A General Relativity Kerr-Newman extremal black hole model of particles

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Abstract

The commonly accepted no-hair conjecture postulates that all black holes can be completely characterized by three and only three externally observable classical parameters: mass, electrical charge, and angular momentum. The Kerr–Newman metric describes the spacetime geometry in the region surrounding a charged, rotating mass. These three parameters are also the basic parameters of many subatomic particles. In light of the similarities between the black holes and the subatomic particles, this paper applies the Kerr–Newman metric to investigate the spacetime properties of a spinning Planck mass particle carrying an angular momentum of one half Planck constant. Depending on the angular frequency of the rotation of the particle, the results exhibit a group of particles with properties similar to those of the stable subatomic particles, including the neutrino, electron, position, proton, and anti-proton. The highly curved spacetime surrounding the particle in Planck scale, together with the rotation of the particle, make the Planck mass particle to appear as a laboratory mass similar to the mass of the respective particle. The laboratory measurable size of these particles is in the same order of their respective Compton wavelengths. Interacting forces between these particles in the Planck scale exhibit strengths similar in magnitudes to the strong force, electrical force, weak force and the gravitational force depending on the spacetime curvature at the point of interaction. This preliminary attempt of investigating the "spinning Planck mass" using the Kerr-Newman metric has resulted with an interesting model that resembles many particles in nature and raised two interesting questions:

(1) Are there any relationships between the fundamental particles and the "spinning Planck masses"?

(2) Could the particle-particle interacting forces be expressed in terms of the interactions of spacetime curvatures in Planck scale?

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Background and Introduction:

One hundred years ago, in November of 1915, Einstein presented what are now known as the Einstein field equations.

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{C^4} T_{\mu\nu} \qquad (1)$$

These equations specify how the geometry of space and time is influenced by whatever matter and radiation are present, and form the core of Einstein's general theory of relativity (1) (2) (3). By the end of 1915, the astrophysicist Karl Schwarzschild found the first non-trivial exact solution to the Einstein field equations, the so-called Schwarzschild metric (4)

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi) \qquad (2)$$

where $r_s = \frac{2MG}{c^2}$.

In the following years, generalizing Schwarzschild's solution to include electrical charge resulted in the Reissner–Nordström solution (5)

$$ds^2 = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

where $r_Q^2 = \frac{Q^2 G}{4\pi\varepsilon_0 c^4}$, and $d\Omega^2 = \mathrm{d}\theta^2 + sin^2 d\phi^2$ (3)

In 1917, Einstein applied his theory to the universe as a whole, initiating the field of relativistic cosmology. In 1965, Ezra "Ted" Newman found the axisymmetric solution of Einstein's field equation for a black hole, which is both rotating and electrically charged. This formula for the metric tensor $g_{\mu\nu}$ is now the Kerr–Newman metric. It is a generalization of the Kerr metric for an uncharged spinning point-mass, which had been discovered by Roy Kerr two years earlier. The Kerr–Newman metric (6) describes the geometry of space-time in the vicinity of a rotating mass M with charge Q. The formula for this metric depends upon what coordinates or coordinate conditions are selected. In spherical coordinates,[4] (Boyer–Lindquist coordinates):

$$c^2 d\tau^2 = -\left(\frac{dr^2}{\Delta} + d\vartheta^2\right)\rho^2 + (cdt - asin^2\vartheta d\phi)^2 \frac{\Delta}{\rho^2} - ((r^2 + a^2)d\phi - acdt)^2 \frac{sin^2\vartheta}{\rho^2} \qquad (4)$$

Where the coordinates (r, ϑ, ϕ) are standard spherical coordinate system, and the length-scales:

$$a \equiv \frac{J}{Mc}$$

$$\rho^2 \equiv r^2 + a^2 + r_Q^2$$

$$\Delta \equiv r^2 - r_s r + a^2 + r_Q^2$$

Here r_s is the Schwarzschild radius (in meters) of the massive body, which is related to its mass M by

$$r_s \equiv \frac{2GM}{c^2}$$

Where G is the gravitational constant, and r_Q is a length-scale corresponding to the electric charge Q of the mass

$$r_Q^2 \equiv \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4}$$

Where $1/4\pi\varepsilon_0$ is Coulomb's force constant

An alternative metric form of the Kerr Newman Metric can also be written as:

$$c^2 d\tau^2 = \frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} c^2 dt^2 - \left(\frac{\rho^2}{\Delta}\right) dr^2 - \rho^2 d\vartheta^2 + (a^2\Delta sin^2\vartheta - r^4 - 2r^2a^2 - a^4)\frac{sin^2\vartheta d\phi^2}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a sin^2\vartheta c dt d\phi}{\rho^2} \quad (5)$$

All these equations and metrics are widely used for describing massive astronomical scale objects from the size of the earth, the sun, neutron stars, quasars, and black holes.

Application of Space Time Metrics to Planck scale particles:

The Planck constant h is one of the fundamental quantities of nature. The energy of the electromagnetic wave, light, is $E = h\upsilon$, or $\hbar\omega$, where υ is the frequency and $\hbar$ is the reduced Planck constant $\hbar = \frac{h}{2\pi}$ and $\omega = 2\pi\upsilon$ is the angular frequency. Together with the velocity of light c, the gravitational constant G, there are three fundamental units that are naturally composed from these constants: $\sqrt{\frac{\hbar G}{c^3}} = l_p$ is the Planck length ($1.61619926 \times 10^{-35}$ meters); $\sqrt{\frac{\hbar G}{c^5}} = \tau_p$ is the Planck Time ($\sim 5.39106x10^{-44}$ sec), and $\sqrt{\frac{\hbar c}{G}} = m_p$ is the Planck Mass ($2.17651(13) \times 10^{-8}$ kg). When these fundamental units are used in the space-time solutions of the Einstein's Equations, some interesting results have followed. An object with the mass of one half Planck Mass, M = ½ $m_p = \frac{1}{2}\sqrt{\frac{\hbar c}{G}}$ has a Schwarzschild radius of one Planck Length $\sqrt{\frac{G\hbar}{c^3}} = l_p$

$$r_s \equiv \frac{2MG}{c^2} = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \frac{\sqrt{G\hbar c}}{c^2} = \sqrt{\frac{G\hbar}{c^3}} = l_p.$$

The curvature term of Schwarzschild Equation (2), $\left(1-\frac{r_s}{r}\right)$ becomes zero and $(1-\frac{r_s}{r})^{-1}$ term becomes infinite for a half Planck Mass object at the Schwarzschild radius of l_p. At the distances approaching this l_p radius, space-time is highly curved just like an astronomical black hole. It has all the properties just like a "micro-black hole". The local time element, $d\tau$, at a distance r away from the object is slowed down in comparison to the far-away time dt. The local line element in the radial direction is lengthened in comparison to the far away dr according to the following relationship:

$$d\tau(local) = (1-\frac{2MG}{rc^2})^{\frac{1}{2}}dt, \quad \text{and} \quad ds(\text{local}) = \left(1-\frac{2MG}{rc^2}\right)^{-\frac{1}{2}}dr. \qquad (6)$$

Also, a "probe" particle of mass m, interacting in this field has a constant energy to mass ratio, E/m [7] of

$$\frac{E}{mc^2} = \left(1-\frac{2MG}{rc^2}\right)\frac{dt}{d\tau} = 1 \qquad (7)$$

i.e. with a space-time curvature of $\sigma = \left(1-\frac{2MG}{rc^2}\right)$. For a particle with mass equal to one half Planck mass $\frac{1}{2}\sqrt{\frac{\hbar c}{G}}$, σ becomes zero at the Schwarzschild radius of one Planck length $\sqrt{\frac{\hbar G}{c^3}}$ and the reciprocal of this space-time curvature term is infinite at l_p (a shell of singularity).

