Neutrosophic Vague Set Theory

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Abstract

In 1993, Gau and Buehrer proposed the theory of vague sets as an extension of fuzzy set theory. Vague sets are regarded as a special case of context-dependent fuzzy sets. In 1995, Smarandache talked for the first time about neutrosophy, and he defined the neutrosophic set theory as a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. In this paper, we define the concept of a neutrosophic vague set as a combination of neutrosophic set and vague set. We also define and study the operations and properties of neutrosophic vague set and give some examples.

Keywords

Vague set, Neutrosophy, Neutrosophic set, Neutrosophic vague set.

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1 Introduction

Many scientists wish to find appropriate solutions to some mathematical problems that cannot be solved by traditional methods. These problems lie in the fact that traditional methods cannot solve the problems of uncertainty in economy, engineering, medicine, problems of decision-making, and others. There have been a great amount of research and applications in the literature concerning some special tools like probability theory, fuzzy set theory [13], rough set theory [19], vague set theory [18], intuitionistic fuzzy set theory [10, 12] and interval mathematics [11, 14].

Since Zadeh published his classical paper almost fifty years ago, fuzzy set theory has received more and more attention from researchers in a wide range of scientific areas, especially in the past few years.

The difference between a binary set and a fuzzy set is that in a "normal" set every element is either a member or a non-member of the set; it either has to be A or not A.

In a fuzzy set, an element can be a member of a set to some degree and at the same time a non-member of the same set to some degree. In classical set theory, the membership of elements in a set is assessed in binary terms: according to a bivalent condition, an element either belongs or does not belong to the set.

By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the closed unit interval [0, 1].

Fuzzy sets generalise classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the later only take values 0 or 1. Therefore, a fuzzy set A in an universe of discourse X is a function $A: X \rightarrow [0, 1]$, and usually this function is referred to as the membership function and denoted by $\mu_{A(x)}$.

The theory of vague sets was first proposed by Gau and Buehrer [18] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets.

A vague set is defined by a truth-membership function t_v and a false-membership function f_v, where $t_v(x)$ is a lower bound on the grade of membership of x derived from the evidence for x, and $f_v(x)$ is a lower bound on the negation of x derived from the evidence against x. The values of $t_v(x)$ and $f_v(x)$ are both defined on the closed interval $[0, 1]$ with each point in a basic set X, where $t_v(x) + f_v(x) \leq 1$.

For more information, see [1, 2, 3, 7, 15, 16, 19].

In 1995, Smarandache talked for the first time about neutrosophy, and in 1999 and 2005 [4, 6] defined the neutrosophic set theory, one of the most important new mathematical tools for handling problems involving imprecise, indeterminacy, and inconsistent data.

In this paper, we define the concept of a neutrosophic vague set as a combination of neutrosophic set and vague set. We also define and study the operations and properties of neutrosophic vague set and give examples.

2 Preliminaries

In this section, we recall some basic notions in vague set theory and neutrosophic set theory. Gau and Buehrer have introduced the following definitions concerning its operations, which will be useful to understand the subsequent discussion.

Definition 2.1 ([18]). Let x be a vague value, $x = [t_x, 1-f_x]$, where $t_x \in [0,1]$, $f_x \in [0,1]$, and $0 \le t_x \le 1-f_x \le 1$. If $t_x = 1$ and $f_x = 0$ (i.e., $x = [1,1]$), then x is called a unit vague value. If $t_x = 0$ and $f_x = 1$ (i.e., $x = [0,0]$), then x is called a zero vague value.

Definition 2.2 ([18]). Let x and y be two vague values, where $x = [t_x, 1-f_x]$ and $y = \left[t_y, 1-f_y\right]$. If $t_x = t_y$ and $f_x = f_y$, then vague values x and y are called equal (i.e. $[t_x, 1-f_x] = \left[t_y, 1-f_y\right]$).

Definition 2.3 ([18]). Let A be a vague set of the universe U. If $\forall u_i \in U$, $t_A(u_i) = 1$ and $f_A(u_i) = 0$, then A is called a unit vague set, where $1 \le i \le n$. If $\forall u_i \in U$, $t_A(u_i) = 0$ and $f_A(u_i) = 1$, then A is called a zero vague set, where $1 \le i \le n$.

