FROM ELEMENTARY PARTICLES TO EARLY UNIVERSE IN THE ULTRA RELATIVISTIC LIMITS

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Abstract

Using a phenomenological model in the ultra-relativistic limits we suggest that elementary particles including photons transforms in to micro black holes subjected to the following critical conditions: (i) When the de Broglie wavelength of elementary particles becomes equal to its Schwarzschild radius its energy reaches an upper limit (E_m) given by the relation: $E_m = hc^3/2Gm$ where m is the rest mass of the elementary particle. (ii) Particle black holes will have a mass equal to the limiting relativistic mass of the elementary particles and Schwarzschild radius equal to Compton wavelength of these particles. Lorentz invariance of the Compton wavelength of elementary particles in the trans-Planckian scales is suggested from this result. Photon black holes are found to be similar to massive elementary particles resembling the Planck particles discussed in cosmology. We find that the known elementary particle physical properties may be a window to the early universe since it provides clues about density distribution and nature of primordial black holes formed during the post Planck era after the Big Bang.

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1. Introduction

Finding upper limits to the energy of the elementary particles is of fundamental importance in Physics. There are no previous attempts to find the same except for the determination of the upper limit to the energy of photon [1-2]. The stability of elementary particles and physics of the early universe are relevant in this context. Ideas of formation of elementary particle black holes (EP black holes here after) can be a basis for finding the limits to the energy of elementary particles and also relevant to cosmology.

Let us consider the well-known de-Broglie relation for elementary particle,

$$\lambda = \frac{h}{p} \tag{1}$$

As the elementary particle is accelerated its mass as well as its momentum increases subjected to the conditions of special relativity. As p $\rightarrow \infty$ where v $\rightarrow c$ we have $\lambda \rightarrow 0$. This condition cannot be realized physically. The minimum value of λ cannot be zero because we cannot localize quantum particles to an infinitesimally small point. Further v cannot become c for material particles like proton electron etc. (photon is an exception here). So we can accelerate an elementary particle up to a stage when de-Broglie wavelength reaches a minimum but non zero value. The second point to be clarified in this context is can we accelerate an elementary particle beyond the Planck scale? [3] There is enough room for acceleration beyond this scale if we consider principle of relativity and known physics of early universe. The nature of quantum physics beyond the Planck scale will be completely known only when we develop a rigorous and robust theory of quantum gravity which is in agreement with cosmology.

The third point worth consideration here is the ultimate fate of apparently stable elementary particles when it goes to ultra-relativistic limits. One possibility is that the elementary particle can transform in to a micro black hole [4] or singularity subjected to Schwarzschild conditions in General Relativity.

In this paper we have found some critical conditions for the transformation of elementary particles into micro black holes in the ultra-relativistic limits. We have found a general expression for finding upper limit to energy/relativistic mass of elementary particles. Both photons and material particles (eg proton, electron etc.) are found to obey similar equation of energy and conditions for formation of particle black holes. These unified physical results for elementary particles is shown to provide new insights in to the mass-energy equivalence in the ultra-relativistic limits, Elementary particle physical data is used to infer density variations and characteristics of primordial black holes formed during the very early universe.

2. Expressions for upper limits of energy for elementary particles and properties of particle black holes

We assume that maximum momentum (p_m) is achieved by an elementary particle when it attains minimum de Broglie wave length (λ_{min}) being equal to Schwarzschild radius (r_s).

$$\lambda_{min} = r_s = \frac{h}{p_m} = \frac{2Gm}{c^2} \tag{2}$$

$$p_m = \frac{hc^2}{2Gm} \tag{3}$$

In the ultra-relativistic limit $E = pc$ and the upper limit to energy of elementary particles is given by

$$E = p_m c = \frac{hc^3}{2Gm} \tag{4}$$

We can also write

$$E_m = Mc^2 \tag{5}$$

where M is the limiting (maximum) relativistic mass of the particle.