Particle with angular moment:

For an object spinning with an angular momentum of J and carrying a charge Q, we can use the Alternative form of Kerr-Newman Metric [6].

$$c^2 d\tau^2 = \frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2}c^2dt^2 - \left(\frac{\rho^2}{\Delta}\right)dr^2 - \rho^2 d\vartheta^2 +$$
$$(a^2\Delta sin^2\vartheta - r^4 - 2r^2a^2 - a^4)\frac{sin^2\vartheta d\phi^2}{\rho^2} - (\Delta - r^2 - a^2)\frac{2asin^2\vartheta cdtd\phi}{\rho^2}$$

Where $a \equiv \frac{J}{Mc}$, $\rho^2 \equiv r^2 + a^2cos^2\vartheta$, $\Delta \equiv r^2 - r_s r + a^2 + r_Q^2$, and $r_Q^2 \equiv \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4}$

By re-grouping the time dependent terms, [8] we have

$$c^2d\tau^2 = \frac{(\Delta - a^2sin^2\vartheta)}{\rho^2}c^2dt^2 - (\Delta - r^2 - a^2)\frac{2asin^2\vartheta cdtd\phi}{\rho^2} + (\text{terms without t}) \qquad (8)$$

By replacing $d\phi$ with ωdt where $\omega \equiv \frac{d\phi}{dt}$

$$c^2d\tau^2 = \frac{(\Delta - a^2sin^2\vartheta)}{\rho^2}c^2dt^2 - (\Delta - r^2 - a^2)\frac{2ac\omega sin^2\vartheta dt^2}{\rho^2} + (\text{terms without t})$$

$$c^2 d\tau^2 = \left[\frac{(\Delta - a^2sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{c\rho^2}\right]c^2dt^2 + (terms\ without\ t)$$

A small segment of τ can then be written as

$$\tau = \{\left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho^2 c}\right] t^2 + (terms\ without\ t)\}^{\frac{1}{2}} \quad (8a)$$

If a τ is divided into two sub-segments $\tau = \tau_A + \tau_B$ and the respective r's from M is written as $r = r_A + r_B$, together with Δ_A, ρ_A, for the respective Δ, and ρ. Let T be the total elapsed time so that if t is for the segment A, and (T-t) is the elapsed time for segment B. For segment A,

$$\tau_A = \{\left[\frac{(\Delta_A - a^2 sin^2\vartheta)}{\rho_A{}^2} - (\Delta_A - r_A{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_A{}^2 c}\right] t^2 + (terms\ without\ t)\}^{\frac{1}{2}}$$

The time differentiation of the above for the segment of A will take the form of:

$$\frac{d\tau_A}{dt} = \frac{\left[\frac{(\Delta_A - a^2 sin^2\vartheta)}{\rho_A{}^2} - (\Delta_A - r_A{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_A{}^2 c}\right] t}{\tau_A}$$

For segment B,

$$\tau_B = \{\left[\frac{(\Delta_B - a^2 sin^2\vartheta)}{\rho_B{}^2} - (\Delta_B - r_B{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_B{}^2 c}\right] (T-t)^2 + (terms\ without\ t)\}^{\frac{1}{2}}$$

The time differentiation of the segment B will take the form of:

$$\frac{d\tau_B}{dt} = \frac{-\left[\frac{(\Delta_B - a^2 sin^2\vartheta)}{\rho_B{}^2} - (\Delta_B - r_B{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_B{}^2 c}\right] (T-t)}{\tau_B}$$

Since $\tau = \tau_A + \tau_B$, and the Principle of Extremal Aging [(2)] says that the segment A and B must yield an extremum time τ, therefore,

$$\frac{d\tau}{dt} = \frac{d\tau_A}{dt} + \frac{d\tau_{\boldsymbol{B}}}{dt}$$

$$= \frac{\left[\frac{(\Delta_A - a^2 sin^2\vartheta)}{\rho_A{}^2} - (\Delta_A - r_A{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_A{}^2 c}\right] t}{\tau_A} + \frac{-\left[\frac{(\Delta_B - a^2 sin^2\vartheta)}{\rho_B{}^2} - (\Delta_B - r_B{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_B{}^2 c}\right] (T-t)}{\tau_B} = 0$$

i.e. $$\frac{\left[\frac{(\Delta_A - a^2 sin^2\vartheta)}{\rho_A{}^2} - (\Delta_A - r_A{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_A{}^2 c}\right] t_A}{\tau_A} = \frac{\left[\frac{(\Delta_B - a^2 sin^2\vartheta)}{\rho_B{}^2} - (\Delta_B - r_B{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_B{}^2 c}\right] (T-t)}{\tau_B} \quad (8b)$$

Set t = t_A, and $(T-t) = t_{\boldsymbol{B}}$, then

$$\left[\frac{(\Delta_A - a^2 sin^2\vartheta)}{\rho_A{}^2} - (\Delta_A - r_A{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_A{}^2 c}\right]\frac{t_A}{\tau_A}$$
$$= \left[\frac{(\Delta_B - a^2 sin^2\vartheta)}{\rho_B{}^2} - (\Delta_B - r_B{}^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho_B{}^2 c}\right]\frac{t_{\boldsymbol{B}}}{\tau_B} \quad (8c)$$

Each part of Equation 8c depends only on the parameters of the respective segment. It displays a quantity that is constant from one segment of the path to another. The value of either side of this equation must be independent of which segment we choose to look at. We have therefore a constant of the motion, the same for all segments. This is the same differential notation to identify the constant of motion as the energy. It is related to the relativistic expression for the total energy of the particle.

In the flat spacetime of special relativity $\frac{E}{mc^2} = \frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

In Schwarzschild geometry, $\frac{E}{mc^2} = (1 - \frac{2MG}{rc^2})\frac{dt}{d\tau}$

In Kerr-Newman Metric, $\frac{E}{mc^2} = \left[\frac{(\Delta - a^2 sin^2\vartheta)}{\rho^2} - (\Delta - r^2 - a^2)\frac{2a\omega sin^2\vartheta}{\rho^2 c}\right]\frac{dt}{d\tau}$ (9)

By expressing ρ *and* Δ: $\rho^2 \equiv r^2 + a^2cos^2\vartheta$, *and* $\Delta \equiv r^2 - r_s r + a^2 + r_Q^2$

$$\frac{E}{mc^2} = \left[\frac{(r^2 - r_s r + a^2 + r_Q^2 - a^2 \sin^2\vartheta)}{r^2 + a^2\cos^2\vartheta} - (r^2 - r_s r + a^2 + r_Q^2 - r^2 - a^2)\frac{2a\sin^2\vartheta\omega}{(r^2 + a^2\cos^2\vartheta)c}\right]\frac{dt}{d\tau}$$
$$= \left[\frac{(r^2 + a^2(1 - sin^2\vartheta) - r_s r + r_Q^2}{r^2 + a^2cos^2\vartheta} - (r_s r - r_Q^2)\frac{2asin^2\vartheta\omega}{(r^2 + a^2cos^2\vartheta)c}\right]\frac{dt}{d\tau}$$
$$= \left[\frac{(r^2 + a^2(1 - sin^2\vartheta)}{r^2 + a^2cos^2\vartheta} - \frac{r_s r - r_Q^2}{r^2 + a^2cos^2\vartheta} - (r_s r - r_Q^2)\frac{2asin^2\vartheta\omega}{(r^2 + a^2cos^2\vartheta)c}\right]\frac{dt}{d\tau}$$
$$= \left[1 - \frac{r_s r - r_Q^2}{r^2 + a^2cos^2\vartheta}(1 - \frac{2asin^2\vartheta\omega}{c})\right]\frac{dt}{d\tau}$$

