WEP and SR on a global free fall grid used to derive gravitational relativistic corrections to GPS/GNSS clocks on the level of the training of standard GPS/GNSS engineers

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Abstract

Using frequency gauged clocks on a free fall grid we look at gravitational phenomena as they appear for observers on a stationary grid in a central field of gravity. With an approach based on Special Relativity, the Weak Equivalence Principle and Newton's gravitational potential we derive first order correct expressions for the gravitational red shift of stationary clocks and of satellites. We also derive a second order correction of a satellite's clock frequency, related to the geodetic precession. In the derivation of the apparent velocity of light in a field of gravity, a Lorentz symmetry breaking occurred. The derived changed radial velocity of light is at the basis of the Shapiro delay and the gravitational index of refraction, so phenomena connected to the curvature of the metric. The advantage of the free fall grid SR-WEP approach is that it is less advanced and thus far less complicated as compared to the GR approach, but still accurate enough for all GPS purposes for the next few decades. Also important: our approach is never in conflict with GR because we do not introduce additional axioms to the ones already in use in GR. We only use less axioms. For GPS engineers, our approach will give a deeper insight into problems concerning clock synchronization in a grid around the earth, without using the complex mathematics needed in GR. The free fall grid SR-WEP approach can be taught to GPS engineers in an achievable and economically affordable time. It will considerably reduce the communication gap between those engineers and the GR experts.

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I. INTRODUCTION: GPS ENGINEERS AND 'THIS STRANGE THEORY'

In standard courses or workshops that treat GPS relevant relativistic physics build up the Special Relativity corrections from first principles but postulate without fundamental justification the necessary results of General Relativity. A nice example of this is the workshop paper by Haustein from 2009 [1]. From a practical point of view, this is fully justified because Special Relativity can be explained in a rather short period of time to engineers with a thorough training in classical Newtonian and Maxwellian physics but with General Relativity the same is simply not possible. It would demand to much of the time and often also of the intellectual capacities of those engineers to master the involved mathematics and its application in the physics of General Relativity. As a result, the GPS engineers simply have to believe the approximations derived from the results as these are presented to them by General Relativity experts. This is an inevitable and uncomfortable situation. Some engineers react by trying to formulate their own theory of gravity within the context of Newtonian and Maxwellian physics [2].

An anecdote from Taylor and Wheelers course book on General Relativity illustrates the situation [3]:

> Launching the Global Positioning System was an immense military and civilian effort. Most participants were not skilled in general relativity and, indeed, wondered if the academic advisors were right about this strange theory. As one later publication put it: *There was considerable uncertainty among Air Force and contractor personnel designing and building the system whether these effects were being correctly handled, and even, on the part of some, whether the effects were real.* The GPS prototype satellite called Navigation Technological Satellite 2 (NTS-2) was launched into a near-12-hour circular orbit on June 23, 1977, with its single atomic clock initially set (on Earth) to run at the same rate as Earth clocks. However, it had a general relativity on/off switch, leading to two possible modes of operation. In the first mode, with the switch set to off, the satellite clock was simply left to run at the rate at which it had been set on Earth. It ran in this condition for 20 days. The satellite clock drifted, relative to Earth clocks, at the rate predicted by general relativity, well within the accuracy capabilities of the orbiting clock. The NTS-2 satellite validated the general relativity results,

so the on/off switch was turned to on. This changed the satellite clock rate to a pre-arranged 38 700 nanoseconds per day slower than that of the Earth clock, also set before launch when the two clocks were side by side on Earth. Then the gravitational blue shift of the signal from an orbiting overhead satellite raised the frequency of the signal received on Earth to that of the Earth clocks. Since then, every GPS satellite goes into orbit with general relativity built into its design and construction. No more general relativity on/off switch!

In this paper we will derive all the relativistic results needed in the GPS field using Special Relativity and the weak equivalence principle or WEP only, using the environment of a grid of frequency gauged clocks in different reference frames. We will also derive results not directly needed for an accurate GPS but we do this in such a way that the engineer can judge for himself if he is allowed to ignore those effects of gravity on his system.

It is generally accepted that adding WEP to Special Relativity allows the derivation of some of the results later on more fundamentally derived by General Relativity, without causing a conflict of first principles. Einstein went through this path and Schiff in 1960 used the same method [4]. Although WEP and SR together can only deal with some of the weak field results and then mostly in their first order approximation, it is a better approach than just postulating results by GR experts to GPS engineers.

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In this paper we will use a free fall grid of frequency gauged standard clocks to derive the relativistic effects used in GPS due to the velocity and the high of the GPS satellites. We will present second order corrections to these effects, allowing an estimate of how many years these second order effects can still be ignored. We will derive the apparent reduced radial speed of light in a field of gravity, giving rise to the Shapiro time delay and the bending of light. GPS engineers can then calculate the magnitude of these effects and decide if or if not to ignore these effects. All this will be done in the framework of WEP and SR without coming in conflict with GR. Experiencing the differences between SR on the one hand and SR plus WEP on the other hand can even work as a preparation of the mindset towards GR. In my view, it will increase the acceptance by the GPS engineers of the results as given by the GR experts. All the effects we derive coïncide with results from chapter four “Applications of the linear approximation” in the college textbook of Ohanian and Ruffini [5]. Our approach doesn't reach beyond that, which should be no surprise as it is based upon

SR and WEP only. GR in its full approach goes a lot further. But our approach is still way more basic and closer to GPS reasoning than the applications of the linear approximation chapter. Our use of a free fall grid of frequency gauged clocks is very similar to the spirit of all global positioning systems:

> Alternative global navigation systems such as GLONASS, GALILEO, and BEIDOU are all based on concepts of clock synchronization based on a locally inertial reference system freely falling along with the earth [6]

In a certain sense, we use a variant of such a GPS grid to derive relativistic corrections due to velocity and gravity.

II. USING FREQUENCY GAUGED CLOCKS ON A FREE FALL GRID

A. A stationary free space frequency gauged rectangular grid of clocks on Einstein Elevators

The most basis assumption made in this paper is the existence of frequency gauged clocks that can emit and absorb frequency gauged photons. The frequency gauged clocks use the atomic standard of time that is based on a transition between two energy levels of an atom. The frequency gauged photon is emitted or absorbed in this transition process. All clocks and photons in the paper are assumed to be frequency gauged using an equivalent of the 2006 SI standard:

> The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. It follows that the hyperfine splitting in the ground state of the caesium 133 atom is exactly 9 192 631 770 hertz, (hfs Cs) = 9 192 631 770 Hz. [7]

The highest relative accuracy of atomic clocks at the time has a fractional frequency inaccuracy of 10^{-17} [9]. Theoretically, the SI procedure could be applied at this relative accuracy, so all our clocks and photons can in principle be frequency gauged to this accuracy. And because the whole paper is about frequencies, the clocks do not have to be gauged to an absolute flow of time. Only time differences or intervals matter. On a stationary Euclidean grid of Einstein Elevators, all equipped with such a clock, the elevators are capable of continuously sending and receiving photons from other Einstein Elevators on the grid. As long as all stationary free space Einstein Elevator clocks can absorb the photons emitted by all the neighbouring clocks, the grid is practically frequency gauged.

Einstein Elevators are locally Lorentz invariant (LLI) and locally position invariant (LPI) and they are small enough to be able to neglect space variations in possible fields of gravity inside the elevators [8]. Inside Einstein Elevators, Special Relativity is valid and possible fields of gravity can be interpreted as accelerations in free space, due to the homogeneity of the field inside the small elevator. By using a grid of such elevators, we go from local inside the elevator to global on the grid. The grid itself is by no means an Einstein Elevator on a larger scale. The grid is a global system.

