Estimation of Ratio and Product of Two Population Means Using Auxiliary Characters in the Presence of Non Response

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Abstract

The auxiliary information is used in increasing the efficiency of the estimators for the parameters of the populations such as mean, ratio, and product of two population means. In this context, the estimation procedure for the ratio and product of two population means using auxiliary characters in special reference to the non response problem has been discussed.

Keywords Auxiliary variable, MSE, non response, SRS, efficiency.

Introduction

The use of auxiliary information in sample surveys in the estimation of population mean, ratio, and product of two population means has been studied by different authors by using different estimation procedures. The review work in this topic has been given by Tripathi et al. (1994) and Khare (2003). In the present context the problems of estimation of ratio and product of two population means have been considered in different situations especially in the presence of non response.

Estimation of Ratio and product of two population means

Case 1. The Case of Complete Response:

Singh (1965,69), Rao and Pareira (1968), Shahoo and Shahoo (1978), Tripathi (1980), Ray and Singh (1985) and Khare (1987) have proposed estimators of ratio and product of two population means using auxiliary characters with known mean. Singh (1982) has proposed the case of double sampling for the estimation of ratio and product of two population mean. Khare (1991(a)) has proposed a class of estimators for R and P using double sampling scheme, which are given as follows:

$$R^* = f(v,u) \text{ and } P^* = g(w,u) \tag{1}$$

such that $f(R,1) = R$, $g(P,1) = P$, $f_1(R,1) = 1$ and $g_1(P,1) = 1$, where $v = \left(\frac{\bar{y}_1}{\bar{y}_2}\right)$, $w = \bar{y}_1\bar{y}_2$ and $u = \left(\frac{\bar{x}_1'}{\bar{x}_1}\right)$. Here $\bar{y}_1$, $\bar{y}_2$ and $\bar{x}_1$ denote the sample mean of study characters y_1, y_2 and auxiliary character x_1 based on a sub sample of size $n\,(< n')$ and $\bar{x}_1'$ is sample mean of x_1 based on a larger sample of size n' drawn by using SRSWOR method of sampling from the population of size N. The first partial derivatives of $f(v,u)$ and $g(w,u)$ with respect to v and w are denoted by $f_1(v,u)$ and $g_1(w,u)$ respectively. The function $f(v,u)$ and $g(w,u)$ also satisfied some regularity conditions for continuity and existence of the functions. The sample size for first phase and second phase sample which may be from the first phase sample or independent of first phase sample drawn from the remaining part of the population ($N - n'$).

Singh et al. (1994) have extended the class of estimators proposed by Khare (1991(a)) and proposed a new class of estimator for R, which is given as follows:

$$R_g = \hat{R}h(u', v') \tag{2}$$

where $\hat{R} = \frac{\bar{y}_1}{\bar{y}_2}$, $u' = \frac{\bar{x}}{\bar{x}'}$ and $v' = \frac{s_x^2}{s_x'^2}$, where $(\bar{x}, s_x^2)$ and $(\bar{x}', s_x'^2)$ are sample mean and sample mean square of auxiliary character based on n and $n'\,(> n)$ units respectively.

Srivastava et al. (1988,89) have suggested chain ratio estimators for R and P . Which are given as follows:

$$R_1^* = \hat{R}\left(\frac{\bar{y}_3}{\bar{y}_3'}\right)\left(\frac{\bar{y}_4'}{\bar{Y}_4}\right) \quad \text{and} \quad R_2^* = \hat{R}\left(\frac{\bar{y}_3'}{\bar{y}_3}\right)\left(\frac{\bar{Y}_4}{\bar{y}_4'}\right) \tag{3}$$

$$P_1^* = \hat{P}\left(\frac{\bar{y}_3}{\bar{y}_3'}\right)^{\alpha_1}\left(\frac{\bar{y}_4'}{\bar{Y}_4}\right)^{\alpha_2} \quad \text{and} \quad P_2^* = \hat{P}\left(\frac{\bar{y}_3}{\bar{Y}_3}\right)^{\beta_1}\left(\frac{\bar{y}_4'}{\bar{Y}_4}\right)^{\beta_2} \tag{4}$$

Further Singh et al. (1994) have given a general class of estimators

$$\hat{R}_h = h(\hat{R}, u, v) \text{ and } \hat{P}_h = h(\hat{P}, u, v), \tag{5}$$

such that $h(R,1,1) = R$ and $h(P,1,1) = P$, where $u = \left(\frac{\bar{y}_3}{\bar{y}_3'}\right)$ and $v = \left(\frac{\bar{y}_4'}{\bar{Y}_4}\right)$. The functions $h(\hat{R}, u, v)$ and $h(\hat{P}, u, v)$ satisfy the regularity conditions.