So, $$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2 + a^2cos^2\vartheta}(1 - \frac{2asin^2\vartheta\omega}{c})\right]\frac{dt}{d\tau} \quad (10)$$

Case I: For Q = 0, $\left(r_Q^2 = \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4} = 0\right)$, $a = 0$, a non-rotating electrically neutral object:

$$\frac{E}{mc^2} = \left[1 - \frac{r_s}{r}\right]\frac{dt}{d\tau} = [1 - \frac{2MG}{rc^2}]\frac{dt}{d\tau}$$

The space-time curvature term $[1 - \frac{2MG}{rc^2}]$ becomes zero at $r = \frac{2MG}{c^2}$ $\forall M$

For M = $\frac{1}{2}m_p = \frac{1}{2}\sqrt{\frac{\hbar c}{G}}$, the space-time curvature term $[1 - \frac{2MG}{rc^2}]$ becomes zero at Planck length, l_p and the reciprocal of this term becomes infinite and it is similar to a "micro black hole". This result is the same as using the Schwarzschild Metric of Equation (2) above.

Case II: For Q = 0, $\left(r_Q^2 = \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4} = 0\right), a \neq 0,$

An electrically neutral object with an angular momentum,

From Equation (10): $\frac{E}{mc^2} = \left[1 - \frac{r_s r}{r^2 + a^2 cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$

(Case IIA) On the equatorial plane, $\vartheta = \frac{\pi}{2}$ i.e. $cos\vartheta = 0, sin\vartheta = 1$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} \tag{11}$$

If $2a\omega = c,\ then\ \ \frac{E}{mc^2} = [1]\frac{dt}{d\tau} \qquad \forall M\ \ and\ \forall\, r$ (12)

That is to say: If the spinning frequency $\omega = \frac{c}{2a}$, then the space-time curvature is always equal to 1 on the equatorial plane. It is independent of its mass. The spacetime curvature on the equatorial plane is always "flat", just like an object of "zero gravitational mass", i.e. equivalent to M = 0. Nevertheless, this object carries a non-zero angular moment of J $(a = \frac{J}{Mc})$ spinning at a frequency of $\omega = \frac{c}{2a}$. Furthermore, the condition of $2a\omega = c$ can be written as $2\frac{J}{Mc}\omega = c.$

If $J = \frac{\hbar}{2}$, and $if\ \omega = \frac{1}{2\tau_p}$

then M $= \frac{2J\omega}{c^2} = \frac{\frac{\hbar}{2}\sqrt{\frac{c^5}{\hbar G}}}{c^2} = \frac{1}{2}\sqrt{\frac{\hbar c}{G}} = \frac{1}{2}m_p$

The frequency ω_p will be called the Planck Frequency $\left(\omega_p \equiv \frac{1}{\tau_p} = \sqrt{\frac{c^5}{\hbar G}}\right)$ in this article. A particle with mass equal to one half Planck Mass, $\frac{1}{2}m_p$, spinning at the one half Planck Frequency, is carrying an angular momentum of $\frac{\hbar}{2}$. It satisfies the condition of $\left(1 - \frac{2a\omega}{c}\right) = 0$. Also, particle with mass equal to one Planck Mass m_p, spinning at the Planck Frequency, is carrying an angular momentum of $\frac{\hbar}{2}$. It also satisfies the condition of $\left(1 - \frac{2a\omega}{c}\right) = 0$. The spacetime curvature term of such a particle, even though its mass is equal to one half the Planck Mass (or one Planck Mass) will be "observed" as a zero mass M = 0 particle on its equatorial plane. Any force acting on this particle can cause it to travel with the velocity of light along its equatorial plane. Unlike the particle of equation (6) and (7) above, this particle having the mass of one Planck Mass (or one half Planck Mass), and spinning with an angular momentum of $\frac{\hbar}{2}$, does not contain any singularity of curvature in spacetime, and it behaves just like an electrically neutral particle of zero rest mass with spin $\frac{\hbar}{2}$. This zero rest mass particle nevertheless can carry energy and/or transfer an angular momentum of $\frac{\hbar}{2}$ to other interacting particles. Many properties of this particle are very much like those of a neutrino.

(Case IIB) Along the polar axis: $\vartheta = 0, \quad cos\vartheta = 1, \quad sin\vartheta = 0$

Equation (10) $\frac{E}{mc^2} = \left[1 - \frac{r_s r}{r^2+a^2cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$ can be written as

$\frac{E}{mc^2} = \left[1 - \frac{r_s r}{r^2+a^2}\right]\frac{dt}{d\tau}$ with $a^2 \neq 0$, $\left[1 - \frac{r_s r}{r^2+a^2}\right] \neq 0$

Since $a = \frac{J}{Mc}$, for a particle of J = $\frac{\hbar}{2}$ and M = $\frac{1}{2}m_p$,

$$a = \frac{\frac{\hbar}{2}}{\frac{1}{2}c\sqrt{\frac{\hbar c}{G}}} = l_p \ , \qquad \text{and} \qquad r_s = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = l_p$$

$\sigma = \left[1 - \frac{r_s r}{r^2+a^2}\right] = [1 - \frac{l_p}{\left(r+\frac{l_p{}^2}{r}\right)}] = [1 - \frac{n}{n^2+1}]$ for r = nl_p. (see footnote 1) (14)

The space-time curvature term $\left[1 - \frac{r_s r}{r^2+a^2}\right]$ is equal to $\frac{1}{2}$ for n=1 in the polar directions. The space-time curvature term does not have any singularity for all $n \geq 1$ $(\forall\, n \geq 1)$ both in the differential space and time coefficients of the Kerr Newman metric. The spacetime curvature is again flat, or equal to 1, $\forall\, r \gg l_p$ in the polar direction. Since the space-time curvature is not equal to 1 in the polar direction when n is a small number, in Planck scale, this is indeed an object with mass when $\vartheta \neq \frac{\pi}{2}$. This is not a "massless object". However, with the property of M = 0 in the equatorial direction, this particle can move along the equatorial plane with the velocity of light just like as a massless particle when a force having a non-zero component in the $\vartheta = \frac{\pi}{2}$ direction is applied to this particle. For an omnidirectional emitting source, and for any giving direction, only one third of the particles can be observed. If this is indeed the case, could this be the explanation for the "missing neutrino"? For n=1 in the polar direction, the spacetime curvature is equal to ½. The ½ to ½ spacetime curvature between the two particles could bind them together along the polar direction.

Case III: Q ≠ 0, $\left(\mathbf{r_Q^2 \neq 0}\right)$, $\mathbf{a\omega > 0}$
Charge Particle with an Angular Momentum:

From Equation (10), $\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$

(Case IIIA) On the equatorial plane, $\vartheta = \frac{\pi}{2}$ i.e. $cos\vartheta = 0;\ sin\vartheta = 1$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} \qquad (14a)$$

Again, if $2a\omega = c,\ \ then\ \frac{E}{mc^2} = [1]\frac{dt}{d\phi}$ $\qquad \forall M\ \ and\ \forall r$

If this particle has an angular momentum J of $\frac{\hbar}{2}$, and if the ω is equal to $\frac{1}{2}\omega_p$, then $2a\omega = 2\frac{J}{Mc}\frac{1}{2\tau_p} = \frac{\hbar}{2Mc}\sqrt{\frac{c^5}{\hbar G}} = c$, therefore, $M = \frac{1}{2}\sqrt{\frac{G\hbar}{c}} = \frac{1}{2}m_p$ The particle core mass is equal to one half Planck Mass. However the space-time curvature in the equatorial plane is equal to 1 (flat) because of $\left(1 - \frac{2a\omega}{c}\right) = 0$ just like the Case II(A) above.

