \vee \ell_2)$ because of the heredity of L_1, and assuming $\ell_1 \wedge \ell_2 = O_N$ .Thus we have $(A \cap U) - \ell_1 = \ell_2$ and $(A \cap U_p) - \ell_2 = \ell_1$ therefore $(U - \ell_1) \cap A = \ell_2 \in L_2$ and $(U - \ell_2) \cap A = \ell_1 \in L_1$. Hence $p \notin NCA^*\left(L_2, NC\tau^*(L_1)\right)$ or $P \notin NCA^*\left(L_1, NC\tau^*(L_2)\right)$ because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $NCA^*(L_1 \cup L_2,\tau) \geq NCA^*\left(L_1, NC\tau^*(L_1)\right) \cap NCA^*\left(L_2, NC\tau^*(L_2)\right)$. To show the second inclusion, let us assume $P \notin NCA^*\left(L_1, NC\tau^*(L_2)\right)$. This implies that there exist $U \in N(P)$ and $\ell_2 \in L_2$ such that $(U_p - \ell_2) \cap A \in L_1$. By the heredity of L_2, if we assume that $\ell_2 \subseteq A$ and define $\ell_1 = (U - \ell_2) \cap A$. Then we have $A \cap U \in (\ell_1 \cup \ell_2) \in L_1 \cup L_2$. Thus, $NCA^*(L_1 \cup L_2,\tau) \subseteq NCA^*\left(L_1, NC\tau^*(L_1)\right) \cap NCA^*\left(L_2, NC\tau^*(L_2)\right)$ and similarly, we can get $NCA^*(L_1 \cup L_2,\tau) \subseteq NCA^*\left(L_2, \tau^*(L_1)\right)$. This gives the other inclusion, which complete the proof.

4.1 Corollary

.Let (X,τ) be a NCTS with topological neutrosophic crisp ideal L on X. Then

i) $NCA^*(L,\tau) = NCA^*(L,\tau^*)$ and $NC\tau^*(L) = NC(NC\tau^*(L))^*(L)$

ii) $NC\tau^*(L_1 \cup L_2) = \left(NC\tau^*(L_1)\right) \cup \left(NC\tau^*(L_2)\right)$

Proof

Follows by applying the previous statement.

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Soft Neutrosophic Loops and Their Generalization

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic loop,soft neutosophic biloop, soft neutrosophic N -loop with the discuission of some of their characteristics. We also introduced a new type of soft neutrophic loop, the so called soft strong neutrosophic loop which is of pure neutrosophic character. This notion also found in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords: Neutrosophic loop, neutrosophic biloop, neutrosophic N-loop, soft set, soft neutrosophic loop,soft neutrosophic biloop, soft neutrosophic N-loop.

1 Introduction

Florentin Smarandache for the first time intorduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origion, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I , and the percentage of falsity in a subset F . Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interal valued fuzzy set. Neutrosophic logic is used to overcome the problems of imperciseness, indeterminate, and inconsistentness of date etc. The theoy of neutrosophy is so applicable to every field of agebra. W.B Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vectorspaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N -groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrsosophic N -semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N -loops, and so on. Mumtaz ali et.al. introduced nuetosophic LA -semigoups.

Molodtsov intorduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in $\left[2,9,10\right]$. Some properties and algebra may be found in $\left[1\right]$. Feng et.al. introduced soft semirings in $\left[5\right]$. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in $\left[6\right]$. Some other concepts together with fuzzy set and rough set were shown in $\left[7,8\right]$.

This paper is about to introduced soft nuetrosophic loop, soft neutrosphic biloop, and soft neutrosophic N -loop and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic loop, soft neutrosophic strong loop, and some of their properties are discuissed. In the next section, soft neutrosophic biloop are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N -loop and their coresponding strong theory have been constructed with some of their properties.

2 Fundamental Concepts

Neutrosophic Loop

Definition 1. A neutrosophic loop is generated by a loop L and I denoted by $\langle L \cup I \rangle$. A neutrosophic loop in general need not be a loop for $I^2 = I$ and I may not have an inverse but every element in a loop has an inverse.

Definition 2. Let $\langle L \cup I \rangle$ be a neutrosophic loop. A proper subset $\langle P \cup I \rangle$ of $\langle L \cup I \rangle$ is called the neutrosophic subloop, if $\langle P \cup I \rangle$ is itself a neutrosophic loop under the operations of $\langle L \cup I \rangle$.

Definition 3. Let $(\langle L \cup I \rangle, \circ)$ be a neutrosophic loop of finite order. A proper subset P of $\langle L \cup I \rangle$ is said to be Lagrange neutrosophic subloop, if P is a neutrosophic subloop under the operation $\circ$ and $o(P) / \mathrm{o}\langle L \cup I \rangle$.

Definition 4. If every neutrosophic subloop of $\langle L \cup I \rangle$ is Lagrange then we call $\langle L \cup I \rangle$ to be a Lagrange neutrosophic loop.

Definition 5. If $\langle L \cup I \rangle$ has no Lagrange neutrosophic subloop then we call $\langle L \cup I \rangle$ to be a Lagrange free neutrosophic loop.

Definition 6. If $\langle L \cup I \rangle$ has atleast one Lagrange neutrosophic subloop then we call $\langle L \cup I \rangle$ to be a weakly Lagrange neutrosophic loop.

