Entropy Gradient Maximization for Systems with Discoupled Time

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Abstract

The conventional definition of time based on the period of oscillations or fluctuations around equilibrium states is considered in the framework of the local formalism of entropy gradient maximization. This definition is shown to be an arbitrary chioce of the time scale in a special case of systems with discoupled 'clock'-degree of freedom. The relation between the conventional definition of time and the time-reference degree of freedom in the local formalism of entropy gradient maximization is established. In this framework it is entirely a restriction on the special case of the entropy function with the separated time-reference degree of freedom. A transformation to this special form is discussed and illustrated with a standard example.

1 Introduction

1.1 Motivation

In the entropy gradient maximization (EGM) formalism recently developed [1], the subject of time was sufficiently revised, so that it does not exist anymore in an usual sense.

In terms of this formalism the concept of time (especially, since the time is arbitrary eligible), appears to be physically not well-defined and meaningless.

Since the only measurable quantities can be viewed as physically relevant, but a "measurement of time" in EGM is entirely a comparison of substates of two subsystems, then instead of a question for a time interval between

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two states $\{A_1, B_1\}$ and $\{A_2, B_2\}$ of subsystems A, B, the formalism allows for, to compare A, B with each other immediately, avoiding the usage of the third subsystem C (clock) of the same closed system.

In other words, the only physically meaningful question is, instead of asking for a time interval between states A_1 and A_2 in order to compare this interval with the one between states B_1 and B_2, we ask, which substate B_2 corresponds to the total state including the substate A_2, provided the A_1 and B_1 are substates of the same total state.

It has been shown in the present remark that this concept is quite compliant with the usual definition of time and the latter is a particular case of EGM with *discoupled time reference*, in which the conventional view is implemented.

In turn, this case includes a particular formulation of *true hamiltonian ergodicity* condition, which establishes the agreeabilty with a conventional definition of time units based on closed loop-processes like oscillations and fluctuations.

1.2 Formulation of the *entropy-gradient-maximization*

The local formalism of entropy gradient maximization (EGM), recently proposed in[1] produces the dynamical equations. for a *closed* system with $n+1$ degrees of freedom (DoF's).

$$q_i = \{q_1, q_2, ..., q_{n+1} = \tau\}$$

where the states $\{q_i\}$ are ordered with respect to increasing values of scalar function $S(q_i)$ -entropy - a scalar field on the space $\{q_i\}$

the degree of freedom τ is any of the dofs $\{q_1, ..., q_{n+1}\}$ which is time-eligible so that

$$\frac{\partial}{\partial \tau} S := S_\tau \geq 0 \text{ for a continuum} \tag{1}$$

The EGM formalism maximizes the entropy variation δS around the given state $\{\overline{q} = (q_1, q_2, ..., q_{n+1} = \tau)\}$

$$\delta S(q_i) = \sum_{i=1}^{n} S_{q_i} dq_i + S_\tau d\tau + (higher \ \ orders) \tag{2}$$

The maximization of the entropy variation (2) with the additional condition - *ergodicity*

$$h(q_i, \tau;\ dq_i, d\tau) = \varepsilon_0 \tag{3}$$

provides

i) the first order equation (system of equations) for $\frac{dq_i}{d\tau}$,

$$\delta S_\tau = 0 \tag{4}$$
$$\delta S_{q_i} = 0, \tag{5}$$

which defines the *trajectory* in the state space $\{q_i, \tau\}$ and

ii) the set of second order differential inequalities

$$\begin{bmatrix} \delta S_{\tau\tau} & \delta S_{\tau\ q_i} \\ \delta S_{q_k\tau} & \delta S_{q_i\ q_k} \end{bmatrix} \{\leq 0\} \text{ matrix } [n+1 \times n+1] \text{ is non-positive} \tag{6}$$

- causality condition for S and/or h

2 Discoupled time

2.1 EGM compared with the canonical formalism

In the canonical (hamiltonian) formalism the state space is determined by the double set of DoF's q_i, p_i by means of local scalar function

$$H = H(q_i, p_i), -$$

Hamilton function - the key object generating the evolution in the state space, the time dependence - dynamics- is introduced externally by imposing the dynamical equations

$$\frac{dq}{dt} = H_p; \quad \frac{dp}{dt} = -H_q$$

- the *Hamiltonian equations.* Disadvantages of this approach which should be mentioned are

- redundant degrees of freedom (2n instead of n), although p_i are generically connected with time derivatives of q_i;
- time is an external parameter - the time scale of the dynamics is not an intrinsic feature of the system and can be chosen arbitrary.