Assume that we have a grid of frequency gauged clocks inside Einstein Elevators and a set of separate clocks A and B that are not on this grid but move inside Einstein Elevators through this grid, see fig.(1). From the fact that clocks A and B aren't capable of absorbing

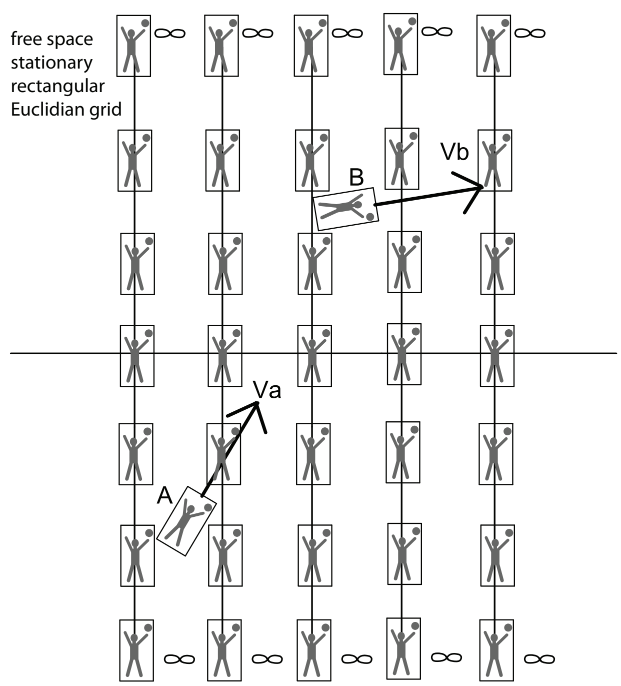

FIG. 1. Einstein Elevators in free space on a rectangular Euclidean grid. The two randomly directed Einstein elevators A and B each have a constant velocity v_a and v_b relative to this grid.

the photons emitted by the stationary clocks on the grid, we know that these clocks A and B are operating on a different frequency, although they apply the same standard clocks.

Randomly moving standard clocks aren't frequency gauged.

Suppose we do not know how the clocks A and B are ticking relative to each other but we do have a procedure of how to relate the frequencies of the individual clocks A and B to the grid. Then we are also able to compare the clocks A and B relative to each other, as seen from the perspective of the grid. So let the frequency of all the clocks on our grid be ν_g and let there be the two clocks A and B that are not on our grid and have frequencies ν_a and ν_b. Suppose we know how to relate the frequency of clocks on the grid to the frequencies of clocks A and B as in

$$\frac{\Delta\nu_{ag}}{\nu_a} = \frac{\nu_a - \nu_g}{\nu_a} = 1 - \frac{\nu_g}{\nu_a} \tag{1}$$

and

$$\frac{\Delta\nu_{bg}}{\nu_b} = \frac{\nu_b - \nu_g}{\nu_b} = 1 - \frac{\nu_g}{\nu_b}. \tag{2}$$

Then we also know how to relate the frequency of clock A directly to the one of clock B, as deduced from the grid perspective. We have

$$\begin{aligned} \frac{\Delta\nu_{ag}}{\nu_a} - \frac{\Delta\nu_{bg}}{\nu_b} &= \frac{\nu_a - \nu_g}{\nu_a} - \frac{\nu_b - \nu_g}{\nu_b} = \frac{\nu_b(\nu_a - \nu_g) - \nu_a(\nu_b - \nu_g)}{\nu_a\nu_b} \\ &= \frac{\nu_b\nu_a - \nu_b\nu_g - \nu_a\nu_b + \nu_a\nu_g}{\nu_a\nu_b} = \frac{\nu_a - \nu_b}{\nu_a} \cdot \frac{\nu_g}{\nu_b} = \frac{\Delta\nu_{ab}}{\nu_a} \cdot \frac{\nu_g}{\nu_b}, \end{aligned} \tag{3}$$

leading to the general formula to relate the frequency shifts of two clocks relative to each other using the frequency shifts of each of these clocks relative to the frequency of clocks on a grid of frequency gauged clocks:

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\frac{\Delta\nu_{ag}}{\nu_a} - \frac{\Delta\nu_{bg}}{\nu_b}}{\frac{\nu_g}{\nu_b}}. \tag{4}$$

Next, imagine that we know how to Lorentz boost clocks from the grid to a position at rest right next to the off-the-grid clocks A and B with frequencies ν_a and ν_b. For such a Lorentz boost, we need to know the velocities v_a and v_b of the elevators A and B relative to the grid and then the Lorentz boost factors γ_a and γ_b can be calculated using

$$\gamma_a = \frac{1}{\sqrt{1 - \frac{v_a^2}{c^2}}}. \tag{5}$$

From Special Relativity we then have

$$\nu_a = \frac{1}{\gamma_a}\nu_g \tag{6}$$

leading to

$$\frac{\Delta\nu_{ag}}{\nu_a} = \frac{\nu_a - \nu_g}{\nu_a} = 1 - \frac{\nu_g}{\nu_a} = 1 - \gamma_a. \tag{7}$$

For the clock B we have equal equations relative to our grid. We insert this Lorentz boost knowledge into Eqn.(4) to get

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\frac{\Delta\nu_{ag}}{\nu_a} - \frac{\Delta\nu_{bg}}{\nu_b}}{\frac{\nu_g}{\nu_b}} = \frac{(1-\gamma_a) - (1-\gamma_b)}{\gamma_b} = \frac{\gamma_b - \gamma_a}{\gamma_b}. \tag{8}$$

Of course Eqn.(6) for clocks A and B can be used to arrive faster at the same result as

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\nu_a - \nu_b}{\nu_a} = \frac{\frac{1}{\gamma_a}\nu_g - \frac{1}{\gamma_b}\nu_g}{\frac{1}{\gamma_a}\nu_g} = \frac{\frac{1}{\gamma_a} - \frac{1}{\gamma_b}}{\frac{1}{\gamma_a}} = \frac{\gamma_b - \gamma_a}{\gamma_b}. \tag{9}$$

In short, the elementary result that will be used in this paper is

$$\boxed{\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\gamma_b - \gamma_a}{\gamma_b}} \tag{10}$$

with Lorentz boosts γ_a and γ_b both relative to the grid. All we need to use eqn.(10) is a frequency gauged grid of standard clocks on the one hand and two non-gauged standard clocks with known Lorentz boosts relative to this grid on the other hand.

If, from the perspective of the grid, we know the relativistic kinetic energy of clocks A and B, then the Lorentz boost connection between these clocks and the grid is also known through the SR definition

$$U_k = (\gamma - 1)U_0 \tag{11}$$

with rest energy $U_0 = m_0c^2$. Then it is not to difficult to calculate the frequency shift between clocks A and B, as seen from the perspective of the grid.

B. From a stationary free space frequency gauged central grid of clocks on Einstein Elevators to a grid around a central mass M

This procedure also works using a central grid in free space, see fig.(2). This is the basic grid used by the GPS, and A and B can be satellites, airplanes, cars or shoppers with a smart-phone searching a specific store. But once we add a central mass M to this grid, see fig.(3), clocks at a different radius from the center of M cannot absorb each others photons any more so the system cases to be a frequency gauged grid. This means that Special Relativity cannot be used on this grid around a central mass, because in Special Relativity only velocity can influence the frequency of standard clocks and these standard clocks are all stationary.

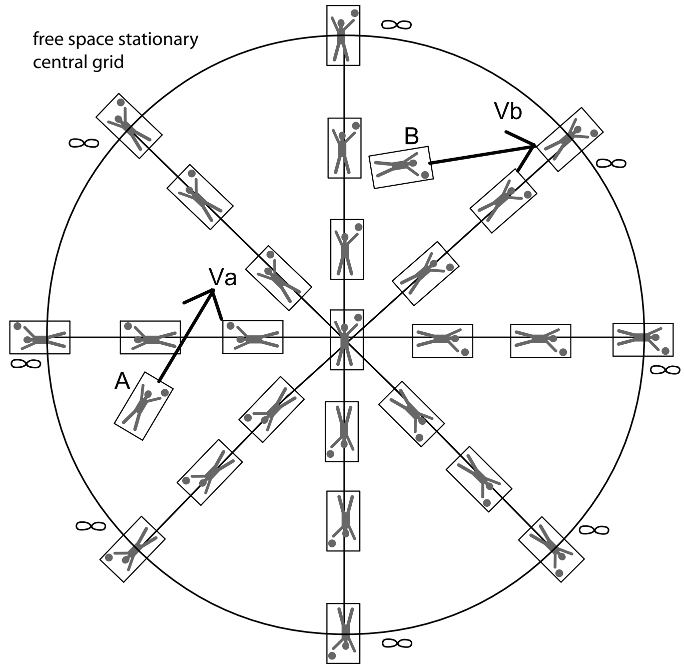

FIG. 2. Einstein Elevators in free space on a central Euclidean grid with two randomly directed Einstein elevators A and B. Such a grid is used by the GPS.

