HOW TO CONSTRUCT SELF/ANTI-SELF CHARGE CONJUGATE STATES FOR HIGHER SPINS? SIGNIFICANCE OF THE SPIN BASES, MASS DIMENSION, AND ALL THAT*

Valeriy V. Dvoeglazov

Universidad Autónoma de Zacatecas, Unidad Académica de Física, México
vdvoeglazov@yahoo.com.mx

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We construct self/anti-self charge conjugate (Majorana-like) states for the $(1/2,0)\oplus(0,1/2)$ representation of the Lorentz group, and their analogs for higher spins within the quantum field theory. The problems of the basis rotations and that of the selection of phases in the Dirac-like and Majorana-like field operators are considered. The discrete symmetries properties (P,C,T) are studied. The corresponding dynamical equations are presented. In the $(1/2,0)\oplus(0,1/2)$ representation they obey the Dirac-like equation with eight components, which has been first introduced by Markov. Thus, the Fock space for corresponding quantum fields is doubled (as shown by Ziino). The particular attention has been paid to the questions of chirality and helicity (two concepts which are frequently confused in the literature) for Dirac and Majorana states, and to the normalization ("the mass dimension"). We further review several experimental consequences which follow from the previous works of M. Kirchbach *et al.* on neutrinoless double beta decay, and G.J. Ni *et al.* on meson lifetimes. The results are generalized for spins 1, 3/2 and 2.

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1. Majorana-like spinors

During the 20th century various authors introduced *self/anti-self* charge-conjugate 4-spinors (including in the momentum representation), see, *e.g.*, [1–9]. The authors found corresponding dynamical equations, gauge transformations and other specific features of them. On using $C = -e^{i\theta}\gamma^2\mathcal{K}$, the anti-linear operator of charge conjugation, we define the *self/anti-self* charge-conjugate 4-spinors in the momentum space $C\lambda^{\mathrm{S,A}}(\boldsymbol{p}) = \pm\lambda^{\mathrm{S,A}}(\boldsymbol{p})$,

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$C\rho^{\rm S,A}(\boldsymbol{p}) = \pm\rho^{\rm S,A}(\boldsymbol{p})$. Such definitions of 4-spinors differ, of course, from the original Majorana definition in the x-representation $\nu(x) = \frac{1}{\sqrt{2}}(\Psi_D(x) + \Psi_D^c(x))$, $C\nu(x) = \nu(x)$ that represents the positive real C-parity only. However, Kirchbach [8] noted "a non-trivial impact of Majorana-[like] framework in experiments with polarized sources".

The rest λ- and ρ- differ from the Dirac case in relative phases

$$\lambda^{\rm S}_{\uparrow}(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}0\\ i\\ 1\\ 0\end{pmatrix}, \qquad \lambda^{\rm S}_{\downarrow}(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}-i\\ 0\\ 0\\ 1\end{pmatrix}, \tag{1}$$

$$\begin{aligned}\lambda^{\rm A}_{\uparrow}(\mathbf{0}) &= -\gamma_5\lambda^{\rm S}_{\uparrow}(\mathbf{0}), \qquad \lambda^{\rm A}_{\downarrow}(\mathbf{0}) = -\gamma_5\lambda^{\rm S}_{\downarrow}(\mathbf{0}),\\ \rho^{\rm S}_{\uparrow\downarrow}(\mathbf{0}) &= \mp i\lambda^{\rm A}_{\downarrow\uparrow}(\mathbf{0}), \qquad \rho^{\rm A}_{\uparrow\downarrow}(\mathbf{0}) = \pm i\lambda^{\rm S}_{\downarrow\uparrow}(\mathbf{0}).\end{aligned} \tag{2}$$

The right and left parts can be boosted with $\Lambda_{\rm R,L}$. As claimed in [4], λ and ρ 4-spinors are *not* the eigenspinors of the helicity. Moreover, λ and ρ are *not* the eigenspinors of the parity, as opposed to the Dirac case ($P = \gamma^0 R$, $R = (\boldsymbol{x} \to -\boldsymbol{x})$). The indices $\uparrow\downarrow$ should be referred to the chiral helicity quantum number introduced in the 60s [10], $\eta = -\gamma^5 h$. The normalizations of the spinors $\lambda^{\rm S,A}_{\uparrow\downarrow}(\boldsymbol{p})$ and $\rho^{\rm S,A}_{\uparrow\downarrow}(\boldsymbol{p})$ have been given in the previous works.

