

New Newton Mechanics Taking Law of Conservation of Energy as Unique Source Law (**Revised Version 3**)

Fu Yuhua

CNOOC Research Institute

E-mail: fyuh1945@sina.com

Abstract: According to the principle of the uniqueness of truth, this paper presents the New Newton Mechanics (NNM) taking law of conservation of energy as unique source law. Examples show that in some cases other laws may be contradicted with the law of conservation of energy. The original Newton's three laws and the law of gravity, in principle can be derived by the law of conservation of energy. Through the example of free falling body, this paper derives the original Newton's second law **and the original law of gravity by using the law of conservation of energy**; and through the example of a small ball rolls along the inclined plane (belonging to the problem cannot be solved by general relativity that a body is forced to move in flat space), derives improved Newton's second law and improved law of gravity by using law of conservation of energy. Whether or not other conservation laws (such as the law of conservation of momentum and the law of conservation of angular momentum) can be utilized, should be tested by law of conservation of energy. When the original Newton's second law is not correct, then the laws of conservation of momentum and angular momentum are no longer correct; therefore the general forms of improved law of conservation of momentum and improved law of conservation of angular momentum are presented. In the cases that law of conservation of energy cannot be used effectively, New Newton Mechanics will not exclude that according to other theories or accurate experiments to derive the laws or formulas to solve some specific problems. For example, with the help of the result of general relativity, the improved Newton's formula of universal gravitation can be derived, which can be used to solve the problem of advance of planetary perihelion and the problem of deflection of photon around the Sun. Again, according to accurate experimental result, the synthesized gravitational formula (including the effects of other celestial bodies and sunlight pressure) for the problem of deflection of photon around the Sun is presented. Unlike the original Newton Mechanics, in New Newton Mechanics, for different problems, may have different laws of motion, different formulas of gravity, as well as different expressions of energy. For example, for the problem of a small ball rolls along the inclined plane, and the problem of advance of planetary perihelion, the two formulas of gravity are completely different.

Keywords: Uniqueness of truth, law of conservation of energy, unique source law, New Newton Mechanics (NNM)

1 Introduction

One of the development trends of natural science is using fewer laws to solve increasing problems. In this process, some laws will play the increasingly great roles; while others will play the smaller roles, or even disappear from the ranks of laws.

Now we discuss the law of conservation of energy. Its main contents are as follows: In a closed system, the total energy of this system remains unchanged.

Because the law of conservation of energy is the most important one in natural sciences, it should play an increasingly great role. For this reason and according to the principle of the uniqueness of truth, this paper presents the New Newton Mechanics (NNM) taking law of conservation of energy as unique source law.

In the area of Newton Mechanics, there should be one truth only. Other so-called truth, either it can be derived by the unique truth, or we can prove that in certain cases it is not true. As well-known, when Newton founded the classical mechanics, four laws were proposed, they were Newton's three laws and the law of gravity. If the law of conservation of energy is choosing as the unique source law, that in principle, all the Newton's four laws can be derived according to the law of conservation of energy; after studying carefully we found that this may indeed be the real case. In addition, in the areas such as physics, mechanics, engineering and so on, there are three very important laws: the law of conservation of energy, the law of conservation of momentum and the law of conservation of angular momentum. If we believe that the law of conservation of energy is the truth, then for the law of conservation of momentum and the law of conservation of angular momentum, either they can be derived by the law of conservation of energy, or we can prove that in certain cases they are not true. We believe that the true situation is the latter, namely, the law of conservation of momentum and the law of conservation of angular momentum are not true in some cases (or their results are contradicted to the law of conservation of energy). Of course, we can also find that in some cases, these two laws still can be used. Taking the example that a man walks along the car located on the horizontal smooth rail, we can see that at present in the area of Newton mechanics, some people do not notice the case of the contradiction between the law of conservation of energy and the law of conservation of momentum.

2 Taking Law of Conservation of Energy as Unique Source Law

2.1 Deriving Original Newton's Second Law and Original Law of Gravity

2.1.1 Deriving Original Newton's Second Law by Using Law of Conservation of Energy

In this section, only Newton's second law can be derived, but we have to apply the law of gravity at the same time, so we present the general forms of Newton's second law and the law of gravity with undetermined constants firstly.

Assuming that for the law of gravity, the related exponent is unknown, and we only know the form of this formula is as follows

$$F = -\frac{GMm}{r^D}$$

where: D is an undetermined constant, in the next section we will derive that its value is equal to 2.

Similarly, assuming that for Newton's second law, the related exponent is also unknown, and we only know the form of this formula is as follows

$$F = ma^D$$

where: D' is an undetermined constant, in this section we will derive that its value is equal to 1.

As shown in Figure 1, supposing that circle O' denotes the Earth, M denotes its mass; m denotes the mass of the small ball (treated as a mass point P), $A O'$ is a plumb line, and coordinate y is parallel to $A O'$. The length of AC is equal to H , and $O'C$ equals the radius R of the Earth.

We also assume that it does not take into account the motion of the Earth and only considering the free falling of the small ball in the gravitational field of the Earth (from point A to point C).

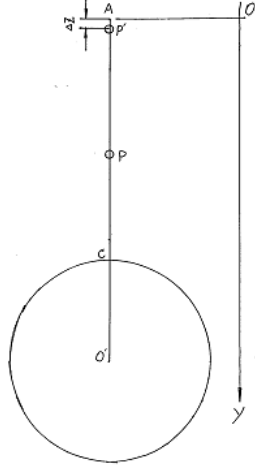


Figure 1 A small ball free falls in the gravitational field of the Earth

For this example, the value of v_P^2 which is the square of the velocity for the small ball located at point P will be investigated. To distinguish the quantities calculated by different methods, we denote the value given by the law of gravity and Newton's second law as v_P^2 , while $v'_P{}^2$ denotes the value given by the law of conservation of energy.