So in a stationary grid with a central mass M, we locally have Einstein Elevators in which Special Relativity can be applied but we cannot use Special Relativity to connect the standard clocks on the Einstein Elevators to each other on the global scale. Then we are not able to relate the frequencies of the standard clocks on the Einstein Elevator Satellite S and Einstein Elevator Earth ground Station E to our grid. Special Relativity brought the downfall of the Newtonian absolute static space and the separate static time, due to the velocity dependence of the frequency of standard clocks, resulting from the constancy of the speed of photons. And now, gravity causes the downfall of Special Relativity, on the global scale.

Of course, GPS engineers can choose to neglect gravity influences and treat the stationary grid around the earth as a Special Relativity frequency gauged grid, but that will greatly reduce the accuracy of the GPS. In practice however, GPS systems use an Euclidean or Newtonian stationary grid in free space attached to the center of the earth as in fig.(2) and then apply corrections due to for example rotation (Sagnac effect), velocity (Special Rela-

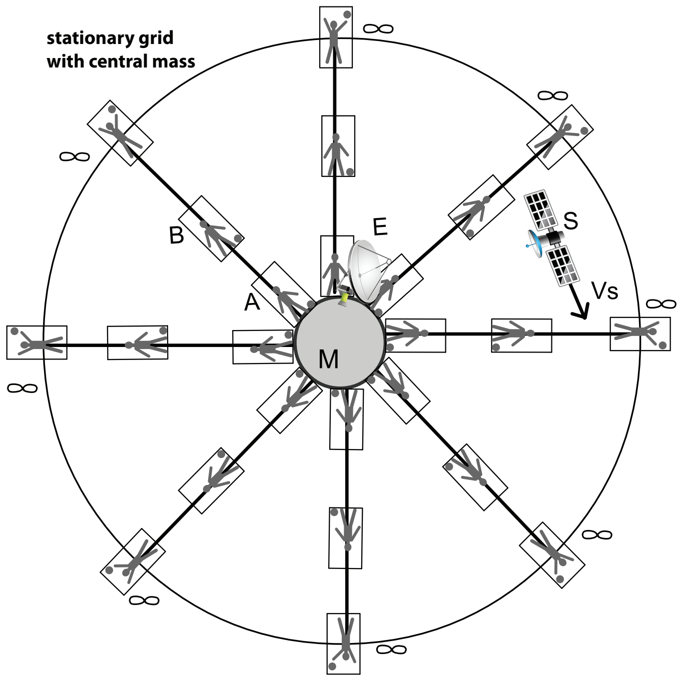

FIG. 3. Einstein Elevators in a stationary grid around a central mass M, including a GPS satellite S and an GPS earthbound station E.

tivity) and gravity (General Relativity) [10]. In this paper, we try to approach the gravity problem relative to GPS accuracy without General Relativity mathematics/geometrics but with use of some of the axioms of Einstein's theory of gravity.

C. Introducing the Universality of Free Fall and a Free Fall Grid

In order to deal with gravity, Special Relativity must be enhanced with a new principle or axiom. This will be the experimental peculiarity of free fall turned into a universal principle as the weak equivalence principle or WEP. According to Ohanian, "the weak principle asserts that *in a given gravitational field all test particles of the same initial velocity fall with the same acceleration*". [11] In such a free-fall, like in an Einstein elevator, we assume the following statement made by Nobili et.al. in 2013:

> Inside a freely falling elevator, as long as the field is uniform (locally), they would be subject to zero total force, which is equivalent to being inside an elevator at rest in empty space (or moving with uniform velocity), in which case there would be no frequency shift. [12]

This implies that an atomic clock placed in an Einstein Elevator at rest in infinity and then set on a free fall trajectory towards a central mass M would all the way down to M remain at the same initial rest system frequency. And according to Will, tests or the WEP have reached accuracies comparable to the accuracy of atomic clocks:

> The Eöt-Wash experiments carried out at the University of Washington used a sophisticated torsion balance tray to compare the accelerations of various materials toward local topography on Earth, movable laboratory masses, the Sun and the galaxy, and have reached levels of $3 \cdot 10^{-13}$. [13]

So we will assume that the atomic clocks in free fall "Einstein Elevators" can in principle remain frequency gauged to a relative accuracy of around 10^{-13}. In this way we can, in principle, establish a free fall grid of highly accurate frequency gauged clocks capable of emitting highly accurate frequency gauged photons. As a consequence of the weak equivalence principle, the laws of Special Relativity remain valid in the Einstein Elevator on a free fall trajectory from infinity towards a central mass M.

> With the Weak Equivalence Principle Einstein has moved from Newton's concept of one global reference frame with gravitational forces and the UFF, to many free falling local frames without gravitational forces.[14]

In this paper we will not use the Strong Equivalence Principle (SEP), understood as the equivalence for an observer in an Einstein Elevator between being at rest in a field of gravity and being in a state of constant acceleration in free space, as a principle on which to construct a global grid. In Schild's short 1960 expression, SEP means that "acceleration is equivalent to a gravitational field". [15] The fact that we do not use the SEP to construct a global grid doesn't imply that we criticize it, it only means that we don't need it in this paper, because we will build a global grid using the free fall weak equivalence principle WEP. General Relativity's geometry approach is based on the SEP to create global grids as for example the Schwartzschild Metric. A global grid based on the WEP can only be a preliminary approach to the problem of gravity, with the General Relativity's SEP geometrification as a full and universal approach. But for GPS purposes, a grid based on the WEP will be sufficient and at the same time introduce the basic circumstances that motivated Einstein to search for a geometrification based on the SEP outside Special Relativity and Newtonian Physics.

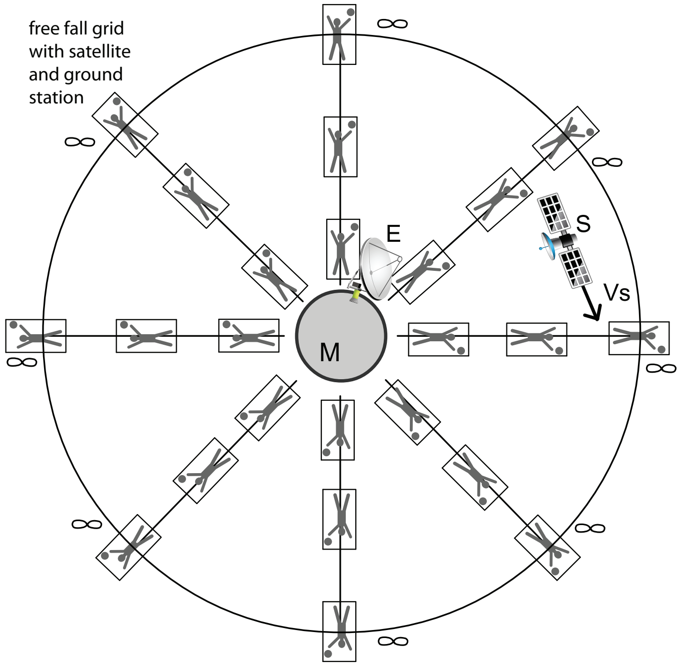

FIG. 4. Elevators in free fall on a global free fall grid, FFG, on which Special Relativity is valid. A GPS satellite in orbit around M and a stationary GPS ground station are also depicted.

We use the universality of free fall and the weak equivalence principle to define a grid of frequency gauged clocks on a free fall grid around a central mass M starting at rest in infinity and stretching all the way to just above the surface of this central mass, see fig.(4). On this grid we have an infinite number of Einstein Elevators in perfect free fall. All the clocks at rest in the Einstein Elevators on this free fall grid (FFG) have rest system frequency $\nu_g = \nu_0 = \nu_\infty$. The free fall of all the elevators on our free fall grid started at rest in infinity, ensuring a shared starting frequency ν_0. Due to the experimental fact of the weak equivalence principle, our clocks on free fall trajectories towards M do not feel any acceleration and thus remain all the way down frequency gauged to the clocks at rest in infinity, because without acceleration there is no Lorentz boost and without Lorentz boost the clock frequency will not change. Thus our FFG of figure (5) constitutes a perfect example of a grid of frequency gauged clocks. On this global grid, Special Relativity remains valid.

