Planck membrane – a domain of co-existence of the GRT and quantum physics, substituting the event horizon of the black hole taking the form of a 'frozen star'

(Part I: The outer region, $r \geqslant 2M$)

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Summary

The GR-concept of the 'frozen star' can be seriously questioned, considering the quantum structure of space. If, however, the parallel validity of GRT and quantum physics becomes a basic assumption, a way out of the problem can be found. The solution – a quantum membrane – fulfilling both criteria, is deduced with the use of the quantum limit for acceleration and the GR-mechanics.

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http://gravityresearchfoundation.org/pdf/awarded/1997/voracek.pdf

1. Introduction to the problem

In the General theory of relativity (GRT), the free fall of a test particle to the event horizon of a black hole takes an infinite time for any outer observer not following the fall. As a consequence, the same conclusion is valid for an astrophysical body undergoing the catastrophic gravitational collapse.

None the less, the spacetime has a quantum structure*, which can complicate the straight picture of the situation as given by the GRT. As an example, let us take the radial free fall from the relative rest at infinity to a source of the Schwarzschild gravitational field. The question is, what time Δt of a distant observer, being at rest relative to the source of the field (the coordinate observer), the test particle needs to fall from level $r = 2M + 1\,\mathrm{m}$ (M is the mass of the black hole) to level $r_s = 2M + (\delta r)_s$, where a local static observer (the standard observer) states that the (standard) thickness

$$(\delta l)_s = (g_{rr})_s^{1/2} (\delta r)_s \tag{1.1}$$

of the layer between level $r = 2M$ and his own level is equal to one Planck length $l_P = 1.616 \times 10^{-35}\,\mathrm{m}$. (The Geometrized system of units, with 1m (not l_P !) as a unit-base, will be used in the subsequent text.)

First, we deduce $(\delta r)_s$. For r approaching $2M$ from above (i.e., for $r = 2M + \delta r$, where $\delta r \to 0^+$), metric coefficient

$$g_{rr} = 2M\, \delta r^{-1} \quad . \tag{1.2}$$

In our example, according to (1.1),

$$(g_{rr})_s^{1/2} (\delta r)_s = l_P \quad ; \tag{1.3}$$

this, together with (1.2), gives

$$(\delta r)_s = l_P^{\,2} (2M)^{-1} \quad . \tag{1.4}$$

(For a detailed deduction see *Adjacent mathematics*.)

Regarding the quantum character of the lengths, the use of formula (1.3) is quite relevant. Notwithstanding, even for the proper length, defined by the relation

* More precisely: The space-lengths and time-intervals behave as if the spacetime had a quantum structure. Empty space and event-less time can not have any structure, since spacetime is not a medium in the sense of the Newtonian concept. There is however the minimal possible material object and the shortest possible time interval betwen two events. (The material/energy-objects can be the virtual particle pairs as well.)

$$\delta\, l_0 = \int_{2M}^{2M+\delta r} \left(g_{rr}\right)^{1/2} \mathrm{d}\, r \tag{1.5}$$

supposing implicitly the smooth structure of space, one obtains the consistent result: If $\delta\, r \to 0^+$, equation (31.9) in Misner *et al.*, 1973, simplifies to

$$\delta\, l_0 = 2\left(2M\,\delta\, r\right)^{1/2} \quad . \tag{1.6}$$

Quantity $\delta\, l$, being the standard thickness defined by formula

$$\delta\, l = \left(g_{rr}\right)^{1/2} \delta\, r \tag{1.7}$$

(used – in fact – already in its applied form (1.1) aimed at our example with the specified standard observer), implies, when used together with equation (1.2), that

$$\delta\, l = \left(2M\,\delta\, r\right)^{1/2} \tag{1.8}$$

and thus,

$$\delta\, l_0 = 2\,\delta\, l \quad . \tag{1.9}$$

We now apply equation (25.38) from Misner *et al.*, 1973, which for our situation, i.e., for $2M \gg \Delta\, r = r - 2M$, simplifies to

$$t = \text{const.} - 2\left(\Delta\, r + M \ln \Delta\, r\right) \quad . \tag{1.10}$$

For the levels in our example, $\Delta\, r$ equals 1 meter and $\left(\delta\, r\right)_s$, respectively.