Khare (1991(b)) have proposed the class of estimators for using multi-auxiliary characters with known means. which are given as follows:

$$R^* = \hat{R}h(u_1, u_2 \ldots u_p) = \hat{R}h(\underline{u}) \text{ and } R^{**} = g(\hat{R}, \underline{u}), \tag{6}$$

such that $h(\underline{e})=1$ and $g(\hat{R},\underline{e})=R$, where $h(\underline{u})$ and $g(\hat{R},\underline{u})$ satisfying some responding conditions.

Further, Khare (1993(a)) has proposed a class of estimators for R using multi-auxiliary characters with unknown means, the class of estimators is given as follows:

$$R_m^* = g(\hat{R},\underline{u'}) , \tag{7}$$

such that $g(R,\underline{e})=1$, where $u_i = \frac{\bar{x}_i{}'}{\bar{x}_i}$, $\underline{u'}=(u_1,u_2 ... u_p)$, $\bar{x}_i$ and $\bar{x}_i{}'$ are sample mean based on n and n' $(>n)$ units for auxiliary characters $x_i, i=1,2,...p.$

Similarly, Khare (1992) have proposed class of estimators for P using p auxiliary characters with known and unknown population mean and studied their properties.

Further, Khare (1990) has proposed a generalized class of estimator for a combination of product and ratio of some population means using multi-auxiliary characters. The parametric combination is given by:

$$\theta = \frac{\bar{Y}_1,\bar{Y}_2,\bar{Y}_3,...,\bar{Y}_m}{\bar{Y}_{m+1},\bar{Y}_{m+2},\bar{Y}_{m+3},...,\bar{Y}_k}, \tag{8}$$

which is the product of first m population means $\bar{Y}_1,\bar{Y}_2,\bar{Y}_3,...,\bar{Y}_m$ divided by product of $k-m$ population means $\bar{Y}_{m+1},\bar{Y}_{m+2},\bar{Y}_{M+3},...,\bar{Y}_k$ respectively. The conventional estimator for θ is given by

$$\hat{\theta} = \frac{\bar{y}_1,\bar{y}_2,\bar{y}_3,...,\bar{y}_m}{\bar{y}_{m+1},\bar{y}_{m+2},\bar{y}_{m+3},...,\bar{y}_k}, \tag{9}$$

It is important to note that for $m=1,k=2; \quad \theta = R$

$m=2,k=2; \quad \theta = P$

$m=1,k=1; \quad \theta = \bar{Y}_1$

$m=k=1; \bar{Y}_1=\bar{Y}_2, \quad \theta=\bar{Y}_1^2,$

$m=k=3; \bar{Y}_1=\bar{Y}_2=\bar{Y}_3, \quad \theta=\bar{Y}_1^3,$

$m=2,k=4; \bar{Y}_1=\bar{Y}_2, \bar{Y}_3=\bar{Y}_4, \quad \theta=\bar{Y}_1^2=R^2,$

Using p auxiliary characters $x_1, x_2, ..., x_p$ with known population means $\bar{X}_1, \bar{X}_2, ..., \bar{X}_p$ the class of estimators θ^* is given by:

$$\theta^* = \hat{\theta}h(\underline{u}), \tag{10}$$

such that $h(\underline{e})=1$, where $\underline{u}'=(u_1,u_2,...u_p)$ and $u_i=\frac{\bar{x}_i}{\bar{X}_i}$, $i=1,2,...,p$.