On this free fall grid, all the clocks are frequency gauged and so all the clocks on the grid can absorb photons emitted by all the other clocks on the grid, although the velocity of the clocks varies with high in the perspective of observers who are stationary relative to the central mass M. Photons on this grid do not fall in or work themselves out of a field of gravity, photons move through this grid with constant frequency and velocity as if in gravity free space. From the perspective of this grid gravity does not exist and the velocity of light is a constant in all directions. Inside these Einstein Elevators it is impossible to observe a difference between a central stationary grid in gravity free space and the free fall grid. Until they crash on the central mass M of course. The difference between the free space central grid and the free fall grid is the addition of gravity and WEP to Special Relativity.

This free fall grid however is not very practical in performing scientific experiments. We quote Rohrlich, who started a 1963 paper on the principle of equivalence with the words:

> Unfortunately, laboratory experiments are not usually performed in falling elevators. (Footnote 1) They are carried out in reference frames which are not inertial, but which are supported in a static gravitational field. Footnote 1: According to general relativity, the special theory of relativity is valid only locally in freely falling reference frames. [16]

Although Special Relativity is valid only locally in freely falling reference frames, with WEP as an extra universal axiom added to Special Relativity a frequency gauged grid of standard clocks can be established on these Einstein Elevators, exactly as in free space Special Relativity circumstances. This grid is not a stationary one but its standard clocks are still frequency gauged to the 'at-infinity' clock frequency as in the central stationary grid in free space. The fact that clocks on this grid with different relative velocities still remain frequency gauged and can absorb each others photons implies that we are beyond Special Relativity on the global scale, although still within Special Relativity on the local scale. As stated already, the difference is made by gravity and the subsequent use of the WEP as an additional axiom.

Most scientific experiments are carried out on a stationary grid in a static gravitational field. We position clocks A and B in Einstein Elevators on such a Stationary Grid (SG), so at rest at a definite radial distance from the center of M, see fig.(3). Observer A is closer to the central mass M than observer B. The observers A and B feel the pull of gravity induced acceleration on their clocks. As they try to exchange photons, they find out that the received photons are not being absorbed, meaning that the photons are frequency shifted relative to their own clocks, although they were send by a similar standard clock at the position of the other one. They conclude that gravity is negatively influencing the procedure of establishing a frequency gauged grid. When they bring their clocks next to each other, either by A going to B or B going to A, the clocks are again absorbing each others photons, thus ensuring the frequency gauged original situation. But as soon as they move up or down again with their clocks, photons case to be absorbed and they cannot achieve a frequency gauged global grid. Whether gravity is influencing the photons or influencing the clocks or both remains an open question for the observers A and B. This is the situation of the experiment performed by Pound and Rebka in 1960 [17].

D. The relative redshift of clocks on the SG, from a Free Fall Grid perspective

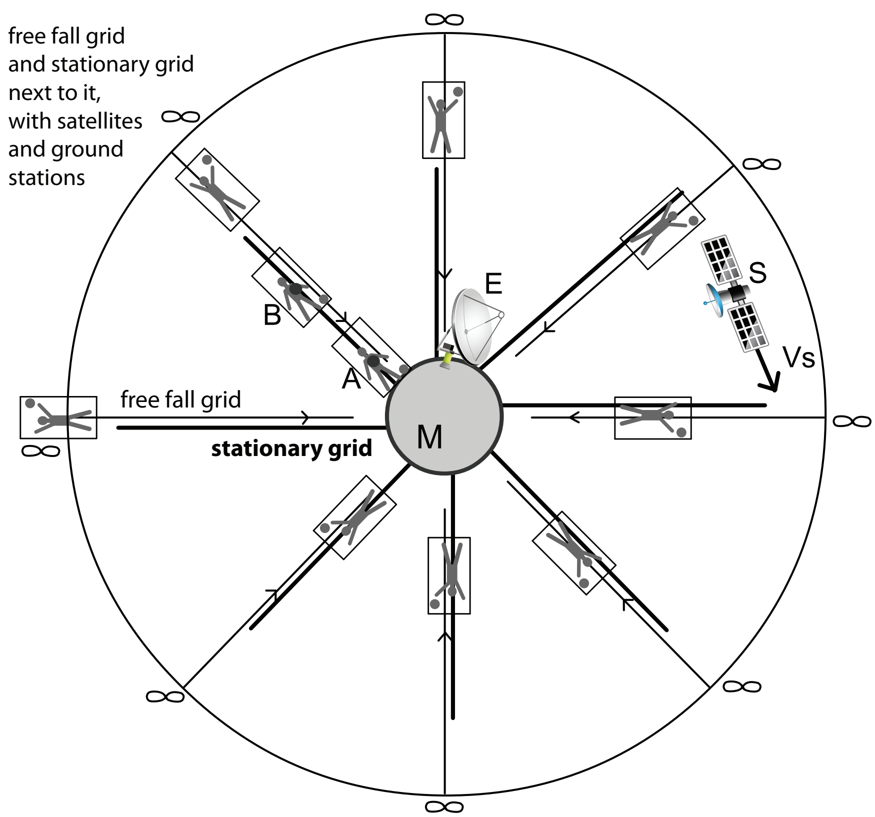

FIG. 5. Elevators in free fall on he FFG and stationary Einstein Elevators A and B and the GPS ground station E on the SG, with an orbiting satellite S on neither grid.

In order to bring some clarity to this situation we insert a free fall grid FFG onto this stationary grid with observers A and B, see fig.(5). Using the conservation of energy and Special Relativity we can relate the clocks A and B on the SG to clocks on the FFG.

Relative to the stationary clocks the Einstein Elevator in free fall has relativistic kinetic $U_k = (\gamma - 1)U_0$ and potential energy $U_\phi = m_0\Phi$ which, due to energy conservation, relate as $U_k = -U_\phi$ because the free fall started at infinity. This results in the Lorentz boost connection between a locally passing by elevator on the FFG and a clock on the SG as

$$\gamma_\phi = 1 - \frac{\Phi}{c^2} \tag{12}$$

A passing by elevator on the FFG can always position or launch a clock from his elevator next to a clock on the SG by such a Lorentz boost. This assures the relations between clocks on the SG and clocks on the FFG, with the latter functioning as a grid of frequency gauged clocks.

A clock at rest in infinity, so on the FFG, then has a frequency shift relative to the clock A on the SG given by

$$\frac{\Delta\nu_{ag}}{\nu_a} = 1 - \gamma_a = \frac{\Phi_a}{c^2} = -\frac{GM}{R_a c^2} < 0 \tag{13}$$

so clock A stationary on the SG at A has a lower frequency than a clock stationary on the FFG at infinity. But being on the FFG at infinity is equivalent to being at the SG at infinity, both are at rest in a zero gravity environment.

The clock at B, located higher in the field, has a similar frequency shift relative to the FFG, with R_b replacing R_a, and thus the frequency shift of clock B relative to clock A is given by inserting eqn(12) for A and B in eqn.(10):

$$\boxed{\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\gamma_b - \gamma_a}{\gamma_b}} = \frac{(1 - \frac{\Phi_b}{c^2}) - (1 - \frac{\Phi_a}{c^2})}{1 - \frac{\Phi_b}{c^2}} = \frac{\frac{\Phi_a}{c^2} - \frac{\Phi_b}{c^2}}{1 - \frac{\Phi_b}{c^2}} = \frac{\frac{\Delta\Phi_{ab}}{c^2}}{1 - \frac{\Phi_b}{c^2}} \approx \frac{\Delta\Phi_{ab}}{c^2}. \tag{14}$$

If we insert Newton's expression for the gravitational potential in Φ we get

$$\Delta\Phi_{ab} = \Phi_a - \Phi_b = -\frac{GM}{R_a} + \frac{GM}{R_b} = GM\left(\frac{R_a - R_b}{R_a R_b}\right) = -\frac{GM}{R_a R_b}h \tag{15}$$

When clocks A and B are close to each other relative to the distance R to the centre of M, we get

$$\frac{\Delta\nu_{ab}}{\nu_a} \approx \frac{\Delta\Phi_{ab}}{c^2} \approx -\frac{gh}{c^2} < 0 \tag{16}$$

with h as the magnitude of the distance between the clocks A and B. As a result, the frequency of the clock at A will be lower than the frequency of the clock B who is positioned less deep in the gravitational field than clock A. At least, from the perspective of the Free Fall Grid. This matches the result obtained by Pound and Rebka in 1959 and the first order prediction of General Relativity.

