Holographic Units of Length, Time and Energy concordant with Atomic Scales are dictated by the Cosmological Constant

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Abstract

Units of length, time and energy calculated by applying the Holographic Principle to the cosmological Event Horizon are concordant with the properties of the atomic nucleus, and remain so over the history of the universe from CMB decoupling into the infinite future. Such units differ from the Planck units by 20 orders of magnitude. The link between cosmology and particle physics is a non-zero cosmological constant.

Introduction

The fundamental units of length, time and energy deduced by dimensional analysis are strikingly incongruous with the scale of phenomena at the atomic level. The Planck length, $l_P = (\hbar G/c^3)^{1/2} = 1.616\times10^{-35}$ m, is 20 orders of magnitude smaller than an atomic nucleus, while the Planck energy, $E_P = (\hbar c^5/G)^{1/2} = 1.956\times10^{9}$ J, is 19 orders larger than the rest mass-energy of a proton.

Not only are these disparities esthetically uncomfortable, they seem deeply disturbing from an informational perspective. If we naïvely consider the world to be about as rich in information as a lattice with one bit per cubic Planck length, reality appears to be severely over-specified. The volume of a proton could contain about 10^{60} bits.

An innovative paradigm, the Holographic Principle [Bousso, 2002], abolishes such informational overkill by focusing on information, rather than energy and matter alone. It imposes a stringent limit on the entropy within a given space, and thus implies a drastic reduction in information and degrees of freedom. Counter-intuitively, this limit scales with the surface area of the boundary, rather than the contained volume.

Applied to a cosmological horizon, the Holographic Principle implies a natural unit of length, l_H, consonant with atomic dimensions [Manley, 2014]. In this paper, we explore further implications of such a re-scaling of fundamental physics. A natural unit of energy, the minimum required to switch a quantum system between orthogonal states in the time l_H/c, can be derived from the theorem of Margolus & Levitin [1998], and is consonant with atomic energy scales.

A Holographic Unit of Length

We consider the information encoded by a uniformly-distributed population of quantum entities (such as spin states) within a spherical region [Fig. 1]. The Covariant Entropy Bound [Bousso, 2002], a precise formulation of the Holographic Principle, limits the enclosed information to one quarter of the boundary surface area, in Planck units.

In the case where this limit is saturated, let us define a holographic length, l_H, such that the number of voxels of side 2 l_H within the 3-dimensional space is equal to the number of pixels of side 2 l_P on the 2-dimensional boundary. With a uniform distribution of information throughout the volume, each voxel could therefore contain 1 bit. Note that the correspondence of 3-dimensional voxels to 2-dimensional pixels need not be a simple mapping (compare an optical hologram whose pattern bears little visible relationship to the scene it records).