Definition 2.4 ([18]). The complement of a vague set A is denoted by A^c and is defined by $\begin{array}{l} t_{A^c} = f_A, \\ 1-f_{A^c} = 1-t_A. \end{array}$

Definition 2.5 ([18]). Let A and B be two vague sets of the universe U. If $\forall u_i \in U$, $\left[t_A(u_i), 1-f_A(u_i)\right] = \left[t_B(u_i), 1-f_B(u_i)\right]$, then the vague set A and B are called equal, where $1 \le i \le n$.

Definition 2.6 ([18]). Let A and B be two vague sets of the universe U. If $\forall u_i \in U$, $t_A(u_i) \le t_B(u_i)$ and $1-f_A(u_i) \le 1-f_B(u_i)$, then the vague set A are included by B, denoted by $A \subseteq B$, where $1 \le i \le n$.

Definition 2.7 ([18]). The union of two vague sets A and B is a vague set C, written as $C = A \cup B$, whose truth-membership and false-membership functions are related to those of A and B by

$$t_C = max(t_A, t_B),\ 1-f_C = max(1-f_A, 1-f_B) = 1-min(f_A, f_B).$$

Definition 2.8 ([18]). The intersection of two vague sets A and B is a vague set C, written as $C = A \cap B$, whose truth-membership and false-membership functions are related to those of A and B by

$$t_C = min(t_A, t_B),\ 1 - f_C = min(1 - f_A, 1 - f_B) = 1 - max(f_A, f_B).$$

In the following, we recall some definitions related to neutrosophic set given by Smarandache. Smarandache defined neutrosophic set in the following way:

Definition 2.9 [6] A neutrosophic set A on the universe of discourse X is defined as

$$A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$$

where $T, I, F : X \rightarrow]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Smarandache explained his concept as it follows: "For example, neutrosophic logic is a generalization of the fuzzy logic. In neutrosophic logic a proposition is $T \equiv true$, $I \equiv indeterminate$, and $F \equiv false$. For example, let's analyze the following proposition: *Pakistan will win against India in the next soccer game*. This proposition can be $(0.6, 0.3, 0.1)$, which means that there is a possibility of $60\% \equiv$ that Pakistan wins, $30\% \equiv$ that Pakistan has a tie game, and $10\% \equiv$ that Pakistan looses in the next game vs. India."

Now we give a brief overview of concepts of neutrosophic set defined in [8, 5, 17]. Let S_1 and S_2 be two real standard or non-standard subsets, then

$$S_1 \oplus S_2 = \{x \mid x = s_1 + s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\},$$
$$\{1^+\} \oplus S_2 = \{x \mid x = 1^+ + s_2,\ s_2 \in S_2\},$$
$$S_1 \overline{\oplus} S_2 = \{x \mid x = s_1 - s_2,\ s_1 \in S_1 \text{ and } s_2 \in S_2\},$$
$$S_1 \odot S_2 = \{x \mid x = s_1 . s_2,\ s_1 \in S_1 \text{ and } s_2 \in S_2\},$$
$$\{1^+\} \overline{\oplus} S_2 = \{x \mid x = 1^+ - s_2,\ s_2 \in S_2\}.$$

Definition 2.10 (Containment) A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$, if and only if

$$\inf T_A(x) \leq \inf T_B(x),\ \sup T_A(x) \leq \sup T_B(x),$$

$$\inf I_A(x) \geq \inf I_B(x),\ \sup I_A(x) \geq \sup I_B(x),$$

$$\inf F_A(x) \geq \inf F_B(x),\ \sup F_A(x) \geq \sup F_B(x),\ \text{for all } x \in X.$$

Definition 2.11 The complement of a neutrosophic set A is denoted by $\overline{A}$ and is defined by

$$T_{\overline{A}}(x)=\{1^+\}\overline{\oplus}\,T_{\hat{A}}(x),$$

$$I_{\overline{A}}(x)=\{1^+\}\overline{\oplus}\,I_{\hat{A}}(x),$$

$$F_{\overline{A}}(x)=\{1^+\}\overline{\oplus}\,F_{\hat{A}}(x),\quad \text{for all } x\in X.$$