E. Relative redshift of photons exchanged on the SG

Now this has been formulated relative to the FFG and using clocks only. If clocks A and B exchange photons, these photons will travel, in the perspective of the observers on the FFG, with velocity c through gravity free space, due to free fall, in between the clocks A and B. According to the FFG observers, the frequency of these photons will not change during the voyage through the space in between A and B and the perceived relative frequency shift is solely due to the different rates of the clocks used to send, absorb or observe these photons.

In the perspective of the stationary observers A and B on the SG however, things might look as if these photons travel into, from B to A, or out of, from A to B, the field of gravity, because the two clocks remain standard clocks. The occurring relative frequency shift of the exchanged photons can be interpreted in terms of energies using Planck's constant in $U = h\nu$ and the apparent Compton mass of photons $m_c c^2 = h\nu$ as

$$\frac{h\Delta\nu_{ab}}{h\nu_a} \approx \frac{\Delta m_c \Delta\Phi_{ab}}{m_a c^2} \approx \frac{gy\Delta m}{m_a c^2} \tag{17}$$

or as

$$h\Delta\nu_{ab} = gy\Delta m, \tag{18}$$

with the relative high now given by the variable y. So photons traveling out of the field from A to B gain gravitational energy and lose photonic energy in the same rate, with, in absolute terms, $\Delta U_\phi = \Delta U_\nu$. For the SG observers who believe in the frequency stability of the standard clocks used, it is as if the field of gravity is performing work upon the apparent Compton mass of the photons traveling between A and B, blue-shifting them while falling into the field from B to A and red-shifting them when moving out of the field from A to B.

This interpretation of the field of gravity performing work on the photons moving in or out of the field has always been a matter of controversy. If one accepts that the clocks sending out these photons are frequency shifted themselves, and accepts the fact that the photons arrive with exactly this clock frequency shift at a higher location in the field, then it is the clocks and not the field that produces the effect on the photons. But a bundle of photons send from A towards B and then moving on uninterrupted towards C higher in the field will be shifted in between B and C with exactly the apparent gravitational energy loss of the photon's Compton mass determined by the high of C relative to B. And in between B and C, the photon has not been in contact with the clock in A. So for local stationary observers on the SG, the gravitational redshift of photons seems to be a localized effect of the gravitational potential on the photons.

Dicke in 1960 concluded that there might be two red shift effects.

> One would be interpreted in the usual way as a light propagation effect. The other, if it exists, would be interpreted as resulting from an intrinsic change in an atom with gravitational potential. [18]

In 1986, Clifford Will states the same dilemma as

> Do the intrinsic rates of the emitter and the receiver or of the clocks change, or is it the light signal that changes frequency during its flight? The answer is that it doesn't matter. Both descriptions are physically equivalent. Put differently there is no operational way to distinguish between the two descriptions. [19]

About one and a half decades later, Okun, Selivanov and Telegdi express the opinion that only one of the descriptions is the right on, and that the other one is the wrong explanation of the red shift of the photon

> On the one hand, the phenomenon is explained through the behavior of clocks which run faster the higher they are located in the potential, whereas the energy and frequency of the propagating photon do not change with height. The light thus appears to be red-shifted relative to the frequency of the clock. On the other hand, the phenomenon is alternatively discussed (even in some authoritative texts) in terms of an energy loss of a photon as it overcomes the gravitational attraction of the massive body. This second approach operates with notions such as the "gravitational mass" or the "potential energy" of a photon and we assert that it is misleading. [22]

So, from 1960 to 2000, the 'normal' explanation of the red shift of photons has shifted almost 180 degrees. In our perspective, the two interpretations are not either or related. From the viewpoint of observers on the FFG the photons are not influenced by gravity, only the clocks in A and B are. On the SG, the influence of the gravity potential on the frequency of photons seems the most natural interpretation of the photon redshift, with a firm believe in the stability of the standard clocks, so we agree with Clifford Will. But on the SG, things will become even more complicated as we focus on the velocity of the photons in the perspective of the SG observers.

An interesting anecdote relates this issue directly to the originally design of the GPS. We quote from Taylor:

> An historical aside: Carroll O. Alley, a consultant to the original GPS project, had a hard time convincing the designers not to apply twice the correction given in (13): first to account for the different rate of time advance on wristwatches located at different altitudes and second to allow for the gravitational blue shift in frequency for the signal sent downward from satellite to Earth. There is only one correction; moreover there is no way to identify uniquely the cause of this correction [3].

It is not strange that Alley had a hard time to convince the GPS engineers, when the general relativity experts themselves are still debating the question as for the cause of the shift: falling photons or slower clocks emitting the photons. My hope is that the perspectives of the different grids used in this paper will clarify the problem also for the engineers: it is a matter of choosing the grid from which perspective to study the problem and then to carefully avoid mixing the arguments derived from different grid-perspectives in the way to the derivation of the conclusion.

The only logical way to conclude that photons shift their frequency because they fall in or work themselves out of a field of gravity is by assuming that the standard clocks are unaffected by the field. The only perspective in which photons unambiguously do not shift frequency while moving in or out a field of gravity is the free fall grid perspective. That is the only grid where standard clocks, and photons alike, are not affected by the field. Mixing these perspectives lies at the source of the debate regarding falling photons. A similar issue arises regarding the speed of photons 'falling' in or 'working themselves out' of a central field of gravity. This issue will be addressed later in this paper.

F. The FFG link between an orbiting satellite and Earth

In figure (5) we also depicted a GPS ground station E and a GPS satellite S. In this section we will use our method to derive the relative frequency shift between the standard clock the satellite S compared to the standard clock on the ground station E. In order to do this using eqn.(10), we need the gamma factors of both the ground station E and the orbiting satellite S relative to our free fall grid. We already used the first gamma factor in the previous derivation: $\gamma_e = 1 - \frac{\Phi_e}{c^2}$. For the second gamma factor I will use a result already arrived at in a previous paper. In [21] I used hyperbolic relativity to derive the Lorentz boost connection between an observer locally passing by on a free fall grid and an orbiting satellite. It was shown that two successive boosts could launch a satellite from the FFG elevator in a stable orbit around M. The first boost gave the satellite an escape amount of kinetic energy U_{esc} relative to the free fall elevator and the second boost gave it an orbital kinetic energy U_{orb}. Using relativistic kinetic energy U_k and the conservation of energy and the energy formulation of the virial theorem we get

$$\gamma_{esc} = 1 - \frac{\Phi}{c^2} \tag{19}$$

from the conservation of energy and from the virial theorem we get

$$\gamma_{orb} = 1 - \frac{\Phi}{2c^2}. \tag{20}$$

Under the condition that the two boosts are perpendicular relative to each other this results in the Lorentz boost connection between the FFG and the satellite as

$$\gamma_{sat} = \gamma_{esc}\gamma_{orb} = \left(1 - \frac{\Phi}{c^2}\right)\left(1 - \frac{\Phi}{2c^2}\right) = 1 - \frac{3\Phi}{2c^2} + \frac{\Phi^2}{2c^4} \approx 1 - \frac{3\Phi}{2c^2}, \tag{21}$$

as has been shown, using standard hyperbolic Special Relativity, in [21].

The relative frequency shift between a stationary earth clock's ν_e and a satellite clock's ν_s is then given by inserting the gamma factors in eqn.(10), to get

$$\boxed{\frac{\Delta\nu_{es}}{\nu_e} = \frac{\gamma_s - \gamma_e}{\gamma_s}} = \frac{(1 - \frac{3\Phi_s}{2c^2}) - (1 - \frac{\Phi_e}{c^2})}{1 - \frac{3\Phi_s}{2c^2}} = \frac{\frac{\Phi_e}{c^2} - \frac{3\Phi_s}{2c^2}}{1 - \frac{3\Phi_s}{2c^2}} \tag{22}$$

We can use the further approximation $1 - \frac{3\Phi_s}{2c^2} \approx 1$ to get

$$\frac{\Delta\nu_{es}}{\nu_e} \approx \frac{\Phi_e}{c^2} - \frac{3\Phi_s}{2c^2} = \frac{\Phi_e}{c^2} - \frac{\Phi_s}{c^2} - \frac{\Phi_s}{2c^2} = \frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2} = \frac{\Delta U_{\phi,s}}{U_0} + \frac{U_{k,s}}{U_0} \tag{23}$$

where in the last step we inserted the rest mass of the satellite and used the expressions for the potential energy and the relativistic kinetic energy of the satellite relative to the stationary Earth observers.