Now we calculate the related quantities according to the law of conservation of energy.

From the law of gravity contained undetermined constant, the potential energy of the small ball located at point P is as follows

$$V = -\frac{GMm}{(D-1)r_{O'P}^{D-1}}$$

According to the law of conservation of energy, we can get

$$-\frac{GMm}{(D-1)r_{O'A}^{D-1}} = \frac{1}{2}mv_P'^2 - \frac{GMm}{(D-1)r_{O'P}^{D-1}}$$

And therefore

$$v_P'^2 = \frac{2GM}{D-1} \left[\frac{1}{r_{O'P}^{D-1}} - \frac{1}{(R+H)^{D-1}} \right]$$

Now we calculate the related quantities according to the law of gravity and Newton's second law.

For the small ball located at any point P , we have

$$dv/dt = a$$

We also have

$$dt = \frac{dy}{v}$$

Therefore

$$v dv = a dy$$

According to the law of gravity contained undetermined constant, along the plumb direction, the force acted on the small ball is as follows

$$F_a = \frac{GMm}{r_{O'P}^D}$$

From the Newton's second law contained undetermined constant, it gives

$$a = \left(\frac{F_a}{m}\right)^{1/D'} = \left(\frac{GM}{r_{O'P}^D}\right)^{1/D'}$$

Then we have

$$v dv = \left\{ \frac{GM}{(R+H-y)^D} \right\}^{1/D'} dy$$

For the two sides of this expression, we run the integral operation from A to P, it gives

$$v_p^2 = 2(GM)^{1/D'} \int_0^{y_p} (R+H-y)^{-D/D'} dy$$

$$v_p^2 = 2(GM)^{1/D'} \left\{ -\frac{1}{1-D/D'} [(R+H-y)^{1-D/D'}] \right\}_0^{y_p}$$

$$v_p^2 = \frac{2(GM)^{1/D'}}{(D/D')-1} \left[\frac{1}{r_{O'P}^{(D/D')-1}} - \frac{1}{(R+H)^{(D/D')-1}} \right]$$

Let $v_p^2 = v_p'^2$, then we should have: $1=1/D'$, and $D-1=(D/D')-1$; these two equations all give: $D'=1$, this means that for free falling problem, by using the law of conservation of energy, we strictly derive the original Newton's second law $F = ma$.

Here, although the original law of gravity cannot be derived (the value of D may be any constant, certainly including the case that D=2), we already prove that the original law of gravity is not contradicted to the law of conservation of energy, or the original law of gravity is tenable accurately.

2.1.2 Deriving Original Law of Gravity by Using Law of Conservation of Energy

In order to really derive the original law of gravity for the example of free falling problem, we should consider the case that a small ball free falls from point A to point P' (point P' is also shown

in Figure1) through a very short distance ΔZ (the two endpoints of the interval ΔZ are point A and point P').

As deriving the original Newton's second law, we already reach

$$v_{P'}^2 = \frac{2GM}{D-1} \left[\frac{1}{(R+H-\Delta Z)^{D-1}} - \frac{1}{(R+H)^{D-1}} \right]$$

where: $R+H-\Delta Z = r_{O'P'}$

For the reason that the distance of ΔZ is very short, and in this interval the gravity can be considered as a linear function, therefore the work W of gravity in this interval can be written as follows

$$W = F_{av} \Delta Z = \frac{GMm}{(R+H-\frac{1}{2}\Delta Z)^D} \Delta Z$$

where, F_{av} is the average value of gravity in this interval ΔZ , namely the value of gravity for the midpoint of interval ΔZ .

Omitting the second order term of ΔZ ($\frac{1}{4}(\Delta Z)^2$), it gives

$$W = \frac{GMm\Delta Z}{(R^2 + H^2 + 2RH - R\Delta Z - H\Delta Z)^{D/2}}$$

As the small ball free falls from point A to point P', its kinetic energy is as follows

$$\frac{1}{2}mv_{P'}^2 = \frac{GMm}{D-1} \left[\frac{(R+H)^{D-1} - (R+H-\Delta Z)^{D-1}}{(R^2 + H^2 + 2RH - R\Delta Z - H\Delta Z)^{D-1}} \right]$$

According to the law of conservation of energy, we have

$$W = \frac{1}{2}mv_{P'}^2$$

Substituting the related quantities into the above expression, it gives

$$\begin{aligned} & \frac{GMm}{D-1} \left[\frac{(R+H)^{D-1} - (R+H-\Delta Z)^{D-1}}{(R^2 + H^2 + 2RH - R\Delta Z - H\Delta Z)^{D-1}} \right] \\ &= \frac{GMm\Delta Z}{(R^2 + H^2 + 2RH - R\Delta Z - H\Delta Z)^{D/2}} \end{aligned}$$

To compare the related terms, we can reach the following three equations

$$D-1=1$$

$$D/2 = D-1$$

$$\Delta Z = (R+H)^{D-1} - (R+H-\Delta Z)^{D-1}$$

All of these three equations will give the following result

$$D=2$$

Thus, we already derive the original law of gravity by using the law of conservation of energy.

2.2 New Three Laws of Motion and New Law of Gravity (Formula) Created By Law of Conservation Of Energy for New Newton Mechanics

The original Newton's three laws of motion are as follows.

Newton's First Law of Motion: Every object in a state of uniform motion (or at rest) tends to remain in that state of motion (or at rest) unless an external force is applied to it. For short rest remains rest, and moving remains moving.

Newton's Second Law of Motion: The relationship between an object's mass m , its acceleration a , and the applied force F is $F = ma$. The direction of the force is the same as the direction of the acceleration.

Newton's Third Law of Motion: For every action there is an equal and opposite reaction.

