The Generators of Quantum Fields

Peter Hickman 15 August 2015

ABSTRACT

G matrices are matrix generators with single entries 1,i. It is shown that the G matrices generate the Cl(p,q), SU(n), SO(n) generators and the matrix representation of CAR and CCR operators. The direct sum of SO(n) and SU(n) results in an expression for the dimension of a SU(N) group. The spatial dimension n is found to be only to 2 or 3 dimensions. It is shown that the unitary representation of SO(3), the spin space of 2x2 SU(2) matrices arises naturally. It follows that there are 3 generations of 8 fermions and 2 complex scalar doublets. The CAR & CCR are invariant under a real scaling of the operators only for a null space-time metric condition on the co-ordinates. Spin conservation requires the generation of spin 1, 2 bosons which is the reason for interactions.

Introduction

G matrices are matrix generators with single entries 1, *i*. It follows that the G matrices generate the Cl(p,q), SU(n), SO(n) generators and the matrix representation of CAR and CCR operators. 2nd quantitisation is avoided by the G matrices forming the quantum algebras - CAR & CCR algebras.

Axiom The infinite set of matrices G_λ called generators with single-entry 1, *i* over the field $\mathbb{R}$ are the generators of quantum fields.

1 Fermions

The generators G_λ satisfy the anti-commutation relation

$$\frac{1}{2}\{G_\lambda^\dagger, G_\mu\} = \delta_{\lambda\mu} M_{\lambda\mu} \quad \textbf{1.1}$$

M are single-entry matrices with entry +1. Form the random state S from the generators G_λ

$$S = \beta_\lambda G_\lambda \quad \textbf{1.2}$$

S will be an element of SU(n) or Cl(p,q) or SO(n) for $\beta = \beta_\lambda$ where λ is in the range of the dimension of the group. Direct sums of the SO(n) and SU(n) generators are linear sums of SU(N) generators where the number of generators d is

$$d = \dim(SO(n)) \dim(SU(n)) \quad \textbf{1.3}$$

$$d = \frac{1}{2} n(n-1)(n^2-1) \quad \textbf{1.4}$$

There are 2 solutions to **1.4**, n=2, n=3. It is conjectured that there are no other solutions to **1.4.** The largest spatial dimension n=3 results in d=24 and N=5, hence space is a maximum of 3d. Generators of SU(3) are 3x3 matrices and since the generators of SU(5) are 5x5 matrices, it follows that the SO(3) matrices are 2x2. The unitary representation of SO(3) are 2x2 spin matrices hence spin s=½ space arises naturally. SU(3) has dimension 8, corresponding to flavour. The 8 spin ½ states are triplets, thus 3 generations of spin ½ fermions.

2 Scalars

For n=2, **1.3** is SO(2), SU(2) hence 2 SU(2) complex scalar doublets – 2 Higgs doublets.

3 Space-Time

The generators G_λ also form matrix representation of fermionic & bosonic operators o_i and the matrices $(x\gamma)_{i\lambda f}$ where γ are gamma matrices. The operator algebras CAR and CCR are invariant under the following real scaling transformation

$$o_i \to \frac{x_{i\lambda f}}{x_0} o_{i\lambda f} \qquad \textbf{3.1}$$

where $x_{i\lambda f}^{\dagger} = x^{i\lambda f}$ are co-ordinates in n-dimensional space and $x_0^{\dagger} = x^0$. The CAR and CCR with the operators given by **3.1** impose the following constraint on the co-ordinates:

$$x_{i\lambda f} x^{i\lambda f} - x_0^2 = 0 \qquad \textbf{3.2}$$

It follows that the co-ordinates $(x_{i\lambda f}, x_0)$ form a (3,1) Lorentzian space i.e. Space-Time.

4 Bosons

Spin conservation for spin s=½ implies that if $\Delta s = \pm 1$, spin $s = \mp 1$ must be generated i.e. spin s=1 bosons must be produced. Similarly the s=1 bosons generate $\Delta s = \pm 2$ which requires the production of spin 2 bosons i.e. gravitons to ensure spin conservation.

SU(5) mixes spin and flavour, so effectively no particles as such. Conjecture – assume this mixing of spin and flavour occurs at GUT or higher energy scales. At lower energy scale, SU(2) spin and SU(3) flavour are not mixed, that is the SU(5) symmetry is broken.

Conclusion

The G matrices form the generators of the Cl(n), SO(n) and SU(n) groups. A direct sum of SO(n) and SU(n) leads to the dimension of space to be 2 or 3. The 3d space has 3 generations of 8 fermions while the 2d space has 2 complex scalar doublets. The conjecture that changes in spin states leads to the production of s=1,2 bosons will be investigated.