($\mathrm{IIIA_n}$) Negative additive frequency

Now, if this $M = \frac{1}{2}m_p$ particle is spinning with an angular momentum J = $\frac{\hbar}{2}$ but having a frequency of $\omega = \frac{1}{2}\omega_p - \frac{1}{2}\omega_e$, where $\omega_e = m_e\frac{c^2}{\hbar}$; m_e being the rest mass of an electron, and if Q is the charge (e) of an electron,

then $\left(1 - \frac{2a\omega}{c}\right) = \left[1 - \frac{\frac{1}{2}\hbar(\omega_p - \omega_e)}{\frac{1}{2}m_p c^2}\right] = \frac{\hbar\omega_e}{m_p c^2} = \frac{m_e}{m_p}$ (15)

Equation (14) becomes

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} = [1 - \frac{r_s r - r_Q^2}{r^2}(\frac{m_e}{m_p})]\frac{dt}{d\tau} \quad (15a)$$

$$\text{since } r_Q^2 \equiv \frac{Q^2}{4\pi\varepsilon_0}\frac{G}{c^4} = \frac{e^2}{4\pi\varepsilon_0}\frac{G}{c^4}\frac{r}{r} = 2\frac{(\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 rc^2})Gr}{c^2} = 2\frac{m_e'}{c^2}Gr \quad (15b)$$

where $m_e' \equiv (\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 rc^2})$. This m_e' is the mass equivalent of the "self energy" of a charge e with a spherical radius of r.

Equation (15a) can be written as

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(\frac{m_e}{m_p}\right)\right]\frac{dt}{d\tau} = \left[1 - \frac{\frac{m_p G}{c^2}r - 2\frac{m_e'}{c^2}Gr}{r^2}\left(\frac{m_e}{m_p}\right)\right]\frac{dt}{d\tau}$$

$$= [1 - \frac{2\left(\frac{1}{2}m_p - m_e'\right)G}{rc^2}\left(\frac{m_e}{m_p}\right)]\frac{dt}{d\tau} \quad (16)$$

the term $\left(\frac{1}{2}m_p - m_e'\right)$ is equal to a gravitating mass of one half Planck Mass minus the equivalent mass of the "self energy of the electrical charge" divided by c^2, with a spherical radius r. Since $m_e' \ll m_p$, $\forall r \geq l_p$, Equation (16) can be written as

$$\frac{E}{mc^2} = [1 - \frac{2\left(\frac{1}{2}m_p - m_e'\right)G}{rc^2}\left(\frac{m_e}{m_p}\right)]\frac{dt}{d\tau} = \left[1 - \frac{m_e G}{rc^2}\right]\frac{dt}{d\tau} \quad (16a)$$

The space-time curvature from the gravitating mass as observed by a "probe" of mass m (or test mass m) is $\left[1-\frac{m_e G}{rc^2}\right]$ $where\ m_e$ is like the rest mass of an electron. The modulation frequency riding on the one half Planck Frequency has a wavelength equal to the Compton wavelength of an electron. The interaction between two of such particles is like two electrons with charge e in each. From the curvature term $\left[1-\frac{m_e G}{rc^2}\right]$, in the $\vartheta=\frac{\pi}{2}$ direction, the net mass of a $\frac{1}{2}m_p$ particle (1.088x10^{-8} kg) is the similar to the mass of an electron m_e, (9.109x10^{-31} kg) when it is spinning with a frequency of $\omega=\frac{1}{2}\omega_p-\frac{1}{2}\omega_e$.

($IIIA_p$) Positive additive frequency

Now, if this $M=\frac{1}{2}m_p$ particle is spinning with an angular momentum J = $\frac{\hbar}{2}$ and with a frequency of $\omega=\frac{1}{2}\omega_p+\frac{1}{2}\omega_e$, where $\omega_e=m_e\frac{c^2}{\hbar}$,

then $$\left(1-\frac{2a\omega}{c}\right)=\left[1-\frac{\frac{1}{2}\hbar\left(\omega_p+\omega_e\right)}{\frac{1}{2}m_p c^2}\right]=-\frac{\hbar\omega_e}{m_p c^2}=-\frac{m_e}{m_p} \qquad (17)$$

This is the same as Equation (15) above with m_e replaced by $-m_e$.
Equation (16) can also be written as

$$\frac{E}{mc^2}=[1-\frac{2\left(\frac{1}{2}m_p-m_e'\right)G}{rc^2}\left(\frac{-m_e}{m_p}\right)]\frac{dt}{d\tau}=\left[1-\frac{(-m_e)G}{rc^2}\right]\frac{dt}{d\tau} \qquad (18)$$

This is the space-time curvature from a mass of $-m_e$.

The electrical interaction between a particle in Case III(A_n) with a particle in Case III(A_p) will be a repulsive force of $F_q=K\frac{e^2}{r}$. However, since the mass of the particle in Case III(A_p) is negative, the acceleration from this "repulsive force" is in the reversed direction, i.e. the interaction between these two particle will be "attractive". This is equivalent to treating the particle in Case III(A_p) (positive added frequency) as a charge of +e and a positive mass of m_e, similar to a ***positron***, and treating the particle in Case III(A_n) (negative added frequency) as a charge of -e with a positive mass of m_e, similar to an electron. These two particles will be attractive to each other and annihilated each other when they are combined.

(IIIB) Along the polar axis: $\vartheta = 0, \quad cos\vartheta = 1, \quad sin\vartheta = 0$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2}\right]\frac{dt}{d\tau} \tag{19}$$

For Q = e, M = $\frac{1}{2}m_p$, and $J = \frac{\hbar}{2}$,

$$r_s = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{\hbar G}{c^3}} = l_p,$$

$$a = \frac{J}{Mc} = \frac{\frac{\hbar}{2}}{\frac{1}{2}m_p c} = l_p \qquad r_Q^2 \equiv \frac{e^2 G}{4\pi\epsilon_0 c^4} = \frac{e^2}{4\pi\epsilon_0 \hbar c}\frac{\hbar G}{c^3} = \alpha\frac{\hbar G}{c^3} = \alpha\, {l_p}^2 \tag{19a}$$

where $\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$ *is the fine structure constant*

Let r be equal to an integer n times l_p, i.e. r = n l_p

Then Equation (19) can be written as

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2}\right]\frac{dt}{d\tau} = [1 - \frac{n{l_p}^2 - \alpha\, {l_p}^2}{(n^2+1){l_p}^2}]\frac{dt}{d\tau} = [1 - \frac{(n-\alpha)}{(n^2+1)}]\frac{dt}{d\tau} \tag{20}$$

where n is the number of Planck lengths away from the pole of the spinning object. For n=1, the space-time curvature term σ = [1- ½ + $\frac{\alpha}{2}$] = $\frac{1+\alpha}{2}$. This is very much similar to that of the Case II B except with the addition of $\alpha/2$ from the electrical charge. For n $\gg 1$, $\sigma \implies 1$ leads to a flat space-time.