From (4) and (5) we can find

$$M = \frac{hc}{2Gm} \tag{6}$$

We can also find that

$$M = \frac{\pi^{1/2} m_p^2}{m} \tag{7}$$

where m_p is the Planck mass.

The values of E_m and M established for common elementary particles is given in Table 1. If we assume that the elementary particle becomes a micro black hole when its relativistic mass is M. The size of the particle black hole is given by

$$R_s = \frac{2GM}{c^2} \tag{8}$$

It is also found that

$$R_s = \frac{h}{mc} = \lambda_c \tag{9}$$

where λ_c is Compton wavelength of elementary particle. The estimated values of λ_c for selected elementary particles are also given in the Table 1.

3. On the nature of photon black holes

If we apply the relation $E_m = \frac{hc^3}{2Gm}$ to photons we have

$$Mc^2 = \frac{hc^3}{2Gm} \tag{10}$$

where M is the maximum equivalent relativistic mass of the photon. Since photons does not have a rest mass, let us assume that the photon transforms in to an elementary particle of mass M in the ultra-relativistic limit so that we can assume that m= M and applying this condition in (6) we get

$$M = \left(\frac{hc}{2G}\right)^{1/2} = \sqrt{\pi} mp \tag{11}$$

where m_p is the Planck mass. For the case of photons also we can write the expression for relativistic energy E as $E = \frac{hc}{\lambda} = pc$ where $\lambda = \frac{h}{p}$. We can find that

$$R_s = \left(\frac{2hG}{c^3}\right)^{1/2} = \sqrt{2} l_p \tag{12}$$

where l_p is the Planck length.
Further we have,

$$\lambda_{min}(photon) = R_s \tag{13}$$

$$R_s = \lambda_{co} = \frac{h}{Mc} \tag{14}$$

where λ_{co} is the Compton wave length of the photon. The relations (12) and (14) found for photons are exactly similar to the expressions used to define the Planck particle [5]. So our initial assumption $m = M$ for photons is justified since photon most likely transforms into a micro black hole resembling a massive elementary particle in the ultra-relativistic limits. Comparison of photon and material particle(electron, proton etc.) black holes are given in Table 2.

4. Elementary particles probably the window to the early universe

The density and temperature of the early universe is understood to be very high [6] and favors the formation of micro black holes [7]. In the previous section it is suggested that all elementary particles will transform in to micro black holes in the ultra-relativistic limits. The density of EP black holes is given by

$$\rho = \frac{M}{V} \tag{15}$$

where M is given by relation (6)

$$V = \frac{4}{3}\pi R_s^3 \tag{16}$$

where R_s is given by the relation (9)

According to Carr[8] the mass(M) of micro black hole formed in early universe is related to time (t) from Big bang as

$$M(kg) = 10^{35}t \tag{17}$$

Our estimates of ρ and t of EP black holes are given in Table 2 using the above equations. The observed time variations of ρ estimated from elementary particle data is found to be in approximate agreement with density-time variations in early universe from 10^{-43} to 10^{-20} seconds as predicted by the Big bang model(www.as.utexas.edu/astronomy/education/301SP09.Ch27 slides.pdf). The temperature of the early universe as a function of time t from Big Bang is given by the expression

$$T_b = 10^{10}/\sqrt{t} \tag{18}$$

here T_b is measured in K and t is in seconds [9]. The Hawking temperature of EP black holes in K is given by the expression

$$T_h = hc/8\pi 2kR \tag{19}$$

From (18) and (19) we have estimated values of T_b and T_h corresponding to time t from EP data and given in Table 2. The values of T_h for all material elementary particles is $<T_b$. For photon black holes however we find

$$T_h = T_b \tag{20}$$

The energy density of early universe (E_d) is found to decrease from $10^{118} Jm^{-3}$ to $10^{72} Jm^{-3}$ during the time interval from 10^{43} s to 10^{-20} s [6] after the Big Bang. The number density of micro black holes (N_b) formed in time t from Big Bang corresponding to each elementary particle is given by $N_b b = E_d/E_m$ where E_m is the upper limit to the energy of elementary particle particles. N_b is inferred to be highest for photon mass black hole (about 10^{108}) and least for electron mass black hole (about 10^{41}). It is interesting to note that there is gap of EP black holes between masses 10^{-8} kg and 10^{11} kg (if we consider long life time particles alone). This may have implications on the physics of the early universe.