The original Newton's law of gravity: The attractive force between two objects is as follows

$$F = -\frac{GMm}{r^2} \quad (1)$$

While for NNM, taking law of conservation of energy as unique source law, then we have the following NNM's three laws of motion and law of gravity.

NNM's First Law of Motion: Every object in a state of uniform motion (or in a state of uniform rotation, or at rest) tends to remain in that state of motion (or in a state of uniform rotation, or at rest) unless an external force is applied to it; otherwise the law of conservation of energy will be destroyed. For short: rest remains rest, moving remains moving, and rotating remains rotating.

NNM's Second Law of Motion: The relationship between an object's mass m , its acceleration a , and the applied force F is a function that should be derived by law of conservation of energy. The direction of the force is the same as the direction of the acceleration. In general, the function can be written as the form of variable dimension fractal: $F = ma^{1+\varepsilon}$, where: ε is a constant or a variable. For different problems, the forms of second law may be different.

NNM's Third Law of Motion: In general, for every action there is an equal and opposite reaction. In special case, the function relationship between action and reaction should be derived by law of conservation of energy. The improved form of the original Newton's third law ($F_{AB} = -F_{BA}$) is as

follows: $F_{AB} = -F_{BA}^{1+\lambda}$, where: λ is a constant or a variable. For different problems, the forms of third law may be different.

NNM's law (formula) of gravity: The attractive force between two objects is a function that should be derived by law of conservation of energy, or experimental data; or derived with the help of other theories. For different problems, the forms of law (formula) of gravity may be different. The results of original Newton's law of gravity are only accurate in the cases that two objects are relative static or running the straight line between one center and another center, and the like; for other cases its results are all approximate. In general, NNM's law (formula) of gravity may be taken as the form that adding the amending term to original Newton's law of gravity, or the following form of variable dimension fractal:

$$F = -\frac{GMm}{r^{2-\delta}} \quad (2)$$

where: δ is a constant or a variable.

Now for an example, a NNM's law (formula) of gravity (an improved Newton's law of gravity) and a NNM's second law of motion (an improved Newton's second law of motion), they are suitable for this example only, are derived simultaneously by law of conservation of energy.

Firstly, the variational principles established by the law of conservation of energy can be given with least squares method (LSM).

Supposing that the initial total energy of a closed system equals $W(0)$, and for time t the total energy equals $W(t)$, then according to the law of conservation of energy:

$$W(0) = W(t) \quad (3)$$

This can be written as:

$$R_w = \frac{W(t)}{W(0)} - 1 = 0 \quad (4)$$

According to LSM, for the interval $[t_1, t_2]$, we can write the following variational principle:

$$\Pi = \int_{t_1}^{t_2} R_w^2 dt = \min_0 \quad (5)$$

Where: \min_0 denotes the minimum value of functional Π and it should be equal to zero.

It should be noted that, in many cases $W(t)$ is approximate, and R_w is not identically equal to zero, therefore Eq.(5) can be used to solve the problem.

Besides the time coordinate, another one can also be used. For example, for interval $[x_1, x_2]$, the following variational principle can be given according to the law of conservation of energy:

$$\Pi = \int_{x_1}^{x_2} R_w^2 dx = \min_0 \quad (6)$$

The above-mentioned principles are established by using the law of conservation of energy directly. Sometimes, a certain principle should be established by using the law of conservation of energy indirectly. For example, a special physical quantity Q may be interested, not only it can be calculated by using the law of conservation of energy, but also can be calculated by using other laws (for this paper they are the law of gravity, and Newton's second law). For distinguishing the values, let's denote the value given by other laws as Q , while denote the value given by the law of conservation of energy as Q' , then the value of R_w can be redefined as follows:

$$R_w = \frac{Q}{Q'} - 1 = 0 \quad (7)$$

Substituting Eq. (7) into Eqs. (5) and (6), as Q' is the result calculated with the law of conservation of energy, it gives the variational principle established by using the law of conservation of energy indirectly. Otherwise, it is clear that the extent of the value of Q accords with Q' .

Substituting the related quantities into Eq. (5) or Eq. (6), the equations derived by the condition of an extremum can be written as follows:

$$\frac{\partial \Pi}{\partial a_i} = \frac{\partial \Pi}{\partial k_i} = 0 \quad (8)$$

After solving these equations, the improved law of gravity, and Newton's second law can be

reached at once. According to the value of Π , the effect of the solution can be judged. The nearer the value of Π is to zero, the better the effect of the solution. It should be noted that besides of solving equations, optimum-seeking methods could also be used for finding the minimum and the constants to be determined. In fact, the optimum seeking method will be used in this paper.

Now we solve an example. As shown in Fig.2, supposing that the small ball rolls along a long incline from A to B. Its initial velocity is zero and the friction and the rotational energy of small ball are neglected.

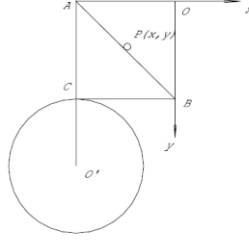


Figure.2 A small ball rolls from A to B

Supposing that circle O' denotes the Earth, M denotes its mass; m denotes the mass of the small ball (treated as a mass point P), $O'A$ is a plumb line, coordinate x is orthogonal to $O'A$, coordinate y is orthogonal to coordinate x (parallel to $O'A$), BC is orthogonal to $O'A$. The lengths of OA , OB , BC , and AC are all equal to H , and $O'C$ equals the radius R of the Earth.

In this example, the value of v_p^2 which is the square of the velocity for the ball located at point P is investigated. To distinguish the quantities, denote the value given by the improved law of gravity and improved Newton's second law as v_p^2 , while v_p^2 denotes the value given by the law of conservation of energy, then Eq. (6) can be written as

$$\Pi = \int_{-H}^0 \left(\frac{v_p^2}{v_p^2} - 1 \right)^2 dx = \min_0 \quad (9)$$

Supposing that the improved law of gravity and improved Newton's second law can be written as the following constant dimension fractal forms

$$F = -\frac{GMm}{r^D} \quad (10)$$

$$F = ma^{1+\varepsilon} \quad (11)$$

where: D and ε are constants.