Case IV: Q ≠ 0, $(\mathbf{r_Q^2 \neq 0})$, $\mathbf{a\omega < 0}$, Charge Particle with Angular Momentum and negative angular velocity: (see footnote 2)

From Equation (10), $\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2cos^2\vartheta}(1 - \frac{2a\omega sin^2\vartheta}{c})\right]\frac{dt}{d\tau}$

(Case IVA) On the equatorial plane, $\vartheta = \frac{\pi}{2}$ i.e. $cos\vartheta = 0, \quad sin\vartheta = 1$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}(1 - \frac{2a\omega}{c})\right]\frac{dt}{d\tau} \tag{21}$$

If $a\omega < 0$, *and if* $-2a\omega = c$

$$\text{Then } \frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}(2)\right]\frac{dt}{d\tau} \qquad \forall M \text{ and } \forall r \tag{22}$$

$$\frac{E}{mc^2} = \left[1 - 2\frac{r_s}{r} + 2\frac{r_Q^2}{r^2}\right]\frac{dt}{d\tau}$$

For a particle of M $=\frac{1}{2}m_p$, the Schwarzschild radius is equal to one Planck length $r_s = l_p$.
At two times the Schwarzschild radius $r = 2r_s = 2l_p$,

$$\frac{E}{mc^2} = \left[2\frac{r_Q^2}{4{l_p}^2}\right]\frac{dt}{d\tau} = \frac{1}{2}\frac{\alpha\, {l_p}^2}{{l_p}^2} = \frac{1}{2}\alpha \qquad (22a)$$

In this Case IV (and for the following two Cases), $\mathbf{a\omega < 0}$ is rotating in a direction opposite to the Case III above. In here, this model has to assume that space-time is not totally symmetrical in rotation. This is similar to being inside the Ergosphere of a rotating black hole where the spacetime is dragged along in the direction of the rotation. The energy of a spinning particle inside the dragged spacetime of the ergosphere depends on the direction of rotation. If the creation (or pair production) of a particle is inside the ergosphere of another rotating object (the host object), e.g. in Case III, when $\omega = \frac{1}{2}\omega_p$ and $a\omega > 0$ with respect to rotation of the "host", the rotational energy cancels the core mass energy. The result is a zero rest mass particle along the equatorial plane. Whereas, in Case IV (and the Cases below), rotational frequency of $\omega = -\frac{1}{2}\omega_p$, or $\mathbf{a\omega < 0}$ with respect to the dragged spacetime inside the ergosphere of the host**,** the rotational energy adds to the core mass energy. The result is a non-zero more massive particle with a very small space-time curvature σ at $r = 2r_s$. Since all particles in this model have a Planck mass at the core, the close vicinity of the rotating core of one particle may serve as a host to the creation or the pair production of another particle.

(CaseIVA$_p$) Positive additive frequency

If the particle is spinning at an angular frequency of

$$\omega = \omega_{m_0} - \frac{1}{2}\omega_p \qquad (23)$$

where $\omega_{m_0} = m_o\frac{c^2}{\hbar}$, and $\lambdabar_o = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$ $\lambdabar_o$ is the reduced Compton wavelength of the particle m_o,

and if Q = e, M $=\frac{1}{2}m_p$ and $J = \frac{\hbar}{2}$,

then $\left(1 - \frac{2a\omega}{c}\right) = 2 - \frac{2m_0}{m_p}$

and $$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(2 - \frac{2m_0}{m_p}\right)\right]\frac{dt}{d\tau} \qquad (24)$$

$$\cong [1 - \frac{2m_pG}{rc^2} + \frac{2m_0G}{rc^2}]\frac{dt}{d\tau} = [1 - \frac{2(m_p - m_0)G}{rc^2}]\frac{dt}{d\tau} \qquad (24a)$$

using $r_Q^2 \ll r_s r$.

For $r = nr_p$, Equation 24 becomes $\frac{E}{mc^2} = [1 - \frac{2(m_p - m_0)G}{nr_p c^2}]\frac{dt}{d\tau}$

At $r = 2l_p$, $\frac{E}{mc^2} = \left[\frac{m_0}{m_p}\right]\frac{dt}{d\tau}$, i.e. the space time curvature at $2l_p$ is $\sigma = \left[\frac{m_0}{m_p}\right]$ (25)

The mass/energy of this object observed (or measured) from a far away distance $r \gg 2l_p$ will be

$$E = \left(m_p - m_0\right)\left[\frac{m_0}{m_p}\right]c^2 = m_0 c^2. \quad (25a)$$

It will appear like a particle of mass m_0, spinning with an angular moment of $J = \frac{\hbar}{2}$, carrying a charge of e. At short distances, the curvature term $[1 - \frac{2(m_p - m_o)G}{rc^2}]$ is the same as the space-time curvature term $\left[1 - \frac{2MG}{rc^2}\right]$ in the Schwarzschild Metric from an object of mass $M = m_p - m_0$. The "gravitational interaction" between two of these masses will be

$$F_g = \frac{G(m_p - m_0)^2}{r^2}. \quad (26)$$

The "electrical interaction" from the charge e will be

$$F_q = \frac{Ke^2}{r^2}, \text{ where } K = \frac{1}{4\pi\varepsilon_o}.$$

The ratio between these two interactions will be

$$\frac{F_g}{F_q} = \frac{G(m_p - m_0)^2}{Ke^2} \cong \frac{Gm_p{}^2}{Ke^2} = \frac{G\frac{\hbar c}{G}}{Ke^2} = \frac{\hbar c}{Ke^2} = \frac{1}{\alpha} \cong 137 \quad (26a)$$

using $m_0 \ll m_p$.

Since this particle is spinning at an angular frequency of $\omega = \omega_{m_0} - \frac{1}{2}\omega_p$, (Equation 23), where $\omega_o = m_o \frac{c^2}{\hbar}$, if $\frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$ is the Compton wavelength of a proton, then the F_g (at short range) that is 137 times stronger than the electrical force F_q. This is very much like the short range "nuclear strong interaction" of a proton. The reduced Compton wavelength of m_o is:

$$\lambda\!\!\!^{-}{}_0 = \frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$$

At short range, when r = $2l_p$, $\sigma = \left[\frac{m_0}{m_p}\right]$. At this space-time curvature, the relativistic distance is lengthened by $\sigma^{-1} = \left[\frac{m_p}{m_0}\right]$ (for extremal Kerr black hole). The energy from the electrical force between two particles at this distance can be written as

$$\mathcal{E}_q = K\frac{e^2}{r_p\sigma^{-1}} = K\frac{e^2}{r_p\frac{m_p}{m_o}} = K\frac{e^2 m_o}{r_p m_p} = K\frac{e^2 m_o}{\sqrt{\frac{G\hbar}{c^3}}\sqrt{\frac{\hbar c}{G}}} = K\frac{e^2 m_o c}{\hbar}$$

$$= \frac{Ke^2}{\hbar c} m_o c^2 = \alpha m_o c^2 \quad (27)$$

where α is the fine structure constant ($\sim 1/137$).
If m_o is the mass of a proton, ($\sim 938\ Mev/c^2$),
then $\mathcal{E}_q \cong 6.85\ Mev$ is approximately equal to the (per nucleon) binding energy of nucleus.

For $r > 2l_p$, the σ changes from an extremely small number of $\frac{m_0}{m_p}$ to $\sim(1-\frac{2}{n})$ for $r = nl_p$. For (n≫ 2) $\sigma \implies 1$ (flat space-time).