Then, owing to (1.4),

$$\Delta\, t \approx 2M \ln\left(2M\, l_P^{-2} \times 1\text{m}\right) \quad . \tag{1.11}$$

For a source of the gravitational field with a mass equal to $4.76\, M_S$ (where M_S is the mass of the Sun), i.e., $M = 7\times 10^3\,\text{m}$, the resulting coordinate time-interval is $\Delta\, t = 2.38\times 10^6\,\text{m}$; expressed in the SI, $\Delta\, t \approx 8\,\text{ms}$. In any case, such a time-interval differs significantly from the duration of eternity. This problem presents a serious cause to doubt the definition of the astrophysical black hole as a ‘frozen star’ (see Misner *et al.*, 1973; Chapter 33).

2. The role of maximal acceleration

We are looking for such a $\left(\delta\, r\right)_a$, where, for a standard observer in the Schwarzschild field, the gravitational acceleration of a test particle at momentaneous relative rest is equal, in its absolute value, to the maximal possible acceleration

$$a_{max} = l_P^{-1} \tag{2.1}$$

(see Voráček, 1989, and references therein). Referring to Zeldovich and Novikov, 1971 (equation (3.2.3) there), or Voráček, 1979a (equation (61) there), the considered gravitational acceleration is

$$a = -Mr^{-2}\left(1-2Mr^{-1}\right)^{-1/2} \quad . \tag{2.2}$$

Since $\left(\delta r\right)_a$ certainly is very small, from (2.1) and (2.2) follows that

$$\left(\delta r\right)_a = l_P^{\,2}\left(8M\right)^{-1} \quad . \tag{2.3}$$

Denoting now with $\left(\delta l\right)_a$ the respective standard distance from the level where $r = 2M$, then, by using relation (1.8), we arrive at the result

$$\left(\delta l\right)_a = \frac{l_P}{2} \quad . \tag{2.4}$$

Such a conclusion is rather astonishing, as

(i) $\left(\delta l\right)_a$ is invariant with respect to mass M, and

(ii) its value $\frac{1}{2} l_P$ suggests that the phenomena in the considered region are of a quantum character.

On the basis of the ascertainments we made, it seems to be reasonable to assume that *there exists a region (r-interval, or layer), close above level $r = 2M$, where the laws of the GRT, as well as those of quantum physics, are valid together at the same time.*

3. Basic facts about the layer close to the level $r = 2M$

Paying regard to the deduction made in Section 2, we consider the level $r_a = 2M + \left(\delta r\right)_a$ to be a reference level in an *r*-interval, of which the upper boundary level (with *r*-coordinate $r_x = 2M + \left(\delta r\right)_x$) still remains to be specified. Referring to the analysis performed by Voráček, 1989, where a didactic deduction is presented, it is possible to require that, for a standard observer on level r_a (the *a*-observer), and everywhere inside the *r*-interval related to the same level of reference as well, the thickness $\left(\delta l\right)_x$ of the considered interval would be equal to $2l_P$. This means that

$$\left(\delta l\right)_x = \left(g_{rr}\right)_a^{1/2}\left(\delta r\right)_x = 2l_P \quad . \tag{3.1}$$

