An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming (GP)

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Abstract. In this paper, finding - a maximal solution is introduced to (V, Λ) fuzzy neutrosophic relation equation. the notion of fuzzy relation equation was first investigated by Sanchez in 1976, while Florentin Smarandache put forward a fuzzy neutrosophic relation equations in 2004 with innovative investigation. This paper is first attempt to establish the structure of solution set on model. The NRE have a wide applications in various real world problems like flow rate in chemical plants, transportation problem, study of bounded labor problem, study of interrelations among HIV/AIDS affected patients and use of genetic algorithms in chemical problems .

Keyword Neutrosophic Logic, Neutrosophic Relation Equations (NRE),Integral Neutrosophic lattices, Fuzzy Integral Neutrosophic Matrices, Maximal Solution, Fuzzy Geometric Programming (FGP).

Introduction

The analysis of most of the real world problems involves the concept of indeterminacy. One cannot establish or cannot rule out the possibility of some relation but says that cannot determine the relation or link; this happens in legal field, medical diagnosis even in the construction of chemical flow in industries and more chiefly in socio economic problems prevailing in various countries.[4], as well as the importance of geometric programming and the fuzzy neutrosophic relation equations in theory and application, I have proposed a new structure for maximum solution in FNRE with geometric programming.

1.1 Definition [4]

Let T, I, F be real standard or non-standard subsets of $]^{-}0,1^{+}[$, with

$sup\, T = t\_sup, inf\, T = t\_inf, sup\, I = i\_sup, inf\, I = i\_inf, sup\, F = f\_sup, inf\, F = f\_inf, and\; n\_sup = t\_sup + i\_sup + f\_sup, n\_inf = t\_inf + i\_inf + f\_inf. Let\; U$

be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set $M\, as\, x(T, I, F)$ and belongs to M in the following way: It is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F. Statically T, I, F are subsets, but dynamically T, I, F are functions operators depending on many known or unknown parameters.

1.2 Physics Example for Neutrosophic Logic [4]

For example the Schrodinger's Theory says that the quantum state of a photon can basically be in more than one place in the same time , which translated to the neutrosophic set means that an element (quantum state) belongs and does not belong to a set (one place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory. In Schroedinger's Equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function Psi which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points). Don't we better describe, using the attribute "neutrosophic" than "fuzzy" or any others, a quantum particle that neither exists nor non-exists?

1.3 Application for Neutrosophic Logic [4]

A cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set. (e.g. there are a kind of separated water props, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud). Also, we are not sure where the cloud ends nor where it begins, neither if some elements are or are not in the set. That's why the percent of indeterminacy is required and the neutrosophic probability (using subsets - not numbers - as components) should be used for better modeling: it is a more organic, smooth, and especially accurate estimation. Indeterminacy is the zone of ignorance of a proposition's value, between truth and falsehood. From the intuitionistic logic, paraconsistent logic, dialetheism, fallibilism, paradoxes, pseudoparadoxes, and tautologies we transfer the "adjectives" to the sets, i.e. to intuitionistic set (set incompletely known), paraconsistent set, dialetheist set, faillibilist set (each element has a percentage of indeterminacy), paradoxist set (an element may belong and may not belong in the same time to the set), pseudoparadoxist set, and tautological set respectively. hence, the neutrosophic set generalizes:

• the intuitionistic set, which supports incomplete set theories $(for\ 0 < n < 1, 0 [\ t, i, f\ [\ 1)$ and incomplete known elements belonging to a set;

• the fuzzy set (for $n = 1\ and\ i = 0, and\ 0 [\ t, i, f\ [\ 1)$;
• the classical set (for $n = 1\ and\ i = 0$, with $t, f\ either\ 0\ or\ 1$);
• the paraconsistent set (for $n > 1, with\ all\ t, i\ , f < 1^{+}$);
• the faillibilist set ($i > 0$);
• the dialetheist set, a set M whose at least one of its elements also belongs to its complement $C(M)$; thus, the intersection of some disjoint sets is not empty
• the paradoxist set ($t = f = 1$);
• the pseudoparadoxist set ($0 < i < 1, t = 1\ and\ f > 0\ or\ t > 0\ and\ f = 1$);
• the tautological set ($i\ , f < 0$).