The result

$$\frac{\Delta\nu_{es}}{\nu_e} \approx \frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2} = \frac{\Delta U_{\phi,s}}{U_0} + \frac{U_{k,s}}{U_0} \tag{24}$$

is the basis relativistic correction used in GPS clock frequencies, with the first as the gravity effect or gravitational potential correction and the second as the velocity effect or the correction due to Special Relativity [10].

G. Limitations of the FFG approach

Our approach based on the use of the free fall grid as a grid of perfectly frequency gauged clocks is not a fundamental theory but a semi-phenomenological approach with inherent limitations. The approach isn't stronger than the assumptions on which it is constructed, a global frequency gauged free fall grid based on the WEP and Special Relativity.

We can make a prediction based on our approach that we expect to be falsified, that should be falsified if General Relativity, with its SEP based geometry approach, is correct. According to our analysis we should have a gravitational redshift between two stationary clocks at different heights of

$$\frac{\Delta\nu_{ab}}{\nu_a} = \frac{\gamma_b - \gamma_a}{\gamma_b} = \frac{\frac{\Phi_a}{c^2} - \frac{\Phi_b}{c^2}}{1 - \frac{\Phi_b}{c^2}} = \frac{\frac{\Delta\Phi_{ab}}{c^2}}{1 - \frac{\Phi_b}{c^2}} \approx \left(1 + \frac{\Phi_b}{c^2}\right)\frac{\Delta\Phi_{ab}}{c^2} = (1+\alpha)\,\frac{\Delta\Phi_{ab}}{c^2}. \tag{25}$$

But according to General Relativity, the factor α in the last expression should be identical zero. In the Earth bound situation, $\alpha \approx 10^{-10}$ according to our analysis. Present day accuracy of this redshift goes to $\alpha < 10^{-6}$. Within some decades the accuracy of the stationary redshift measurements should reach the 10^{-10} relative accuracy and we expect the limitations of a *global* WEB based free fall grid to become apparent.

For the redshift of a satellite in orbit relative to a stationary ground station, we have the interesting

$$\frac{\Delta\nu_{es}}{\nu_e} \approx \frac{\frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2}}{1 - \frac{3\Phi_s}{2c^2}} \approx \left(1 + \frac{3\Phi_s}{2c^2}\right)\left(\frac{\Delta\Phi_{es}}{c^2} - \frac{\Phi_s}{2c^2}\right) = (1+\alpha)\left(\frac{\Delta U_{\phi,s}}{U_0} + \frac{U_{k,s}}{U_0}\right) \tag{26}$$

and in this case the α term could be interpreted as the de Sitter correction term in the redshift due to the curvature of the orbit of the satellite. In paper [21] we derived the geodetic precession or the de Sitter precession using the free fall grid approach as

$$\Omega_G = (\gamma_s - 1)\Omega_s \approx -\frac{3\Phi_s}{2c^2}\Omega_s \tag{27}$$

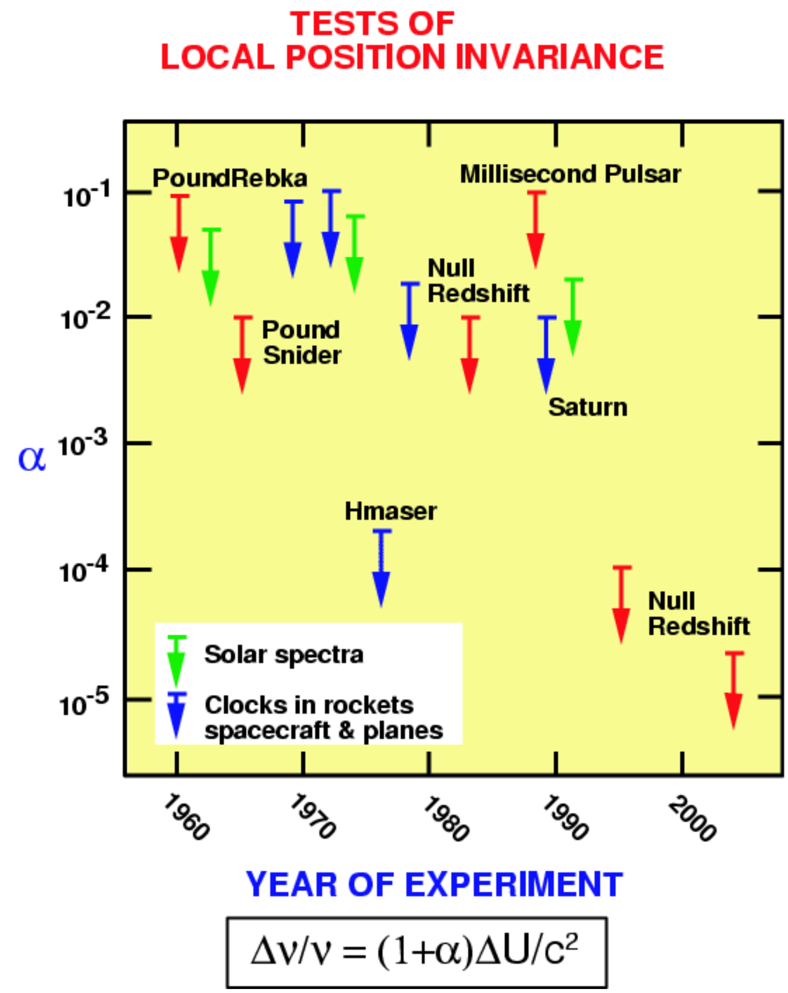

FIG. 6. Selected tests of local position invariance via gravitational redshift experiments, showing bounds on , which measures degree of deviation of redshift from the formula $\frac{\Delta\nu}{\nu} = \frac{\Delta\Phi}{c^2}$. In null redshift experiments, the bound is on the difference in between different kinds of clocks. From: www.livingreviews.org [13].

which makes the interpretation of the α term as a de Sitter correction for a satellite redshift consistent.

As for our assumptions, we used the weak equivalence principle and the related universality of free fall principle to create a *global* grid. These principles have been experimentally tested *locally* with a 10^{-12} relative accuracy. We also used the kinetic energy in its Special Relativity formulation as $U_k = (\gamma - 1)U_0$. Then we assumed the gravitational mass to be equal to the rest mass in our energy considerations, by using $U_\phi = m_0\Phi$, thus ignoring all gravitational self energy complications. We used Newton's gravitational scalar potential, where in more complex situations tensor potentials, metric tensors, are needed. We assumed the virial theorem for a satellite to remain valid in relativistic contexts, by keeping its energy formulation classical. We quote from Taylor and Wheeler's chapter on GPS [3]:

> We assume in this chapter that the radius rH of the circular orbit of the satellite and the speed v of the satellite in that orbit are both computed accurately enough using Newtonian mechanics. In contrast, Chapter 8 carries out the Schwarzschild analysis of circular orbits. When you have completed that chapter, you will be able to show that Newtonian values of orbit radius and speed are sufficiently accurate to describe the orbit of a GPS satellite.

So, we remain dependent on General Relativity experts for some aspects of our approach. This is no problem because our whole approach is intended to be a pragmatic intermediary theory between SR and GR, based on SR and the WEP. So, somewhere on the line towards higher accuracy, higher velocities or stronger fields of gravity parts of these assumptions have to fail. It should be weird if they wouldn't fail. Nevertheless, our pragmatic approach has the value of simplicity. It will be enough for almost all of GPS's purposes, as related to relativity corrections. And it has the advantage of using WEP and SR only, but still producing the GR first approximation results.

III. THE VELOCITY OF LIGHT ON THE STATIONARY GRID AND SPACE CURVATURE

A. The velocity of light on the free fall grid

The velocity of light on the free fall grid is by definition the Special Relativity velocity of light c_0 because the Einstein Elevator has started its free fall from an at rest in Minkowskian free space in infinity position. During the free fall, all the laws of SR remain valid within the local area of the Einstein Elevator because it starts as and remains an inertial system.

