



# Misfire Fault Diagnosis Method of Gasoline Engines Using the Cosine Similarity Measure of Neutrosophic Numbers

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**Abstract.** This paper proposes a distance measure of neutrosophic numbers and a similarity measure based on cosine function, and then develops the misfire fault diagnosis method of gasoline engines by using the cosine similarity measure of neutrosophic numbers. In the fault diagnosis, by the cosine similarity measure between the fault knowledge (fault patterns) and required diagnosis-testing sample with neutrosophic number information and its relation indices, the proposed fault diagnosis

method can indicate the main fault type and fault trends. Then, the misfire fault diagnosis results of gasoline engines demonstrate the effectiveness and rationality of the proposed fault diagnosis method. The proposed misfire fault diagnosis method not only gives the main fault types of the engine, but also provides useful information for future fault trends. The proposed method is effective and reasonable in the misfire fault diagnosis of gasoline engines.

**Keywords:** Neutrosophic number, distance measure, cosine similarity measure, misfire fault diagnosis, gasoline engine.

## 1 Introduction

Misfire fault is usually produced in gasoline engines [1]. However, it can descend its power, increase its fuel consumption and aggravate its pollution of exhaust emission when the burning quality of mixture gases descends in the combustion chamber of gasoline engines. Therefore, to keep better operating performance of the engine, we have to find out and eliminate the affected factors of low burning quality in the engine. Then, the exhaust emission in gasoline engines mainly contains the components of HC, NO<sub>x</sub>, CO, CO<sub>2</sub>, O<sub>2</sub>, water vapor etc, which can affect the burning quality of mixture gases in the engine. Under different burning conditions in the engine, the content of the components can be changed in some range as the change of operating status or the occurrences of various mechanical and electronic faults in the engine. Hence, one can indicate the operating status of the engine by analyzing the change of exhaust emission content [1].

However, fault diagnosis is an important topic in engineering areas. In many real situations, the fault data cannot provide deterministic values because the fault testing data obtained by experts are usually imprecise or uncertain due to a lack of data, time pressure, measurement errors, or the experts' limited attention and knowledge. In real situations, the fault testing data usually contain the determinate information and the indeterminate information. While neutrosophic numbers proposed originally by Smarandache [2-4] may express it since a neutrosophic number consists of its determinate part and its indeterminate part. Therefore, it is a better tool for

expressing incomplete and indeterminate information. The neutrosophic number can be represented as  $N = a + bI$ , which consists of its determinate part  $a$  and its indeterminate part  $bI$ . In the worst scenario,  $N$  can be unknown, i.e.  $N = bI$ . When there is no indeterminacy related to  $N$ , in the best scenario, there is only its determinate part  $N = a$ . Obviously, it is very suitable for the expression of incomplete and indeterminate information in fault diagnosis problems. Therefore, the neutrosophic number can effectively represent the fault data with incomplete and indeterminate information. Although the neutrosophic numbers have been defined in neutrosophic probability since 1996 [2], since then, little progress has been made for processing indeterminate problems by neutrosophic numbers in scientific and engineering applications. In order to break through the applied predicament, this paper proposes a distance measure of neutrosophic numbers and a similarity measure of neutrosophic numbers based on cosine function (so-called cosine similarity measure) for handling the misfire fault diagnosis problems of gasoline engines under neutrosophic number environment.

The remainder of the paper is organized as follows. In Section 2, we introduce some basic concepts related to neutrosophic numbers and some basic operational relations of neutrosophic numbers. Section 3 proposes a distance measure and a cosine similarity measure for neutrosophic numbers. Section 4 develops a fault diagnosis method using the cosine similarity measure for the misfire fault diagnosis problems of gasoline engines under neutrosophic number environment and demonstrates the effectiveness and rationality of the misfire fault diagnosis method. Section 5 gives the conclusions and future directions of

research.