5. Discussion

Relativity theory developed in the first two decades of twentieth century made two important contributions relevant to the high energy physics.(i)Mass energy equivalence and (ii) Schwarzschild solution to Einstein's General relativity equations. In a series of papers co- authored by Einstein between 1929-1945 we can find the ideas of microscopic black holes [10]. This was further developed by others later which included concepts of quantum black holes. Three processes are described in research literature for the formation of micro-black holes: (i) Vacuum or density fluctuations in the early universe [7] (ii) High energy particle collisions [11] and (iii) Transformation of an elementary particle in to a black hole in the ultra-relativistic limits [2]. In this paper we have explicitly considered the third possibility of formation of elementary particle sized black holes in a phenomenological model. This has lead as to new insights in to the mass energy equivalence at the ultra-relativistic limits. The validity of basic physical laws at or beyond the Planck scale (Lorentz invariance, fundamental length and uncertainty principle), particle acceleration mechanisms beyond the Planck energy, possibility of formation of micro black holes in the TeV energy scales, stability of particle black holes against Hawking radiation etc. are problems which worth consideration in the present context [12-17]. But they remain as open physical problems until a full quantum gravity theory consistent with cosmology is developed. Further our results are self-consistent and presented in a semi-classical phenomenological context. The physical implications of the new results proposed in this paper will be discussed below.

Our first result is introduced initially as a postulate from which other results can be obtained in a self-consistent manner. It is from equation (2), λ_{min}=r_s. This equation sets the lower limit to the de Broglie wavelength of the elementary particles in the ultra-relativistic limits where r_s is the Schwarzschild radius of the elementary particles. The values of λ_{min} for common elementary particles such as proton, electron etc. is found to be at least twenty orders lower than the Planck length. In the ultra-relativistic limits the contribution from rest-mass energy to total energy (E) of elementary particles is negligible and is determined mainly by its momentum (p) so that

$$E = pc \tag{21}$$

We have found general expressions for the upper limits to relativistic mass (M) and energy (E_m) of elementary particles $M = hc/2Gm = \sqrt{\pi}mp^2/m$ (as followed from equation 6 and 7) where m_p is the Planck mass. From equation 4, $E_m = hc^3/2Gm$. For photons we define, m=M and we find from equation 11,

$$M_{ph} = \sqrt{\left(\frac{hc}{2G}\right)} = \sqrt{\pi}m_p \tag{22}$$

A similar relation for Mph is found in an earlier paper [1-2] in which the photon energy in the ultra-relativistic limits is assumed to be contributed mainly by its rotational (spin) kinetic energy.

The estimated values of M and E_m for common elementary particles with rest mass is found to be about twenty orders higher than the Planck energy pointing to again a possible trans-Planckian physical scale for the elementary particles. The validity of trans-Planckian physics [14,18] is supported by recent theories on quantum gravity and the emerging theories on micro black holes which are likely to have formed during the early universe.

In the ultra-relativistic limits all elementary particles is suggested to transform in to micro black hole whose size or Schwarzschild radius is found as (followed from equations 8 and 9)$R_s = \lambda_c = 2GM/c^2$ where $\lambda_c = h/mc$ and M is the maximum relativistic mass and λ_c is the Compton wavelength of the elementary particle. From equation 14, for photons we have $\lambda_c(photons) = h/Mc$ Equations (2) and (9) are critical conditions found in this study for the transformation of an elementary particle in to a micro black hole. They are not reported in general form previously even though ideas of photon [2] and proton black holes [19] are explicitly discussed in literature. We find that photon black holes are similar to Planck particles discussed in cosmological theories [20].