Neutrosophic Biloops

Definition 6. Let $(\langle B \cup I \rangle, *_1, *_2)$ be a non-empty neutrosophic set with two binary operations $*_1, *_2$, $\langle B \cup I \rangle$ is a neutrosophic biloop if the following conditions are satisfied.

1. $\langle B \cup I \rangle = P_1 \cup P_2$ where P_1 and P_2 are proper subsets of $\langle B \cup I \rangle$.
2. $(P_1, *_1)$ is a neutrosophic loop.
3. $(P_2, *_2)$ is a group or a loop.

Definition 7. Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop. A proper subset P of $\langle B \cup I \rangle$ is said to be a neutrosophic subbiloop of $\langle B \cup I \rangle$ if $(P_1, *_1, *_2)$ is itself a neutrosophic biloop under the operations of $\langle B \cup I \rangle$.

Definition 8. Let $B = (B_1 \cup B_2, *_1, *_2)$ be a finite neutrosophic biloop. Let $P = (\mathrm{P}_1 \cup P_2, *_1, *_2)$ be a neutrosophic biloop. If $\mathrm{o(P)} / \mathrm{o(B)}$ then we call P, a Lagrange neutrosophic subbiloop of B.

Definition 9. If every neutrosophic subbiloop of B is Lagrange then we call B to be a Lagrange neutrosophic biloop.

Definition 10. If B has atleast one Lagrange neutrosophic subbiloop then we call B to be a weakly Lagrange neutrosophic biloop.

Definition 11. If B has no Lagrange neutrosophic subbiloops then we call B to be a Lagrange free neutrosophic biloop.

Neutrosophic N-loop

Definition 12. Let $S(B) = \{S(B_1) \cup S(B_2) \cup ... \cup S(B_n), *_1, *_2, ..., *_N\}$ be a non-empty neutrosophic set with N-binary operations. $S(B)$ is a neutrosophic N-loop if $S(B) = S(B_1) \cup S(B_2) \cup ... \cup S(B_n)$, $S(B_i)$ are proper subsets of $S(B)$ for $1 \le i \le N$ and some of $S(B_i)$ are neutrosophic loops and some of the $S(B_i)$ are groups.

Definition 13. Let $S(B) = \{S(B_1) \cup S(B_2) \cup ... \cup S(B_n), *_1, *_2, ..., *_N\}$ be a neutrosophic N-loop. A proper subset $(\mathrm{P}, *_1, *_2, ..., *_N)$ of $S(B)$ is said to be a neutrosophic sub N-loop of $S(B)$ if P itself is a neutrosophic N-loop under the operations of $S(B)$.

Definition 14. Let $(L = L_1 \cup L_2 \cup ... \cup L_N, *_1, *_2, ..., *_N)$ be a neutrosophic N-loop of finite order. Suppose P is a proper subset of L, which is a neutrosophic sub N-loop. If $o(P) / o(L)$ then we call P a Lagrange neutrosophic sub N-loop.

Definition 15. If every neutrosophic sub N-loop is Lagrange then we call L to be a Lagrange neutrosophic N-loop.

Definition 16. If L has atleast one Lagrange neutrosophic sub N -loop then we call L to be a weakly Lagrange neutrosophic N -loop.

Definition 17. If L has no Lagrange neutrosophic sub N -loop then we call L to be a Lagrange free neutrosophic N -loop.

Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 7. A pair (F,A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(\mathrm{a})$ may be considered as the set of a -elements of the soft set (F,A), or as the set of a -approximate elements of the soft set.

Example 1. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentence. Let

$$E = \begin{Bmatrix} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{Bmatrix}.$$

Let us consider a soft set (F,A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $A = \{a_1, a_2, a_3\}$ be the set of parameters where

a_1 stands for the parameter 'high rent,

a_2 stands for the parameter 'normal rent,

a_3 stands for the parameter 'in good condition.

Suppose that

$$F(a_1) = \{s_1, s_4\},$$
$$F(a_2) = \{s_2, s_5\},$$
$$F(a_3) = \{s_3\}.$$

The soft set (F,A) is an approximated family $\{F(a_i), i = 1,2,3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Then (F,A) is a soft set as a collection of approximations over U, where

$$F(a_1) = high\ rent = \{s_1, s_2\},$$
$$F(a_2) = normal\ rent = \{s_2, s_5\},$$
$$F(a_3) = in\ good\ condition = \{s_3\}.$$

Definition 8. For two soft sets (F,A) and (H,C) over U, (F,A) is called a soft subset of (H,C) if

1. $A \subseteq C$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F,A) \subset (H,\mathrm{C})$. Similarly (F,A) is called a soft superset of (H,C) if (H,C) is a soft subset of (F,A) which is denoted by $(F,A) \supset (H,\mathrm{C})$.

Definition 9. Two soft sets (F,A) and (H,C) over U are called soft equal if (F,A) is a soft subset of (H,C) and (H,C) is a soft subset of (F,A).

Definition 10. Let (F,A) and (K,C) be two soft sets over a common universe U such that $A \cap C \neq \phi$. Then their restricted intersection is denoted by $(F,A) \cap_R (K,\mathrm{C}) = (H,\mathrm{D})$ where (H,D) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in \mathrm{D} = A \cap C$.

Definition 11. The extended intersection of two soft sets (F,A) and (K,C) over a common universe U is the soft set (H,D), where $D = A \cup C$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - C, \\ \mathrm{K}(c) & \text{if } c \in C - A, \\ F(c) \cap \mathrm{K}(c) & \text{if } c \in A \cap C. \end{cases}$$

We write $(F,A) \cap_\varepsilon (K,C) = (H,\mathrm{D})$.