Now we calculate the related quantities according to the law of conservation of energy.

From Eq.(10), the potential energy of the small ball located at point P is

$$V = -\frac{GMm}{(D-1)r_{O'P}^{D-1}} \quad (12)$$

According to the law of conservation of energy, we can get

$$-\frac{GMm}{(D-1)r_{O'A}^{D-1}} = \frac{1}{2}mv_P'^2 - \frac{GMm}{(D-1)r_{O'P}^{D-1}} \quad (13)$$

And therefore

$$v_P'^2 = \frac{2GM}{D-1} \left[\frac{1}{r_{O'P}^{D-1}} - \frac{1}{(R+H)^{D-1}} \right] \quad (14)$$

Now we calculate the related quantities according to the improved law of gravity and improved Newton's second law.

Supposing that the equation of rolling line is

$$y = x + H \quad (15)$$

For the ball located at point P,

$$dv/dt = a \quad (16)$$

Because

$$dt = \frac{ds}{v} = \frac{\sqrt{2}dx}{v}$$

$$\text{Therefore } vdv = a\sqrt{2}dx \quad (17)$$

According to the improved law of gravity, the force along to the tangent is

$$F_a = \frac{GMm}{r_{O'P}^D} \frac{1}{\sqrt{2}} \quad (18)$$

According to the improved Newton's second law, for point P, the acceleration along to the tangent is

$$a = \left(\frac{F_a}{m}\right)^{1/1+\varepsilon} = \left(\frac{GM}{r_{O'P}^D \sqrt{2}}\right)^{1/1+\varepsilon} \quad (19)$$

From Eq. (17) , it gives

$$v dv = \left\{ \frac{GM}{[(H+x)^2 + (R+H-y)^2]^{D/2} \sqrt{2}} \right\}^{1/1+\varepsilon} \sqrt{2} dx \quad (20)$$

Substituting Eq.(15) into Eq.(20), and for the two sides, we run the integral operation from A to P, it gives

$$v_P^2 = 2 \int_{-H}^{x_P} \left\{ \frac{GM}{[(H+x)^2 + (R-x)^2]^{D/2}} \right\}^{1/1+\varepsilon} (\sqrt{2})^{\varepsilon/1+\varepsilon} dx \quad (21)$$

Then the value can be calculated by a method of numerical integral.

The given data are assumed to be: for Earth, $GM=3.99 \times 10^{14} \text{m}^3/\text{s}^2$; the radius of the Earth $R=6.37 \times 10^6 \text{m}$, $H=R/10$, try to solve the problem shown in Fig. 1, find the solution for the value of v_B^2 , and derive the improved law of gravity and the improved Newton's second law.

Firstly, according to the original law of gravity, the original Newton's second law (i.e., let $D=2$ in Eq.(10), $\varepsilon=0$ in Eq.(11)) and the law of conservation of energy, all the related quantities can be calculated, then substitute them into Eq.(9), it gives

$$\Pi_0 = 571.4215$$

Here, according to the law of conservation of energy, it gives $v_B^2 = 1.0767 \times 10^7$, while according to the original law of gravity, and the original Newton's second law, it gives $v_B^2 = 1.1351 \times 10^7$, the difference is about 5.4 %. For the reason that the value of Π_0 is not equal to zero, then the values

of D and ε can be decided by the optimum seeking method. At present all the optimum seeking methods can be divided into two types, one type may not depend on the initial values which program may be complicated, and another type requires the better initial values which program is simple. One method of the second type, namely the searching method will be used in this paper.

Firstly, the value of D is fixed so let $D=2$, then search the value of ε , as $\varepsilon=0.0146$, the value of Π reaches the minimum 139.3429; then the value of ε is fixed, and search the value of D , as $D=1.99989$, the value of Π reaches the minimum 137.3238; then the value of D is fixed, and search the value of ε , as $\varepsilon=0.01458$, the value of Π reaches minimum 137.3231. Because the last two results are highly close, the searching can be stopped, and the final results are as follows

$$D=1.99989, \quad \varepsilon=0.01458, \quad \Pi=137.3231$$

Here the value of Π is only 24% of Π_0 . While according to the law of conservation of energy, it gives $v_B^2=1.0785 \times 10^7$, according to the improved law of gravity and the improved Newton's second law, it gives $v_B^2=1.1073 \times 10^7$, the difference is about 2.7 % only.

The results suitable for this example with the constant dimension fractal form are as follows
The improved law of gravity reads

$$F = -\frac{GMm}{r^{1.99989}} \quad (22)$$

The improved Newton's second law reads

$$F = ma^{1.01458} \quad (23)$$

The above mentioned results have been published on reference [1].

According to the above results, it can be said that we could not rely on any experimental data, only apply the law of conservation of energy to derive the improved law of gravity, and improved Newton's second law; and demonstrate that the original Newton's law of gravity and Newton's second law are all tenable approximately for this example.

For the example shown in Fig.2 that a small ball rolls along the inclined plane, in order to obtain the better results, we discuss the variable dimension fractal solution with Eq.(4) that is established by the law of conservation of energy directly.

Supposing that the improved Newton's second law and the improved law of gravity with the form

of variable dimension fractal can be written as follows: $F = ma^{1+\varepsilon}$, $\varepsilon = k_1u$; $F = -GMm/r^{2-\delta}$, $\delta = k_2u$; where: u is the horizon distance that the small ball rolls ($u = x + H$).