There are several studies which associate elementary particles with micro black holes. One idea is the formation of elementary particle sized' black holes which are supposed to have formed during early universe. Vacuum fluctuations are proposed as a mechanism for formation of such tiny black holes. The physical properties of the early universe such as matter, density, temperature are shown to decide the properties such as mass of primordial micro black holes. Apart from this several cosmological theories speak about formation of massive elementary particles during early universe. Planck mass particles are prominent among the same. In this paper we have considered the transformation of an elementary particle in to a micro black hole in the ultra relativistic limits. We find that every elementary particle is associated with a micro-black hole of unique physical characteristics which are determined by basic physical properties of the elementary particle such as its maximum relativistic mass and Compton wavelength.

Another interesting possibility of significance which emerges from the present study is the possible Lorentz invariance of the Compton wavelength of elementary particles till its energy reaches an upper limit (E_m) in the trans-Planckian scales! Rest mass of an elementary particle is a Lorentz invariant hence we can expect the upper limit mass of the same M defined by equation (9) is also a Lorentz invariant which may have implications on the physics of the primordial black holes formed in very early universe as discussed below.

The EP black hole mass density distribution mimics the same of early Universe as predicted by Big Bang model. This result suggest that EP particle black holes can form in early universe where the energy density is several orders of magnitude higher than the upper limit energy found for the different elementary particles. The very early universe as known from current cosmological models generally supports trans-Planckian physics. The properties of EP black holes found in this paper will have implications on the characteristics of PMB's formed in early universe as summarized in Table 4. It is shown that from elementary particle rest mass we can find the upper limits to the relativistic mass M of these particles (this is the mass of EP black hole formed in the ultra-relativistic limits) and density of early universe corresponding to the time (t in second from the Big Bang) prescribed for M. n several papers it is suggested that mass spectrum of elementary particles follow some definite patterns and arrangements [21-22] pointing out the discrete and non- random nature of the rest mass m of fundamental particles. In one recent paper an empirical relation is suggested for the calculation of the rest mass of the elementary particles [23] as

$$m = Nm_e/2\alpha \tag{23}$$

Here N is an arbitrary integer ranging generally between 1-200, m_e is the rest mass of the electron and α is the fine structure constant. With in accuracy of 1% we can estimate the rest mass m of several stable weakly and strongly interacting elementary particles. If such a relation is valid then it implies grand unification of strong, weak and electromagnetic forces .This will have implications on the formation of primordial black holes formed in early universe in the post Planck era of Big bang which are likely to be quantum black holes [24] whose mass spectrum may obey some specific quantum principles. Further the possiblity of Lorentz invariance of the EP black hole mass is also a result of relevance in this context.

Table 1: Estimated values of Schwarzschild radius (r_s), upper limits of relativistic mass (M) and energy (E_m) of elementary particles where Compton wavelengths (λ_c) of these particles are also given.