Definition 12. The restricted union of two soft sets (F,A) and (K,C) over a common universe U is the soft set (H,D), where $D = A \cup C$, and for all $c \in D$, $H(c)$ is defined as $H(c) = F(c) \cup \mathrm{K}(c)$ for all $c \in D$. We write it as

$(F,A) \cup_R (K,\mathrm{C}) = (H,\mathrm{D})$.

Definition 13. The extended union of two soft sets (F,A) and (K,C) over a common universe U is the soft set (H,D), where $D = A \cup C$, and for all $c \in D$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - C, \\ \mathrm{K}(c) & \text{if } c \in C - A, \\ F(c) \cup K(c) & \text{if } c \in A \cap C. \end{cases}$$

We write $(F,A) \cup_\varepsilon (K,\mathrm{C}) = (H,\mathrm{D})$.

3 Soft Neutrosophic Loop

Definition 14. Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L \cup I \rangle$. Then (F,A) is called soft neutrosophic loop if and only if $F(a)$ is neutrosophic subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Example 2. Let $\langle L \cup I \rangle = \langle L_7(4) \cup I \rangle$ be a neutrosophic loop where $L_7(4)$ is a loop. Then (F,A) is a soft neutrosophic loop over $\langle L \cup I \rangle$, where

$$F(a_1) = \{\langle e, eI, 2, 2I \rangle\}, F(a_2) = \{\langle e, 3 \rangle\},$$
$$\mathrm{F}(a_3) = \{\langle e, eI \rangle\}.$$

Theorem 1. Every soft neutrosophic loop over $\langle L \cup I \rangle$ contains a soft loop over L.

Proof. The proof is straightforward.

Theorem 2. Let (F,A) and (H,A) be two soft neutrosophic loops over $\langle L \cup I \rangle$. Then their intersection $(F,A) \cap (H,A)$ is again soft neutrosophic loop over $\langle L \cup I \rangle$.

Proof. The proof is staightforward.

Theorem 3. Let (F,A) and (H,C) be two soft neutrosophic loops over $\langle L \cup I \rangle$. If $A \cap C = \phi$, then $(F,A) \cup (H,\mathrm{C})$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Remark 1. The extended union of two soft neutrosophic loops (F,A) and (K,C) over $\langle L \cup I \rangle$ is not a soft neutrosophic loop over $\langle L \cup I \rangle$.

With the help of example we can easily check the above remark.

Proposition 1. The extended intersection of two soft neutrosophic loopps over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Remark 2. The restricted union of two soft neutrosophic loops (F,A) and (K,C) over $\langle L \cup I \rangle$ is not a soft neutrosophic loop over $\langle L \cup I \rangle$.

One can easily check it by the help of example.

Proposition 2. The restricted intersection of two soft neutrosophic loops over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Proposition 3. The AND operation of two soft neutrosophic loops over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Remark 3. The OR operation of two soft neutosophic loops over $\langle L \cup I \rangle$ may not be a soft nuetrosophic loop over $\langle L \cup I \rangle$.

Definition 15. Let $\langle L_n(m) \cup I \rangle = \{e, 1, 2, ..., n, eI, 1I, 2I, ..., nI\}$ be a new class of neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L_n(m) \cup I \rangle$. Then (F,A) is called soft new class neutrosophic loop if $F(a)$ is a neutrosophic subloop of $\langle L_n(m) \cup I \rangle$ for all $a \in A$.

Example 3. Let

$\langle L_5(3) \cup I \rangle = \{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a new class of neutrosophic loop. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ be a set of parameters. Then (F, A) is soft new class neutrosophic loop over $\langle L_5(3) \cup I \rangle$, where

$$F(a_1) = \{e, eI, 1, 1I\}, F(a_2) = \{e, eI, 2, 2I\},$$
$$F(a_3) = \{e, eI, 3, 3I\}, F(a_3) = \{e, eI, 4, 4I\},$$
$$F(a_5) = \{e, eI, 5, 5I\}.$$

Theorem 4. Every soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$ is a soft neutrosophic loop over $\langle L_n(m) \cup I \rangle$ but the converse is not true.

Proposition 4. Let (F, A) and (K, C) be two soft new class neutrosophic loops over $\langle L_n(m) \cup I \rangle$. Then

1) Their extended intersection $(F, A) \cap_E (K, C)$ is a soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
2) Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft new classes neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
3) Their AND operation $(F, A) \wedge (K, C)$ is a soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.

Remark 4. Let (F, A) and (K, C) be two soft new class neutrosophic loops over $\langle L_n(m) \cup I \rangle$. Then

1) Their extended union $(F, A) \cup_E (K, C)$ is not a soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
2) Their restricted union $(F, A) \cup_R (K, C)$ is not a soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
3) Their OR operation $(F, A) \vee (K, C)$ is not a soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.

One can easily verify $(1), (2),$ and (3) by the help of examples.

Definition 16. Let (F, A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F, A) is called the identity soft neutrosophic loop over $\langle L \cup I \rangle$ if $F(a) = \{e\}$ for all $a \in A$, where e is the identity element of $\langle L \cup I \rangle$.

Definition 17. Let (F, A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F, A) is called an absolute soft neutrosophic loop over $\langle L \cup I \rangle$ if $F(a) = \langle L \cup I \rangle$ for all $a \in A$.