This means that a photon bouncing between the ceiling and the floor of the Einstein Elevator on the free fall grid will be observed by persons in the elevator as moving with constant velocity of light c_0.

From this FFG perspective, photons moving in free space in or out a field of gravity do this without being affected by this field. It is sufficient to imagine two small holes in the ceiling and the floor of the elevator, through which photons pass and move through the elevator while other photons in the elevator move between a mirror on the ceiling and the floor. There should be no difference in their velocities, the bouncing photon and the passing through photon should travel at the same speed in the elevator. This coïncides with Okun's viewpoint in the matter: photons do not fall under the influence of gravity [22]. A photon is not an apple.

B. The velocity of the bouncing photon in the perspective of the stationary grid observer

What will be the outcome when an observer on the stationary grid determines the velocity of the bouncing photon on the locally passing by free fall elevator on the FFG? Well, on the elevator the velocity of the bouncing photons is determined by photon travel time interval and elevator length as

$$c_0 = \frac{\Delta L_0}{\Delta T_0} \tag{28}$$

For the stationary observer, the passing by free fall elevator has Lorentz boost factor γ_ϕ and will be perceived with the usual Lorentz contraction as having contracted length

$$\Delta L_\phi = \frac{1}{\gamma_\phi}\Delta L_0. \tag{29}$$

In a Special Relativity context, the clocks on the elevator would be slowed down by the same Lorentz boost factor and the stationary clock would run faster than the clock on the moving by elevator. In Minkowski space-time we would have for the stationary observer

$$\Delta T_\phi = \frac{1}{\gamma_\phi}\Delta T_0. \tag{30}$$

so a time that would seem contracted relative to the passing by elevator time, resulting in a velocity of light observed by the stationary guy as

$$c_\phi = \frac{\frac{1}{\gamma_\phi}\Delta L_0}{\frac{1}{\gamma_\phi}\Delta T_0} = \frac{\Delta L_0}{\Delta T_0} = c_0. \tag{31}$$

But now gravity destroys the symmetry and it is the clock on the stationary grid that is moving slower relative to the clock on elevator falling on the free fall grid. See figure (7). Gravitational time dilation breaks the Minkowskian Lorentz symmetry of time dilation and

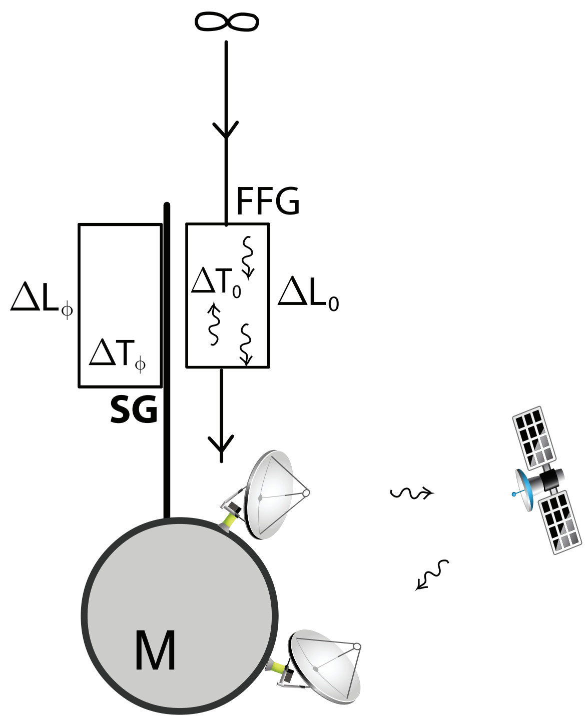

FIG. 7. A bouncing photon in the free fall elevator observed by a stationary grid observer, with at the right an addition Shapiro time delay measurement set up.

length contraction, resulting in an apparent velocity of light as perceived by the observer on the stationary grid as

$$c_\phi = \frac{\frac{1}{\gamma_\phi}\Delta L_0}{\gamma_\phi \Delta T_0} = \frac{1}{\gamma_\phi^2}\frac{\Delta L_0}{\Delta T_0} = \frac{1}{\gamma_\phi^2}c_0. \tag{32}$$

And with

$$\gamma_\phi = 1 - \frac{\Phi}{c^2} \tag{33}$$

we get

$$\gamma_\phi^2 = \left(1 - \frac{\Phi}{c^2}\right)^2 = 1 - \frac{2\Phi}{c^2} + \frac{\Phi^2}{c^4} \approx 1 - \frac{2\Phi}{c^2} \tag{34}$$

and

$$\frac{1}{\gamma_\phi^2} \approx \frac{1}{1 - \frac{2\Phi}{c^2}} \approx 1 + \frac{2\Phi}{c^2} = 1 - \frac{2GM}{Rc^2} \tag{35}$$

resulting in an apparent velocity of light for the observer on the stationary grid as

$$c_\phi = \frac{1}{\gamma_\phi^2}c_0 \approx \left(1 + \frac{2\Phi}{c^2}\right)c_0 = \left(1 - \frac{2GM}{Rc^2}\right)c_0 < c_0 \tag{36}$$

This leads to the Shapiro delay and to the gravitational index of refraction. The last is then given by

$$n_\phi = \frac{c_0}{c_\phi} = \gamma_\phi^2 \approx 1 + \frac{2GM}{Rc^2} > 1 \tag{37}$$

explaining the bending of light rays that pass by close to the sun. For a derivation of the Shapiro delay and the gravitational bending of light based on $c_\phi = \left(1 - \frac{2GM}{Rc^2}\right) c_0$, see Okun [22].

In GPS these effects exist too, but can be ignored due to the smallness of the effects in the context of GPS ground stations and GPS satellites in the weak field of earth gravity. A detail has to be mentioned: because our analysis only applies to the radial distance, the change in light velocity in a field of gravity is also radial, not tangential, to this field.

C. Interpretation troubles concerning the speed of light in gravity fields.

So the apparent velocity of light produces real effects, subsequently ascribed to space-time curvature. But in the perspective of the observers in the elevators on the free fall grid, light will not be bend by the sun, nor will it experience a Shapiro delay. The question might be, which observer has the better access to the real world, the one on the free fall grid or the one on the stationary grid? Neither might be the correct answer: both perspectives are useful but decisions concerning reality claims are beyond our reach. This gets interesting when we realize that the famous astronomical Einstein Lensing depends on the gravitational index of refraction and thus on the apparent velocity of light in such a field. Observers in free fall from infinity towards the center of an astronomical Einstein Lensing system will not observe Einstein rings. All other observers will in some extend observe these Einstein rings. At least according to our analysis based on the free fall grid and SR plus WEP. In other words, Einstein rings are relativistic phenomena, depending on the relative motion between observer and Einstein Lensing system. In the same way, when you are in radial free fall towards the sun from an initial position at rest relative to the sun and far away from the sun, you will not observe gravitational lensing when light from a distant star passes close by the surface of the sun. At least, according to our analysis that states that light will not be bent in the perspective of the free fall grid. But for all those not in a radial free fall from a position at rest in infinity, light will be bent when it passes close by the surface of the sun.