The function $h(u_1,u_2,...u_p)=h(\underline{u})$ satisfied the following regularity conditions:

a) Whatever be the sample chosen $(\underline{u})$, assume values in abounded closed convex sub set G of p dimensional real space containing the point $\underline{u}=\underline{e}$.

b) In G, the function $h(\underline{u})$ is continuous and bounded.

c) The first and second partial derivatives of $h(\underline{u})$ exists and are continuous and bounded in G.

For two auxiliary variables it is found that the lower bond of the variance of the class of estimators θ^* is same as given by the estimators proposed by Singh (1969) and Shah and Shah (1978). Hence it is remarked that the class of estimators θ^* will attain lower bound for mean square error if the specified and regularity conditions are satisfied.

Further, Khare (1993b) have proposed the class of two phase sampling estimators for the combination of product and ratio of some population means using multi-auxiliary characters with unknown population means, which is given as follows:

$$\theta^{**}=\hat{\theta}h(\underline{v}), \tag{11}$$

where $\underline{v}=(v_1,v_2,...v_p)'$, $v_i=\frac{\bar{x}_i'}{\bar{x}_i}$, $i=1,2,...p$.

Such that $h(\underline{e})=1$ and $h(\underline{v})$ satisfies some regularly conditions.

Case 2. Incomplete Response in the Sample due to Non-response:

In case of non-response on some units selected in the sample, Hansen and Hurwitz (1946) have suggested the method of sub sampling from non-respondents and proposed the estimator for population mean. Further, Khare et al. (2014) have proposed some new estimators in this situation of sub sampling from non-respondents.

Khare & Pandey (2000) and Khare & Sinha (2010) have proposed the class of estimators for ratio and product of two population means using auxiliary character with known population mean in the presence of non-response on the study characters, which is given as follows:

$$R_i^*=R^*h(u_i) \text{ and } P_i^*=P^*h(u_i),\, i=1,2, \tag{12}$$

such that $h(1)=1$, where $R^*=\frac{\bar{y}_1^*}{\bar{y}_2^*}$, $P^*=\bar{y}_1^*\bar{y}_2^*$, $u_1=\frac{\bar{x}^*}{\bar{X}}$, $u_2=\frac{\bar{x}}{\bar{X}}$ and $\bar{y}_1^*$, $\bar{y}_2^*$ and $\bar{x}^*$ are sample means for y_1, y_2 and x characters proposed by Hansen and Hurwitz (1946) based on n_1+r units and $\bar{x}$ is the sample mean based on n units. Khare & Sinha (2012) have proposed a combined class of estimators for ratio and product of two population mean in the presence of non-response with known population mean $\bar{X}$. This is a more general class of estimators for R and P under some specified and regularity conditions. Khare et al. (2013 (a)) have proposed an improved class of estimators for R. In this case, the improved class of estimators for R using auxiliary character with known population mean $\bar{X}$ in the presence of non response is given as follows:

$$R_i = g(v, u_i)\; i = 1,2, \tag{13}$$

such that $g(R,1) = R$, $g_1(R,1)$ and $g_{12}(R,1) = R^{-1} g_2(R,1)$, where $v = \frac{\bar{y}_1^*}{\bar{y}_2^*}$, $u_1 = \bar{x}^*$ and $u_2 = \bar{x}$. The function $g(v, u_i)\; i = 1,2$ assumes positive values in a real line containing the point $(R,1)$. The function $g(v, u_i)$ is assumed to be continuous and bounded in a real line and its first and second order partial derivatives exists. The first partial derivative of $g(v, u_i)\; i = 1,2$ at the point $(R,1)$ with respect to v and u_i is denoted by $g_1(R,1)$ and $g_2(R,1)$. The second order partial derivative of $g(v, u_i)\; i = 1,2$ with respect to v, v and u_i, and u_i at the point $(R,1)$ is denoted by $g_{11}(R,1)$, $g_{12}(R,1)$ and $g_{22}(R,1)$ respectively. Some members of the class of estimators R_i are given as follows:

$$C_1 = w_0 v u_i^{\alpha_i},\; C_2 = w_1 v + w_2 u_i,\; C_3 = w_1' v + w_2' v u_i^{\beta_i},\; i = 1,2, \tag{14}$$

where w_0, w_1, w_2, w_1', w_2', α_i and β_i $i = 1,2$, are constants. Further the class of estimator proposed by Khare and Sinha (2013) is more efficient than the estimator proposed by Khare and Pandey (2000).

