# Point Mass Model for Predicting 2-D Particles Motion Modes in Vertical Rotation Drum

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**Abstract**. Motion modes of pseudo 2-d granular particles in vertical rotation drum is modeled using point mass motion considering only friction, normal, and gravitation forces. Two schemes in evaluating the forces are used, i.e. dynamics in 2-d linear and 1-d angular motions. Finite difference method implementing Euler scheme is used to solve the equation of motion.

**Keywords**: granular particles, vertical rotation, drum, kiln, motion modes.

**PACS**:

## Introduction

Vertical rotation drums or kilns are apparatus for mixing, granulation, segregation, coating, and drying granular particles [Laio, 2009], which induces various modes of particles motion inside it, e.g. slipping, slumping, rolling, cascading, cataracting, and centrifuging [Henein, 1983]. Particles mass and mixture concentration can shift the motion modes as observed in pseudo-2d system for steel and plastic spheres [Yulia, 2015]. Particles motion modes that are normally shown in parameter space of number of particles $N$ and angular velocity $\omega$ triggers this work to understand the relation between $N$ and $\omega$ from theoretical background of point mass motion.

## Theory

Supposed that there is a cluster of particles with mass $m$ is sitting on the inner side of vertical rotation drum. It is located at angular position $\theta$ measured from direction of gravity $g$ in counter clockwise direction. The drum is rotating also in counter clockwise direction with angular velocity $\omega_D$. Centripetal force concept in radial direction will give

$$N = mR\omega^2 + mg\cos\theta , \quad (1)$$

while Newton second law of motion in angular direction will produce

$$mR^2\alpha = Rf - Rmg\sin\theta . \quad (2)$$

Friction force $f$ depends on angular position $\theta$ to fall between static friction and dynamic friction cases. In the former case it is always fulfilled that

$$f = mg\sin\theta \quad (3)$$

and as consequences $\omega = \omega_D$, while in the later one

$$f = \mu_k mg\cos\theta . \quad (4)$$

Critical angular position $\theta_S$, which separates both cases, is determined through

$$\theta_S = \arctan\mu_s . \quad (5)$$

There is also maximum angle $\theta_N$ where cluster of particles still stick to the inner side of the drum. The value can be derived from Equation (1) for $N > 0$

$$\cos\theta_N > -\frac{R\omega^2}{g} . \quad (6)$$

Equation (6) can be clearer understood when it is represented in sine function instead of cosine function due to value of cosine which is negative in the second quadrant ($\pi/2 < \theta_N < \pi$)

$$\sin\left(\frac{\pi}{2} - \theta_N\right) > -\frac{R\omega^2}{g}$$

then

$$\theta_N < \frac{\pi}{2} + \arcsin\left(\frac{R\omega^2}{g}\right). \quad (7)$$

Equation of motion of cluster of particles can be resumed from previous equations as

$$\frac{d^2\theta}{dt^2} = \begin{cases} 0, & 0 < \theta < \theta_S, \\ -\frac{g}{R}\left(\sin\theta - \mu_k \cos\theta\right), & \theta_S < \theta < \theta_N. \end{cases} \tag{8}$$

with initial condition $\omega(0) = \omega_D$. Position of cluster of particles is Cartesian coordinate system is

$$x = x_D + R\sin\theta \tag{9}$$

and

$$y = y_D + R\cos\theta \tag{10}$$

with $(x_D, y_D)$ is position of rotation axis of the drum.

And if $\omega > 0$ at $\theta = \theta_N$ (or $t = t_N$) then cluster of particles will begin to perform parabolic motion that obeys

$$x(t) = \left(x_D + R\sin\theta_N\right) + \omega R\cos\theta_N\left(t - t_N\right), \tag{11}$$

and

$$y(t) = \left(y_D - R\cos\theta_N\right) + \omega R\cos\theta_N\left(t - t_N\right) - \frac{1}{2}g\left(t - t_N\right)^2 \tag{12}$$

for $t \geq t_N$. The motion should be terminated at $t = t_E$ that fulfills the end condition

$$\left[x(t_E) - x_D\right]^2 + \left[y(t_E) - y_D\right]^2 > R^2, \tag{13}$$

since it already leaves the drum.

