Oscillations and Superluminosity of Neutrinos

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Abstract

Two conflicting theoretical structures, one using the non-relativistic Schroedinger equation and the other using relativistic energy, are used simulataneously to derive the expression, for example for $P_{\nu_\mu \to \nu_\tau}(t)$, to study the neutrino oscillations. This has been confirmed experimentally. Here we try to resolve the above theoretical inconsistency. We show that this can be done in a single consistent theoretical framework which demands that the neutrinos be superluminal. We therefor predict that in the neutrino appearance experiments (for example $P_{\nu_\mu \to \nu_\tau}(t)$) the neutrinos shall be seen to travel with velocities which are faster than that of light. The experimentalists are urged to try to confirm this prediction.

Keywords: Neutrino oscillations, appearance experiments, phase velocity, superluminal neutrino, faster than light neutrino, Legendre transformation

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Neutrino oscillation is a well confimed phenomenon. As to mass, what is measured empirically is $\mid \Delta m^2 \mid = \mid {m_1}^2 - {m_2}^2 \mid$, i. e. the difference of mass-squared of neutrino oscillation from flavour-1 to flavour-2.

The derivation of the theoretical expression for neutrino oscillations between different flavours, for example $P_{\nu_\mu \to \nu_\tau}(t)$ is well defined in text books. Below we point out an inconsistency in these derivations, for example in $P_{\nu_\mu \to \nu_\tau}(t)$ and which we discuss in detail below.

Given three generations of particles, we have three flavours of neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ with mass eigenstates (ν_1, ν_2, ν_3). For the sake of simplicity we confine ourselves to the two flavours (ν_μ, ν_τ). An orthogonal transormation links it with mass eigenstates (ν_2, ν_3) as

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \tag{1}$$

We work non-relativistically with the Schroedinger equation giving the energy eigenstates as

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = H \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} E_2 & 0 \\ 0 & E_3 \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \tag{2}$$

Then the time evolution of these states is given by

$$\mid \nu_\mu \rangle_t = cos\theta \; e^{iE_2 t)} \mid \nu_2 \rangle + sin\theta \; e^{iE_3 t)} \mid \nu_3 \rangle \tag{3}$$

$$\mid \nu_\tau \rangle_t = -sin\theta \; e^{iE_2 t)} \mid \nu_2 \rangle + cos\theta \; e^{iE_3 t)} \mid \nu_3 \rangle \tag{4}$$

The probability of obtaining a ν_τ at some later time (t > 0) from an initial (t=0) pure ν_μ beam is

$$P_{\nu_\mu \to \nu_\tau}(t) = \mid \langle \nu_\tau \mid \nu_\mu \rangle \mid^2 = sin^2(2\theta) sin^2(\frac{1}{2}(E_2 - E_3)t) \tag{5}$$

The next step is done by "everyone" but we quote ref. [1. p.487], "Now I will make an approximation that goes beyond what I have assumed in using the non-relativistic Schroedinger equation (above), but which is valid for high-energy neutrinos, namely the rest-mass energy of neutrino is small compared to the total energy". In the first breath one takes the non-relativistic approximation and in the next breath (below) one uses the relativistic energy as:

$$E_2 = \sqrt{(p^2c^2 + {m_2}^2c^4)} \rightarrow\sim pc(1 + \frac{{m_2}^2c^4}{2p^2c^2}) \tag{6}$$

Next we assume (and this too is a major assumption) that at high energies all the neutrinos have the same momenta and then

$$E_2 - E_3 = \frac{c^4}{2pc}({m_2}^2 - {m_3}^{@}) = \frac{c^4}{2pc}\Delta(m^2) = \frac{\Delta(m^2c^4)}{2E} \tag{7}$$

where E is the average energy of the neutrinos. For time t the path travelled is L/c and so

$$P_{\nu_\mu \rightarrow \nu_\tau}(t) = sin^2(2\theta)sin^2(\frac{\mid \Delta(m^2c^4)L \mid}{4E\hbar c}) \tag{8}$$

The above equation has been well tested experimentally and has allowed the extraction of mass-squared difference of neutrinos. However the fact renains that we have clearly mixed up conflicting aspects of the non-relativistic and the relativistic realities in the same formalism. It apparently works well, but clearly it is a quick-fix of some kind and points that somewhere something is amiss in our understanding of the phenomenon. What could that be?