Further, Khare and Sinha (2002(a, b)) have proposed two phase sampling estimators for ratio and product of two population means in the presence of non-response. Khare and Sinha (2004(a,b)) have proposed a more general class of two phase sampling estimators for R and P. which are given as follows:

$$T_i = g(v, u_i),\; i = 1,2, \tag{15}$$

such that $g(R,1) = R$ and $g_1(R,1) = 1$, where $v = \frac{\bar{y}_1^*}{\bar{y}_2^*}$, $u_1 = \frac{\bar{x}^*}{\bar{x}'}$, $u_2 = \frac{\bar{x}}{\bar{x}'}$ and $\bar{x}'$ is sample mean based on n' $(> n)$ units. The function $g(v, u_i)$ satisfy some regularly conditions.

$$T_i^* = g(w, u_i),\; i = 1,2, \tag{16}$$

such that $g(P,1) = 1$ and $g_1(P,1) = 1$, where $w = \bar{y}_1^* \bar{y}_2^*$, $u_1 = \frac{\bar{x}^*}{\bar{x}'}$, $u_2 = \frac{\bar{x}}{\bar{x}'}$ and $g(w, u_i)$ satisfy some regularly conditions.

Khare et al. (2012) have proposed two generalized chain type estimators T_{g1} and T_{g2} for R using auxiliary characters in the presence of non-response, which are given as follows:

$$T_{g1} = \hat{R}\left(\frac{\bar{x}^*}{\bar{x}'}\right)^{\alpha_1}\left(\frac{\bar{z}'}{\bar{\bar{Z}}}\right)^{\alpha 2} \text{ and } T_{g2} = \hat{R}\left(\frac{\bar{x}}{\bar{x}'}\right)^{\beta_1}\left(\frac{\bar{z}'}{\bar{\bar{Z}}}\right)^{\beta 2}, \tag{17}$$

where $\hat{R} = \frac{\bar{y}_1^*}{\bar{y}_2^*}$ and (α_1, α_2) and (β_1, β_2) are suitable constants. It has been observed that due to use of additional auxiliary character with known population mean along with the main auxiliary character, the proposed class of estimators T_{g1} and T_{g2} are more efficient than the corresponding generalized estimators for R using the main auxiliary character only in the case of two phase sampling in the presence of non response for fixed sample sizes (n', n) and also for fixed cost ($C \le C_0$). It is also seen that less cost is incurred for T_{g1} and T_{g2} than the cost incurred in the generalized estimator for R in the case of two phase sampling in the presence of non response for specified precision ($V = V_0$).

Further, generalized chain estimators for ratio and product of two population means have been improved by putting $R^* = k_1\hat{R}$ and $P^* = k_1\hat{P}$ in place of $\hat{R}$ and $\hat{P}$ in the proposed estimators of R and P. Further, Khare et al. (2013 (b)) have proposed the improved class of chain type estimators for ratio of two population means using two auxiliary characters in the presence of non-response. The class of estimators is given as follows:

$$R_{ci} = f\left(\hat{R}, u_i, v\right),\ i = 1,2, \tag{18}$$

such that $f(R,1,1) = 1$ and $f_1(R,1,1) = 1$, where $\hat{R} = \frac{\bar{y}_1^*}{\bar{y}_2^*}$, $u_1 = \frac{\bar{x}^*}{\bar{x}'}$, $u_2 = \frac{\bar{x}}{\bar{x}'}$ and $v = \frac{\bar{z}'}{\bar{Z}}$. The function $f\left(\hat{R}, u_i, v\right),\ i = 1,2$ satisfies some regularity conditions.