**Simulation**

Second part of Equation (8) can be solved easily. Finite Difference method implementing Euler scheme will be used to get the numeric solution using

$$\omega(t + \Delta t) = \omega(t) + \alpha(t)\Delta t \tag{14}$$

and

$$\theta(t + \Delta t) = \theta(t) + \omega(t)\Delta t \tag{15}$$

with time step $\Delta t$.

**Results and discussion**

Not all modes in vertical rotation drum can be shown using this model since it considers only one point mass representing a cluster of particle. Observed modes are shown in Figure 1.

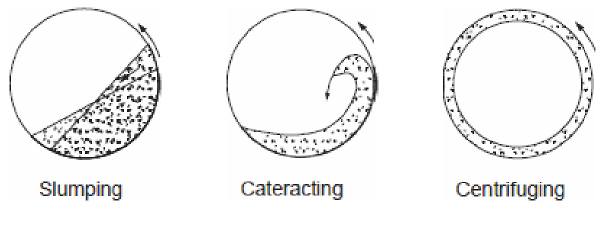
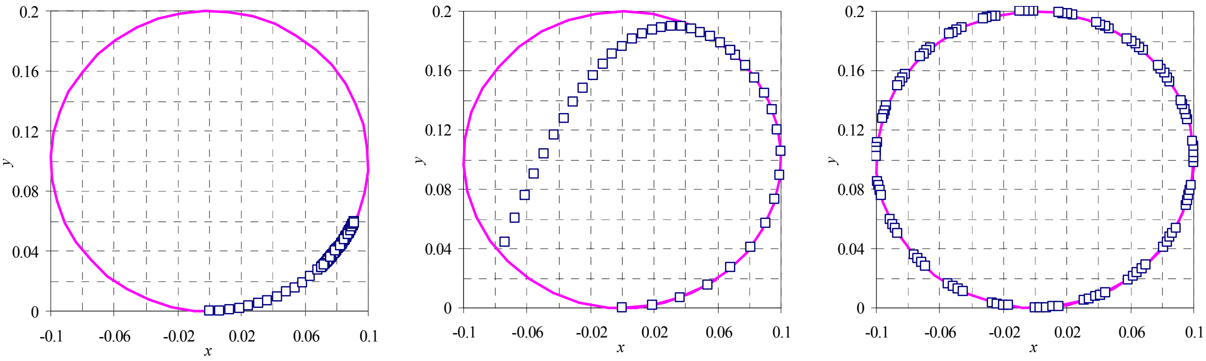



Figure 1. Three motion modes [Ingram, 2005] that can be accommodated by point mass model.

A cluster of particle has mass $m$ = 0.15 kg. Gravity is set to 9.81 m/s$^2$. Three different drum angular velocities $\omega_D$ are used 2π rad/s, 4π rad/s, and 10π rad/s. Drum radius $R_D$ is 10 cm. The accommodated modes are shown in Figure 2 and as comparison related modes from experiment [Yulia, 2015] are also shown in Figure 3.




(a) (b) (c)

Figure 2. Three pronounced motion modes for $m$ = 0.15 kg: (a) slumping with $\omega_D = 2\pi$ rad/s, (b) cateracting with $\omega_D = 4\pi$ rad/s, and (c) centrifuging with $\omega_D = 10\pi$ rad/s.

(a) (b) (c)

Figure 3. Three motion modes for plastic marbles [Yulia, 2015]: (a) slumping ($N$ = 50, $\omega$ = 15 rpm), (b) cataracting ($N$ = 250; $\omega$ = 120 rpm), and (c) centrifuging ($N$ = 600; $\omega$ = 120 rpm).