2.2 Definition of a conventinal time as a discoupled degree of freedom

The EGM formalism outlined above claims in paricular, a closed system in an equilibrium cannot posess any eigentime, since the local maximum of entropy as equilibrium condition

$$\frac{\partial}{\partial q_i} S(q_i) = 0, \text{ for all } q_i$$

excludes an existence of a degree of freedom $\tau := q_t$ satisfying the time eligibility

$$\frac{\partial}{\partial q_t} S(q_i) > 0$$

For oscillations, which are closed paths in the state space around the stable equilibrium states (attractors), only partial intervals of values q_t can be chosen, where this condition is satisfied. Thus the *internal* eigentime of the closed system can be defined for example piecewiese like

$$\tau := \begin{cases} q_t & \frac{\partial}{\partial q_t} S > 0 \\ -q_t & \frac{\partial}{\partial q_t} S < 0 \end{cases}$$

The following helpful definition is in order:
degree of freedom τ is called *discoupled* (from other degrees of freedom q_i) in the total entropy $S(\tau, q_i)$, if

$$S(\tau, q_i) = \sigma(\tau) + s(q_i) \tag{7}$$

This condition is equivalent to the requirement for all second derivatives mixed with τ to disappear:

$$S_{q_i \tau} = 0, \quad i = 1, \ ..., \ n$$

If additionally, the ergodicity does not depend on $\tau, d\tau$ (in extended global and local sense):

$$h_{d\tau} = h_\tau = 0$$

then the system can be called *true hamiltonian* by comparison with the canonical form.

The definition of unbounded time scale in a conventional sense in units of fluctuation periods around some equilibrium state, mentioned above, is explained for this case as follows:

Suppose, the external degree of freedom τ of the clock-subsystem is *discoupled* in the total entropy $S(\tau, x, y)$ of e.g. three degrees of freedom,

$$S(\tau, x, y) = \sigma(\tau) + s(x, y) \tag{8}$$

while the degree of freedom τ is also not contained in the ergodicity condition

$$h = h(x, y) \tag{9}$$

(the ergodicity for $x(\tau), y(\tau)$ is *true hamiltonian.*

Then the differentiation according to the *entropy gradient maximization formalism* [1] provides:

$$t: \; = \; -\sigma'(\tau) = s_x(x,y)\dot{x} + s_y(x,y)\dot{y} \tag{10}$$

$$\frac{dx}{dy} \; = \; \frac{\dot{x}}{\dot{y}} = -\frac{h_y(x,y)}{h_x(x,y)} \tag{11}$$

(the dot denotes $\frac{\partial}{\partial\tau}$, the prime $\frac{d}{d\tau}$ respectively)

The latter itself provides the path $\mathcal{P}(x, y) = 0$ - a trajectory in the state subspace $\{x, y\}$ uniquely (together with an initial condition $\mathcal{P}(x_0, y_0) = 0$).

and the former is only the parametrization of the latter, thus its solutions are parametrizations of $\mathcal{P}(x, y) = 0$.

The existence of closed paths is therefore only a property of **ergodicity function** $h(x, y)$, like a **Hamilton function** for harmonic oscillator

$$H(p, q) = p^2 + q^2 = \varepsilon_0$$

A 'time' period is assigned only by the parametrization τ, which is not unique.

Thus the definition of time $\sigma(\tau)$ in terms of oscillation or fluctuation periods taken for time units, by means of (8) is a pure clock choice, which does not depend on the geometry of state space or entropy structure, unless the clock degree of freedom is not discoupled

$$\text{(in that general case} \quad \frac{\partial S(\tau, q_i)}{\partial \tau} = -\frac{\partial S(\tau, q_i)}{\partial q_i}\dot{q}_i \text{)}$$

2.3 Separable time-refernce degree of freedom

The special case considered above gives rise to the following reverse question: For a dynamical system

$$S = S(p, q)$$
$$\varepsilon_0 = h(p, q)$$

Does a transformation

$$\{p, q\} \to \{x, \tau\}$$

exists, such that

$$S(p, q) = \tilde{S}(x, \tau) = \Sigma(x) + \sigma(\tau)$$
$$h(p, q) = \tilde{h}(x, \tau)$$

i.e. in terms of transformed degrees of freedom it becomes a system with the separated time τ. In such a case the system can be called a *system with the separable time*

Example of a time separation

For a system with 2 degrees of freedom $\{q, p\}$ with global ergodicity

$$h(q, p) = q^2 - p^2$$

it is natural if the entropy is a linear function of the volume of state space:

$$S(q, p) = L(qp), \quad L - \text{ some linear function.}$$

The transformation

$$n = \frac{q - p}{2}, \quad \tau = \frac{p + q}{2}$$
$$\text{and } q = \tau + n; \quad p = \tau - n$$

brings the system to the form

$$\tilde{S}(n, \tau) = L(\tau^2) - L(n^2)$$
$$\tilde{h}(n, \tau) = 4n\tau$$

with the separated time τ, e.g. a quantum system with the occupation number n, where the ergodicity function transporms to the *action*, and the entropy corresponds the quantum system with the external clock of the macroscopic *labor*-system.

3 Conclusions

As it has been shown in the note, the formalism proposed in [1] includes also the definition of *external* time as a special case of *discoupled clock* (8).

By the way, this case is physically non-realisable, on the basis of the fact alone, that both subsystems - an observable system $\{q_i\}$ and the external clock $\{\tau\}$ - should be compared with each other in a total frame of reference; therefore, the information exchange means an interaction between them. For instance, it can be never neglected in a quantum case, since getting any information of a system changes the quantum state.

References

[1] I.V. Drozdov
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