Particle	**Symbol**	**m(Kg)**	**r(m)**	**E(J)**	**M(Kg)**	**R(m)**
Photon				3.47E+09	3.86E-08	5.73E-35
Electron	(e-)	9.08E-31	1.35E-57	1.48E+32	1.64E+15	2.43E-12
Muon	(μ-)	1.88E-28	2.78E-55	7.15E+29	7.94E+12	1.18E-14
Neutral Pion	π0	2.40E-28	3.56E-55	5.59E+29	6.21E+12	9.21E-15
Charged Pion	π+	2.48E-28	3.68E-55	5.41E+29	6.01E+12	8.90E-15
Charged Kaon	K+	8.78E-28	1.30E-54	1.53E+29	1.70E+12	2.52E-15
Neutral Kaon	K0	8.85E-28	1.31E-54	1.52E+29	1.69E+12	2.50E-15
Eta	H	9.74E-28	1.44E-54	1.38E+29	1.53E+12	2.27E-15
Proton	P	1.67E-27	2.47E-54	8.04E+28	8.94E+11	1.32E-15
Neutron	N	1.67E-27	2.48E-54	8.03E+28	8.93E+11	1.32E-15
Eta Prime	η'	1.70E-27	2.52E-54	7.88E+28	8.76E+11	1.30E-15
Lambda	Λ0	1.98E-27	2.94E-54	6.77E+28	7.52E+11	1.11E-15
Sigma	Σ+	2.11E-27	3.13E-54	6.35E+28	7.05E+11	1.05E-15
Sigma	Σ0	2.12E-27	3.14E-54	6.33E+28	7.03E+11	1.04E-15
Sigma	Σ-	2.13E-27	3.16E-54	6.30E+28	7.00E+11	1.04E-15
Delta	Δ++	2.19E-27	3.25E-54	6.13E+28	6.81E+11	1.01E-15
Delta	Δ+	2.19E-27	3.25E-54	6.13E+28	6.81E+11	1.01E-15
Delta	Δ0	2.19E-27	3.25E-54	6.13E+28	6.81E+11	1.01E-15
Delta	Δ-	2.19E-27	3.25E-54	6.13E+28	6.81E+11	1.01E-15
Xi	Ξ0	2.34E-27	3.47E-54	5.74E+28	6.38E+11	9.45E-16
Xi	Ξ-	2.35E-27	3.48E-54	5.71E+28	6.35E+11	9.41E-16
Omega	Ω-	2.97E-27	4.41E-54	4.51E+28	5.02E+11	7.43E-16
Tau	(τ-)	3.16E-27	4.68E-54	4.25E+28	4.72E+11	7.00E-16

Table 2: Comparison of photon and material particle black hole (BH) properties where M is the maximum relativistic mass, m is the rest mass of the particles, m_p is the Planck mass, λ_{min} is the minimum de Broglie wavelength and λ_c is the Compton wavelength of the elementary particle.

Particle Black hole Property	**Protons/Electrons etc.**	**Photons**
1. Size or Schwarzschild radius of Black holes (R_s)	$R_s = \lambda_c = \frac{h}{mc}$	$R_s = \lambda_c = \frac{h}{mc}$
2. Mass	$M = \frac{\pi m_p^2}{m}$	$M = \sqrt{\pi} m_p$
3. Critical conditions	i) $\lambda_{min} = r_s = \frac{2Gm}{c^2}$ ii) $\lambda_c = R_s = \frac{2GM}{c^2}$	$\lambda_{min} = \lambda_c = R_s = \frac{2GM}{c^2}$

Table 3: Estimating density, Hawking's temperature and time of formation in early universe of elementary particle black holes.

Elementary Particle	**Symbol**	**Upper limit Mass** (Kg)	**Compton Wave length** (m)	**Volume** (m^3)	**Density** (Kgm^{-3})	**Time from big bang** (s)	**Hawk. Temp.** (K)	**Big bang Temp.** (K)
Photon		3.86E-08	5.73E-35	7.87E-103	4.9E+94	3.86E-43	3.19E+30	1.68E+32
Electron	(e-)	1.64E+15	2.43E-12	6.03E-35	2.72E+49	1.64E-20	7.51E+07	7.14E+54
Muon	(μ-)	7.94E+12	1.18E-14	6.83E-42	1.16E+54	7.94E-23	1.55E+10	3.45E+52
Neutral Pion	π0	6.21E+12	9.21E-15	3.27E-42	1.9E+54	6.21E-23	1.98E+10	2.70E+52
Charged Pion	π+	6.01E+12	8.90E-15	2.96E-42	2.03E+54	6.01E-23	2.05E+10	2.61E+52
Charged Kaon	K+	1.70E+12	2.52E-15	6.68E-44	2.54E+55	1.7E-23	7.25E+10	7.39E+51
Neutral Kaon	K0	1.69E+12	2.50E-15	6.52E-44	2.58E+55	1.69E-23	7.31E+10	7.33E+51
Eta	η	1.53E+12	2.27E-15	4.89E-44	3.13E+55	1.53E-23	8.05E+10	6.66E+51
Proton	p	8.94E+11	1.32E-15	9.74E-45	9.18E+55	8.94E-24	1.38E+11	3.89E+51
Neutron	n	8.93E+11	1.32E-15	9.70E-45	9.21E+55	8.93E-24	1.38E+11	3.88E+51
Eta Prime	η'	8.76E+11	1.30E-15	9.16E-45	9.56E+55	8.76E-24	1.41E+11	3.81E+51