**2 Neutrosophic numbers and their basic operational relations**

Smarandache [2-4] firstly proposed a concept of a neutrosophic number, which consists of the determinate part and the indeterminate part. It is usually denoted as  $N = a + bI$ , where  $a$  and  $b$  are real numbers, and  $I$  is indeterminacy, such that  $I^2 = I, 0 \cdot I = 0$ , and  $I/I = \text{undefined}$ .

For example, a neutrosophic number is  $N = 3 + 2I$ . If  $I \in [0, 0.2]$ , it is equivalent to  $N \in [3, 3.4]$  for sure  $N \geq 3$ , this means that the determinate part of  $N$  is 3, while the indeterminate part of  $N$  is  $2I$  and  $I \in [0, 0.2]$ , which means the possibility for number “ $N$ ” to be a little bigger than 3.

Let  $N_1 = a_1 + b_1I$  and  $N_2 = a_2 + b_2I$  be two neutrosophic numbers. Then, Smarandache [2-4] gave the following operational relations of neutrosophic numbers:

- (1)  $N_1 + N_2 = a_1 + a_2 + (b_1 + b_2)I$ ;
- (2)  $N_1 - N_2 = a_1 - a_2 + (b_1 - b_2)I$ ;
- (3)  $N_1 \times N_2 = a_1a_2 + (a_1b_2 + b_1a_2 + b_1b_2)I$ ;
- (4)  $N_1^2 = (a_1 + b_1I)^2 = a_1^2 + (2a_1b_1 + b_1^2)I$ ;
- (5)  $\frac{N_1}{N_2} = \frac{a_1 + b_1I}{a_2 + b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)} \cdot I$  for  $a_2 \neq 0$  and  $a_2 \neq -b_2$ ;

$$(6) \sqrt{N_1} = \sqrt{a_1 + b_1I} = \begin{cases} \sqrt{a_1} - (\sqrt{a_1} + \sqrt{a_1 + b_1})I \\ \sqrt{a_1} - (\sqrt{a_1} - \sqrt{a_1 + b_1})I \\ -\sqrt{a_1} + (\sqrt{a_1} + \sqrt{a_1 + b_1})I \\ -\sqrt{a_1} + (\sqrt{a_1} - \sqrt{a_1 + b_1})I \end{cases}$$

**3 Distance measure and cosine similarity measure between neutrosophic numbers**

In this section, we propose a distance measure of neutrosophic numbers and a similarity measure between neutrosophic numbers based on cosine function.

**Definition 1.** Let  $A = \{N_{A1}, N_{A2}, \dots, N_{An}\}$  and  $B = \{N_{B1}, N_{B2}, \dots, N_{Bn}\}$  be two sets of neutrosophic numbers, where  $N_{Aj} = a_{Aj} + b_{Aj}I$  and  $N_{Bj} = a_{Bj} + b_{Bj}I$  ( $j = 1, 2, \dots, n$ ) for  $a_{Aj}, b_{Aj}, a_{Bj}, b_{Bj} \in R$  ( $R$  is all real numbers). Then, a distance measure between  $A$  and  $B$  is defined as

$$D(A, B) = \frac{1}{2n} \sum_{j=1}^n \left( \left| a_{Aj} + \inf(b_{Aj}I) - a_{Bj} - \inf(b_{Bj}I) \right| + \left| a_{Aj} + \sup(b_{Aj}I) - a_{Bj} - \sup(b_{Bj}I) \right| \right) \quad (1)$$

Obviously, the distance measure should satisfy the following properties (D1-D3):

- (D1)  $D(A, B) \geq 0$ ;
- (D2)  $D(A, B) = 0$  if  $A = B$ ;
- (D3)  $D(A, B) = D(B, A)$ .

However, when we considers the importance of each element in the set of neutrosophic numbers, the weight of each element  $w_j$  ( $j = 1, 2, \dots, n$ ) can be introduced with  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ . Thus, we have the following weighted distance measure between  $A$  and  $B$ :

$$D_w(A, B) = \frac{1}{2} \sum_{j=1}^n w_j \left( \left| a_{Aj} + \inf(b_{Aj}I) - a_{Bj} - \inf(b_{Bj}I) \right| + \left| a_{Aj} + \sup(b_{Aj}I) - a_{Bj} - \sup(b_{Bj}I) \right| \right) \quad (2)$$

Obviously, the weighted distance measure also satisfies the above properties (D1-D3).