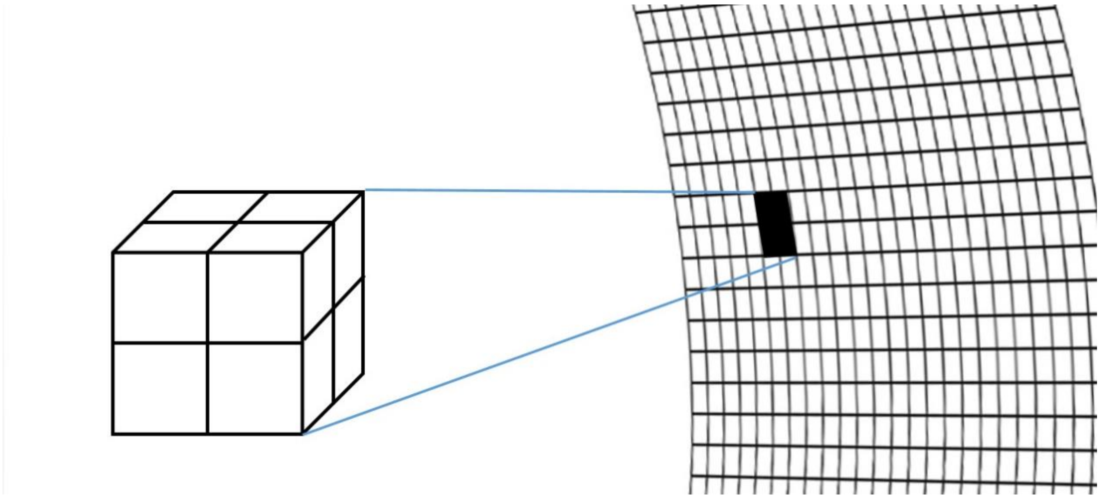

Fig. 1. Encoding 1 bit of information, an area $4\ l_P^2$ on the 2-dimensional boundary surface corresponds to a 3-dimensional cube of volume $8\ l_H^3$ within the enclosed space. When the radius of the enclosed space is large, $l_H >> l_P$

A sphere of radius R, with a surface area of $4\pi R^2$, can enclose $\pi R^2/l_P^2$ bits within its volume of $^4/_3\ \pi R^3$ at saturation of the entropy bound. Solving for the holographic unit of length, l_H, where each bit occupies 8 l_H^3, we find

$$l_H = (R\ l_P^2/6)^{1/3} \tag{1}$$

The length scale, l_H, set by the holographic entropy bound, increases as a function of the radius, R, of the enclosing boundary. On macroscopic scales, $l_H >> l_P$.

The largest radius which is meaningful in our universe is clearly a cosmological horizon, the greatest distance of objects with which we can have causal connection. Such horizons are of the order of the radius of the Hubble sphere, c/H_0, the distance light could travel in an inertial frame in the time since the Big Bang, *circa* 14 Glyr, or 1.3×10^{26} m. Applying this value to R in equation (1), we find $l_H \approx 1.8\times10^{-15}$ m, a value close to the diameter of the proton [twice the r.m.s. electromagnetic radius of 0.8775×10^{-15} m, i.e. 1.755×10^{-15} m; CODATA, 2010].

For a precise estimate of l_H, we must identify the relevant cosmological horizon and calculate its radius, R, as accurately as the evidence permits. The Hubble sphere itself is not, in fact, a horizon [Lineweaver & Egan, 2010]. Instead, the horizon constraining the entropy and information of our present reality lies at the slightly greater distance from which light emitted today will eventually be able to reach us in a universe subject to accelerating expansion, according to the standard cosmological model, ΛCDM. This is the Cosmic Event Horizon, whose radius, R_{CEH}, is estimated by integrating the Friedmann equation for the spatial scale factor, $a(t)$, from the present into the infinite future [Lineweaver & Egan, 2010].

$$R_{CEH}(t) = a(t)\int_t^{\infty} \frac{c}{a(t)}dt \tag{2}$$

$$\frac{da}{dt} = \left(\frac{\Omega_R}{a^2} + \frac{\Omega_M}{a} + \frac{\Omega_\Lambda}{a^{-2}}\right)^{1/2} \tag{3}$$

$$H_t \equiv \frac{da/dt}{a(t)} \tag{4}$$

On current evidence, our universe is spatially flat, the Ω terms summing to unity {subscript R denoting radiation (presently negligible), M matter (baryonic and Cold Dark), and Λ the dark energy or cosmological constant}. The integral is rendered finite by the accelerating expansion driven by the dominant Ω_Λ (≈ 0.7 in the present epoch). The calculation yields $R_{CEH} \approx 16$ Glyr $\approx 1.5\times10^{26}$ m, about 10% larger than the Hubble radius, c/H_0.

Uncertainty in cosmological parameters affects the estimate of R_{CEH}, and therefore l_H. The value is relatively insensitive to variations in Ω_M but is significantly influenced by H_0, the present value of the Hubble parameter, over the range of contemporary estimates [Fig. 2].