With the similar searching method, the values of k_1, k_2 can be determined, and the results are as follows

$$\varepsilon = 8.85 \times 10^{-8}u, \quad \delta = 2.71 \times 10^{-13}u$$

The results of variable dimension fractal are much better than that of constant dimension fractal. For example, the final $\Pi = 5.8662 \times 10^{-4}$, it is only 0.019% of Π_0 (3.1207). While according to the law of conservation of energy, it gives $v_B^2 = 1.0767 \times 10^7$, according to the improved law of gravity and the improved Newton's second law, it gives $v_B^2 = 1.0777 \times 10^7$, the difference is about 0.093 % only.

The results suitable for this example with the variable dimension fractal form are as follows

The improved law of gravity reads

$$F = -\frac{GMm}{r^{2-2.71 \times 10^{-13}u}} \quad (24)$$

The improved Newton's second law reads

$$F = ma^{1+8.85 \times 10^{-8}u} \quad (25)$$

where: u is the horizon distance that the small ball rolls ($u = x + H$).

There is another problem should also be discussed. That is the improved kinetic energy formula. As well-known, the kinetic energy formula has been modified in the theory of relativity, now we improve the kinetic energy formula with the law of conservation of energy.

Supposing that the improved kinetic energy formula is $E_d = \frac{1}{2}mv^{2-\lambda}$, $\lambda = k_3u$; where: u is the horizon distance that the small ball rolls ($u = x + H$).

With the similar searching method, we can get: $k_3 = 9.95 \times 10^{-13}$, then the improved kinetic energy formula with variable dimension fractal form reads

$$E_d = \frac{1}{2} m v^{2-9.95 \times 10^{-13} u}$$

Because the effect of improvement is very small (the value of II is only improved from 5.8662×10^{-4} into 5.8634×10^{-4}), therefore these results should be for reference only.

3 With the Help of General Relativity and Accurate Experimental Data to Derive the Improved Newton's Formula of Universal Gravitation

Prof. Hu Ning derived an equation according to general relativity, with the help of Hu's equation and Binet's formula, we get the following improved Newton's formula of universal gravitation^[2]

$$F = -\frac{GMm}{r^2} - \frac{3G^2 M^2 mp}{c^2 r^4} \quad (26)$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object m moving around the object M along with a curve, and the value of p is given by: $p = a(1-e^2)$ (for ellipse), $p = a(e^2-1)$ (for hyperbola), $p = y^2/2x$ (for parabola).

It should be noted that, this improved Newton's formula of universal gravitation can also be written as the form of variable dimension fractal.

Suppose

$$-\frac{GMm}{r^D} = -\frac{GMm}{r^2} - \frac{3G^2 M^2 mp}{c^2 r^4}$$

It gives $D = -\ln\left(\frac{1}{r^2} + \frac{3GMp}{c^2 r^4}\right) / \ln r$

For the problem of gravitational deflection of a photon orbit around the Sun, $M=1.99 \times 10^{30}$ kg, $r_0=6.96 \times 10^8$ m, $c=2.9979 \times 10^8$ m/s, then we have: $1.954997 \leq D \leq 2$.

The improved Newton's universal gravitation formula (Eq.(26)) can give the same results as given by general relativity for the problem of planetary advance of perihelion and the problem of gravitational deflection of a photon orbit around the Sun.

For the problem of planetary advance of perihelion, the improved Newton's universal gravitation formula reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2ma(1-e^2)}{c^2r^4} \quad (27)$$

For the problem of gravitational deflection of a photon orbit around the Sun, the improved Newton's universal gravitation formula reads

$$F = -\frac{GMm}{r^2} - \frac{1.5GMmr_0^2}{r^4} \quad (28)$$

where: r_0 is the shortest distance between the light and the Sun, if the light and the Sun is tangent, it is equal to the radius of the Sun.

The funny thing is that, for this problem, the maximum gravitational force given by the improved Newton's universal gravitation formula is 2.5 times of that given by the original Newton's law of gravity.

Although the deflection angles given by Eq. (26) and Eq. (28) are all exactly the same as given by general relativity, they have still slight deviations with the precise astronomical observations. What are the reasons? The answer is that the deflection angle not only is depended on the gravitational effect of the Sun, but also depended on the gravitational effects of other celestial bodies, as well as the influences of sunlight pressure and so on. If all factors are taken into account, not only general relativity can do nothing for this problem, but also for a long time it could not be solved by theoretical method. Therefore, at present the only way to solve this problem is based on the precise observations to derive the synthesized gravitational formula (including the effects of other celestial bodies and sunlight pressure) for the problem of deflection of photon around the Sun.

As well-known, the deflection angle ϕ_0 given by general relativity or the improved Newton's formula of universal gravitation is as follows

$$\phi_0 = 1.75''$$

Adding an additional term to Eq.(28), it gives the synthesized gravitational formula between the photon and the Sun as follows

$$F = -\frac{GMm}{r^2} \left(1 + \frac{3GMp}{c^2r^2} + \frac{wG^2M^2p^2}{c^4r^4} \right) \quad (29)$$

where: w is a constant to be determined.

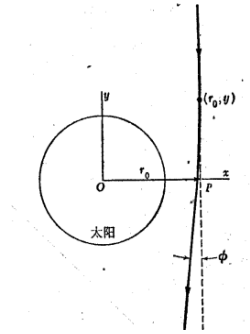


Figure 3. Deflection of photon around the Sun

Now We Determine The Value Of w According To Accurate Experimental Data.

Firstly the problem of deflection of photon around the Sun as shown in **Fig.3** will be solved with Eq.(29). The method to be used is the same as presented in references [2] and [3].

Supposing that m represents the mass of photon. Because the deflection angle is very small, we can assume that $x=r_0$; thus on point (x, y) , its coordinate can be written as (r_0, y) , then the force acted on photon reads

$$F_x = \frac{Fr_0}{(r_0^2 + y^2)^{1/2}} \quad (30)$$

Where: The value of F is given by Eq.(29).