Khare and Sinha (2007) have proposed estimator for R using multi-auxiliary characters with known population mean in the presence of non-response. The class of estimators t_i is given as follows:

$$t_i = \hat{R} g_i(\underline{u_i'}),\ i = 1, 2, \tag{19}$$

such that $g_i(\underline{e_i'}) = 1$, where $\underline{u_i}$ and $\underline{e_i}$ denote the column vectors $(u_{i1}, u_{i2}, \dots, u_{ip})'$ and $(1, 1, \dots, 1)'$, $u_{1j} = \frac{\bar{x}_j^*}{\bar{X}_j}$ and $u_{2j} = \frac{\bar{x}_j}{\bar{X}_j}$ $j = 1, 2, \dots, p$.

An improved under class of estimators for R using multi-auxiliary variables using double sampling scheme in the presence of non-response has been proposed by Khare and Sinha (2012) and studies their properties.

Khare and Sinha (2014) have extended the class of estimator proposed by Khare and Sinha (2012) and proposed a wider class of two phase sampling estimators for R using multi-auxiliary characters in the presence of non-response.

References

1. Hansen, M. H. and Hurwitz, W. N. (1946): The problem of non-response in sample surveys. Jour. Amer. Stat. Assoc., 41, 517-529.

2. Khare, B. B. (1987): On modified class of estimators of ratio and product of two population means using auxiliary character. Proc. Math. Soc, B.H.U. 3, 131-137.

3. Khare, B. B. (1990): A generalized class of estimators for combination of products and ratio of some population means using multi-auxiliary characters. J. Stat. Res., 24, 1-8.

4. Khare, B. B. (1991 (a)): Determination of sample sizes for a class of two phase sampling estimators for ratio and product of two population means using auxiliary character. Metron (Italy), XLIX, (1-4), 185-197.

5. Khare, B. B. (1991 (b)): On generalized class of estimators for ratio of two population means using multi-auxiliary characters. Aligarh J. Stat. 11, 81-90.

6. Khare, B. B. (1992): On class of estimators for product of two population means using multi-auxiliary characters with known and unknown means. Ind. J. Appl. Stat., 1, 56-67.

7. Khare, B. B. (1993(a)): A class of two phase sampling estimators for the combination of product and ratio of several population means using multi-auxiliary characters. Proc. Nat. Acad. Sci., India, 63 (4), Pt. II, 391-397.

8. Khare, B. B. (1993(b)): On a class of two phase sampling estimators for ratio of two population means using multi-auxiliary characters. Proc. Nat. Acad. Sci., India, 63(a), III, 513-520.

9. Khare, B. B. (2014): Estimation of population parameters using the technique of sub sampling from non respondents in sample surveys- A Review. Proc. Nat. Acad. Sci. Sec A, 84 (3), 337-343.

10. Khare, B. B. (2014-15): Applications of statistics in bio-medical sciences. Prajna, Special Issue on Science & Technology, Vol. - 60 (2),.

11. Khare, B. B. and Pandey, S. K. (2000): A class of estimators for ratio of two population means using auxiliary character in presence of non-response. J. Sc. Res., 50, 115-124.

12. Khare, B. B. and Sinha, R. R. (2002a): Estimation of the ratio of two populations means using auxiliary character with unknown population mean in presence of non response. Prog. Maths. Vol.- 36. No. (1, 2), 337-348.

13. Khare, B. B. and Sinha, R. R. (2002b): On class of two phase sampling estimators for the product of two population means using auxiliary character in presence of non response. Proc. of Vth international symposium on optimization and Statistics held at AMU, Aligarah, 221-232.

14. Khare, B. B. and Sinha, R. R. (2004 (a)): Estimation of finite population ratio using two phase sampling in presence of non response. Aligarh J. Stat. 24, 43-56.

15. Khare, B. B. and Sinha, R. R. (2004 (b)): On the general class of two phase sampling estimators for the product of two population means using the auxiliary characters in the presence of non-response. Ind.J. Appl. Statistics., 8, 1-14.

16. Khare, B. B. and Sinha, R. R. (2007): Estimation of the ratio of the two population means using multi- auxiliary characters in presence of non-response. In “Statistical techniques in life testing, reliability, sampling theory and quality control” edited by B. N. Pandey Narosa publishing house, New Delhi, 163-171.

17. Khare, B. B. and Sinha, R. R. (2010): On class of estimators for the product of two population means using auxiliary character in presence of non-response. Inter. Trans. Appl. Sci., 2(4), 841-846.