Definition 2.12 (Intersection) The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x)=T_A(x)\odot T_B(x),$$

$$I_C(x)=I_A(x)\odot I_B(x),$$

$$F_C(x)=F_A(x)\odot F_B(x),\quad \text{for all } x\in X.$$

Definition 2.11 (Union) The union of two neutrosophic sets A and B is a neutrosophic set C written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x)=T_A(x)\oplus T_B(x)\overline{\oplus}\,T_A(x)\odot T_B(x),$$

$$I_C(x)=I_A(x)\oplus I_B(x)\overline{\oplus}\,I_A(x)\odot I_B(x),$$

$$F_C(x)=F_A(x)\oplus F_B(x)\overline{\oplus}\,F_A(x)\odot F_B(x),\quad \text{for all } x\in X.$$

3 Neutrosophic Vague Set

A vague set over U is characterized by a truth-membership function t_v and a false-membership function f_v , $t_v : U \to [0,1]$ and $f_v : U \to [0,1]$ respectively where $t_v(u_i)$ is a lower bound on the grade of membership of u_i which is derived from the evidence for u_i , $f_v(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i and $t_v(u_i)+f_v(u_i)\le 1$. The grade of membership of u_i in the vague set is bounded to a subinterval $\left[t_v(u_i),1-f_v(u_i)\right]$ of $[0,1]$. The vague value $\left[t_v(u_i),1-f_v(u_i)\right]$ indicates that the exact grade of membership $\mu_v(u_i)$ of u_i maybe unknown, but it is bounded by $t_v(u_i) \le \mu_v(u_i) \le f_v(u_i)$ where $t_v(u_i) + f_v(u_i) \le 1$. Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets (N-sets) A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(u)$; $I_A(u)$ and $F_A(u)$ are real standard or nonstandard subsets of $[0,\ 1]$. It can be written as:

$$A = \{< u,\ (T_A(u),\ I_A(u),\ F_A(u))\ >: u \in U, T_A(u),\ I_A(u),\ F_A(u) \in [0,1]\}.$$

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so:

$$0 \le \sup T_A(u) + \sup I_A(u) + \sup F_A(u) \le 3.$$

By using the above information and by adding the restriction of vague set to neutrosophic set, we define the concept of neutrosophic vague set as it follows.

Definition 3.1 A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as

$$A_{NV} = \{< x, \hat{T}_{A_{NV}}(x), \hat{I}_{A_{NV}}(x), F_{A_{NV}}(x) >, x \in X\}$$

whose truth-membership, indeterminacy-membership and falsity-membership functions is defined as

$$\hat{T}_{A_{NV}}(x) = [T^-, T^+],\ \hat{I}_{A_{NV}}(x) = [I^-, I^+],\ \hat{F}_{A_{NV}}(x) = [F^-, F^+],$$

where

$$T^+ = 1 - F^-,\ F^+ = 1 - T^-, \text{ and}$$

$$^-0 \le T^- + I^- + F^- \le 2^+,$$

when X is continuous, a NVS A_{NV} can be written as

$$A_{NV} = \int_X < x, \hat{T}_{A_{NV}}(x), \hat{I}_{A_{NV}}(x), \hat{F}_{A_{NV}}(x) > / x,\ x \in X.$$

When X is discrete, a NVS A_{NV} can be written as

$$A_{NV} = \sum_{i=1}^{n} < x, \hat{T}_{A_{NV}}(x_i), \hat{I}_{A_{NV}}(x_i), \hat{F}_{A_{NV}}(x_i) > / x_i,\ x_i \in X.$$

In neutrosophic logic, a proposition is $T \equiv true$, $I \equiv indeterminate$, and $F \equiv false$ such that:

$$0 \le \sup T_{A_N}(u) + \sup I_{A_N}(u) + \sup F_{A_N}(u) \le 3.$$

Also, vague logic is a generalization of the fuzzy logic where a proposition is $T \equiv true$ and $F \equiv false$, such that: $t_v(u_i) + f_v(u_i) \leq 1$, he exact grade of membership $\mu_v(u_i)$ of u_i maybe unknown, but it is bounded by

$$t_v(u_i) \leq \mu_v(u_i) \leq f_v(u_i).$$

For example, let's analyze the Smarandache's proposition using our new concept: *Pakistan will win against India in the next soccer game*. This proposition can be as it follows:

$$\widehat{T}_{A_{NV}} = [0.6, 0.9],\ \widehat{I}_{A_{NV}} = [0.3, 0.4] \text{ and } \widehat{F}_{A_{NV}} = [0.4, 0.6],$$

which means that there is possibility of $60\%\ to\ 90\% \equiv$ that Pakistan wins, $30\%\ to\ 40\% \equiv$ that Pakistan has a tie game, and $40\%\ to\ 60\% \equiv$ that Pakistan looses in the next game vs. India.