Because

$$mv_x = \int F_x dt = \int F_x \frac{dy}{v_y} \approx \frac{1}{c} \int F_x dy \quad (31)$$

Hence

$$v_x \approx -\frac{2GMr_0}{c} \int_0^{\infty} \frac{dy}{(r_0^2 + y^2)^{3/2}} - \frac{6G^2 M^2 pr_0}{c^3} \int_0^{\infty} \frac{dy}{(r_0^2 + y^2)^{5/2}} - \frac{2wG^3 M^3 p^2 r_0}{c^5} \int_0^{\infty} \frac{dy}{(r_0^2 + y^2)^{7/2}} \quad (32)$$

Because

$$\int_0^{\infty} \frac{dy}{(r_0^2 + y^2)^{3/2}} = \frac{1}{r_0^2}, \quad \int_0^{\infty} \frac{dy}{(r_0^2 + y^2)^{5/2}} = \frac{2}{3r_0^4}, \quad \int_0^{\infty} \frac{dy}{(r_0^2 + y^2)^{7/2}} = \frac{8}{15r_0^6}$$

Therefore

$$v_x \approx -\frac{2GM}{cr_0} - \frac{4G^2 M^2 p}{c^3 r_0^3} - \frac{16wG^3 M^3 p^2}{15c^5 r_0^5}$$

Because

$$\phi \approx \text{tg}\phi \approx \frac{|v_x|}{c}$$

By using the half normal chord given in reference [2], it gives

$$p = \frac{c^2 r_0^2}{2GM}$$

Then the deflection angle is as follows

$$\phi = \frac{4GM}{c^2 r_0} \left| 1 + \frac{w}{15} \right| \quad (33)$$

Where: r_0 is the radius of Sun.

Because

$$\phi_0 = \frac{4GM}{c^2 r_0} \quad (34)$$

Then, it gives

$$\phi = \phi_0 \left(1 + \frac{w}{15} \right) \quad (35)$$

Thus the value of w can be solved as follows

$$w = 15\left(\frac{\phi}{\phi_0} - 1\right) \quad (36)$$

Now we can determine the value of w according to the experimental data.

Table 1 shows the experimental data of radio astronomy for the deflection angle of photon around the Sun (taken from reference [4]).

Table 1. The experimental data of radio astronomy for the deflection angle of photon around the Sun

Year	Observer	Observed value / "
1969	G.A.Seielstud et al	1.77±0.20
1969	D.O.Muhleman et al	1.82 ^{+0.24} _{-0.17}
1969	I.I.Shapiro	1.80±0.2
1970	R.A.Sramak	1.57±0.08
1970	J.M.Hill	1.87±0.3
1972		1.82±0.14
1974		1.73±0.05
1975		1.78±0.02

Now we choose the experimental data in 1975, it gives

$$1.76 \leq \phi \leq 1.80$$

Then, we have

$$0.08571 \leq w \leq 0.42857$$

Taking the average value, it gives

$$w = 0.25714$$

Thus, according to the experimental data, the synthesized gravitational formula can be decided.

4 Contradiction between the Law of Conservation Of Energy and the Law of Conservation Of Momentum As Well As the Law of Conservation of Angular Momentum

As well-known, unlike the law of conservation of energy, the law of conservation of momentum and the law of conservation of angular momentum are only correct under certain conditions. For example, considering friction force and the like, these two laws will not be correct.

Now we point out further that for NNM the law of conservation of momentum as well as the law of conservation of angular momentum will be not correct under certain conditions (or their results contradict with the law of conservation of energy).

As well-known, in order to prove the law of conservation of momentum as well as the law of conservation of angular momentum, the original Newton's second law should be applied. However, as we have made clear, the original Newton's second law will not be correct under certain conditions, for such cases, these two laws also will not correct.

Here we find another problem, if the original three conservation laws are all correct, therefore for certain issues, the law of conservation of energy and the other two conservation laws could be combined to apply. While for NNM, if the other two conservation laws cannot be applied, how to complement the new formulas to replace these two conservation laws? The solution is very simple: according to the law of conservation of energy, for any time, the derivatives of total energy $W(t)$ should be all equal to zero, then we have

$$\frac{d^n W(t)}{dt^n} = 0 \quad n = 1, 2, 3, \dots \quad (37)$$

In addition, running the integral operations to the both sides of Eq.(3), it gives

$$W(0)t = \int_0^t W(t)dt \quad (38)$$

Now we illustrate that, because there is one truth only, even within the scope of original classical mechanics, the contradiction could also appear between the law of conservation of energy and the law of conservation of momentum.

As shown in Fig.4, a man walks along the car located on the horizontal smooth rail, the length of the car equals L , the mass of the man is m_1 and the car is m_2 . At beginning the man and the car are all at rest, then the man walks from one end to the other end of the car, try to decide the moving distances of the man and the car. This example is taken from references [5].

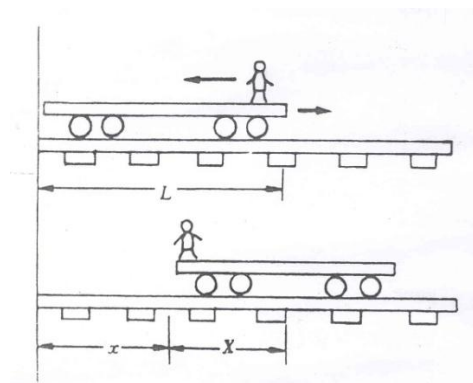


Figure 4 A Man Walks along the Car Located On the Horizontal Smooth Rail

As solving this problem by using the original classical mechanics, the law of conservation of momentum will be used, it gives

$$m_1v_1 + m_2v_2 = 0$$

However, at beginning the man and the car are all at rest, the total energy of the system is equal to zero; while once they are moving, they will have speeds, and the total energy of the system is not equal to zero; thus the law of conservation of energy will be destroyed. For this paradox, the original classical mechanics looks without seeing. In fact, considering the lost energy of the man and applying the law of conservation of energy, the completely different result will be reached.