Table 3 (Contd.)

Elementary Particle	**Symbol**	**Upper limit Mass** (Kg)	**Compton Wave length** (m)	**Volume** (m^3)	**Density** (Kgm^{-3})	**Time from big bang** (s)	**Hawk. Temp.** (K)	**Big bang Temp.** (K)
Lambda	Λ0	7.52E+11	1.11E-15	5.79E-45	1.3E+56	7.52E-24	1.64E+11	3.27E+51
Sigma	Σ+	7.05E+11	1.05E-15	4.78E-45	1.48E+56	7.05E-24	1.75E+11	3.07E+51
Sigma	Σ0	7.03E+11	1.04E-15	4.74E-45	1.48E+56	7.03E-24	1.75E+11	3.06E+51
Sigma	Σ-	7.00E+11	1.04E-15	4.68E-45	1.5E+56	7E-24	1.76E+11	3.05E+51
Delta	Δ++	6.81E+11	1.01E-15	4.30E-45	1.58E+56	6.81E-24	1.81E+11	2.96E+51
Delta	Δ+	6.81E+11	1.01E-15	4.30E-45	1.58E+56	6.81E-24	1.81E+11	2.96E+51
Delta	Δ0	6.81E+11	1.01E-15	4.30E-45	1.58E+56	6.81E-24	1.81E+11	2.96E+51
Delta	Δ-	6.81E+11	1.01E-15	4.30E-45	1.58E+56	6.81E-24	1.81E+11	2.96E+51
Xi	Ξ0	6.38E+11	9.45E-16	3.54E-45	1.8E+56	6.38E-24	1.93E+11	2.77E+51
Xi	Ξ-	6.35E+11	9.41E-16	3.49E-45	1.82E+56	6.35E-24	1.94E+11	2.76E+51
Omega	Ω-	5.02E+11	7.43E-16	1.72E-45	2.92E+56	5.02E-24	2.46E+11	2.18E+51
Tau	(τ-)	4.72E+11	7.00E-16	1.43E-45	3.29E+56	4.72E-24	2.61E+11	2.05E+51

Table 4: Properties of Elementary particle black holes and its implications in early universe.

No.	Property of elementary particle (EP) Black Hole	Implications for the early Universe
1	(a)Mass of elementary particle black hole ranges from 10 kg to 10 kg (b)Size of elementary particle black hole ranges from 10 m to 10m (c)No elementary particle is found to have a confirmed rest mass below the electron rest mass	These properties will reflect on the PBH characteristics formed in the post Planck era of Big Bang (10^{-45} s to 10^{-20} s) Present maximum value of the mass of EP Black hole (electron black hole)may pose limit to the mass of PBH formed during very early universe
2	Density of EP black holes is to found to vary from 10 for photon black hole to 10 for tau black hole	These density variations approximately correspond with density variations in early universe as predicted by big bang model from 10^{-45} s to 10^{-20} s
3	Number density of EP Black hole formed in early universe is inferred to be highest for photon Black hole and least for electron black hole.	The probability of formation of PBH in early universe possibly decreases with time.
4	Possible quantization of elementary particle rest mass	If this is true it implies that PBH formed in early universe in the post Planck era may be quantum black holes

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