According to (1.2) and (2.3)

$$\left(g_{rr}\right)_a^{1/2} = 4M\, l_P^{-1} \quad , \tag{3.2}$$

thus, relation (3.1) gives the result

$$\left(\delta r\right)_x = l_P^{\,2}\left(2M\right)^{-1} \quad . \tag{3.3}$$

Then, comparing result (1.4) with the last one, we see that $\left(\delta r\right)_x$ and $\left(\delta r\right)_s$ are equivalent. Quite compatibly with (1.7): The fact that $\left(\delta l\right)_x = 2l_P$, while $\left(\delta l\right)_s = \left(g_{rr}\right)_s^{1/2}\left(\delta r\right)_s = l_P$, is connected to circumstance that the standard observer in Section 1 (*s*-observer) was already placed on another level of reference, where, in accordance with (1.2) and (1.4),

$$\left(g_{rr}\right)_s^{1/2} = 2M\, l_P^{-1} \quad . \tag{3.4}$$

In order to avoid any confusion, it seems to be useful to present these results in a diagram (Figure 1).

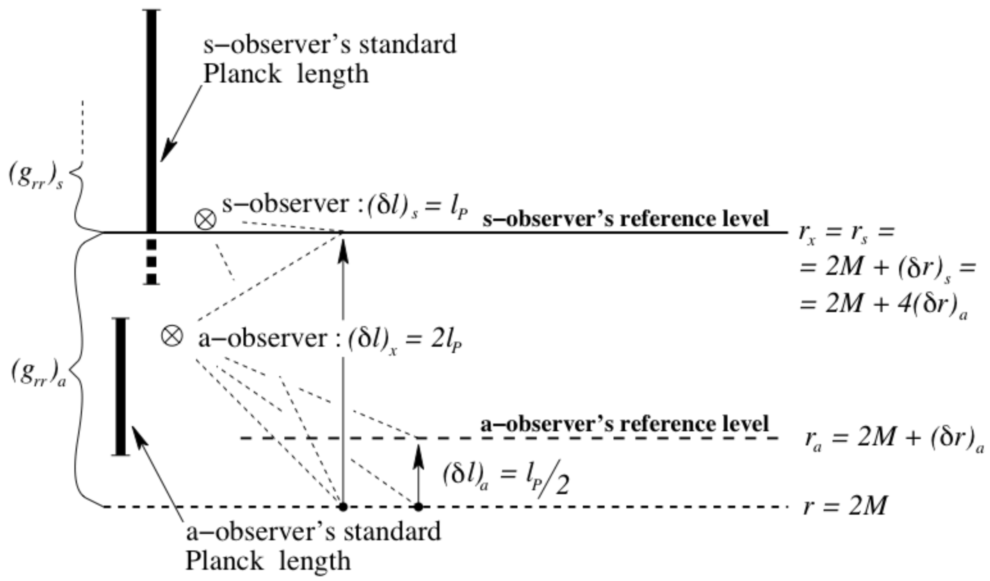

Figure 1. For the standard a-observer anywhere inside the interval $\left(2M\,,\,r_s\right)$, the thickness of respective layer is equal to $2l_P$; the reference metric coefficient g_{rr} inside the whole mentioned region is $\left(g_{rr}\right)_a$, as the space has a quantum character. A standard observer on level r_s already has to use metric coefficient $\left(g_{rr}\right)_s$, and the length $\left(\delta l\right)_s$ is related to him; however, using the value $\left(g_{rr}\right)_s$, he can not descend under level r_s in order to perform a practical measurement there. The quantity $\left(\delta l\right)_s$ is thus physically rather irrelevant as it can not be measured directly. Since quantities $\left(\delta l\right)_x$ and $\left(\delta l\right)_a$ are related to the same radial length-interval (corresponding to the layer), with a sole reference metric coefficient $\left(g_{rr}\right)_a$, they are not linked together through relation $\delta l \propto \delta r^{1/2}$ as expressed by formula (1.8), being valid for $\delta r \to 0^+$ exclusively under the assumption of a smooth structure of space, not for a space with a quantum character; instead, formula (1.7) is to be used, where g_{rr} is considered to be constant within the whole layer, and, consequently, $\delta l \propto \delta r$ when comparing $\left(\delta l\right)_x$ with $\left(\delta l\right)_a$. On the other hand, the proportionality $\delta l \propto \delta r^{1/2}$ is relevant for $\left(\delta l\right)_s$ when compared with $\left(\delta l\right)_a$. Relation $\left(\delta l\right)_x = 2\left(\delta l\right)_s$ is then consistent with formula (1.9), corresponding to the actual situation with its proper quantistic logic, when the 'fuzzy character' of level $r_x = r_s$ is considered.

