Energy Conservation in Monopole Theories

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Abstract: The paper discusses a monopole theory based on the assumption that electromagnetic fields of charges are identical to electromagnetic fields of monopoles. It proves that this theory violates energy conservation. This result is consistent with the absence of a regular Lagrangian density for this theory as well as with the systematic failure of experiments aiming to detect such monopoles. By contrast, a monopole theory that is derived from a regular Lagrangian density is free of these problems.

1. Introduction

The general proof of energy-momentum conservation of a field theory uses a Lagrangian density from which the system's equations of motion are derived. If this Lagrangian density does not depend explicitly on the space-time coordinates then energy-momentum conservation is obtained (see [1], p. 82). This work examines the problem of energy conservation in a classical theory of electric charges and magnetic monopoles.

One approach to the charge-monopole problem implicitly assumes that the electromagnetic fields of charges and the electromagnetic fields of monopoles are identical physical objects (see [2] and [3], pp. 251-260). Hereafter, this approach is called the identical fields assumption. An alternative approach to this problem assumes that the theory should be derived from a regular Lagrangian density [4]. Hereafter, this approach is called the Lagrangian assumption.

Both approaches use the same duality relations between electromagnetic fields

$$\mathbf{E} \to \mathbf{B}, \quad \mathbf{B} \to -\mathbf{E} \tag{1}$$

and

$$e \to g, \quad g \to -e, \tag{2}$$

where g denotes the monopole strength. As a matter of fact, these relations are duality rotations by $\pi/2$ (see [3], p. 252; [5], p. 1363).

The two theories mentioned above agree on the following issue. Consider the present structure of Maxwellian electrodynamics which describes the motion of systems of electromagnetic fields and electric charges. This system contains no monopoles. Applying the duality transformations (1) and (2) to this system, one obtains a system of electromagnetic fields and monopoles. This system contains no charges. Evidently, the dual system has properties which are analogous to those of ordinary electrodynamics. This issue is used below. It turns out that the two theories differ in the way of unifying these systems and obtaining a self-consistent charge-monopole theory.

The purpose of this work is to examine the self-consistency of the identical fields assumption. This task is done by an examination of energy conservation in a closed cycle of a specific device. The second section proves that the theory based on the identical fields assumption does not conserve energy. The results are discussed in the third section.

2. A Test of the Identical Fields Assumption

The following device is used for testing the consistency of the identical fields assumption. Consider two parallel plates made of a material which is a monopole insulator. The plates are covered uniformly with positive and negative monopoles, respectively. This device is a monopole analog of a capacitor and **B** denote the magnetic field between the plates. A rod made of a material whose permeability $\mu > 1$ is put between the plates. Here A, B denote two points on the rod's axis and C, D denote two points outside the rod (see fig. 1).

Let us take a test monopole g where $g > 0$ and see how it interacts with the

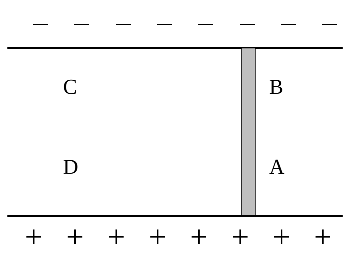

Figure 1: *A monopole moves along the rectangular closed path $A - B - C - D - A$ (see text).*

system. At the beginning, the monopole is at point A. Consider its motion along the closed path $A - B - C - D - A$. Evidently, the final state is the same as the initial state. The rod and the magnetic monopole capacitor do not move. Due to the monopole analog of electrodynamics, the interaction of the test monopole with the capacitor conserves energy. The field of the uniformly magnetized rod is the same as that of a solenoid (see [6], pp. 13-5, 13-6). Let us divide the rod into three parts: that which is below A, the section $A - B$ and the part which is above B. The closed loop $A - B - C - D - A$ does not enter the inner part of the first and the third sections of the rod. Therefore, its interaction with their field conserves energy. On the other hand, the test monopole moves along a closed trajectory that passes through a magnet between the points A, B and the trajectory is closed outside this magnet. Therefore, the test monopole acquires energy that is not balanced. This experiment proves that the identical fields assumption is inconsistent with energy conservation.

As pointed out in the introduction, if a theory is derived from a regular Lagrangian density which is invariant under space-time translation then this theory conserves energy and momentum. Therefore, the contradiction obtained herein is consistent with the fact that textbooks do not show an explicit form of a regular Lagrangian density for a charge-monopole theory which is based on the identical fields assumption.

3. Discussion

The absence of a regular Lagrangian density for the theory based on the identical fields assumption means that this theory is *not* equivalent to the theory which is derived from the Lagrangian assumption. Furthermore, the variational principle is regarded as a fundamental principle for constructing quantum theories [7, 8]. It means that the standard way of constructing quantum theories is inapplicable for a theory which is based on the identical fields assumption.

It is shown in [4] that a self-consistent charge-monopole theory can be constructed on the basis of the Lagrangian assumption. The form of its construction guarantees energy conservation in general and the specific process described above makes no exception. Indeed, the fields of the theory derived in [4] have the following properties: charges do not interact with bound fields of monopoles; monopoles do not interact with bound fields of charges; radiation fields of the systems are identical and charges as well as monopoles interact with them. Evidently, electrons carry electric charge and are free of monopoles and the permeability of the rod of fig. 1 depends on its electronic state. It means that the rod does not interact with the bound field of the monopoles that cover the plates. Therefore, the magnetic field between the plates is uniform and the energy of the motion of the test monopole along the closed cycle $A-B-C-D-A$ is balanced.

Beside solving the specific problem which is described above, this theory provides an explanation for many hadronic properties [9, 10, 11]. By contrast, many experimental attempts to detect a monopole of the identical fields assumption have been carried out for more than half a century. An authorized report on these attempts [12] describes the present situation in these words: "To date there have been no confirmed observations of exotic particles possessing magnetic charge." It is interesting to point out that this systematic failure was predicted long ago [13, 14].

The monopole case can be regarded as another example of the validity of the following fundamental principle: Nature does not respect an inconsistent theory; Nature may respect a self-consistent theory.

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