Definition 18. Let (F, A) and (H, C) be two soft neutrosophic loops over $\langle L \cup I \rangle$. Then (H, C) is callsed soft neutrosophic subloop of (F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic subloop of $F(a)$ for all $a \in A$.

Example 4. Consider the neutrosophic loop

$$\langle L_{15}(2) \cup I \rangle = \{e,1,2,3,4,...,15,eI,1I,2I,...,14I,15I\}$$

of order 32. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) is a soft neutrosophic loop over $\langle L_{15}(2) \cup I \rangle$, where

$$F(a_1) = \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\},$$
$$F(a_2) = \{\mathrm{e}, 2, 5, 8, 11, 14\},$$
$$\mathrm{F}(a_3) = \{e, 3, eI, 3I\}.$$

Thus (H, C) is a soft neutrosophic subloop of (F, A) over $\langle L_{15}(2) \cup I \rangle$, where

$$H(a_1) = \{e, eI, 2I, 5I, 8I, 11I, 14I\},$$
$$H(a_2) = \{e, 3\}.$$

Theorem 5. Every soft loop over L is a soft neutrosophic subloop over $\langle L \cup I \rangle$.

Definition 19. Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F, A) be a soft set over $\langle L \cup I \rangle$. Then (F, A) is called soft normal neutrosophic loop if and only if $F(a)$ is normal neutrosophic subloop of $\langle L \cup I \rangle$ for all

$a \in A$.

Example 5. Let $\langle L_5(3) \cup I \rangle = \{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a neutrosophic loop. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then clearly (F,A) is soft normal neutrosophic loop over $\langle L_5(3) \cup I \rangle$, where

$$F(a_1) = \{e, eI, 1, 1I\}, F(a_2) = \{\mathrm{e}, \mathrm{eI}, 2, 2\mathrm{I}\},$$
$$F(a_3) = \{\mathrm{e}, \mathrm{eI}, 3, 3\mathrm{I}\}.$$

Theorem 6. Every soft normal neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Proposition 5. Let (F,A) and (K,C) be two soft normal neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,\mathrm{C})$ is a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A) \cap_R (K,\mathrm{C})$ is a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A) \wedge (K,\mathrm{C})$ is a soft normal neutrosophic loop over $\langle L \cup I \rangle$.

Remark 5. Let (F,A) and (K,C) be two soft normal neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended union $(F,A) \cup_E (K,\mathrm{C})$ is not a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted union $(F,A) \cup_R (K,\mathrm{C})$ is not a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
3. Their OR operation $(F,A) \vee (K,\mathrm{C})$ is not a soft normal neutrosophic loop over $\langle L \cup I \rangle$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 20. Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft Lagrange neutrosophic loop if $F(a)$ is a Lagrange neutrosophic subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Example 6. In Example (1), (F,A) is a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.

Theorem 7. Every soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 8. If $\langle L \cup I \rangle$ is a Lagrange neutrosophic loop, then (F,A) over $\langle L \cup I \rangle$ is a soft Lagrange neutrosophic loop but the converse is not true.

Remark 6. Let (F,A) and (K,C) be two soft Lagrange neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,\mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A) \cap_R (K,\mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A) \wedge (K,\mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A) \cup_E (K,\mathrm{C})$ is not a soft Lagrnage neutrosophic loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A) \cup_R (K,\mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F,A) \vee (K,\mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 21. Let $\langle L \cup I \rangle$ be a neutrosophic loop and

(F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called soft weak Lagrange neutrosophic loop if atleast one $F(a)$ is not a Lagrange neutrosophic subloop of $\langle L\cup I\rangle$ for some $a\in A$.

Example 7. Consider the neutrosophic loop

$$\langle L_{15}(2)\cup I\rangle=\{e,1,2,3,4,...,15,eI,1I,2I,...,14I,15I\}$$

of order 32. Let $A=\{a_1,a_2,a_3\}$ be a set of parameters. Then (F,A) is a soft weakly Lagrange neutrosophic loop over $\langle L_{15}(2)\cup I\rangle$, where

$$F(a_1)=\{e,2,5,8,11,14,eI,2I,5I,8I,11I,14I\},$$
$$F(a_2)=\{e,2,5,8,11,14\},$$
$$F(a_3)=\{e,3,eI,3I\}.$$

Theorem 9. Every soft weak Lagrange neutrosophic loop over $\langle L\cup I\rangle$ is a soft neutrosophic loop over $\langle L\cup I\rangle$ but the converse is not true.

Theorem 10. If $\langle L\cup I\rangle$ is weak Lagrange neutrosophic loop, then (F,A) over $\langle L\cup I\rangle$ is also soft weak Lagrange neutrosophic loop but the converse is not true.

Remark 7. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic loops over $\langle L\cup I\rangle$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weak Lagrnage neutrosophic loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 22. Let $\langle L\cup I\rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange free neutrosophic loop if $F(a)$ is not a lagrange neutrosophic subloop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 11. Every soft Lagrange free neutrosophic loop over $\langle L\cup I\rangle$ is a soft neutrosophic loop over $\langle L\cup I\rangle$ but the converse is not true.

Theorem 12. If $\langle L\cup I\rangle$ is a Lagrange free neutrosophic loop, then (F,A) over $\langle L\cup I\rangle$ is also a soft Lagrange free neutrosophic loop but the converse is not true.

Remark 8. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic loops over $\langle L\cup I\rangle$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange soft neutrosophic loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange soft neutrosophic loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage soft neutrosophic loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L\cup I\rangle$.