According to this model, at far away distance, the net mass of this $\frac{1}{2}m_p$ particle ($1.088\text{x}10^{-8}$ kg) is observed according to Equation (25a) as the mass of a proton ($1.6726\text{x}10^{-27}$ kg) when it is spinning with a frequency of $\omega = \omega_{m_0} - \frac{1}{2}\omega_p$. If the two particles are separated by 2 Planck length along the equatorial plane, the (attractive) force between them will have the magnitude of a "strong force" (Equation 26). At this distance, the electrical force (repulsive) between them is 1/137 times weaker than this attractive strong force (Equation 26a). The storage energy from the electrical potential is of 6.85 Mev. (Equation 27) When the separation of these two particles is more than 2 Planck lengths, the repulsive electrical force will overcome the attractive force. The two particles will fly apart releasing the 6.85 Mev energy.

(CaseIVA$_n$) Negative additive frequency

If the particle is spinning at an angular frequency of

$$\omega = -\omega_{m_0} - \frac{1}{2}\omega_p \tag{28}$$

where $\omega_{m_0} = m_o \frac{c^2}{\hbar}$, and $\frac{\lambda_0}{2\pi} = \frac{c}{\omega_{m_0}} = \frac{\hbar}{m_0 c}$, ƛ is the Compton wavelength of the particle m_o,

and if Q = e, M = $\frac{1}{2}m_p$ and $J = \frac{\hbar}{2}$,

then $\left(1 - \frac{2a\omega}{c}\right) = 2 + \frac{2m_0}{m_p}$

and $\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2}\left(2 + \frac{2m_0}{m_p}\right)\right]\frac{dt}{d\tau}$

$$\cong [1 - \frac{2m_pG}{rc^2} - \frac{2m_0G}{rc^2}]\frac{dt}{d\tau} = [1 - \frac{2(m_p+m_0)G}{rc^2}]\frac{dt}{d\tau} \tag{29}$$

using $r_Q^2 \ll r_s r$.

Together with the angular momentum of $J = \frac{\hbar}{2}$ the space-time curvature of this M = $\frac{1}{2}m_p$ object is like an object of mass $M = m_p + m_0$ for $r \geq 2l_p$. At $r = 2l_p$, $\frac{E}{mc^2} = \left[\frac{-m_0}{m_p}\right]\frac{dt}{d\tau} = \left[\frac{m_0}{m_p}\right]\frac{dt}{-d\tau}$,

i.e. the space time curvature at $2l_p$ is $\sigma = \left[\frac{m_0}{m_p}\right]$ $with\ -d\tau$,

i.e., local time of the particle is in reversed direction:

similar to an ***Anti-particle***.

The mass/energy of this object as seen (or measured) from a far away distance $r \gg 2l_p$ will be $E = \left(m_p + m_0\right)\left[\frac{-m_0}{m_p}\right]c^2 \cong -m_0c^2$, just like an anti-particle of mass m_0, spinning with an angular moment of $J = \frac{\hbar}{2}$, carrying a charge of e. At short distances, the curvature term $[1 - \frac{2(m_p+m_o)G}{rc^2}]$ is the same as the space-time curvature term $\left[1 - \frac{2MG}{rc^2}\right]$ in the Schwarzschild Metric from an object with mass $M = m_p + m_0$. The "gravitational interaction" between two of these masses will be $F_g = \frac{G(m_p+m_0)^2}{r^2}$, and the "electrical interaction" from the charge e will be $F_q = \frac{Ke^2}{r^2}$, where $K = \frac{1}{4\pi\varepsilon_o}$. The ratio between these two interactions will be $\frac{F_g}{F_q} = \frac{G(m_p+m_0)^2}{Ke^2} \cong \frac{Gm_p{}^2}{Ke^2} = \frac{G\frac{\hbar c}{G}}{Ke^2} = \frac{\hbar c}{Ke^2} = \frac{1}{\alpha} \cong 137$ using $m_o \ll m_p$.

(CaseIVB) Along the polar axis: $\vartheta = 0, \quad cos\vartheta = 1, \quad sin\vartheta = 0$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2}\right]\frac{dt}{d\tau}$$

For Q = e, $\quad$ M $= \frac{1}{2}m_p, \qquad J = \frac{\hbar}{2}$,

$$r_s = \frac{Gm_p}{c^2} = \frac{G\sqrt{\frac{\hbar c}{G}}}{c^2} = \sqrt{\frac{\hbar G}{c^3}} = l_p,$$

$$a = \frac{J}{Mc} = \frac{\frac{\hbar}{2}}{\frac{1}{2}m_p c} = l_p, \quad r_Q^2 = \frac{e^2 G}{4\pi\epsilon_0 c^4} = \alpha\hbar\frac{G}{c^3} = \alpha\, l_p{}^2$$

where $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$ $\textit{is the fine structure constant.}$

If $r = n\, l_p,\ then$

$$\frac{E}{mc^2} = \left[1 - \frac{r_s r - r_Q^2}{r^2+a^2}\right]\frac{dt}{d\tau} = \left[1 - \frac{nl_p{}^2 - \alpha l_p{}^2}{n^2 l_p{}^2 + l_p{}^2}\right]\frac{dt}{d\tau} = \left[1 - \frac{(n-\alpha)}{(n^2+1)}\right]\frac{dt}{d\tau} \qquad (30)$$

Summery, extension and interesting questions:

Based on the similarity of the basic mass, charge, and angular momentum properties of black holes and fundamental particles, when the Kerr-Newman solution to the Einstein Field equation is applied to a spin ½ particles Planck mass, many interesting space-time properties resulted.

Other than the non-spinning Planck mass of case I, the space-time curvature of all the spinning particles in the equatorial plane is different from the curvature in the polar directions.

For Case II and Case III, the space-time curvature on the equatorial plane is equal to one or just slightly different from one because of the mass equivalent from the energy of the electrical charge of the particle.

The properties of the particles in case II and III are very much like those of leptons.

However, equatorial plane curvature (σ) for particles in Case IV is very small but not zero. At Planck length, the gravitational interaction of two such particles (Gm_pm_p) is very strong. When the particles are separated by a large distance, (n >>1), the observable mass is σ times the Planck Mass and becomes m_0. This is similar to the observed mass m_0 of the particle as measured in the lab. The gravitational interaction will simply be proportional to (Gm_0m_0). At Planck length, (Gm_pm_p) has a similar magnitude as the "strong force"

Along the polar direction, all the particles from Case II, III, and IV have similar curvature terms of $(1-\frac{n-\alpha}{n^2+1})$, or $(1-\frac{n}{n^2+1})$ when there is no electrical charge. At the distance of one Planck length (n=1), the curvature is practically equal to ½. A mass of m_0 at infinity will have a relativistic mass of $2m_0$ at Plank length (n=1) from the interacting mass of ½m_p along the polar direction. The gravitational interaction between these two masses will simply be proportional to (Gm_0m_p). This magnitude is in between (Gm_pm_p) and (Gm_0m_0) similar to the "weak interation".

Furthermore, the r_Q^2 term in Equation 10 arises from an energy equivalent of α times the rotational energy of the particle. Since this fine structure constant α is closely related to electrical charge, in many ways, the rotation-rotation interaction, such as spin-spin interaction of the two rotational objects resembles the electrical charge including the sign of the additive frequency and the sign of the charge.

Should this be the case, then, could all four interactions in nature be expressed in terms of the curvature interactions of the spacetime geometry?

This also leads to two interesting questions:

(1) Are there any relationships between the fundamental particles and the "spinning Planck mass"?
(2) Could the particle-particle interacting forces be expressed in terms of the interactions of spacetime curvatures in Planck scale?

