The Pentaquark and the Pauli Exclusion Principle

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Abstract

A subtle but very real difference in how the Pauli Exclusion Principle is applicable to baryons in three-quark systems and to those in multiquark systems is presented. This distinction creates no important physical manifestations for structures with light quarks, as in case of the SU(3)-flavour group with (u,d,s)-quarks. However it does produce significant effects for multiquark systems containing one or more heavy quarks like the c- and b-quarks. In fact, these consequences permit us to comprehend the structure of the two pentaquark states at 4.38 and 4.45 Gev, which were discovered recently by the LHCb Collaboration at CERN. This model makes a unique prediction of the existence of similar two new pentaquarks with structure (uudb$\bar{b}$) and with similar spin assignments as above.

Keywords: Exotic multiquark states, Pentaquark, Pauli Exclusion Principle, Light quarks, Heavy quarks, Charm quark, Beauty Quark, QCD, SU(n)-flavor symmetry

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Recently the LHCb collaboration at CERN has produced strong evidence in favour of the long sought after pentaquark. They have determined two states at 4.38 and 4.45 GeV with preferred spin assignments of $3/2^-$ and $5/2^+$ respectively [1]. In this paper we attempt to comprehend and explain this very significant discovery.

We commence by following the clear presentation of multiquark systems provided by Matveev and Sorba [2].

Let us start with the SU(3)-flavour symmetry of (u,d,s)-quarks. The requirement that multiquark states be color singlets, coupled with the requirements of the Pauli Exclusion Principle, leads to the possibility of assigning these states to be completely antisymmetric representations of the group $SU(18)_{FSC} \supset SU(6)_{FS} \otimes SU(3)_c$ The significance of being able to do so consistently for a multiquark system shall become clear below.

For a baryon with three quarks there is no need to talk of the bigger group SU(18) above. The antisymmetry of the three quarks in $SU(3)_c$ is enough to demand symmetry of the structure in the group $SU(6)_{FS}$ as these correspond to conjugate Young Diagrams of each other with respect to the group $SU(6)_{FS} \otimes SU(3)_c$.

But the product group above is not good enough to provide consistent antisymmetric states for a multiquark system. To understand this we follow Matveev and Sorba [2] and for the time being confine ourselves to the 2-flavor case for which the groups are : $SU(12)_{FSC} \supset SU(4)_{FS} \otimes SU(3)_c$.

Let us consider a 6-q state made of light (u,d) quarks. The fully antisymmetric state could be obtained only from the combination of two conjugate Young Diagrams for $SU(4)_{FS} \otimes SU(3)_c$ with dimensions (50 of $SU(4)_{FS}$) $\otimes$ (1 of $SU(3)_c$).

However here we come across an ambiguity. One finds that in the product 4x4x4x4x4x4 the above 50-dimensional representation occurs five times. The same number of states arise as singlets of the color group in the product 3x3x3x3x3x3. Thus surprisingly the product group $SU(4)_{FS} \otimes SU(3)_c$, which was sufficient and good enough to provide consistent antisymmetric states for 3-q through corresponding conjugate Young Diagrams, fails to provide unique and consistent antisymmetric states through appropriate conjugate Young Diagrams for the 6-q system.

In order to remedy this deficiency, one must move beyond the group structure $SU(4)_{FS} \otimes SU(3)_c$. The larger symmetry group $SU(12)_{FSC} \supset SU(4)_{FS} \otimes SU(3)_c$ is a potential candidate. This embeds the product group

$SU(4)_{FS} \otimes SU(3)_c$ in the larger $SU(12)_{FSC}$ group to allow the Pauli Exclusion Principle to be implemented properly for the multiquark system.

Given 12 as the fundamental representation of the group $SU(12)_{FSC}$, consider the product 12x12x12x12x12x12. The completely antisymmetric representation $[1^6]$ of this group has dimensions 924 and its reduction with respect to the group $SU(4)_{FS} \otimes SU(3)_c$ is

$$924 = (50, 1) + (64, 8) + (6, 27) + (10, 10) + (\bar{10}, \bar{10}) \tag{1}$$

It is now evident that the 924-representation contains (50,1) once and once only. And this is precisely what is required. Hence the need for this larger symmetry group $SU(12)_{FSC}$ which demands the fully antisymmetric state of dimension 924 and its embedding of the smaller product group $SU(4)_{FS} \otimes SU(3)_c$ ensures that the 6-q state is actually the properly antisymmetrized state as (50,1).

Note that the existence of the larger symmetry group $SU(12)_{FSC}$ is a special demand which need not be automatically satisfied. The $SU(2)_F$ is a pretty good symmetry group as the corresponding (u,d)-quarks are light and thus the bigger group $SU(12)_{FSC}$ is a reasonable extension. Note that, on account of it being part of the bigger group $SU(12)_{FSC}$, the colour property loses its uniqueness. This uniqueness in $SU(3)_c$ being an exact symmetry, is lost in the bigger group. Now colour, being part of the fundamental representation, is analogous to the other structures of flavor and spin for the bigger SU(12) group.