Note that a Lagrangian is given as L(q,$\dot{q}$) in terms of the generalized coordinate q and its time derivative as $\dot{q}$. The Hamiltonian H(q,p) is given in terms of q and the generalized momenta p. The H and L are related through the Legendre transformation defined as

$$H(q,p) = p\ \dot{q} - L(q,\dot{q}) \tag{9}$$

With equation of motion

$$\dot{q} = \frac{\partial H}{\partial p}\ ,\ \dot{p} = -\frac{\partial H}{\partial q} \tag{10}$$

We know that the relativistic energy can be written in terms of phase velocity as

$$E = p\ v_p \tag{11}$$

where

$$v_p = c\sqrt{1 + \frac{{m_0}^2 c^4}{p^2 c^2}} \tag{12}$$

Let us assume that our neutrinos obey the above relationship and as we know that the phase velocity $v_p >$ c, they are superluminal. Next we use the standard approximation that for these high energy neutrinos, the rest mass is much smaller than its energy and thus

$$v_p \sim c(1 + \frac{{m_0}^2 c^4}{2p^2 c^2}) = c + v_f \tag{13}$$

where

$$v_f = \frac{{m_0}^2 c^4}{2p^2 c^2} c \tag{14}$$

Here v_f stands for the velocity of neutrino in excess of the velocity of light c ($v_f <<$ c)

And so

$$E = pc + pv_f \tag{15}$$

$$E - pc = pv_f \tag{16}$$

Next we assume that there exists a Lagrangian L(q,v_f) where $\dot{q} = v_f$. Then as per Legendre transformation above we define an Hamiltonian

$$H(q,p) = pv_f - L(q, v_f) \tag{17}$$

and so

$$(E - pc) - L(q, v_f) = pv_f - L(q, v_f) = H(q,p) \tag{18}$$

Now from equation of motion

$$v_f = \dot{q} = \frac{\partial H}{\partial p} \tag{19}$$

This is good and consistent. Next we require that

$$\dot{p} = -\frac{\partial H}{\partial q} = \frac{\partial L}{\partial q} = 0 \tag{20}$$

Now we use the virtue of having two first order equation of motion for an Hamiltonian to a single second order equation of motion for a Lagrangian. As we see below, it allows/demands first order superluminal Hamiltonian equations of motion while none of the second order. In the above the Hamiltonian structure holds even if the Lagrangian is set to zero or is a constant. A motivation for this may be as follows. For a non-zero (or constant) Lagrangian, with the Lagrange equation of motion, in the canonical manner, this would mean that the linear momentum is conserved. This in turn would mean that the space is homogeneous. Thus if the above were true, then it would mean that the above may hold for any homogeneous space, including ours, and then superluminosity would hold for all the particles. But this is not so. And hence for eqn (20) to hold, then we have to demand that the Lagrangian itself is zero or a constant. Then the above equation will hold but would not demand homogeneity of space. It will just mean that all the neutrinos will have the same and conserved momentum for all. Remember that above we had assumed that at high energies all the neutrinos would have the same momenta (see statement before eqn. (7)). Now it is this, that is demanded by the above equation. Thus

$$H(q,p) = pv_f - L(constant) \tag{21}$$

Where L(constant) may be set equal to zero also. Thus we have obtained a proper Hamitonian which gives the correct energy difference in eqn. (7) and thus the above good results for neutrino oscillation in a consistent manner. What is important is that the neutrino is superluminal and that the Hamiltonian depends upon v_f the velocity above the velocity of light c. Hence superluminosity of neutrinos is a clear prediction of our model here. The OPERA experiment which observes the appearance experiment $P_{\nu_\mu \to \nu_\tau}(t)$, should be able to observe the neutrinos travelling with velocities faster than that of the light. These and others doing similar experements, or studying $P_{\nu_e \to \nu_\mu}$(t), are urged to confirm this clearcut prediction of our model.

References

1. R. Mann, "An Introduction to Particle Physics and the Standard Model", CRC Press, Florida, 2010