Compared with all other types of sets, in the neutrosophic set each element has three components which are subsets (not numbers as in fuzzy set) and considers a subset, similarly to intuitionistic fuzzy set, of "indeterminacy" - due to unexpected parameters hidden in some sets, and let the superior limits of the components to even boil over 1 (over flooded) and the inferior limits of the components to even freeze under 0 (under dried). For example: an element in some tautological sets may have $t > 1$, called "over included". Similarly, an element in a set may be "over indeterminate" (for $i > 1$, in some paradoxist sets), "over excluded" (for $f > 1$, in some unconditionally false appurtenances); or "under true" (for $t < 0$, in some unconditionally false appurtenances), "under indeterminate" (for $i < 0$, in some unconditionally true or false appurtenances), "under fals some unconditionally true appurtenances). This is because we should make a distinction between unconditionally true $(t > 1, and\ f < 0\ or\ i < 0)$ and conditionally true appurtenances $(t\ [\ 1, and\ f\ [\ 1\ or\ i\ [\ 1)$. In a rough set RS, an element on its boundary-line cannot be classified neither as a member of RS nor of its complement with certainty. In the neutrosophic set a such element may be characterized by $x(T, I, F)$, with corresponding set-values for T, I, F]-0,1+[. One first presents the evolution of sets from fuzzy set to neutrosophic set. Then one introduces the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where]-0,1+[is the non-standard unit interval, and thus one defines the neutrosophic set.[4]

2 Basic Concepts for NREs

2.1 Definition [3].

A Brouwerian lattice L in which, for any given elements a & b the set of all $x \in L$ such that $a \Lambda x \leq b$ contains a greatest element, denoted $a \propto b$, the relative pseudocomplement of $a\ in\ b$ [san].

2.2 Remark [4]

If $L = [0, 1]$, then it is easy to see that for any given $a, b \in L$,

$$a \propto b = \begin{cases} 1 & a \leq b \\ b & a > b \end{cases}$$

2.3 Definition [4]

Let $N = L \cup \{I\}$ *where L is any lattice and I an indeterminate.*

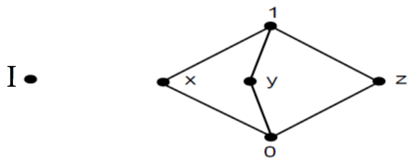
Define the max, min operation on N as follows

$Max\ \{x, I\} = I$ for $all\ x \in L \setminus \{1\}$

$Max\{1, I\} = 1$
$Min\{x, I\} = I$ for $all\ x \in L \setminus \{0\}$
$Min\{0, I\} = 0$
We know if $x, y \in L$ then max and min are well defined in L. N is called the integral neutrosophic lattice.

2.3 Example [4]

Let $N = L \cup \{I\}$ given by the following diagram:



Clearly [4]

$Min\{x, I\} = I \quad for\ all\ x \in L \setminus \{0\}$
$Min\{0, I\} = 0$
$Max\{x, I\} = I \quad for\ all\ x \in L \setminus \{1\}$
$Max\{1, I\} = 1$
We see N is an integral neutrosophic lattice and clearly the order of N is 6.

2.4 Remark [4]

1- If L is a lattice of order n and $N = L \cup \{I\}$ be an integral neutrosophic lattice then order of N is $n + 1$.
2. For an integral neutrosophic lattice N also $\{0\}$ is the minimal element and $\{1\}$ is the maximal element of N.