6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

4 Soft Neutrosophic Strong Loop

Definition 23. Let $\langle L\cup I\rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft neutrosophic strong loop if and only if $F(a)$ is a strong neutrosophic subloop of $\langle L\cup I\rangle$ for all $a\in A$.

Proposition 6. Let (F,A) and (K,C) be two soft neutrosophic strong loops over $\langle L\cup I\rangle$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is a soft neutrosophic strong loop over $\langle L\cup I\rangle$.

Remark 9. Let (F,A) and (K,C) be two soft neutrosophic strong loops over $\langle L\cup I\rangle$. Then

1. Their extended union $(F,A)\cup_E(K,C)$ is not a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
2. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
3. Their OR operation $(F,A)\vee(K,C)$ is not a soft neutrosophic strong loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 24. Let (F,A) and (H,C) be two soft neutrosophic strong loops over $\langle L\cup I\rangle$. Then (H,C) is called soft neutrosophic strong subloop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic strong subloop of $F(a)$ for all $a\in A$.

Definition 25. Let $\langle L\cup I\rangle$ be a neutrosophic strong loop and (F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange neutrosophic strong loop if $F(a)$ is a Lagrange neutrosophic strong subloop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 13. Every soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$ is a soft neutrosophic loop over $\langle L\cup I\rangle$ but the converse is not true.

Theorem 14. If $\langle L\cup I\rangle$ is a Lagrange neutrosophic strong loop, then (F,A) over $\langle L\cup I\rangle$ is a soft Lagrange neutrosophic loop but the converse is not true.

Remark 10. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong loops over $\langle L\cup I\rangle$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange strong neutrosophic loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage neutrosophic strong loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic strong loop over

$\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 26. Let $\langle L \cup I \rangle$ be a neutrosophic strong loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft weak Lagrange neutrosophic strong loop if atleast one $F(a)$ is not a Lagrange neutrosophic strong subloop of $\langle L \cup I \rangle$ for some $a \in A$.

Theorem 15. Every soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 16. If $\langle L \cup I \rangle$ is weak Lagrange neutrosophic strong loop, then (F,A) over $\langle L \cup I \rangle$ is also soft weak Lagrange neutrosophic strong loop but the converse is not true.

Remark 11. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft weak Lagrnage neutrosophic strong loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 27. Let $\langle L \cup I \rangle$ be a neutrosophic strong loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft Lagrange free neutrosophic strong loop if $F(a)$ is not a lagrange neutrosophic strong subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Theorem 17. Every soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 18. If $\langle L \cup I \rangle$ is a Lagrange free neutrosophic strong loop, then (F,A) over $\langle L \cup I \rangle$ is also a soft Lagrange free neutrosophic strong loop but the converse is not true.

Remark 12. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft Lagrnage free strong neutrosophic strong loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic strong loop over

$\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic Biloop

Definition 27. Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop and (F, A) be a soft set over $(\langle B \cup I \rangle, *_1, *_2)$. Then (F, A) is called soft neutrosophic biloop if and only if $F(a)$ is a neutrosophic subbiloop of $(\langle B \cup I \rangle, *_1, *_2)$ for all $a \in A$.

Example 8. Let $(\langle B \cup I \rangle, *_1, *_2) = (\{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\} \cup \{g : g^6 = e\}$ be a neutrosophic biloop. Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is clearly soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$, where

$$F(a_1) = \{e, 2, eI, 2I\} \cup \{g^2, g^4, e\},$$
$$F(a_2) = \{e, 3, eI, 3I\} \cup \{g^3, e\}.$$

Theorem 19. Let (F, A) and (H, A) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Proof. Straightforward.

Theorem 20. Let (F, A) and (H, C) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$ such that $A \cap C = \phi$. Then their union is soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Proof. Straightforward.

Proposition 7. Let (F, A) and (K, C) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
3. Their AND operation $(F, A) \wedge (K, C)$ is a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Remark 13. Let (F, A) and (K, C) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
3. Their OR operation $(F, A) \vee (K, C)$ is not a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 28. Let $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ be a new class neutrosophic biloop and (F, A) be a soft set over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is called soft new class neutrosophic subbiloop if and only if $F(a)$ is a neutrosophic subbiloop of $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ for all $a \in A$.

Example 9. Let $B = (B_1 \cup B_2, *_1, *_2)$ be a new class neutrosophic biloop, where $B_1 = \langle L_5(3) \cup I \rangle = \{e,1,2,3,4,5,eI,2I,3I,4I,5I\}$ be a new class of neutrosophic loop and $B_2 = \{g : g^{12} = e\}$ is a group.

$\{e, eI, 1, 1I\} \cup \{1, g^6\}$,
$\{e, eI, 2, 2I\} \cup \{1, g^2, g^4, g^6, g^8, g^{10}\}$,
$\{e, eI, 3, 3I\} \cup \{1, g^3, g^6, g^9\}$,

$\{e, eI, 4, 4I\} \cup \{1, g^4, g^8\}$ are neutrosophic subloops of B. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then (F, A) is soft new class neutrosophic biloop over B, where

$$F(a_1) = \{e, eI, 1, 1I\} \cup \{e, g^6\},$$
$$F(a_2) = \{\mathrm{e}, \mathrm{eI}, 2, 2\mathrm{I}\} \cup \{e, g^2, g^4, g^6, g^8, g^{10}\},$$
$$F(a_3) = \{e, eI, 3, 3I\} \cup \{e, g^3, g^6, g^6\},$$
$$F(a_4) = \{e, eI, 4, 4I\} \cup \{e, g^4, g^8\}.$$

Theorem 21. Every soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is trivially a soft neutrosophic biloop over but the converse is not true.