4. The spacetime metric in the region between $r = 2M$ and $r_s = 2M + l_P{}^2 (2M)^{-1}$

In order to fulfill both the criteria of quantum physics and those of the GRT in the given region, such a model of metric seems to be an optimal solution, which, for a standard observer there, assures the timelessness of the region. Such a requirement is then expressed mathematically as

$$(g_{tt})_a = 0 \quad \text{(effectively)}, \tag{4.1}$$

despite the fact that the event horizon does not exist. The representative metric coefficient $(g_{rr})_a$ valid there was already specified in the previous Section (see formula (3.2)). These requirements are well compatible with the fact that inside the considered region it is not possible to define a radial motion, since its standard (as well as proper) thickness equals $2l_P$. Consequently, the quantum limit for acceleration is never reached.

At the same time, however, the rule must be valid, that the mass-energy density there must not surmount the Planck density $\rho_P = 3\left(8\pi\, l_P{}^2\right)^{-1}$ (see, e.g., Voráček, 1989, and references therein). In its own consequence, such a criterion could indirectly compensate well the absence of time-quantity. We will call the considered region the *Planck membrane* (PM) by definition.

Moreover, we can not see any contradiction in the fact that the function of the above described mechanism is well conserved in spite of an accretionary growth of the mass of the PM (and of mass M, as a direct consequence); while $2M$ is, in a trivial way, conformally changing with M, level $r = 2M$ remains always to be a level pertaining to the lower boundary of the enlarging PM. Perhaps it could work so, due to the possibility that in the PM the metric coefficient g_{tt} is effectively zero only in the radial direction, while it equals $-(g_{rr})_a^{-1}$ in the tangential (horizontal) direction. The lumps of matter falling into the PM are thus perfectly dissolved there in a process conserving their (coordinate) mass-energy, very probably resulting in the only permitted movements – those in all possible horizontal directions within the PM, so that the sum of their momenta there is statistically zero. Since their radial momenta also must be conserved, while any radial movement does not exist in the PM, the membrane, being an integral entity, must behave – in some way – conformally to the laws of physics, possibly through vibrations being successively damped as a consequence of the stochasticity of the directions of the radial momenta vectors (of the in-falling lumps) over the whole surface of the PM. At the same time, the incompatibility problem of the notion ‘frozen star’ with the accretive growth of its asymptotic gravitational radius $r_g = 2M$ (see Tsitsing, 1995, and references therein) is eliminated, since – thanks to Machian induced forces (Voráček, 1979b) – the PM is effectively influencing/determining the metric in its neighborhood, even if it is growing and/or vibrating. (In the very closeness of the PM, quantum mechanics will certainly be working compatibly with the GRT.)

5. The Planck membrane: Some consequences

In our model of the PM introduced above, the notion of the *surface of last influence* (defined by Misner *et al.*; Chapter 33) would not be useful anymore, while the concept of 'frozen star' can be conserved.

Another consequence would be that the conditions for the origination of the Hawking radiation (Hawking, 1974 and 1975) do not exist and thus, the mechanism of quantum evaporation of the black holes can not work, which could explain the observational absence of the final gamma-radiation bursts as predicted by the theory*.

The introduction of the PM has however one positive impact: It answers the query what happens to information that falls into the black hole. In our model, the information is not destroyed in the PM. Similarily, the world-lines of infalling particles do not end, since their space-components (*i.e.* their trajectories) finish forever in the PM over the event horizon.