The dynamical coordinate-space equations can be written in the 8-component form. Similar formulations have been presented by Markov [11], and by Barut and Ziino [3]. The group-theoretical basis for such doubling has been given in the papers by Gelfand, Tsetlin and Sokolik [12], who first presented the theory in the 2-dimensional representation of the inversion group in 1956. The Lagrangian was written [6]. The connection with the Dirac spinors has been found in [6, 8]. We can see that the two sets are connected by the unitary transformations, and this represents itself the rotation of the spin-parity basis. It was shown in [6] that the covariant derivative (and, hence, the interaction) can be introduced in this construct in the following way $\partial_\mu \to \nabla_\mu = \partial_\mu - ig\, Ł^5 A_\mu$, where $Ł^5 = \mathrm{diag}(\gamma^5, -\gamma^5)$, the 8×8 matrix. The proposed Lagrangian [6] remains to be invariant. This tells us that while self/anti-self charge conjugate states have zero eigenvalues of the ordinary (scalar) charge operator but they can possess the axial charge. Next, we introduced the Majorana-like field operator ($b^\dagger \equiv a^\dagger$), which admits additional phase (and, in general, normalization) transformations $\nu^{\rm ML\,\prime}(x^\mu) = [c_0 + i(\boldsymbol{\tau}\cdot\boldsymbol{c})]\nu^{\rm ML\,\dagger}(x^\mu)$, where c_α are arbitrary parameters. The $\boldsymbol{\tau}$ matrices are defined over the field of 2×2 matrices. The non-Abelian construct is permitted, and it is based on the spinors of the Lorentz group only. It is interesting to note that $[\nu^{\rm ML}(x^\mu) \pm C\nu^{\rm ML\,\dagger}(x^\mu)]/2$ lead naturally to the Ziino–Barut scheme of massive chiral fields [3], if the former are composed from $\lambda^{\rm S,A}$ spinors. Recently, the interest to these models raised again [9, 13].

2. Chirality and helicity

Ahluwalia [4] claimed Incompatibility of Self-Charge Conjugation with Helicity Eignestates and Gauge Interactions. I showed that the gauge interactions of λ and ρ 4-spinors are different. Z.-Q. Shi and G.J. Ni promote a very extreme standpoint. Namely, "the spin states, the helicity states and the chirality states of fermions in Relativistic Quantum Mechanics ... are entirely different ... the polarization of fermions in flight must be described by the helicity states" (see also his Conclusion Section [14]). In fact, they showed experimental consequences of their statement. Markov wrote long ago [11] *two* Dirac equations with opposite signs at the mass term. He added and subtracted them. His χ and η solutions can be presented as some superpositions of the Dirac 4-spinors $u-$ and $v-$. The concept of the doubling of the Fock space has been developed in Ziino works (*cf.* [12, 15]). In their case, their charge conjugate states are at the same time the eigenstates of the chirality.

Let us analyse the above statements. It is known [16] that one can transform $\mathcal{U}_1(\boldsymbol{\sigma}\cdot\boldsymbol{a})\mathcal{U}_1^{-1} = \sigma_3|\boldsymbol{a}|$. One has

$$\begin{aligned}
\mathcal{U}_1 &= \begin{pmatrix} 1 & p_l/(p+p_3) \\ -p_r/(p+p_3) & 1 \end{pmatrix}, \\
U_1 &= \begin{pmatrix} \mathcal{U}_1 & 0 \\ 0 & \mathcal{U}_1 \end{pmatrix}, \qquad U_1\hat{h}U_1^{-1} = \left|\frac{\boldsymbol{n}}{2}\right| \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}.
\end{aligned} \tag{3}$$

Then, applying other unitary matrix U_3

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{4}$$

we transform to the basis, where helicity is equal (within the factor $\frac{1}{2}$) to γ^5, the chirality operator.