To easily apply neutrosophic numbers to fault diagnosis problems in this paper, we propose the similarity measure of neutrosophic numbers based on cosine function.

**Definition 2.** Let  $A = \{N_{A1}, N_{A2}, \dots, N_{An}\}$  and  $B = \{N_{B1}, N_{B2}, \dots, N_{Bn}\}$  be two sets of neutrosophic numbers, where  $N_{Aj} = a_{Aj} + b_{Aj}I \subseteq [0, 1]$  and  $N_{Bj} = a_{Bj} + b_{Bj}I \subseteq [0, 1]$  ( $j = 1, 2, \dots, n$ ) for  $a_{Aj}, b_{Aj}, a_{Bj}, b_{Bj} \geq 0$ . Then, a cosine similarity measure between  $A$  and  $B$  is defined as follows:

$$C(A, B) = \sum_{j=1}^n w_j \cos \left\{ \frac{\pi}{4} \left( \left| a_{Aj} + \inf(b_{Aj}I) - a_{Bj} - \inf(b_{Bj}I) \right| + \left| a_{Aj} + \sup(b_{Aj}I) - a_{Bj} - \sup(b_{Bj}I) \right| \right) \right\} \quad (3)$$

where  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ . Obviously, the cosine similarity measure should satisfy the following properties (P1-P3):

- (P1)  $0 \leq C(A, B) \leq 1$ ;
- (P2)  $C(A, B) = 1$  if  $A = B$ ;
- (P3)  $C(A, B) = C(B, A)$ .

**4 Misfire fault diagnosis method of gasoline engines using the cosine similarity measure**

**4.1 Fault diagnosis method**

For a fault diagnosis problem, assume that there are a set of  $m$  fault patterns (fault knowledge)  $P = \{P_1, P_2, \dots, P_m\}$  and a set of  $n$  characteristics (attributes)  $Q = \{Q_1, Q_2, \dots, Q_n\}$ . Then the fault information of a fault pattern  $P_k$  ( $k = 1, 2, \dots, m$ ) with respect to a characteristic  $Q_j$  ( $j = 1,$

2, ..., n) is represented by a set of neutrosophic numbers  $P_k = \{N_{k1}, N_{k2}, \dots, N_{kn}\}$ , where  $N_{kj} = a_{kj} + b_{kj}I \subseteq [0, 1]$  for  $a_{kj}, b_{kj} \geq 0$  ( $k = 1, 2, \dots, m; j = 1, 2, \dots, n$ ). Then, the information of a testing sample is represented by a set of neutrosophic numbers  $P_t = \{N_{t1}, N_{t2}, \dots, N_{tm}\}$ , where  $N_{tj} = a_{tj} + b_{tj}I \subseteq [0, 1]$  for  $a_{tj}, b_{tj} \geq 0$  ( $t = 1, 2, \dots, s; j = 1, 2, \dots, n$ ).

The similarity measure value  $v_k$  ( $k = 1, 2, \dots, m$ ) can be obtained by the following cosine similarity measure between  $P_t$  and  $P_k$ :

$$v_k = C(P_t, P_k) = \sum_{j=1}^n w_j \cos \left\{ \frac{\pi}{4} \left( \left| a_{tj} + \inf(b_{tj}I) - a_{kj} - \inf(b_{kj}I) \right| + \left| a_{tj} + \sup(b_{tj}I) - a_{kj} - \sup(b_{kj}I) \right| \right) \right\}. \quad (4)$$

For convenient fault diagnosis, the cosine values of  $v_k$  ( $k = 1, 2, \dots, m$ ) are normalized into the relation indices within the interval  $[-1, 1]$  by the following formula:

$$\delta_k = \frac{2v_k - v_{\min} - v_{\max}}{v_{\max} - v_{\min}}, \quad (5)$$

where  $v_{\max} = \max_{1 \leq k \leq m} \{v_k\}$ ,  $v_{\min} = \min_{1 \leq k \leq m} \{v_k\}$  and  $\delta_k \in [-1, 1]$ .