As the original law of conservation of momentum ($P_t = P_0 = Const$) and the law of conservation of angular momentum ($L_t = L_0 = Const$) are not correct, we can propose their improved forms of variable dimension fractal. The improved law of conservation of momentum: $P_t = P_0^{1+\delta}$ (δ is a constant or a variable), and the improved law of conservation of angular momentum: $L_t = L_0^{1+\varepsilon}$ (ε is a constant or a variable).

References

1. Fu Yuhua, Deriving Improved Newton's Second Law and the Law of Gravity at One Time with Form of Fractal Formula, Engineering Science. 2003, Vol5, No.6, 55-58
2. Fu Yuhua, Improved Newton's formula of universal gravitation, Ziranazhi (Nature Journal), 2001(1), 58-59
3. C. Kittel et al, Translated into Chinese by Chen Bingqian et al, Mechanics, Beijing: Science Press, 1979, 535-537
4. Liu Liao, General relativity, Beijing: Higher education press, 1987, 202
5. Xu Hexing, Mechanics (revised edition), Shanghai: East China Normal University Press, 1998, 175-176

Appendix:

Solving Problems of Advance of Mercury's Perihelion and Deflection of

Photon Around the Sun with New Newton's Formula of Gravity

Fu Yuhua

(CNOOC Research Institute, E-mail:fuyh1945@sina.com)

Abstract: According to the new Newton's formula of gravity (the original law of gravity plus a correction term), i.e., improved Newton's formula of gravity, applying the methods of classical mechanics to solve the problem of advance of Mercury's perihelion and the problem of deflection

of photon around the Sun respectively, and the results are the same as given by general relativity. Pointing out that the further topic is based on deriving original law of gravity and original Newton's second law with law of conservation of energy to derive the new Newton's formula of gravity (the improved Newton's formula of gravity) with law of conservation of energy. To realize the purpose that partially replacing relativity and solving some problems that cannot be solved by relativity with the methods of classical mechanics.

Key words: Law of gravity, new Newton's formula of gravity (improved Newton's formula of gravity), advance of Mercury's perihelion, deflection of photon around the Sun, law of conservation of energy

Introduction

In references [1, 2], the new Newton's formula of gravity (the original law of gravity plus a correction term), i.e., the improved Newton's formula of gravity, was presented; but it did not present the detailed process to solve the problem of advance of Mercury's perihelion and the problem of deflection of photon around the Sun respectively with the methods of classical mechanics. While, in this paper the detailed process will be given (the results are the same as given by general relativity).

The improved Newton's formula of gravity is as follows

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \quad (1)$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object with mass m moving around the object with mass M along a curve, and the value of p is given by: $p = a(1-e^2)$ (for ellipse), $p = a(e^2-1)$ (for hyperbola), $p = y^2/2x$ (for parabola).

1 Solving the problem of advance of Mercury's perihelion with new Newton's formula of gravity

In classical mechanics, acted by the central force, the orbit differential equation (Binet's formula) reads

$$h^2u^2(u''+u) = -\frac{F}{m} \quad (2)$$

where: $u = \frac{1}{r}$.

As deriving Eq.(1), it already gives

$$h^2 = GMp \quad (3)$$

Substituting Eq.(1) and Eq.(3) into Eq.(2), we have the following equation of planet's movement around the Sun

$$u''+u = \frac{1}{p} + \frac{3GMu^2}{c^2} \quad (4)$$

For ellipse, $p = a(e^2-1)$, thus the approximate solution for Eq.(4) is as follows

$$u \approx \frac{GM}{a(1-e^2)c^2} \left[1 + e \cos \left(1 - \frac{3GM}{a(1-e^2)c^2} \right) \varphi \right] \quad (5)$$

Hence, the value of ε for advance of planetary perihelion for one circuit is as follows

$$\varepsilon = \frac{24\pi^3 a^2}{T^2 c^2 (1-e^2)} \quad (6)$$

where: T, a, and e are orbital period, semi-major axis and eccentricity respectively.

Obviously, this result is the same as given by general relativity.

In addition, according to Eq.(1), for problem of planetary motion around the Sun, the improved Newton's formula of gravity reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2 M^2 m a (1-e^2)}{c^2 r^4}$$

2 Solving the problem of deflection of photon around the Sun with new Newton's formula of gravity

As solving this problem by using the improved formula of gravity, the method to be used is the same as presented in references [3], in which the original law of gravity was used.

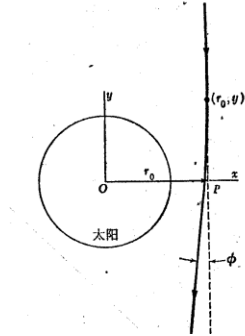


Fig. 1 Deflection of photon around the Sun

Supposing that m represents the mass of photon, for the reason that it will be eliminated, so it is not necessary to give its value. As shown in Fig.1, r_0 represents the nearest distance between the photon and the center of the Sun, for the reason that the deflection is very small, so actually the value of r_0 is the same as the photon is not deflected. When the photon is located at (r_0, y) (the value of y is measured from point P in Fig.1), the force acted on photon is as follows

$$F_x = \frac{Fr_0}{(r_0^2 + y^2)^{1/2}} \quad (7)$$

where: $F = -\frac{GMm}{r_0^2 + y^2} - \frac{3G^2 M^2 m p}{c^2 (r_0^2 + y^2)^2}$

Because

$$mv_x = \int F_x dt = \int F_x \frac{dy}{v_y} \approx \frac{1}{c} \int F_x dy$$

Therefore

$$v_x \approx -\frac{2GMr_0}{c} \int_0^\infty \frac{dy}{(r_0^2 + y^2)^{3/2}} - \frac{6G^2M^2pr_0}{c^3} \int_0^\infty \frac{dy}{(r_0^2 + y^2)^{5/2}}$$

After calculating, it gives

$$v_x \approx -\frac{2GM}{cr_0} - \frac{4G^2M^2p}{c^3r_0^3} \quad (8)$$

Hence, the deflection angle ϕ is as follows

$$\phi \approx \text{tg}\phi \approx \frac{|v_x|}{c} = \frac{2GM}{c^2r_0} + \frac{4G^2M^2p}{c^4r_0^3} \quad (9)$$

While, the value of ϕ should be determined by iteration method.