Example 3.1 Let $U = \{u_1, u_2, u_3\}$ be a set of universe we define the NVS A_{NV} as follows:

$$A_{NV} = \left\{ \frac{u_1}{\langle [0.3, 0.5], [0.5, 0.5], [0.5, 0.7] \rangle}, \frac{u_2}{\langle [0.4, 0.7], [0.6, 0.6], [0.3, 0.6] \rangle}, \frac{u_3}{\langle [0.1, 0.5], [0.5, 0.5], [0.5, 0.9] \rangle} \right\}.$$

Definition 3.2 Let Ψ_{NV} be a NVS of the universe U where $\forall u_i \in U$,

$$\widehat{T}_{\Psi_{NV}}(x) = [1,1],\ \widehat{I}_{\Psi_{NV}}(x) = [0,0],\ \widehat{F}_{\Psi_{NV}}(x) = [0,0],$$

then Ψ_{NV} is called a unit NVS, where $1 \leq i \leq n$.

Let Φ_{NV} be a NVS of the universe U where $\forall u_i \in U$,

$$\widehat{T}_{\Phi_{NV}}(x) = [0,0],\ \widehat{I}_{\Phi_{NV}}(x) = [1,1],\ \widehat{F}_{\Phi_{NV}}(x) = [1,1],$$

then Φ_{NV} is called a zero NVS, where $1 \leq i \leq n$.

Definition 3.3 The complement of a NVS A_{NV} is denoted by A^c and is defined by

$$\widehat{T}^c_{A_{NV}}(x) = [1 - T^+, 1 - T^-],$$
$$\widehat{I}^c_{A_{NV}}(x) = [1 - I^+, 1 - I^-],$$
$$\widehat{F}^c_{A_{NV}}(x) = [1 - F^+, 1 - F^-],$$

Example 3.2 Considering *Example 3.1,* we have:

$$A^{c}{}_{NV}=\left\{\frac{u_1}{\langle[0.5,0.7],[0.5,0.5],[0.3,0.5]\rangle},\frac{u_2}{\langle[0.3,0.6],[0.4,0.4],[0.4,0.7]\rangle},\frac{u_3}{\langle[0.5,0.9],[0.5,0.5],[0.1,0.5]\rangle}\right\}.$$

Definition 3.5 Let A_{NV} and B_{NV} be two NVSs of the universe U . If $\forall u_i \in U$,

$$\hat{T}_{A_{NV}}(u_i)=\hat{T}_{B_{NV}}(u_i),\ \hat{I}_{A_{NV}}(u_i)=\hat{I}_{B_{NV}}(u_i) \text{ and } \hat{F}_{A_{NV}}(u_i)=\hat{F}_{B_{NV}}(u_i),$$

then the NVS A_{NV} and B_{NV} are called equal, where $1 \le i \le n$.

Definition 3.6 Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$,

$$\hat{T}_{A_{NV}}(u_i)\le\hat{T}_{B_{NV}}(u_i),\ \hat{I}_{A_{NV}}(u_i)\ge\hat{I}_{B_{NV}}(u_i) \text{ and } \hat{F}_{A_{NV}}(u_i)\ge\hat{F}_{B_{NV}}(u_i),$$

then the NVS A_{NV} are included by B_{NV} , denoted by $A_{NV}\subseteq B_{NV}$, where $1 \le i \le n$.