Increase of drum angular velocity will change particles motion modes from slumping to cascading and then later to centrifuging. It is also interesting to investigate influence of mass of the particle cluster $m$ at the same value of drum angular velocity $\omega_D$. Unfortunately, this model do not include influence of mass $m$ in gaining angular velocity $\omega$. Stationary state is always considered as it can be seen in Equations (3) and (4), where there is no transition defined. And as predicted, the modes depend only to $\omega_D$ as shown in Figure 4.

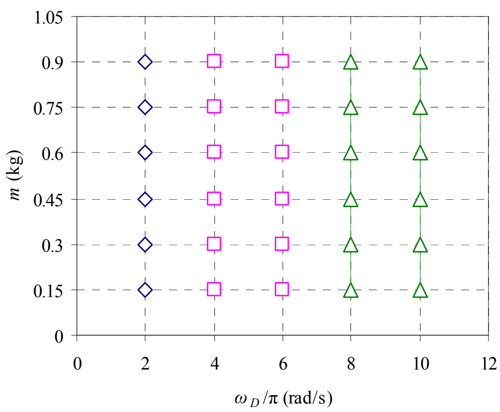



Figure 4. Motion modes as function of drum angular velocity $\omega_D$ and mass of particles cluster $m$: slipping (◊), cataracting (□), and centrifuging (Δ).

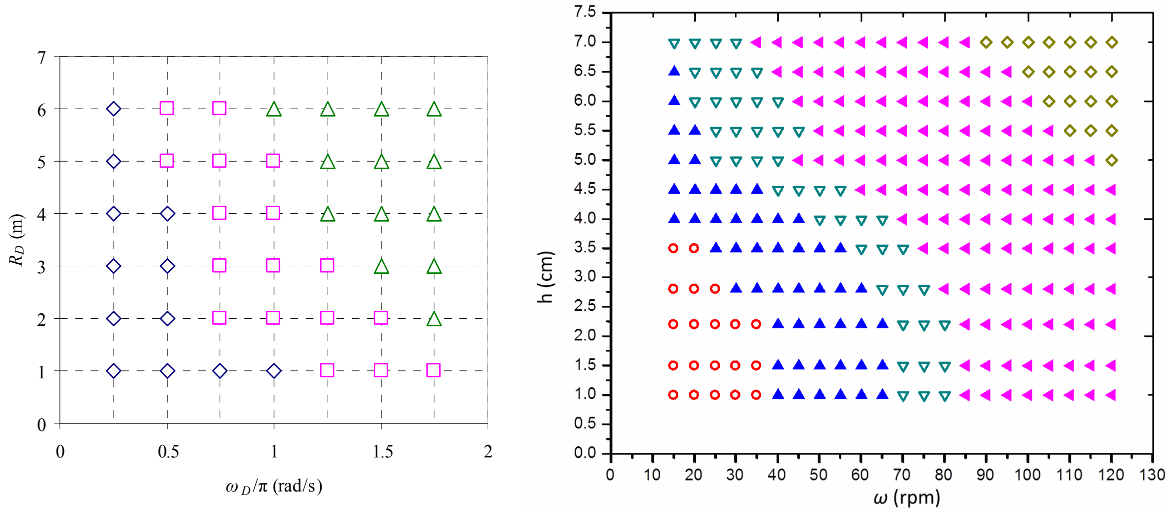



Figure 5. Left: Motion modes as function of drum angular velocity $\omega_D$ and drum radius $R_D$ from point mass model: slumping (◊), cataracting (□), and centrifuging (Δ). Right: Motion modes from experiment [Yulia, 2015]: slumping (o), rolling (▲), cascading ( ∇ ), cataracting (◀), and centrifuging (◊).

Radius of the drum $R_D$ can be seen as depth of granular particles since the model only consider a cluster of particles. It requires assumption that there is a $R_{D,\min}$, then for certain value of $R_D$

$$h = R_D - R_{D,\min}, \quad (16)$$

or simply $h \sim R_D$.

## Conclusion

A simple point mass model has been developed and it can predict three motion modes of particles in vertical rotation drum: slumping, cataracting, and centrifuging. The increase of drum angular velocity will change the mode from slumping to cataracting, and later to centrifuging with the same modes order as observed in experiment.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