Before determining the value of ϕ , firstly we will validate that the value of the second term in Eq.(9) is equal to the value of the first term, that means that the deflection given by the second term in Eq.(9) is equal to that given by the first term.

As solving problem of deflection of photon around the Sun with general relativity, the photon's orbit is a hyperbola, and its equation is as follows

$$u = u_0 \cos\phi + \frac{GMu_0^2(1 + \sin^2\phi)}{c^2} \quad (10)$$

where: $u_0 = \frac{1}{r_0}$

Hence, the reciprocal of the half normal chord p is as follows

$$\frac{1}{p} = u|_{\phi=\pi/2} = \frac{2GM}{c^2r_0^2} \quad (11)$$

Substituting this value of the half normal chord p into Eq. (9), it gives

$$\phi = \frac{4GM}{c^2r_0} = \frac{4GM}{c^2R_s} \quad (12)$$

where: R_s is the radius of the Sun.

Thus, the value of the second term in Eq.(9) is really equal to the value of the first term.

Now we determine the value of ϕ in Eq.(9) by iteration method.

Suppose

$$\phi = \frac{KGM}{c^2 r_0} \quad (13)$$

In order to apply iteration method, the relationship between ϕ and p should be given.

Considering two straight lines, the first one is $x = r_0$, the second one is passing through the origin O and the first quadrant, and it makes an angle of $\phi/2$ with the positive direction of Y axis; the value of the half normal chord p is equal to the distance between the origin O and the intersection of the two straight lines. Supposing that the intersection of the two straight lines is the point P_1 (it is not shown in Fig.1), then its coordinates are $(r_0, \sqrt{p^2 - r_0^2})$.

From the triangle formed by the three points of origin O , P , and P_1 , it gives

$$\frac{r_0}{p} = \sin \frac{\phi}{2} \approx \frac{\phi}{2}$$

$$p = \frac{2r_0}{\phi} \quad (14)$$

Now, considering the result of the value of ϕ given by the original law of gravity, it gives

$$K_0 = 2$$

Here, the deflection angle is as follows

$$\phi_0 = \frac{2GM}{c^2 r_0}$$

From Eq. (14), its corresponding half normal chord p_0 is as follows

$$p_0 = \frac{c^2 r_0^2}{GM}$$

Substituting the value of p_0 into Eq. (9), it gives

$$\phi_1 = \frac{6GM}{c^2 r_0}$$

Namely

$$K_1 = 6$$

Similarly, the values of K_2 , K_3 , and the like are as follows: 3.3333, 4.4000, 3.8182, 4.0952, 3.9535, 4.0234, 3.9883, 4.0059, 3.9971, 4.0015, 3.9993, 4.0004, 3.9998, 4.0001, 4.0000, 4.0000; finally it gives

$$K = 4$$

This result is also the same as given by general relativity.

According to Eq.(1), for problem of deflection of photon around the Sun, the improved Newton's formula of gravity reads

$$F = -\frac{GMm}{r^2} - \frac{1.5GMmr_0^2}{r^4}$$

where: r_0 is the shortest distance between the photon and the Sun, if the light and the Sun is tangent, it is equal to the radius of the Sun.

The interesting fact is that, for this problem, the maximum gravitational force given by the improved Newton's formula of gravity is 2.5 times of that given by the original Newton's law of gravity.

3 Further topic

In references [4-6], the original law of gravity and the original Newton's second law have been derived with law of conservation of energy, based on this, the further topic is to derive the new Newton's formula of gravity (the improved Newton's formula of gravity), i.e., Eq.(1), with law of conservation of energy. And, finally, to realize the purpose that partially replacing relativity and solving some problems that cannot be solved by relativity with the methods of classical mechanics.

References

- 1 Fu Yuhua, Improved Newton's formula of universal gravitation, *Ziranzazhi (Nature Journal)*, 2001(1), 58-59
- 2 Florentin Smarandache, V. Christianto, Fu Yuhua, R. Khrapko, J. Hutchison. *Unfolding the Labyrinth: Open Problems in Physics, Mathematics, Astrophysics, and Other Areas of Science*. Hexis-Phoenix, 2006, 85-99
- 3 Kittel C., Knight W. D. and Ruderman M. A., *Mechanics*. Berkeley Physics Course Vol.1, McGraw-Hill, 1973.
- 4 Fu Yuhua, Deriving Improved Newton's Second Law and the Law of Gravity at One Time with Form of Fractal Formula, *Engineering Science*. 2003, Vol.5, No.6, 55-58
- 5 Fu Yuhua, Expanding Newton Mechanics with Neutrosophy and Quad-stage Method —New Newton Mechanics Taking Law of Conservation of Energy as Unique Source Law, *Neutrosophic Sets and Systems*, Vol.3, 2014
- 6 Fu Yuhua, New Newton Mechanics Taking Law of Conservation of Energy as Unique Source Law, *Science Journal of Physics*, Volume 2015, Article ID sjp-130, 12 Pages, 2015, doi: 10.7237/sjp/130