Then, we can rank the relation indices and determine the fault type or predict possible fault trends for the tested equipment. If there is the maximum relation index  $\delta_k = 1$ , then we can determine that the testing sample  $P_t$  should belong to the fault pattern  $P_k$ .

#### 4.2 Misfire fault diagnosis of gasoline engines

We apply the fault diagnosis method based on the cosine similarity measure to the misfire fault diagnosis of gasoline engines.

Let us investigate the misfire fault diagnosis problem of the gasoline engine EQ6102. Generally speaking, the misfire faults of the engine can be classified into three fault types: no misfire (normal work), slight misfire and severe misfire to indicate the operating status of the engine. Here, the slight misfire indicates the decline in the performance of ignition capacitance or the ignition delay, or the spark plug misfire in a cylinder of six cylinders, and then the severe misfire indicates the spark plug misfire in two cylinders of six cylinders. According to field-testing data [1], we can obtain the fault knowledge of the three fault types, i.e. a set of three fault patterns  $P = \{P_1, P_2, P_3\}$  with respect to a set of five characteristics (five components)  $Q = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$ , as shown in Table 1.

Table 1 Three fault patterns of misfire faults for the engine EQ6102

$P_k$ (Fault patterns)	$Q_1$ ( $\varphi_{HC} \times 10^{-2}$ )	$Q_2$ ( $\varphi_{CO_2}$ )	$Q_3$ ( $\varphi_{NO_x} \times 10$ )	$Q_4$ ( $\varphi_{CO} \times 10^{-1}$ )	$Q_5$ ( $\varphi_{O_2}$ )
$P_1$ (Normal work)	[0.03, 0.08]	[0.51, 0.93]	[0.03, 0.08]	[0.3, 0.5]	[0.062, 0.09]
$P_2$ (Slight misfire)	[0.01, 0.046]	[0.426, 0.84]	[0.04, 0.12]	[0.29, 0.5]	[0.04, 0.11]
$P_3$ (Severe misfire)	[0.2, 0.5]	[0.3, 0.7]	[0.1, 0.3]	[0.1, 0.3]	[0.07, 0.15]

Table 2 Fault knowledge expressed by neutrosophic numbers

$P_k$ (Fault knowledge)	$Q_1$ ( $\varphi_{HC} \times 10^{-2}$ )	$Q_2$ ( $\varphi_{CO_2}$ )	$Q_3$ ( $\varphi_{NO_x} \times 10$ )	$Q_4$ ( $\varphi_{CO} \times 10^{-1}$ )	$Q_5$ ( $\varphi_{O_2}$ )
$P_1$ (Normal work)	0.03+1.7857I	0.51+15I	0.03+1.7857I	0.3+7.1429I	0.062+I
$P_2$ (Slight misfire)	0.01+1.2857I	0.426+14.7857I	0.04+2.8571I	0.29+7.5I	0.04+2.5I
$P_3$ (Severe misfire)	0.2+10.7143I	0.3+14.2857I	0.1+7.1429I	0.1+7.1429I	0.07+2.8571I

Table 3 Tasting samples of exhaust emission

Number of tasting samples ( $P_t$ )	$Q_1$ ( $\varphi_{HC} \times 10^{-2}$ )	$Q_2$ ( $\varphi_{CO_2}$ )	$Q_3$ ( $\varphi_{NO_x} \times 10$ )	$Q_4$ ( $\varphi_{CO} \times 10^{-1}$ )	$Q_5$ ( $\varphi_{O_2}$ )	Actual fault types
1	0.0455	0.047	0.033	0.48	0.0527	$P_2$
2	0.0572	0.075	0.062	0.42	0.0751	$P_1$
3	0.0261	0.065	0.086	0.453	0.0431	$P_2$
4	0.0312	0.062	0.051	0.287	0.1064	$P_2$
5	0.3761	0.045	0.139	0.179	0.1025	$P_3$
6	0.4220	0.052	0.188	0.194	0.0931	$P_3$
7	0.0189	0.081	0.091	0.459	0.0377	$P_2$
8	0.0555	0.086	0.057	0.39	0.0736	$P_1$
9	0.0551	0.085	0.050	0.386	0.0789	$P_1$