Next, we take the SU(3)-flavour group with (u,d,s)-quarks as being a reasonably good symmetry too. All the quarks are still light enough. So an extension of the above logic for two flavours is easily and consistently accomplished for the three-flavour case as $SU(18)_{FSC} \supset SU(6)_{FS} \otimes SU(3)_c$. The same arguments as above thus hold true here for the issue of consistent antisymmetry of the three-quark and the multiquark cases.

Now let us consider the case of including the heavy charm quark. We take (u,d,s,c) as provinding the fundamental representation of the SU(4)-flavour group. The c-quark is much heavier than the other three light quarks. It being so heavy, the question arises as to whether we can still classify the states of charmed baryons and mesons as per the SU(4) group structure. The baryons, with and without charm as a part of the SU(4) representation, are still fully symmetric. This occurs for three quarks in $SU(4)_F$ as the relevant group is the product group $SU(8)_{FS} \otimes SU(3)_c$. The colour group provides the antisymmetric state in an exact manner. This then ensures, through the conjugate Young Diagram for the group $SU(8)_{FS}$, a fully symmetric state. Hence as per this perspective, in spite of the fact that the $SU(4)_F$ symmetry is badly broken, it is the dominating role of the colour group that the proper symmetric structure holds for baryons in this group. However, this does not necessarily imply the following larger group structure $SU(24)_{FSC} \supset SU(8)_{FS} \otimes SU(3)_c \supset SU(4)_F \otimes SU(2)_S \otimes SU(3)_c$ There is no justification for this larger SU(24) group. There exists no reason for the good synmmetry of the colour group to merge with the badly broken 4-flavour symmetry group. Hence, there exists no 24-dimensional fundamental representaion which would play this role for the larger SU(24) group above.

To belabour the point, the $SU(12)_{FSC}$ and the $SU(18)_{FSC}$ above were justifiable larger groups with the ansatz that the corresponding colour group representation be as "good" as that of the 2- and 3-flavour groups respectively, with the colour seamlessly merging itself with the spin and flavour properties to provide the proper representation for the larger groups. However, the same cannot be achieved within the $SU(4)_F$ group, whence the larger group $SU(24)_{FSC}$ is not viable physically. Thus the c-quark in any multiquark state shall not be constrained by the Pauli Exclusion Principle with respect the other u-, d- and s-quarks (which though, within themselves, obey this particular constraint). Hence for the configuration (uudc) the three (uud) are subject to the Pauli Exclusion Principle, but the c-quark is not. What does this mean physically?

One novel method is to resort to the study of hypernuclear systems. In that field it is known that a nucleus which is comprised of protons and neutrons is subject to the Generalized Pauli Exclusion Principle (GPEP). As per the GPEP the nuclear wave function is antisymmetric with respect to the exchange of any pair of nucleons. However, the λ - hyperon is not subject to this GPEP and thus may enter the nucleus as a λ to form a hypernucleus. The accepted modus is for this particle to migrate right to the lowest orbital of the nucleus. In other words, it sits at the very centre of the nucleus itself. This is an empirically confirmed fact for low and medium mass hypernuclei.

Using this parallel as a suitable guide, one can now better comprehend the (uudc$\bar{c}$) pentaquark state discovered by the LHCb group at CERN. As an initial first step, the four quarks (uudc) form a single multiquark state. As such, as discussed above, three of these (uud) form an antisymmetrized multiquark state, while the c particle does not partake of this state. However, when the c-quark approaches this state (which is required as per the behaviour of the λ in a hypernucleus) it navigates right to the lowest orbital and just sits at the centre of the (uud) state. Therefor we may visualize the c-quark surrounded by a cloud of (uud) quarks. Hence we may treat this c-quark as a "quasi-quark". This quasi-c-quark is in colour 3 representation. It seeks a $\bar{c}$ with colour representation $\bar{3}$ to form a state analogous to the J/ψ state, but with the distinction that the c-quark is a fattened "quasi-quark".

The next stringent requirement is to match the particular spins of the two discovered pentaquarks. We assume that the three (uud) quarks sit in the lowest excited states of the well known resonances N(1440) and N(1520) with spins $1/2^+$ and $3/2^-$ respectively. The quasi-c-quark spin 1/2 attaches itself to the above states with spin states ($1^+ and 0^+$) and ($2^- and 1^-$) respectively. Taking the maximal stretched states (which may be justified from the effect of the spin-orbit force that renders the $p_{3/2}$ more bound than the $p_{1/2}$ state in the nucleus) above for the quasi-c-quark, and combining it with the $\bar{c}$, we finally obtain the spin states $3/2^-$ and $5/2^+$ respectively. These are the two experimentally observed states at 4.38 and 4.45 GeV respectively.

Now given the fact that corrresponding to the above ($c\bar{c}$) J/ψ state there is a ($b\bar{b}$) γ (9.46 GeV) state, this model makes a unique prediction of the existence of similar two new pentaquarks with structure (uudb$\bar{b}$) and with similar spin assignments as above.

References

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