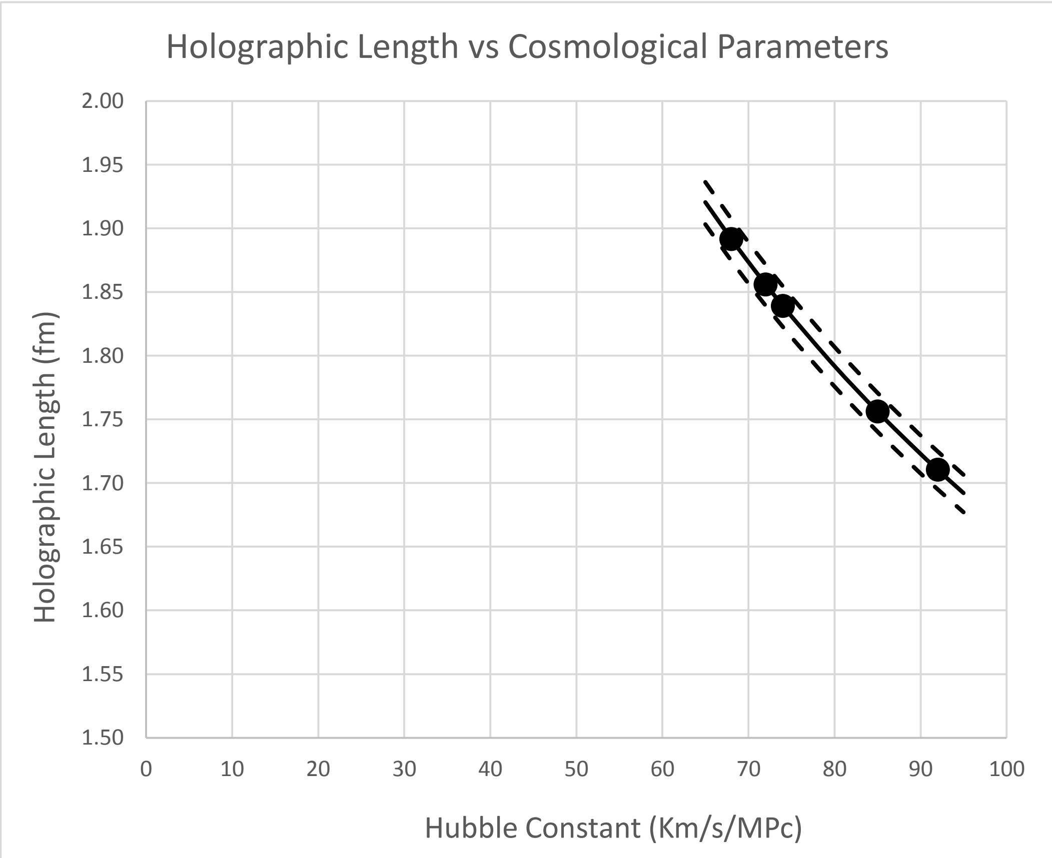

Fig. 2. Cosmological parameters influence the estimate of the Holographic Length, l_H. The solid line shows the relationship between the Hubble Constant, H_0, and l_H in a flat universe with $\Omega_M = 0.30$, $\Omega_\Lambda = 1 - \Omega_M$. Observational estimates of H_0 are shown by filled circles. Values (from top left to bottom right) were: 68 from the Planck Collaboration [2013]; 72 from the Hubble Space Telescope project [Freedman *et al.*, 2000], 74 from a mid-IR calibration [Freedman *et al.*, 2012], 85 and 92 by different calculation methods on observations of Cepheid variables within 20 MPc [Willick & Batra, 2001]. The dotted lines are for Ω_M = 0.25 (lower) and Ω_M = 0.35 (upper). Units for H_0 are traditionally Km/s/Mpc, reflecting the observational basis of the data, but the dimensions reduce to reciprocal time: $1/H_0$ approximates the time since the Big Bang, *circa* 14 Gyr.