Okun categorizes c_ϕ as being the coordinate or world velocity and c_0 as the velocity of light in a local inertial frame of reference [22]. In our terms that would be the global velocity of light for c_ϕ and the local velocity of light for c_0. But according to General Relativity, an Einstein Elevator stationary located in a central field of gravity still represents a local frame of reference due to the fact that its experienced field of gravity can also be explained by a constant acceleration in free space. And a mere constant acceleration doesn't change the speed of light, only its observed direction if that direction isn't parallel to the acceleration. So in our view, the categorizations 'global' and 'local' for c_ϕ and c_0 are not as such valid in our analysis. In the free fall grid, the velocity of light is always c_0, on the global and on the local scale of things. And our derivation of the changed radial speed of light for a stationary observer in a central field of gravity was based on a Einstein Elevator local analysis and only afterwards extended globally. Thus in our stationary grid, the observed radial speed of light is always c_ϕ, locally and globally and in our free fall grid the observed radial speed of light is always c_0, locally and globally. In our view, Okun's treatment mixes our two grid perspectives. So his derivation and our derivation are mutually exclusive, although they both lead to the same effects. One could state that what Okun splits using 'global' and 'local' I split using 'stationary grid' and 'free fall grid' but the two ways do not overlap, they cross and thus cannot be smoothly connected. In our view, in the stationary grid the velocity of light is not a universal constant and thus Special Relativity is not valid in that grid. In the free fall grid on the other hand, the velocity of light is universal constant and thus Special Relativity can be applied in that grid, although with special care due to the additional WEP axiom. All the results of this paper are based on knowing the situation on the free fall grid and knowing the Lorentz boost connection with an observer on the stationary grid. A more general approach would be to replace Special Relativity with something comparable but also valid on the stationary grid. That approach is General Relativity. So in General Relativity, the speed of light might be not a constant any more but a function of the gravitational potential. That however would be very confusing and the approach of General Relativity is to keep the velocity of light a universal constant and to bend the metric instead. The path from Special Relativity to Special Relativity plus WEP in a free fall metric to General Relativity is not a smooth one, but involves a discontinuous change of paradigms. And the biggest paradigm jump, or change of mind set, is from SR plus WEP to GR, that strange theory created by weird academics. In my view, most of the related paradoxes discussed in the literature involve mix-ups of these paradigm jumps. Most of these paradigm mix-up discussions cannot be avoided because they involve the necessary translation of GR results in SR and Newtonian languages needed for experimental use. So where in the SR plus WEP perspective light in a field of gravity has a reduced speed for stationary observers, GR just compresses space, curves it, and keeps the velocity of light a constant for stationary observers. The resulting time delay for photons is the same in both approaches, at least for present day accuracies. Discussions regarding 'falling photons' have a tendency to be never-ending because the participants usually live in different mindsets or paradigms. The 'falling photons' case is a nice example of an attempt to translate SR plus WEP or GR results or outcomes into a Newtonian mindset. Only one item is really a fundamental problem in this context, not just a matter of different mindsets but equal outcomes, and that is the 'matter waves' issue, because GR really doesn't know how to deal with them. That is the Quantum Gravity issue. That then implies that GR is not sacrosanct. Nevertheless, in the field of gravity, GR is still no. 1 in town.

IV. WHICH GRID FOR GLOBAL POSITIONING SYSTEMS?

The American GPS system is based on a ECI reference system, using a fixed orientation in space relative to the distant stars as the primary grid and adjusting the standard clocks on the satellites as much as possible already on earth in order to have them at their orbiting trajectory frequency gauged to the earth standard clocks. This comes closest to our free space stationary central grid of frequency gauged clocks.

The European Galileo space navigation system will be different, according to Ashby.

> Since no factory frequency offset is applied to atomic clocks in the GALILEO satellites, relativity effects will cause satellite clock time to ramp away from TAI and will require large correction terms to be transmitted to users. [6]

This means that the Galileo satellite clocks will be much like our stationary grid in a central field of gravity, where standard clocks are by definition of being standard all frequency gauged while all being stationary at the same location. Once these standard clocks are given a velocity and a different high, they aren't frequency gauged any more, but because we know the original frequency, the offset can always be measured. The American GPS satellite clock approach has the advantage of being already corrected in advance. The European approach has the advantage of less complications in the standardization of its satellite clocks and the advantage of being more flexible in applying and developing clock-offset corrections because they have to be implemented in the easy accessible and renewable devices on earth.

So, all present day global positioning systems use a Newtonian positioning system and apply relativistic corrections in order to arrive at the greatest possible accuracy. Could this be done differently? We quote from the site of ESA, the European Space Agency [23]:

> Einsteins general theory of relativity, which deals with gravity, could be used to improve global navigation systems in the future. ESA's Advanced Concepts Team has collaborated with the University of Ljubljana to research this possibility.
>
> Present navigation satellite systems, such as Galileo and GPS, employ Newtonian trigonometry to determine positions, using Earth stations as reference points. This approach would perform ideally if all the satellites and the receiver were at rest and far from Earth. However, this is only correct as a first approximation because of the level of precision needed by a GNSS, the distortions that Earth causes in nearby space and time (space-time curvature) and the effects of the relative motions between the satellites and the user (relativistic inertial effects) both have to be considered. These are accounted for by introducing relativistic corrections to the Newtonian theory. For a ground user, these corrections can be as large as 12 km after one day.
>
> A simple way to avoid having to deal with the defects of Newtonian theory is to change the paradigm. Instead of modeling the system in a Newtonian framework and adding relativistic corrections, the positioning system could be modeled directly in general relativity. Drawing inspiration from a paper published by Bartolomé Coll of Systèmes de Référence Temps-Espace at the Observatoire de Paris, a study was conducted to introduce such relativistic coordinates for the definition of a global reference frame that can be used for positioning and navigation.
>
> A local Schwarzschild frame was designed, based on the clock signals originating from four satellites. Algorithms to read the users local Schwarzschild coordinates from the four satellites signals were developed, implemented and tested. The new reference frame is based on the dynamics of the satellites instead of relying on

> the location of terrestrial ground stations. If the constellation of satellites were to be equipped with inter-satellite communications, then each satellite could be a user of its own positioning system. The first results promise an increase in the accuracy and stability by using the new reference frame. The question whether it can avoid the need for terrestrial reference frames is still under study.

At a 2012 ESA workshop, titled 'Relativistic Positioning Systems and their Scientific Applications' the physicist Coll stated:

> The idea of a relativistic positioning system appeared almost simultaneously and in all likelihood independently, in Bahder, Coll and Rovelli [24].

The idea of Coll and others is rather straightforward:

> Many relativistic corrections are applied to the Global Navigation Satellite Systems (GNSS). Neil Ashby presents in Physics Today (May 2002) a good account of how these relativistic corrections are applied, why, and which are their orders of magnitude. Unfortunately, it is generally proposed that relativity is only a correction to be applied to Newtonian physics. We rather believe that there is a fully relativistic way to understand a GNSS system, this leading to a new way of operating it. As gravitation has to be taken into account, it is inside the framework of general relativity that the theory must be developed [25].

The idea is sound, but if one then studies the papers of the three mentioned authors, Bahder, Coll and Rovelli, the impression arises that a functioning relativistic positioning system in the framework of general relativity, in a non-Newtonian grid provided by general relativity experts, is not that straightforward.

Our grid of free falling Einstein Elevators is the most simple on, because all standard clocks on this grid have the unchanged standard clock frequency, but of course only theoretical because they eventually all crash on the central mass M, the Earth, and they have to be launched at infinity. Using our grid, we don't need General Relativity in order to calculate the necessary corrections for the Newtonian grids. In our opinion, the free fall grid is at the moment the most practical form of a theoretical relativistic positioning system to be useful in the implementation of highly accurate Newtonian global positioning systems. It competes with the Schwartzschild metric approach of General Relativity as a provider of corrections to the Newtonian metric used in global positioning systems. The advantage of the free fall grid SR-WEP approach is that it is less advanced and thus far less complicated as compared to the GR approach, but still accurate enough for all GPS purposes for the next few decades.

V. CONCLUSION

With our approach based on Special Relativity, the Weak Equivalence Principle and Newton's gravitational potential we could derive first order correct expressions for the gravitational red shift of stationary clocks and of satellites. We could also derive first order correct expressions for the geodetic precession, the Shapiro delay and the gravitational index of refraction, so phenomena connected to the curvature of the metric.

We did not derive an expression for the Lense-Thirring precession or drag of the metric by a rotating central mass M. Our free fall grid and the stationary grid were constructed around a stationary central mass M so we already excluded the Lense-Thirring effect in the construction phase of our model.

Our approach leads to the same first order results as the derivations based on the Schwartzschild solution of the Einstein Equations. But we did not formulate a fundamental theory of gravity. Our approach was opportunistic and ad-hoc because based upon a set of assumptions with limited reach. It would be interesting to find out at what point our approach will start to fail, so when our assumptions are no longer pragmatically useful. It is most likely that our assumptions will not fail all at once and a detailed analysis of its actual falsification would be interesting. It is to be expected that increased accuracy of clocks will bring the α factor, incorporating second order effects as for example the de Sitter precession correction, within measurable reach somewhere in the next 40-50 years.

For GPS engineers, our approach will give a deeper insight into problems concerning clock synchronization in a grid around the earth, without using the complex mathematics needed in GR. Our approach based on WEP and SR is within the reach of the standard GPS engineer. Above all, the free fall SR-WEP approach can be taught to GPS engineers in an achievable and affordable way. It will considerably reduce the communication gap between those engineers and the GR experts.

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