6. The Planck membrane: Arising queries**

A plethora of questions connected to the physics of black holes must be answered in relation to the model of PM we introduced.

a) *The PM as a 'one way gravitational conductor'*

   Consistently with the GRT, an event horizon does not exist, but every particle falling towards the source of the gravitational field is eternally trapped in the PM, for any outer observer not following the fall. Thus, even the gravitational information/graviton is trapped there (see Voráček, 1983, p.153). Still, owing to Birkhoff's theorem, the gravitational information/gravitons must tunnel through the PM from the source-side out. Such a conclusion would be in accordance with the opinion of R. Feynman, that the gravitational information is mediated by virtual gravitons tunneling out. We prefer, instead of the gravitons, to speak about 'gravity-quanta', analogous to the wave-trains.

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* If it were acceptable that the Hawking radiation would originate, thanks to the strong gravitational tidal gradient within a certain radial interval over the PM, then it is possible to deduce that the total coordinate evaporation power of a black hole would be proportional to M^3 or to M, for the isotropized respective anisotropic metric.

** We do not present complex solutions here (even if known to the author), since the necessary foundations have not yet been published.

b) *The inner structure of a black hole*

It is possible to construct a model of the inner structure of the matter within the cavity of the PM, where laws of quantum physics, delay of gravitational information, the GRT-law of mass-energy conservation*, and Birkhoff's theorem are employed, so that the existence of a singularity or of some 'inner GR-event horizon(s)' are avoided. The question of the isotropy or anisotropy of the metric of spacetime is one the key principles of the model.

c) *The past and future of black holes*

The development and the destiny of black holes depends in a special way on the boundary conditions, which are determined cosmologically. (The questions which must be answered before, were presented already in 1982 by Królak.) Actually, the boundary condition is, with a high degree of precision, the flat spacetime. In the beginning of the final stage of the collapse, the coordinate mass-energy of the PM is very low and the inner component within its shell, isolated by the PM from the outer gravitational information, is dominating. The situation results in the kinematics determined by the *Principle of dynamical indifference* (introduced by Voráček, 1983): The black hole becomes effectively indifferent to the dynamical gravitational information coming from the outside, and the PM must follow the body in its cavity. When the coordinate mass-energy of the PM becomes dominating (thanks to the accretion), the behavior of the black hole changes: It can be the component of a binary or of a multiple system, while the PM is determining the mechanics in its cavity (possibly even thanks to the robustness of Birkhoff's theorem). The same mechanism can be the cause of the scenario where a star collapsed into a black hole, which consequently abandoned the binary or multiple system of gravitationally bounded cosmic bodies. The negative energy-defect successively arisen, can be compensated in the terms of classical celestial mechanics of the system or – possibly – even by a gravitational wave bearing a negative energy. The phenomena observed by H. Arp (2002, and references therein), where the quasars abandoned the galaxies in the relatively young Universe, together with the 'bridges' between them and the respective galaxy, can perhaps be explained by using the same mechanism: Such quasars, in their early phase of development, would be dynamically indifferent (and thus, they can leave the galaxy where they originated), while they would be gravitationally active (and could 'draw the bridges' out from the galaxies. Similar – yet more complex – configurations get analyzed by Bell (2002). The same mechanism can be used in order to explain why the components of the dynamically bound multiple star system can be ejected. Important details related to the observed cosmological redshift combined with the Doppler

* Including the solution of the potential energy problem

shift, in connection with mentioned phenomena, were published by Basu (2006). As we still sustain the standard model of the closed universe and the validity of the Mach principle along with the GRT (Voráček, 2008), the destiny of black holes in the final stage of the *world-end* (big crunch) will be their dissolvation; the massive black holes will dissolve first, those smaller, consequently, later and later, while for the hypothetic inner black hole observer inside the PM only a very short time interval will pass between the collapse and the final dissolvation. Our description of the scenario is – as a whole – very similar to that presented by Hawking and Penrose (1969) or by Christodolou and Ruffini (1971), but having taken into consideration the Mach principle, our picture of the process becomes rather more complex and rich in physical details.

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