The author of [10] and others introduced the *chiral* helicity $\eta = -\gamma_5 h$, which is equal (within the sign and the factor $\frac{1}{2}$) to the well-known matrix $\boldsymbol{\alpha}$ multiplied by $\boldsymbol{n}$. Again, $U_1(\boldsymbol{\alpha}\cdot\boldsymbol{n})U_1^{-1} = \alpha_3|\boldsymbol{n}|$ with the same matrix U_1. Applying the second unitary transformation

$$U_2\alpha_3U_2^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \alpha_3 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{5}$$

we again come to the γ_5 matrix. The determinants are: $\mathrm{Det}U_1 = 1 \neq 0$, $\mathrm{Det}U_{2,3} = -1 \neq 0$. Thus, helicity, chirality and chiral helicity are connected by the unitary transformations. To say that the 4-spinor is the eigenspinor of the *chiral helicity*, and, at the same time, it is *not (!)* the eigenspinor of the helicity operator (and that the physical results would depend on this) signifies the same as to say that rotations have physical significance on the fundamental level.

3. Charge conjugation and parity for $S = 1$

Several formalisms have been used for higher spin fields, *e.g.*, [17, 18]. "For spin-1 … the requirement of self/anti-self charge conjugacy *cannot* be satisfied. That is, there does not exist a ζ [the phase factors between right- and left- 3-'spinors'] that can satisfy the spin-1 … requirement $S^c_{[1]}\,\lambda(p^\mu) = \pm\lambda(p^\mu)$, $S^c_{[1]}\,\rho(p^\mu) = \pm\rho(p^\mu)$", Ref. [4]. This is due to the fact that $C^2 = -1$ within this definition of the charge conjugation operator. "We find, however, that the requirement of self/anti-self conjugacy under charge conjugation can be replaced by the requirement of self/anti-self conjugacy under the operation of $\Gamma^5\,S^c_{[1]}$ [precisely, which was used by Weinberg [18] due to the different choice of the equation for the negative-frequency 6-'bispinors']". The covariant equations for λ- and ρ-objects in the $(1,0)\oplus(0,1)$ representation have been obtained in [6] under the certain choice of the phase factors in the definitions of left and right 3-objects.

Da Rocha *et al.* [13] investigated properties of the (anti)commutation of C and P operators, *cf.* with the previous works of the 50s–60s [19]. It is this case which has been attributed to the $Q = 0$ eigenvalues (the truly neutral particles). You may compare these results with those of Refs. [4, 7, 20]. The acronym "ELKO" is the synonym for the self/anti-self charge conjugated states (the Majorana-like spinors). It is easy to find the correspondence between "the new notation" [13, 21] and the previous one. Namely, $\lambda^{\mathrm{S,A}}_{\uparrow} \to \lambda^{\mathrm{S,A}}_{-,+}$, $\lambda^{\mathrm{S,A}}_{\downarrow} \to \lambda^{\mathrm{S,A}}_{+,-}$. So, why the difference appeared in the da Rocha formulas comparing with my previous results on the classical level? In my papers, see, *e.g.*, [6, 7, 20], I presented the explicit forms of the λ- and ρ-4-spinors in the basis $\hat{S}_3\xi(\mathbf{0}) = \pm\frac{1}{2}\xi(\mathbf{0})$. The corresponding properties with respect to the parity (on the classical level) are different

$$\gamma^0\lambda^{\mathrm{S}}_{\uparrow\downarrow}\left(p^{\mu'}\right) = \pm i\lambda^{\mathrm{S}}_{\downarrow\uparrow}(p^\mu)\,, \qquad \gamma^0\lambda^{\mathrm{A}}_{\uparrow\downarrow}\left(p^{\mu'}\right) = \mp i\lambda^{\mathrm{A}}_{\downarrow\uparrow}(p^\mu)\,. \tag{6}$$