Definition 3.7 The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV} , written as $C_{NV}=A_{NV}\cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} by

$$\hat{T}_{C_{NV}}(x)=\left[max\left(\hat{T}^{-}_{A_{NVx}},\hat{T}^{-}_{B_{NVx}}\right),max\left(\hat{T}^{+}_{A_{NVx}},\hat{T}^{+}_{B_{NVx}}\right)\right],$$

$$\hat{I}_{C_{NV}}(x)=\left[min\left(\hat{I}^{-}_{A_{NVx}},\hat{I}^{-}_{B_{NVx}}\right),min\left(\hat{I}^{+}_{A_{NVx}},\hat{I}^{+}_{B_{NVx}}\right)\right],$$

$$\hat{F}_{C_{NV}}(x)=\left[min\left(\hat{F}^{-}_{A_{NVx}},\hat{F}^{-}_{B_{NVx}}\right),min\left(\hat{F}^{+}_{A_{NVx}},\hat{F}^{+}_{B_{NVx}}\right)\right].$$

Definition 3.8 The intersection of two NVSs A_{NV} and B_{NV} is a NVS H_{NV} , written as $H_{NV}=A_{NV}\cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} by

$$\hat{T}_{H_{NV}}(x)=\left[min\left(\hat{T}^{-}_{A_{NVx}},\hat{T}^{-}_{B_{NVx}}\right),min\left(\hat{T}^{+}_{A_{NVx}},\hat{T}^{+}_{B_{NVx}}\right)\right],$$

$$\hat{I}_{H_{NV}}(x)=\left[max\left(\hat{I}^{-}_{A_{NVx}},\hat{I}^{-}_{B_{NVx}}\right),max\left(\hat{I}^{+}_{A_{NVx}},\hat{I}^{+}_{B_{NVx}}\right)\right],$$

$$\hat{F}_{H_{NV}}(x)=\left[max\left(\hat{F}^{-}_{A_{NVx}},\hat{F}^{-}_{B_{NVx}}\right),max\left(\hat{F}^{+}_{A_{NVx}},\hat{F}^{+}_{B_{NVx}}\right)\right].$$

Example 3.3 Let $U=\{u_1,u_2,u_3\}$ be a set of universe and let NVS A_{NV} and B_{NV} define as follows:

$$A_{NV} = \left\{ \frac{u_1}{\langle [0.3,0.5],[0.7,0.8],[0.5,0.7] \rangle}, \frac{u_2}{\langle [0.4,0.7],[0.6,0.8],[0.3,0.6] \rangle}, \frac{u_3}{\langle [0.1,0.5],[0.3,0.6],[0.5,0.9] \rangle} \right\}.$$

$$B_{NV} = \left\{ \frac{u_1}{\langle [0.7,0.8],[0.3,0.5],[0.2,0.3] \rangle}, \frac{u_2}{\langle [0.2,0.4],[0.2,0.4],[0.6,0.8] \rangle}, \frac{u_3}{\langle [0.9,1],[0.6,0.7],[0,0.1] \rangle} \right\}.$$

Then we have $C_{NV} = A_{NV} \cup B_{NV}$ where

$$C_{NV} = \left\{ \frac{u_1}{\langle [0.7,0.8],[0.3,0.5],[0.2,0.3] \rangle}, \frac{u_2}{\langle [0.4,0.7],[0.2,0.4],[0.3,0.6] \rangle}, \frac{u_3}{\langle [0.9,1],[0.3,0.6],[0,0.1] \rangle} \right\}.$$

Moreover, we have $H_{NV} = A_{NV} \cap B_{NV}$ where

$$H_{NV} = \left\{ \frac{u_1}{\langle [0.3,0.5],[0.7,0.8],[0.5,0.7] \rangle}, \frac{u_2}{\langle [0.2,0.4],[0.6,0.8],[0.6,0.8] \rangle}, \frac{u_3}{\langle [0.1,0.5],[0.6,0.7],[0.5,0.9] \rangle} \right\}.$$

Theorem 3.1 Let P be the power set of all NVS defined in the universe X. Then $\langle P; \cup_{NV}, \cap_{NV} \rangle$ is a distributive lattice.

Proof Let A, B, C be the arbitrary NVSs defined on X. It is easy to verify that

$A \cap A = A, A \cup A = A$ (idempotency),

$A \cap B = B \cap A, A \cup B = B \cup A$ (commutativity),

$(A \cap B) \cap C = A \cap (B \cap C), (A \cup B) \cup C = A \cup (B \cup C)$ (associativity), and

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C), A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$ (distributivity).

4 Conclusion

In this paper, we have defined and studied the concept of a neutrosophic vague set, as well as its properties, and its operations, giving some examples.

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