2.5 Conventions About Neutrosophic Sets [4]

Let $A, B \in N(X)$ *i.e.*,
$A : X \to [0,1] \cup I$, $B : X \to [0,1] \cup I$
$(A \cap B)(x) = min\{A(x), B(x)\}$,if $A(x) = I$ or $B(x) = I$ then $(A \cap B)(x)$ is defined to be I i.e., $min\{A(x), B(x))\} = I_9$
$I(A \cup B)(x) = max\{A(x), B(x)\}$ if one of $A(x) = I\ or\ B(x) = I\ then\ (A \cup B)(x) = I$ i.e., $max\{A(x), B(x)\} = I$.
Thus it is pertinent to mention here that if one of $A(x) = I\ or\ B(x) = I$ then $(A \cup B)(x) = (A \cap B)(x)$. i.e., is the existence of indeterminacy $max\{A(x), B(x)\} = min\{A(x), B(x)\} = I$
$\bar{A}(x) = 1 - A(x);\ if\ A(x) = I$
$then\quad \bar{A}(x) = A(x) = I$.

2.6 Definition [4]

Let $N = [0,1] \cup I$ where I is the indeterminacy. The $m \times n$ matrices $M_{m\times n} = \{(a_{ij}) / a_{ij} \in [0,1] \cup I\}$ is called the fuzzy integral neutrosophic matrices. Clearly the class of $m \times n$ matrices is contained in the class of fuzzy integral neutrosophic matrices.

2.7 Example [4]

Let $A = \begin{pmatrix} I & 0.1 & 0 \\ 0.9 & 1 & I \end{pmatrix}$

A is a 2×3 integral fuzzy neutrosophic matrix. We define operation on these matrices. An integral fuzzy neutrosophic row vector is a $1 \times n$ integral fuzzy neutrosophic matrix. Similarly an integral fuzzy neutrosophic column vector is a $m \times 1$ integral fuzzy neutrosophic matrix.

2.8 Example [4]

$A = (0.1, 0.3, 1, 0, 0, 0.7, I, 0.002, 0.01, I, 1, 0.12)$ is a integral row vector or a 1×12, integral fuzzy neutrosophic matrix.

2.9 Example [4]

$B = (1, 0.2, 0.111, I, 0.32, 0.001, I, 0, 1)^T$ is an integral neutrosophic column vector or B is a 9×1 integral fuzzy neutrosophic matrix.
We would be using the concept of fuzzy neutrosophic column or row vector in our study.

2.10 Definition [4]

Let $P = (p_{ij})$ be a $m \times n$ integral fuzzy neutrosophic matrix and $Q = (q_{ij})$ be a $n \times p$ integral fuzzy neutrosophic matrix. The composition map P o Q is defined by R (r_{ij}) which is a $m \times p$ matrix where $\left(r_{ij} = \max_k \min(p_{ik}\, q_{kj})\right)$ with the assumption $\max(p_{ij}, I) = I$ and $\min(p_{ij}, I) = I$ where pij $\in (0,1)$. $\min(0, I) = 0$ and $\max(1, I) = 1$.

2.11 Example [4]

Let $p = \begin{bmatrix} 0.3 & I & 1 \\ 0 & 0.9 & 0.2 \\ 0.7 & 0 & 0.4 \end{bmatrix}$, $Q = (0.1, I, 0)^T$
be two integral fuzzy neutrosophic matrices.

$$P \text{ o } Q = \begin{bmatrix} 0.3 & I & 1 \\ 0 & 0.9 & 0.2 \\ 0.7 & 0 & 0.4 \end{bmatrix} \text{o} \begin{bmatrix} 0.1 \\ I \\ 0 \end{bmatrix} =$$
$(I\ , I\ , 0.1)^T$

3 Structure of the Maximal Solution Set

B.Y.Cao proposed the structure set of maximal and minimal solution for FRGP (fuzzy relation geometric programming) with $(\vee, \wedge)$ operator [optimal models & meth with fuzzy quantities 2010] [1-2]. Its useful and necessary to call back the following ideas