As in [22], Ahluwalia, Grumiller and da Rocha have chosen the well-known helicity basis (*cf.* [22, 23]). In this basis, the parity transformation ($\theta \to \pi - \theta$, $\phi \to \pi + \phi$) leads to the properties

$$R\phi_{\rm L}^{-}(\mathbf{0}) = -ie^{i(\theta_2-\theta_1)}\phi_{\rm L}^{+}(\mathbf{0})\,, \qquad R\phi_{\rm L}^{+}(\mathbf{0}) = -ie^{i(\theta_1-\theta_2)}\phi_{\rm L}^{-}(\mathbf{0})\,, \tag{7}$$

$$R\Theta(\phi_{\rm L}^{-}(\mathbf{0}))^{*} = -ie^{-2i\theta_2}\phi_{\rm L}^{-}(\mathbf{0})\,, \qquad R\Theta(\phi_{\rm L}^{+}(\mathbf{0}))^{*} = +ie^{-2i\theta_1}\phi_{\rm L}^{+}(\mathbf{0})\,. \tag{8}$$

This opposes to the spinorial basis, where, for instance: $R\phi_{\rm L}^{\pm}(\mathbf{0}) = \phi_{\rm L}^{\pm}(\mathbf{0})$. Further calculations are straightforward, and, indeed, they can lead to $[C,P]_{-}=0$ when acting on the "ELKO" states, due to $[C,\gamma^5]_{+}=0$.

In the $(1,0)\oplus(0,1)$ representation the situation is similar (see the formulas (31) in [24]). The analogs of (7,8) are

$$R\phi_{\rm L}^{-}(\mathbf{0}) = +e^{i(\theta_{-}-\theta_{+})}\phi_{\rm L}^{+}(\mathbf{0})\,, \qquad R\phi_{\rm L}^{+}(\mathbf{0}) = +e^{i(\theta_{+}-\theta_{-})}\phi_{\rm L}^{-}(\mathbf{0})\,, \tag{9}$$

$$R\Theta(\phi_{\rm L}^{-}(\mathbf{0}))^{*} = -e^{-2i\theta_{-}}\phi_{\rm L}^{-}(\mathbf{0})\,, \qquad R\Theta(\phi_{\rm L}^{+}(\mathbf{0}))^{*} = -e^{-2i\theta_{+}}\phi_{\rm L}^{+}(\mathbf{0})\,, \tag{10}$$

$$R\phi_{\rm L}^{0}(\mathbf{0}) = -\phi_{\rm L}^{0}(\mathbf{0})\,, \qquad R\Theta(\phi_{\rm L}^{0}(\mathbf{0}))^{*} = +e^{-2i\theta_0}\phi_{\rm L}^{0}(\mathbf{0})\,. \tag{11}$$

If we would like to extend the Nigam–Foldy conclusion, Ref. [19] (about $[C,P]_{-}=0$ corresponds to the neutral particles even in the higher spin case (?)) then we should use the helicity basis on the classical level. However, on the level of the quantum-field theory (the "secondary" quantization) the situation is self-consistent. As shown in 1997 [7, 20], we can obtain easily *both* cases (commutation and anti-commutation) on using $\lambda^{\rm S,A}$ 4-spinors, which have been used earlier (in the basis *column*(1 0) *column*(0 1)).

4. Conclusions

We presented a review of the formalism for the momentum-space Majorana-like particles in the $(S,0)\oplus(0,S)$ representation of the Lorentz Group. The λ and ρ 4-spinors satisfy the 8-component analogue of the Dirac equation. Moreover, they have different gauge transformations comparing with the usual Dirac 4-spinors. Their helicity, chirality and chiral helicity properties have been investigated in detail. These operators are connected by the given unitary transformations. At the same time, we showed that the Majorana-like 4-spinors can be obtained by the rotation of the spin-parity basis. Meanwhile, several authors have claimed that the physical results would be different on using calculations with these Majorana-like spinors. Thus, the $(S,0)\oplus(0,S)$ representation space (even in the case of $S=1/2$) has additional mathematical structures leading to deep physical consequences, which have not yet been explored. However, several claims made by other researchers concerned with chirality, helicity, chiral helicity should not be considered to be true until the time when experiments confirm them. Next, we discussed the $[C,P]_{\pm}=0$ dilemma for neutral and charged particles on using the analysis of the basis rotations and phases. The questions of the normalization have been considered in several papers of ours, *e.g.*, Ref. [25].

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