In Table 1,  $\varphi_{HC} \times 10^{-2}$ ,  $\varphi_{CO_2}$ ,  $\varphi_{NO_x} \times 10$ ,  $\varphi_{CO} \times 10^{-1}$  and  $\varphi_{O_2}$  in the characteristic set  $Q = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$  indicate the exhaust emission concentration of the five components

HC, CO<sub>2</sub>, NO<sub>x</sub>, CO and O<sub>2</sub> expressed by volume percentage, and also we can consider the characteristic values of  $Q_j$  ( $j = 1, 2, 3, 4, 5$ ) as interval values. Then, the

interval values can be transformed into the neutrosophic numbers by the indeterminacy  $I \in [0, 0.028]$ , which express the characteristics of  $Q_j$  ( $j = 1, 2, \dots, n$ ), as shown in Table 2.

To illustrate the effectiveness of the misfire fault diagnosis of the engine, we introduce the nine sets of field-testing samples for the engine EQ6102 from [1], which are shown in Table 3.

Then, the importance of the five characteristics (five components) is considered by the weight vector  $W = (w_1, w_2, w_3, w_4, w_5)^T = (0.05, 0.35, 0.3, 0.2, 0.1)^T$  [1]. By using Eqs. (4) and (5), the diagnosis results are shown in Table 4. From Tables 3 and 4, the fault diagnosis results are in accordance with all the actual fault types.

Meanwhile, it is very easy to diagnose or predict fault types of the engine EQ6102 from Table 4. For example,

for Number 9, since the relation index of  $P_1$  is equal to 1, it indicates the fault type  $P_1$  (no misfire). Then one can predict that the engine has the slight misfire trend since the relation index of  $P_2$  is 0.923 and the fault type  $P_3$  has a very low possibility of severe misfire due to the negative relation index (-1). Similarly, one can also diagnose and predict fault types according to the relation indices for other testing samples in Table 4. Therefore, the proposed fault diagnosis method for the engine can not only diagnose the main fault type but also predict the future fault trend by the relation indices. Compared with the fault diagnosis method for the engine in [1], the fault diagnosis method proposed in this paper is simpler and easier than the fault diagnosis method by using extension set theory [1].

Table 4 Results of the relation indices and fault diagnoses

Number of tasting samples ( $P_i$ )	Relation indices ( $\delta_k$ )			Fault diagnosis results
	$P_1$	$P_2$	$P_3$	
1	0.5135	1.0000	-1.0000	$P_2$
2	1.0000	0.9850	-1.0000	$P_1$
3	0.9957	1.0000	-1.0000	$P_2$
4	0.9291	1.0000	-1.0000	$P_2$
5	-1.0000	-0.0077	1.0000	$P_3$
6	-1.0000	-0.7964	1.0000	$P_3$
7	0.9903	1.0000	-1.0000	$P_2$
8	1.0000	0.8578	-1.0000	$P_1$
9	1.0000	0.9230	-1.0000	$P_1$

**5 Conclusion**

This paper proposed a distance measure and a cosine similarity measure between neutrosophic numbers. Then, the fault diagnosis method based on the cosine similarity measure was proposed and was applied to the misfire fault diagnosis of gasoline engines under neutrosophic number environment. The fault diagnosis results of the engine demonstrated the effectiveness and rationality of the proposed fault diagnosis method. This fault diagnosis method can not only determinate the main fault type of engines but also predict future fault trends according to the relation indices, and then it is simpler and easier than the fault diagnosis method based on extension theory. The method proposed in this paper extends existing fault diagnosis methods and provides a useful way for fault diagnoses of gasoline engines. In the future, the developed diagnosis method will be extended to other fault diagnoses, such as vibration faults of turbines, aircraft engines and gearboxes.

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