The properties of the spinning ½m_p entities resemble many of the basic and stable subatomic particles:

(1) Neutrino (Case II above): This particle carries an angular moment of $\frac{\hbar}{2}$. Spinning at one half Planck frequency $\frac{\omega_p}{2}$. It is electrically neutral; it may carry energy and has a zero rest mass. It travels with the speed of light along the equatorial plane. It can interact with other particle with a "weak force" along the polar direction. Since it can only travel along the equatorial plane, only 1/3 of them can be detected from any isotropic emitter. Could this be the reason for the "missing neutrinos" from the Sun or from any neutrino source on Earth?

(2) Electron (Case IIIA$_n$ above): This particle carries an angular moment of $\frac{\hbar}{2}$. It is spinning with a frequency $-\frac{\omega_e}{2}$ less than one half of Planck frequency ("negative side band") where $\omega_e = \frac{m_e c^2}{\hbar}$ is the Compton frequency of an electron. The size of this particle arriving from the "side band" is in the order of the Compton wavelength of an electron. It carries a unit charge of -e and interact with other charge particles with the coupling constant of k, where $\frac{Ke^2}{\hbar c}$ is the fine structure constant α. In the polar direction, it also interacts with other particles with "weak interaction" in additional to the interaction from electrical charge.

(3) Positron (Case IIIA$_p$ above): With spinning frequency$+\frac{\omega_e}{2}$ more than $\frac{\omega_p}{2}$ ("positive side band"), this particle carries a positive charge of +e. It can be considered as –e with $-m_e$ just like an anti-particle of electron. The gyromagnetic dipole properties of electron (or positron) in Kerr-Newman metric has also been discussed by other physicists. [(10)]

(4) Proton (Case IVA$_p$ above): With spinning frequency $\frac{\omega_{m_0}}{2} - \frac{\omega_p}{2}$ where ω_{m_0} (positive side band) is the Compton frequency $\omega_o = \frac{m_o c^2}{\hbar}$ of a proton, this spin ½ particle carries a positive charge of +e. At 2 Planck length $(2l_p)$, the gravitational force $(Gm_p m_p)$ between two of these particles is 137 times stronger, $(\frac{\hbar c}{Ke^2}\ times\ stronger)$, than the electrical force (Ke^2) just like the "nuclear strong force". The space-time curvature σ at $2l_p$ is $\frac{m_0}{m_p}$. Therefore, when the second particle is moved from $2l_p$ to infinity, (σ =1), the relativistic observed mass become m_o, i.e., a proton mass.

(5) Anti-proton (Case IVA$_n$ above): With spinning frequency $-\frac{\omega_{m_0}}{2} - \frac{\omega_p}{2}$ (with a negative side band), this spin ½ particle carries a negative charge of –e, (or +e with a negative mass), and just like the anti-particle of a proton.

The coupled composite of one spinning ½m_p particle with another one or more spinning ½m_p particle(s) also has properties that resembles many of the unstable subatomic particles. These composites have finite lifetime and often decay to the decay products consistent of its components.

(6) Neutron (Could this be a composite particle of a proton, an electron, and a neutrino?): The space-time curvature σ of a proton in the polar direction is equal to $1-\frac{(n-\alpha)}{(n^2+1)}$ or (½ $+\frac{\alpha}{2}$) for n = 1 (one Planck length). At this distance, gravitational force between a spin one half, ½m_p particles can be held by the "weak force" from the polar to polar direction space time curvature of ½ on both sides. A positively charged proton, a negatively charged electron and a neutrino can than be held by both the electrical force and the "weak force" from both sides and exhibited as a spin ½ particle with neutral electrical charge. The time period of the electron at the space-time curvature of one half Planck mass ½m_p will be dilated by a factor of $\frac{2\omega_e}{\omega_p}$, i.e. $\tau \cong \frac{2\omega_e}{\omega_p}\tau_e$. Numerically, $\tau \cong 607$ seconds, matching the half-life of a free neutron (~ 10 minutes). This composite particle is unstable by itself and it decays into an electron, a proton and a neutrino $n \Rightarrow p+e+\bar{\upsilon}$ with a half-life of about 10 minutes. τ_e here is the period of Compton wave length of an electron.

(7) Pion (Could this be a composite particle of Case IVA, and Case II, or Case IIIAn or Case IIIAp?): The space-time curvature of Case IVA in the equatorial plane at one Planck length is $\frac{\alpha}{2}$. A composite of this with an electron or positron will have a space-time curvature of $(\frac{2}{\alpha}-1)$, and have a mass of $(\frac{2}{\alpha}-1)m_e$. (The composite curvature is subtractive because one of the components is an anti-particle). This composite particle belongs to the group of "strong interaction" particle as well as "weak interaction" particle.

(π^o) Could a composite particle with a Case II (neutrino) held together in the polar direction be a spin zero neutral pion? This particle interacts with both "weak interaction" and "strong interaction" like a neutral pion π^o.

(π^+) Could a composite particle with a Case IIIAp (positron) held together in the polar direction be a spin zero positively charged particle, pion plus? The mass of this particle will be $\sim(\frac{2}{\alpha}-1)m_e$. The numerical value is 139.54 MeV/c^2 very close to the measured value of 139.57018(35) MeV/c^2.

(π^-) Could a composite particle with a Case IIIAn (electron) held together in the polar direction be a spin zero negatively charged particle like a pion minus. The mass of this particle will be $\sim(\frac{2}{\alpha}-1)m_e$. The numerical value is 139.54 MeV/c^2 very close to the measured value of 139.57018(35) MeV/c^2 [(9)].

(8) Kaon (?): Similar to Case IV: In this case, M= m_p instead of M=$\frac{1}{2}m_p$. If the angular frequency is $\omega = \omega_{m_0} - \omega_p$, then, at r = 2 l_p the space-time curvature will be $\frac{m_0}{2m_p}$. Together with a neutrino, this will be a spin zero particle with a mass about one half of a proton mass like a K^+ . For $\omega = -\omega_{m_0} - \omega_p$, the composite particle is like a K^- . K^0 is like a particle of $\omega = -\omega_p$.

So far, this model has led to a number of interesting facts below. Could these be just "numerical coincidences" or could these be some indication of the validity of this model?

(a) Neutrino is massless and nevertheless carries angular momentum and energy. It travels at the velocity of light just like the Case II particle.

(b) 1/3 of solar neutrino is missing: Case II particle only travels along the equatorial plane. (could this be an alternative to neutrino oscillations)

(c) Only "left handed" neutrino is observed: $\mathbf{a\omega} > \mathbf{0}$ and J = $\frac{\hbar}{2}$ are required to be massless spin ½ particle.

(d) The mass of $(\frac{2}{\alpha} - 1)m_e$ 139.54 MeV/c^2 is very close to the measured value of 139.57018(35) MeV/c^2 for pion plus or pion minus

(e) The free neutron half-life of 10 minutes is very close to $\tau \cong \frac{2\omega_e}{\omega_p}\tau_e$. Where ω_e is the Compton frequency of electron, τ_e is the period of this frequency, and ω_p is the Planck frequency. Numerically, $\tau \cong 607$ sec, just about 10 minutes.

(f) The ratio between these strong force and the electrical force is similar to

$$\frac{G(m_p - m_0)^2}{Ke^2} \cong \frac{G{m_p}^2}{Ke^2} = \frac{1}{\alpha} \cong 137$$

(g) The binding energy between two nucleon is $\alpha m_o c^2$, where m_o is the mass of a proton. $\alpha m_o c^2$ has a numerical value of 6.85 Mev.