18. Khare, B. B. and Sinha, R. R. (2012 (a)): Combined class of estimators for ratio and product of two population means in presence of non-response. Int. Jour. Stats. and Eco., 8(S12), 86-95.

19. Khare, B. B. and Sinha, R. R. (2012 (b)): Improved classes of ratio of two population means with double sampling the non-respondents. Statistika-Statistics & Economy Jour. 49(3), 75-83.

20. Khare, B. B. and Sinha, R. R. (2014): A class of two phase sampling estimator for ratio of two populations means using multi-auxiliary characters in the presence of non response. Stat. in Trans. New series, 15, (3), 389-402.

21. Khare, B. B. and Srivastava, S. R. (1999): A class of estimators for ratio of two population means and means of two populations using auxiliary character. J. Nat. Acad. Math. 13, 100-104.

22. Khare, B. B. and Srivastava, S. Rani. (1998): Combined generalized chain estimators for ratio and product of two population means using auxiliary characters. Metron (Italy) LVI (3-4), 109-116.

23. Khare, B. B., Jha, P. S. and Kumar, K. (2014): Improved generalized chain estimators for ratio and product of two population means using two auxiliary characters in the presence of non-response. International J. Stats & Economics, 13(1), 108-121.

24. Khare, B. B., Pandey, S. K. and Kumar, A. (2013 (a)): Improved class of estimators for ratio of two population means using auxiliary character in presence of non-response. Proc. Nat. Acad. Sci. India, 83(1), 33-38.

25. Khare, B. B., Kumar, K. and Srivastava, U. (2013 (b)): Improved classes of chain type estimators for ratio of two population means using two auxiliary characters in the presence of non-response. Int. Jour. Adv. Stats. & Prob. 1 (3), 53-63.

26. Khare, B. B., Srivastava, U. and Kumar K. (2012): Chain type estimators for ratio of two population means using auxiliary characters in the presence of non response. J. Sc. Res., BHU, 56, 183-196.

27. Khare, B. B., Srivastava, U. and Kumar K. (2013): Generalized chain type estimators for ratio of two population means using two auxiliary characters in the presence of non-response. International J. Stats & Economics, 10(1), 51-64.

28. Rao, J. N. K. and Pareira, N. P. (1968): On double ratio estimators. Sankhya Ser. A., 30, 83-90.

29. Ray, S. K. and Singh, R. K. (1985): Some estimators for the ratio and product of population parameters. Jour. Ind. Soc. Agri. Stat., 37, 1-10.

30. Shah, S. M. and Shah, D. N. (1978): Ratio cum product estimators for estimating ratio (product) of two population parameters. Sankhya Ser. *C.*, 40, 156-166.

31. Singh, M. P. (1969): Comparison of some ratio cum product estimators. Sankhya, Ser. B, 31, 375-378.

32. Singh, R. K. (1982b): On estimating ratio and product of population parameters. Calcutta Stat. Assoc., 20, 39-49.

33. Singh, V. K., Singh, Hari P. and Singh, Housila P. (1994): Estimation of ratio and product of two finite population means in two phase sampling. J. Stat. Plan. Inf. 41, 163-171.

34. Singh, V. K., Singh, Hari P., Singh, Housila P. and Shukla, D. (1994): A general class of chain type estimators for ratio and product of two population means of a finite population. Commun. Stat.- TM. 23 (5), 1341-1355.

35. Srivastava, Rani S., Khare, B. B. and Srivastava, S.R. (1988): On generalized chain estimator for ratio and product of two population means using auxiliary characters. Assam Stat. Review, 2(1), 21-29.

36. Srivastava, Rani S., Srivastava, S. R. and Khare, B. B. (1989): Chain ratio type estimator for ratio of two population means using auxiliary characters. Commun. Stat. Theory Math.(USA), 18(10), 3917-3926.

37. Tripathi, T. P. (1980): A general class of estimators for population ratio. Sankhya Ser. C., 42, 63-75.

38. Tripathi, T. P., Das, A. K. and Khare, B. B. (1994): Use of auxiliary information in sample surveys - A review. Aligarh J. Stat., 14, 79-134.