Proposition 8. Let (F, A) and (K, C) be two soft new class neutrosophic biloops over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ is a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{C})$ is a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{C})$ is a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.

Remark 14. Let (F, A) and (K, C) be two soft new class neutrosophic biloops over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then

1. Their extended union $(F, A) \cup_E (K, \mathrm{C})$ is not a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
2. Their restricted union $(F, A) \cup_R (K, \mathrm{C})$ is not a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
3. Their OR operation $(F, A) \vee (K, \mathrm{C})$ is not a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.

One can easily verify $(1), (2)$, and (3) by the help of examples.

Definition 29. Let (F, A) be a soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then (F, A) is called the identity soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ if $F(a) = \{e_1, e_2\}$ for all $a \in A$, where e_1, e_2 are the identities of $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ respectively.

Definition 30. Let (F, A) be a soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then (F, A) is called an absolute-soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ if $F(a) = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ for all $a \in A$.

Definition 31. Let (F, A) and (H, C) be two soft neutrosophic biloops over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then (H, C) is called soft neutrosophic subbiloop of (F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic subbiloop of $F(a)$ for all $a \in A$.

Example 10. Let $B = (B_1 \cup B_2, *_1, *_2)$ be a neutrosophic biloop, where $B_1 = \langle L_5(3) \cup I \rangle = \{e, 1, 2, 3, 4, 5, eI, 2I, 3I, 4I, 5\mathrm{I}\}$ be a new class of neutrosophic loop and $B_2 = \{g : g^{12} = e\}$ is a group. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then (F, A) is soft neutrosophic biloop over B, where

$$F(a_1) = \{e, eI, 1, 1I\} \cup \{e, g^6\},$$
$$F(a_2) = \{\mathrm{e}, \mathrm{eI}, 2, 2\mathrm{I}\} \cup \{e, g^2, g^4, g^6, g^8, g^{10}\},$$
$$F(a_3) = \{e, eI, 3, 3I\} \cup \{e, g^3, g^6, g^6\},$$
$$F(a_4) = \{e, eI, 4, 4I\} \cup \{e, g^4, g^8\}.$$

Then (H, C) is soft neutrosophic subbiloop of (F, A), where

$$H(a_1)=\{e,2)\cup\{e,g^2\},$$
$$H(a_2)=\{\mathrm{e},\mathrm{eI},3,3I\}\cup\{e,g^6\}.$$

Definition 32. Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft Lagrange neutrosophic biloop if and only if $F(a)$ is Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$ for all $a\in A$.

Example 11. Let $B=(\mathrm{B}_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=\{g:g^8=e\}$. Then clearly (F,A) is a soft Lagrange soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$, where

$$F(a_1)=\{\mathrm{e},\mathrm{eI},2,2\,\mathrm{I}\}\cup\{\mathrm{e}\},$$
$$\mathrm{F}(a_2)=\{e,eI,3,3I\}\cup\{e\}.$$

Theorem 22. Every soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Remark 15. Let (F,A) and (K,C) be two soft Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft Lagrnage neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 33. Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft weakly Lagrange neutrosophic biloop if atleast one $F(a)$ is not a Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$ for some $a\in A$.

Example 12. Let $B=(\mathrm{B}_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=\{g:g^8=e\}$. Then clearly (F,A) is a soft weakly Lagrange neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$, where

$$F(a_1)=\{\mathrm{e},\mathrm{eI},2,2\,\mathrm{I}\}\cup\{\mathrm{e}\},$$
$$\mathrm{F}(a_2)=\{e,eI,3,3I\}\cup\{\mathrm{e},\mathrm{g}^4\}.$$

Theorem 23. Every soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 24. If $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a weakly Lagrange neutrosophic biloop, then (F,A) over B is also soft weakly Lagrange neutrosophic biloop but the converse is not holds.

Remark 16. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$

is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrnage neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 34. Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft Lagrange free neutrosophic biloop if and only if $F(a)$ is not a Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$ for all $a\in A$.

Example 13. Let $B=(\mathrm{B}_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=\{g:g^8=e\}$. Then clearly (F,A) is a soft Lagrange free neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$, where

$$F(a_1)=\{\mathrm{e},\mathrm{eI},2,2\mathrm{I}\}\cup\{\mathrm{e},\mathrm{g}^2,\mathrm{g}^4,\mathrm{g}^6\},$$
$$\mathrm{F}(a_2)=\{e,eI,3,3I\}\cup\{\mathrm{e},\mathrm{g}^4\}.$$

Theorem 25. Every soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 26. If $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a Lagrange free neutrosophic biloop, then (F,A) over B is also soft Lagrange free neutrosophic biloop but the converse is not holds.

Remark 17. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic Strong Biloop

Definition 35. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop where B_1 is a neutrosopphic biloop and B_2 is a neutrosophic group and (F,A) be soft set over B. Then (F,A) over B is called soft neutrosophic strong biloop if and only if $F(a)$ is a neutrosopchic strong subbiloop of B for all $a\in A$.

