Two conjectures of generalization of Feuerbach-Luchterhand theorem

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Abstract

In Euclidean geometry, Feuerbach-Luchterhand theorem is a generalization of Pythagorean theorem, Stewart theorem and the British Flag theorem.....In this note, I propose two conjectures of generalization of Feuerbach-Luchterhand theorem.

Theorem 1 (Feuerbach-Luchterhand)**.** *Let $ABCD$ be a cyclic quadrilateral, P be a point on the plane then:*

$$PA^2.DB.BC.CD - PB^2.AC.CD.DA + PC^2.BD.DA.AB - PD^2.CA.AB.BC = 0 \quad (1)$$

Conjecture 2 (A generalization of Feuerbach-Luchterhand)**.** *Let 2n-convex cyclic polygon $A_1A_2A_3...A_{2n}$, let P be a point on the plane, then:*

$$\sum_{i=1}^{2n}(-1)^{i+1}.PA_i^2.\frac{A_{i-1}A_{i+1}}{A_iA_{i-1}.A_iA_{i+1}} = 0 \quad (2)$$

In a case the polygon is a hexagon, or a octagon, you can check the conjecture 2 in [1][2]. Where $A_0 = A_{2n}$ and $A_{2n+1} = A_1$

Conjecture 3 (A generalization of conjecture 2)**.** *Let two similar 2n-convex cyclic polygon $A_1A_2A_3...A_{2n}$ and $B_1B_2B_3...B_{2n}$, then:*

$$\sum_{i=1}^{2n}(-1)^{i+1}.B_iA_i^2.\frac{A_{i-1}A_{i+1}}{A_iA_{i-1}.A_iA_{i+1}} = 0 \quad (3)$$

Where $A_0 = A_{2n}$ and $A_{2n+1} = A_1$

References

[1] http://tube.geogebra.org/m/1443865

[2] http://tube.geogebra.org/m/1443867

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