3.1 Definition

If $X(A,b) \neq \emptyset$ it can be completely determined by a unique maximum solution and a finite number of minimal solution. The maximum solution can be obtained by applying the following operation:-

$\widehat{x_j} = \wedge\{b_i | b_i < a_{ij}\}\ (1 \le i \le m, 1 \le j \le n)$
Stipulate that set $\{\wedge\emptyset = 1\}$. If $\hat{x}_j = (\hat{x}_1, \hat{x}_{2,n} \ldots, \hat{x})^T$ is a solution to $AoX = b$. then $\hat{x}$ must be the greatest solution.

3.2 An original notion to find maximal solution

The most important question:

What is the structure of the maximum element for any fuzzy neutrosophic relation equations in the interval $[0,1] \cup I$??

We know that

$\max\{0, I\} = I$ & $\max\{x, I\} = I \quad \forall x \in [0,1) \cup I$.

Depending upon the definition (3.1), the stipulation that $\{\wedge\emptyset = 1\}$ will be fixed.

Also by Sanchez (1976) we have

$$a\alpha b = \begin{cases} 1 & a \le b \\ b & a > b \end{cases}$$

Don't forget that α is relative psedo complement of $a\ in\ b$. On the other hand, Florentin was deffind the neutrosophic lattice see ref. [4] p.235

So if we want to establish the maximum solution for any FNRE in the interval $[0,1] \cup I$, we must redefine $a\alpha b$ where $a\alpha b = a \wedge x \le b$; $a, x, b \in [0,1] \cup I$.

Note that, all matrices in this work are an integral fuzzy neutrosophic matrices. It is obvious that a or b are either belonging to $[0,1]$ or equal to I , so we have that following status:

1- If $a \in [0,1]$ & $b \in I$, $a\alpha b = x = I$ where $a \neq 0$ therefore $a \in (0,1]$, here we must remember that $\min(I, x) = I\ \forall x \in (0,1] \cup I$.
2- If $a = I$ & $b \in [0,1]$, then $a\alpha b = x = 0$,here $\min(I, x) = I\ \ \forall x \in (0,1] \cup I$ also $\min(I, 0) = 0$.
3- At $a\&b \in [0,1]$, the solution will back to the same case that stated by Sanchez ,i.e.
$$a\alpha b = \begin{cases} 1 & a \le b \\ b & a > b \end{cases}$$
4- At $a = b = I$,this implies that $a\alpha b = 1$.

Note that , $a \wedge x \le b\ \rightarrow \min(a, x) \le b\ \rightarrow \min(I, x) \le I\ \rightarrow x = 1$

Consequently :

$$\widehat{x_j} = a\alpha b = \begin{cases} 1 & a_{ij} \le b_i\ or\ a_{ij} = b_{ij} = I \\ b_i & a_{ij} > b_i \\ 0 & a_{ij} = I\ and\ b_{ij} = [0,1] \\ I & b_i = I\ and\ a_{ij} = (0,1] \\ not\ comp. & a_{ij} = 0\ and\ b_{ij} = I \end{cases}$$

3.3 Lemma

If $a_{ij} = 0\ and\ b_i = I$ then $AoX = b$ is not compatible.

Proof

Let $a_{ij} = 0$, $b_i = I$

What is the value of $x_j \in [0,1] \cup I$ satisfying

$\vee_{ij}^{n} (a_{ij} \wedge x_j) = b_i \quad \forall\ \ 1 \le i \le m$?

We have

1- $a\alpha b = a_{ij} \wedge x_j \le b_i$
2- $\min(0, x_j) = 0 \quad \forall\ x_j \in [0,1] \cup I$

So $a\alpha b = a_{ij} \wedge x_j = \min(0, x_j) = 0$ not equal nor less than to I

We know that the incomparability occurs only when $x \in FN\ and\ y \in [0,1]$ see ref. [4] p.233

$\therefore AoX = b$ is not compatible

Without loss of generality , suppose $b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n$ when we rearranged the components of b in decreasing order we also adjusted A, x and $f(x)$ accordingly b.