Example 14. Let $B=(B_1\cup B_2,*_1,*_2)$ where $B_1=\langle L_5(2)\cup I\rangle$ is a neutrosophic loop and $B_2=\{0,1,2,3,4,,1\mathrm{I},2\mathrm{I},3\mathrm{I},4\mathrm{I}\}$ under multiplication modulo 5 is a neutrosophic group. Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is soft neutrosophic strong biloop over B, where

$$F(a_1)=\{e,2,eI,2I\}\cup\{1,I,4I\},$$
$$F(a_2)=\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}\cup\{1,\mathrm{I},4\mathrm{I}\}.$$

Theorem 27. Every soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 28. If $B=(B_1\cup B_2,*_1,*_2)$ is a neutrosophic strong biloop, then (F,A) over B is also soft neutrosophic strong biloop but the converse is not holds.

Proposition 9. Let (F,A) and (K,C) be two soft neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

Remark 18. Let (F,A) and (K,B) be two soft neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 36. Let (F,A) and (H,C) be two soft neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then (H,C) is called soft neutrosophic strong subbiloop of (F,A), if

3. $C\subseteq A$.
4. $H(a)$ is a neutrosophic strong subbiloop of $F(a)$ for all $a\in A$.

Definition 37. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(B_1\cup B_2,*_1,*_2)$. Then (F,A) is called soft Lagrange neutrosophic strong biloop if and only if $F(a)$ is a Lagrange neutrosophic strong subbiloop of $B=(B_1\cup B_2,*_1,*_2)$ for all $a\in A$.

Theorem 29. Every soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Remark 19. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft Lagrnage neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 38. Let $B=(B_1 \cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(B_1 \cup B_2,*_1,*_2)$. Then (F,A) is called soft weakly Lagrange neutrosophic strong biloop if atleast one $F(a)$ is not a Lagrange neutrosophic strong subbiloop of $B=(B_1 \cup B_2,*_1,*_2)$ for some $a \in A$.

Theorem 30. Every soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 31. If $B=(B_1 \cup B_2,*_1,*_2)$ is a weakly Lagrange neutrosophic strong biloop, then (F,A) over B is also soft weakly Lagrange neutrosophic strong biloop but the converse does not holds.

Remark 20. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic strong biloops over $B=(B_1 \cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 39. Let $B=(B_1 \cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(B_1 \cup B_2,*_1,*_2)$. Then (F,A) is called soft Lagrange free neutrosophic strong biloop if and only if $F(a)$ is not a Lagrange neutrosophic subbiloop of $B=(B_1 \cup B_2,*_1,*_2)$ for all $a \in A$.

Theorem 32. Every soft Lagrange free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 33. If $B=(B_1 \cup B_2,*_1,*_2)$ is a Lagrange free neutrosophic strong biloop, then (F,A) over B is also soft strong lagrange free neutrosophic strong biloop but the converse is not true.

Remark 21. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong biloops over $B=(B_1 \cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft Lagrnage free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1 \cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic N-loop

Definition 40. Let $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft neutrosophic N -loop if and only if $F(a)$ is a neutrosopchic sub N -loop of $S(B)$ for all $a\in A$.

Example 15. Let $S(B)=\{S(B_1)\cup S(B_2)\cup S(B_3),*_1,*_2,*_3\}$ be a neutrosophic 3-loop, where $S(B_1)=\langle L_5(3)\cup I\rangle$, $S(B_2)=\{g:g^{12}=e\}$ and $S(B_3)=S_3$. Then (F,A) is sof neutrosophic N -loop over $S(B)$, where

$$F(a_1)=\{\mathrm{e},\mathrm{eI},2,2\mathrm{I}\}\cup\{\mathrm{e},\mathrm{g}^6\}\cup\{\mathrm{e},(12)\},$$
$$\mathrm{F}(a_2)=\{e,eI,3,3I\}\cup\{e,g^4,g^8\}\cup\{e,(13)\}.$$

Theorem 34. Let (F,A) and (H,A) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic N -loop over $S(B)$.

Proof. Straightforward.

Theorem 35. Let (F,A) and (H,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ such that $A\cap C=\phi$. Then their union is soft neutrosophic N -loop over $S(B)$.

Proof. Straightforward.

Proposition 10. Let (F,A) and (K,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is a soft neutrosophic N -loop over $S(B)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is a soft neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic N -loop over $S(B)$.

Remark 22. Let (F,A) and (H,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft neutrosophic N -loop over $S(B)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft neutrosophic N -loop over $S(B)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft neutrosophic N -loop over $S(B)$.

One can easily verify $(1),(2)$, and (3) by the help of examples.

Definition 41. Let (F,A) be a soft neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then (F,A) is called the identity soft neutrosophic N -loop over $S(B)$ if $F(a)=\{e_1,e_2,...,e_N\}$ for all $a\in A$, where $e_1,e_2,...,e_N$ are the identities element of $S(B_1),S(B_2),...,S(B_N)$ respectively.

Definition 42. Let (F,A) be a soft neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then (F,A) is called an absolute-soft neutrosophic N -loop over $S(B)$ if $F(a)=S(B)$ for all $a\in A$.

Definition 43. Let (F,A) and (H,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then (H,C) is called soft neutrosophic sub N -loop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic sub N -loop of $F(a)$ for all $a\in A$.

Definition 45. Let $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft Lagrange neutrosophic N -loop if and only if $F(a)$ is Lagrange neutrosophic sub N -loop of $S(B)$ for all $a\in A$.