(h) In both Case III and Case IV, positive side-band leads to positive charge and negative side-band leads to negative charge.

Table I summarizes the various conditions of this model and the resulting properties that resemble many of the particles in nature.

$M=\frac{1}{2}m_p$	Q	J	$\omega \equiv \frac{d\phi}{dt}$	ϑ	$\sigma\ at\ r$ (σ =1 = flat)	$\sigma\ at\ nl_p$	$\bar{\lambda}$ (size) Compton Wavelength	Mass observed from infinity	Resembling Particle
Case I	0	0	0	$\forall\vartheta$	$[1-\frac{2MG}{rc^2}]$	$(1-\frac{1}{n})$ $\sigma=0$ $@\ n=1$	l_p	$\frac{m_p}{2}$	"Micro Schwarzschild Black hole"
Stable particles									
Case IIA	0	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}$	$\frac{\pi}{2}$	1 ∀ r	1	l_p	0	Neutrino
Case IIB				0		$1-\frac{n}{n^2+1}$			
Case IIIA$_n$	-e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}-\frac{\omega_e}{2}$	$\frac{\pi}{2}$	1 ∀ r	1	$\frac{\hbar}{m_e c}$	m_e	Electron $m_e =$ *electron mass*
Case IIIB				0		$1-\frac{(n-\alpha)}{(n^2+1)}$			
Case IIIA$_p$	+e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}+\frac{\omega_e}{2}$	$\frac{\pi}{2}$	1 ∀ r	1	$\frac{\hbar}{m_e c}$	m_e $d\tau<0$	Positron
Case IVA$_p$	+e	$\frac{\hbar}{2}$	$\frac{\omega_{m_0}}{2}-\frac{\omega_p}{2}$	$\frac{\pi}{2}$	$[1-\frac{2(m_p-m_o)G}{rc^2}]$	$\left[\frac{m_0}{m_p}\right]$ @ n=2	$\frac{\hbar}{m_o c}$	m_o	Proton $m_o =$ *proton mass*
Case IVB				0	$\left[1-\frac{r_s r-r_Q^2}{r^2+a^2}\right]$	$1-\frac{(n-\alpha)}{(n^2+1)}$			
Case IVA$_n$	-e	$\frac{\hbar}{2}$	$-(\frac{\omega_{m_0}}{2}+\frac{\omega_p}{2})$	$\frac{\pi}{2}$	$[1-\frac{2(m_p+m_o)G}{rc^2}]$	$\left[\frac{-m_0}{m_p}\right]$ @ n=2	$\frac{\hbar}{m_o c}$	m_o $d\tau<0$	Anti-Proton
Unstable particles of m = $m_p/2$			**Composite ⊕**						
$M=m_p/2$	Q	J	$\omega \equiv \frac{d\phi}{dt}$	ϑ	$\sigma\ at\ r$ (σ =1 = flat)	$\sigma\ at\ nl_p$	$\bar{\lambda}$ (size) Compton Wavelength	Mass observed from infinity	Resembling Particle
Case IV A$_o$	0	0	$-\frac{\omega_p}{2}\oplus(\frac{\omega_p}{2})$			$\alpha/2$			π^o
Case IV A$_{pe}$	+e	0	$-\frac{\omega_p}{2}$ $\oplus(\frac{\omega_p}{2}+\frac{\omega_e}{2})$			$\alpha/2$	$\frac{\alpha\hbar}{2m_e c}$	$\sim(\frac{2}{\alpha}-1)m_e$	π^+
Case IV A$_{ne}$	-e	0	$-\frac{\omega_p}{2}$ $\oplus(\frac{\omega_p}{2}-\frac{\omega_e}{2})$			$\alpha/2$	$\frac{\alpha\hbar}{2m_e c}$	$\sim\left(\frac{2}{\alpha}-1\right)m_e$ $d\tau<0$	π^-
CaseIIIA$_{un}$	-e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}-\frac{\omega_\mu}{2}$		1 ∀ r	1	$\frac{\hbar}{m_\mu c}$	m_μ	μ^-
CaseIIIA$_{up}$	+e	$\frac{\hbar}{2}$	$\frac{\omega_p}{2}+\frac{\omega_\mu}{2}$		1 ∀ r	1	$\frac{\hbar}{m_\mu c}$	m_μ $d\tau<0$	μ^+

CaseIV A_{pN}	0	$\frac{\hbar}{2}$	$\left(\frac{\omega_{m_0}}{2}-\frac{\omega_p}{2}\right)$ $\oplus\left(\frac{\omega_p}{2}-\frac{\omega_e}{2}\right)$ $\oplus\left(\frac{\omega_p}{2}\right)$		$[1-\frac{2(m_p-m_o)G}{rc^2}]$	$\left[\frac{m_0}{m_p}\right]$ @ n=2	$\sim\frac{\hbar}{m_o c}$	$\sim m_o$	Neutron
Unstable particles of m = m_p			**Composite $\oplus$**						
$M = m_p$	Q	J	$\omega \equiv \frac{d\phi}{dt}$	ϑ	σ *at r* (σ =1 = flat)	σ *at* nl_p	ƛ (size) Compton Wavelength	Mass observed from infinity	Resembling Particle
Case IIA	0	$\frac{\hbar}{2}$	ω_p	$\frac{\pi}{2}$	1 ∀ r	1	l_p	0	Neutrino
Case IIB				0		$1-\frac{8n}{4n^2+1}$			
Case IV Ape	+e	0	$-\omega_p+\frac{\omega_e}{2}$ $\oplus\left(\frac{\omega_p}{2}+\frac{\omega_e}{2}\right)$			$\left[\frac{m_0}{2m_p}\right]$ @ n=2	$\frac{2\hbar}{m_o c}$	$\frac{m_o}{2}$ $+\delta$	K^+
Case IV Ane	-e	0	$-\omega_p$ $\oplus\left(\frac{\omega_p}{2}-\frac{\omega_e}{2}\right)$			$\left[\frac{m_0}{2m_p}\right]$ @ n=2	$\frac{2\hbar}{m_o c}$	$\frac{m_o}{2}$ $+\delta$	K^-
Case IV A_o	0	0	$-\omega_p\oplus\left(\frac{\omega_p}{2}\right)$			$\left[2\frac{m_0}{m_p}\right]$ @ n=2	$\frac{2\hbar}{m_o c}$	$m_o/2$ $+\delta$	K^0

Footnote 1: This model assumes that space is quantized with a minimum length of one Planck length l_p.

Footnote 2: This model assumes that rotation is directional.

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(1) Misner, Charles W.; Thorne, Kip S.; Wheeler, John Archibald (1973). *Gravitation*. San Francisco: W. H. Freeman. ISBN 978-0-7167-0344-0.

(2) Edwin F. Taylor, John Archibald Wheeler "Exploring Black Holes, Introduction to General Relativity" Addison Wesley Longman, ISBN 0-201-38423-X

(3) Richard A. Mould "Basic Relativity" Springer, ISBN 0-387-95210-1

(4) https://en.wikipedia.org/wiki/Schwarzschild_metric

(5) https://en.wikipedia.org/wiki/Reissner%E2%80%93Nordstr%C3%B6m_metric

(6) https://en.wikipedia.org/wiki/Kerr%E2%80%93Newman_metric

(7) Equation [3] in page 4-4 of Reference (2) above.

(8) Equation [5 to 11] in page 4-4 of Reference (2) above.

(9) https://en.wikipedia.org/wiki/Pion

(10) B. Carter, Phys. Rev. 174 1559 (1968)