Formula For Pi

The goal of this paper is to compute the number Pi based on the cosine rule.

by Rodolfo A. Frino

Electronics Engineer
Degree from the National University of Mar del Plata - Argentina
rodolfo_frino@yahoo.com.ar
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The Formula

Let us consider a circle of radius 1 as shown in Fig. 1.

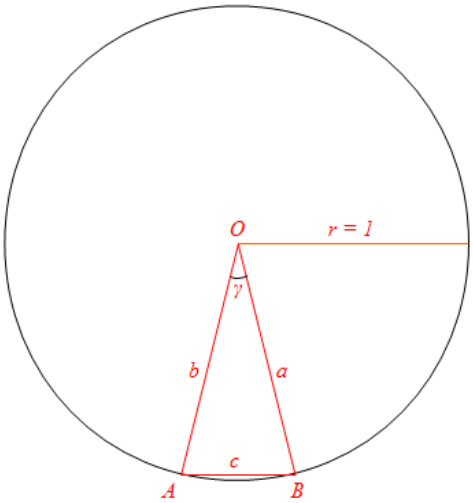

Figure 1

Let us divide the central angle (360°) in *n* equal angles so that the angle γ , which belongs to the isosceles triangle AOB, is

$$\gamma = \frac{360}{n} \tag{1}$$

Let us apply the cosine rule to this triangle

$$c^2 = a^2 + b^2 - 2\,a\,b\cos\gamma \tag{2}$$

Taking into account equation (1) and also the fact that the radius of the circle is 1,

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$$r = a = b = 1 \tag{3}$$

we can solve this equation for c. Thus we get

$$c = \sqrt{1+1-2\cos\frac{360}{n}} \tag{4}$$

$$c = \sqrt{2\left(1-\cos\frac{360}{n}\right)} \tag{5}$$

If n is large enough, we may use the following approximation for the length of the circumference, L, of the circle

$$L = 2\pi r \approx n c \tag{6}$$

But because we have chosen $r = 1$, we may write

$$2\pi \approx n c \tag{7}$$

or

$$\pi \approx \frac{n c}{2} \tag{8}$$

Combining equations (5) and (8) we get

$$\pi \approx \frac{n}{2}\sqrt{2\left(1-\cos\frac{360}{n}\right)} \tag{9}$$

or

$$\pi \approx n\sqrt{\frac{1}{2}\left(1-\cos\frac{360}{n}\right)} \tag{10}$$

When n approaches infinity, the above approximation turns into an equality. Thus we write the final equation as follows

$$\lim_{n\to\infty}\left[n\sqrt{\frac{1}{2}\left(1-\cos\frac{360}{n}\right)}\right] = \pi \tag{11}$$

For practical purposes we use the formula

$$p \approx n\sqrt{\frac{1}{2}\left(1-\cos\frac{360}{n}\right)} \tag{12}$$

It is worthwhile to observe that when n approaches infinity, p approaches π

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Table 1 shows the values of *p* for different values of *n* (see note)

n	*p* *(computed with a PC)*
1	0
10	3.09016994374947
100	3.14107590781283
1,000	3.14158748588072
10,000	3.14159260224008
100,000	3.14159268175395
1,000,000	3.14159277010269
10,000,000	3.14163694416106

Table 1

Note: Since hand held calculators are not accurate enough, calculations should preferably be performed with a PC. The calculations shown here were performed with a PC. However, even PC's cannot compute the cosine of very small angles either. So that the optimum value of Pi will be computed with *n* = 100,000, approximately. By the way, could this formula be used to find out how accurate is your processor's ALU (Arithmetic and Logic Unit)? The value of pi, accurate to the 30 decimal places, is: 3.14 15 92 65 35 89 79 32 38 46 26 43 38 32 79...

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