Theorem 36. Every soft Lagrange neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Remark 23. Let (F,A) and (K,C) be two soft Lagrange neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic N -loop over (B).
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage neutrosophic N -loop over $S(B)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 46. Let $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft weakly Lagrange neutrosophic biloop if atleast one $F(a)$ is not a Lagrange neutrosophic sub N -loop of $S(B)$ for some $a\in A$.

Theorem 37. Every soft weakly Lagrange neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Theorem 38. If $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a weakly Lagrange neutrosophic N -loop, then (F,A) over $S(B)$ is also soft weakly Lagrange neutrosophic N -loop but the converse is not holds.

Remark 24. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrnage neutrosophic N -loop over $S(B)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 47. Let $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over

$S(B)$. Then (F,A) is called soft Lagrange free neutrosophic N-loop if and only if $F(a)$ is not a Lagrange neutrosophic sub N-loop of $S(B)$ for all $a \in A$.

Theorem 39. Every soft Lagrange free neutrosophic N-loop over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a soft neutrosophic biloop but the converse is not true.

Theorem 40. If $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a Lagrange free neutrosophic N-loop, then (F,A) over $S(B)$ is also soft lagrange free neutrosophic N-loop but the converse is not hold.

Remark 25. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic N-loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N-loop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N-loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N-loop over $S(B)$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft Lagrnage free neutrosophic N-loop over $S(B)$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N-loop over $S(B)$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N-loop over $S(B)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic Strong N-loop

Definition 48. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic N-loop and (F,A) be a soft set over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then (F,A) is called soft neutrosophic strong N-loop if and only if $F(a)$ is a neutrosopchic strong sub N-loop of $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ for all $a\in A$.

Example 16. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup L_3,*_1,*_2,*_3\}$ where $L_1=\langle L_5(3)\cup I\rangle, L_2=\langle L_7(3)\cup I\rangle$ and $L_3=\{1,2,1I,2I\}$. Then (F,A) is a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$, where

$$F(a_1)=\{\mathrm{e},2,\mathrm{eI},2\mathrm{I}\}\cup\{\mathrm{e},2,\mathrm{eI},2\mathrm{I}\}\cup\{1,\mathrm{I}\},$$
$$\mathrm{F}(a_2)=\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}\cup\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}\cup\{1,2,2\mathrm{I}\}.$$

Theorem 41. All soft neutrosophic strong N-loops are soft neutrosophic N-loops but the converse is not true.

One can easily see the converse with the help of example.

Proposition 11. Let (F,A) and (K,C) be two soft neutrosophic strong N-loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$.

Remark 26. Let (F,A) and (K,C) be two soft neutrosophic strong N-loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended union $(F,A)\cup_E (K,C)$ is not a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their OR operation $(F,A)\vee(K,C)$ is not a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 49. Let (F,A) and (H,C) be two soft neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$. Then (H,C) is called soft neutrosophic strong sub N -loop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic strong sub N -loop of $F(a)$ for all $a\in A$.

Definition 50. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N -loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange neutrosophic strong N -loop if and only if $F(a)$ is a Lagrange neutrosophic strong sub N -loop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 42. Every soft Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Remark 27. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft Lagrnage neutrosophic strong N -loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 51. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N -loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft weakly Lagrange neutrosophic strong N -loop if atleast one $F(a)$ is not a Lagrange neutrosophic strong sub N -loop of $\langle L\cup I\rangle$ for some $a\in A$.

Theorem 43. Every soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Theorem 44. If $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ is a weakly Lagrange neutrosophic strong N -loop, then (F,A) over $\langle L\cup I\rangle$ is also a soft weakly Lagrange neutrosophic strong N -loop but the converse is not true.

Remark 28. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrnage neutrosophic strong N -loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 52. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,*_2,\ldots,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange free neutrosophic strong N -loop if and only if $F(a)$ is not a Lagrange neutrosophic strong sub N -loop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 45. Every soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,*_2,\ldots,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Theorem 45. If $\langle L\cup I\rangle=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,*_2,\ldots,*_N\}$ is a Lagrange free neutrosophic strong N -loop, then (F,A) over $\langle L\cup I\rangle$ is also a soft Lagrange free neutrosophic strong N -loop but the converse is not true.

Remark 29. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,*_2,\ldots,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Conclusion

This paper is an extension of neutrosphic loop to soft neutrosophic loop. We also extend neutrosophic biloop, neutrosophic N -loop to soft neutrosophic biloop, and soft neutrosophic N -loop. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established within soft neutrosophic loop.

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Abstract

This volume is a collection of seven papers, written by different authors and co-authors (listed in the order of the papers): A. A. Salama, F. Smarandache, Valeri Kroumov, A. A. A. Agboola, S. A. Akinleye, M. Ali, M. Shabir, M. Naz, I. Deli, Y. Toktas, S. Broumi, Z. Zhang, C. Wu, S. A.Alblowi, C. Dyer.

In first paper, the authors proposed Neutrosophic Closed Set and Neutrosophic Continuous Function. Neutrosophic Vector spaces are proposed in the second paper. Neutrosophic Bi-LA-Semigroup and Neutrosophic N-LA-Semigroup is studied in third paper. In fourth paper Neutrosophic Parameterized Soft Relations and Their Applications are introduced. Similarly in fifth paper A novel method for single valued neutrosophic multi-criteria decision making with incomplete weight information are discussed. In paper six New Neutrosophic Crisp Topological Concept is presented by the authors. Soft Neutrosophic Loops and Their Generalization is given in seventh paper.

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