Now, in fuzzy neutrosophic numbers, how can we classify numbers to rearrange them? For more details see ref. [5] page 245.

3.4 Example

Rearranged the following matrices in decreasing order

$$1)\ b = \begin{bmatrix} 0.85 \\ I \\ 0.5 \\ I \end{bmatrix}\ 2)\ b = \begin{bmatrix} I \\ I \\ 0.5 \\ 0.1 \end{bmatrix}\ 3)\ b = \begin{bmatrix} I \\ 0.6 \\ I \\ 0.1 \end{bmatrix}$$

Solution.

$$1)\ b = \begin{bmatrix} I \\ I \\ 0.85 \\ 0.5 \end{bmatrix}\ 2)\ b = \begin{bmatrix} I \\ I \\ 0.5 \\ 0.1 \end{bmatrix}\ 3)\ b = \begin{bmatrix} I \\ I \\ 0.6 \\ 0.1 \end{bmatrix}$$

4 Numerical Examples:-

Find the maximum solution for the following FNREGP problems:-

1)

$$min f(x) = 5x_1^{-.5}x_2^{-1.5}x_3^2x_4^{-2}x_5^{-1}$$

$$s.t. \begin{bmatrix} I & 0.8 & 0.6 & I & 0 \\ 0.8 & 0.7 & 0.8 & 1 & 0.8 \\ 0.6 & 0.9 & 0.8 & 0.9 & 0.5 \\ I & 0.2 & I & 0.6 & 0.2 \\ 0.3 & 0.3 & 0.5 & 0.2 & 0.1 \\ I & 0.1 & 0.2 & 0.3 & I \end{bmatrix} o \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} I \\ I \\ I \\ .5 \\ .5 \\ .4 \end{bmatrix}$$

Solution :- The greatest solution is

$\hat{x}_1 = \wedge\{1, I, I, 0,1,0\} = 0$

$\hat{x}_2 = \wedge\{I, I, I, 1,1,1\} = I$

$\hat{x}_3 = \wedge\{I, I, I, 0,1,1\} = 0$

$\hat{x}_4 = \wedge\{1, I, I, .5,1,1\} = I$

$\hat{x}_5 = \wedge\{not\ comparable, \ldots\ldots.\}$

Therefore by lemma (3.3) the system $Aox = b$ is not comparable.

2)

$$\min f(x) = (1.5I\Lambda x_1^{.5})V(2I\Lambda x_2)V(.8\Lambda x_3^{-.5})V(4\Lambda x_4^{-1})$$

s.t. $Aox = b$ where

$$A = \begin{bmatrix} I & .2 & .85 & .9 \\ .8 & .2 & I & .1 \\ .9 & .1 & I & .6 \\ I & .8 & .1 & I \end{bmatrix}$$

$b = (I, .6, .5, I)$, $0 \leq x_i \leq 1$, $1 \leq i \leq 4$

$\hat{x}_1 = \wedge(1,1,.5,1) = 0.5$

$\hat{x}_2 = \wedge(I, 1,1, I) = I$

$\hat{x}_3 = \wedge(I, 0,0, I) = 0$

$\hat{x}_4 = \wedge(I, 1, .5,1) = I$

$\therefore\ \hat{x} = (0.5, I, 0, I)^T$

Conclusion

In this article, the basic notion for finding maximal solution in a geometric programming subject to a system of fuzzy neotrosophic relational equation with max-min composition was introduced. In 1976,Sanchez gave the formula of the maximal solution for fuzzy relation equation concept and describing in details its structure. Some numerical examples have shown that the proposed method is betimes step to enter in this kind of problems to search for minimal solutions which remains as unfathomable issue.

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