Since the variation in the data reflects different assumptions as well as different observational bases, it seems pointless to calculate formal statistical limits. By inspection, however, we can be reasonably confident of the value $l_H = 1.8 \pm 0.1 \times 10^{-15}$ m. This value is close to the electromagnetic diameter of the proton, and 20 orders of magnitude larger than the Planck length.

A Holographic Unit of Time

A holographic unit of time, t_H, follows trivially from the holographic unit of length:

$$t_H = l_H / c \tag{5}$$

The value, $t_H = 6.0 \times 10^{-24}$ s, is 20 orders of magnitude larger than the Planck time, $t_P = (\hbar G/c^5)^{1/2} = 5.391 \times 10^{-44}$ s.

A Holographic Unit of Energy

Considering a natural unit of energy, we may be tempted to take l_H as a Compton wavelength, giving a mass-energy of 1.10×10^{-10} J or 1.23×10^{-27} Kg, of the same order as the rest mass of the proton [1.673×10^{-27} Kg; CODATA].

The Holographic Principle, however, is the relationship of geometry to information, rather than energy. In this conceptual framework, a natural unit of energy may be deduced from the theorem of Margolus & Levitin [1998], which defines the minimum time necessary for a quantum system of a given energy, E, to switch to an orthogonal state (i.e. to write or erase 1 bit).

$$t = h/4E \tag{6}$$

Setting $t = t_H$, we have $E_H = 2.8 \times 10^{-11}$ J, or 177 MeV, or 3.1×10^{-28} Kg$\times c^2$. Such a value is concordant with atomic energy scales, lying between the rest mass-energy of the electron (0.511 MeV) and that of the proton (938 MeV), and being 20 orders of magnitude less than the Planck energy, $E_P = (\hbar c^5/G)^{1/2} = 1.956 \times 10^9$ J, or 1.221×10^{19} GeV.

Variation of Holographic Units over Cosmological Timescales

Since the radius of the event horizon, R_{CEH}, varies with time, it follows that the holographic length, l_H, must vary. Surprisingly however, this variation is modest through the observable history of the universe [Table 1]. The rate of expansion of the event horizon is initially rapid (formally infinite at $t = 0$), and falls to zero in the future [Margalef-Bentabol, Margalef-Bentabol & Cepa, 2013]. At the time of the earliest directly observable phenomenon, the decoupling of the Cosmic Microwave Background ($t = 0.380$ Myr), l_H is already within one order of magnitude of its present value.

TABLE 1

Epoch	Time (Gyr)	Event Horizon ($m\times10^{25}$)	Holographic Length ($m\times10^{-15}$)
Infinite future	∞	16	1.9
Present	14	15	1.8
$z = 6$	0.9	5.9	1.4
CMB	0.0004	0.05	0.3
Planck Era	$\approx10^{-60}$	$\approx10^{-60}$	$\approx10^{-20}$

Table 1: Evolution of the Holographic Length scale over cosmological time. Note the redshift value z = 6 represents approximately the limit of contemporary telescopic observations in the Hubble deep field and Sloan digital sky survey. The CMB, observed at millimeter wavelengths, decoupled from matter at only 0.003% of the present age of the universe. The Planck time, 5.39×10^{-44} s = 1.708×10^{-60} Gyr.

Discussion

A system of natural units, independent of anthropogenic artefacts, emerges simply from dimensional analysis of the fundamental constants of physics. Planck's constant, $\hbar$, has dimensions $L^2\ M^1\ T^{-1}$, Newton's gravitational constant, G, has $L^3\ M^{-1}\ T^{-2}$, and the speed of light in a vacuum, c, has $L^1\ M^0\ T^{-1}$. From simple combinations of these constants, we derive the Planck units of length, mass, time and energy. The literature of theoretical physics removes the clutter of constants from equations by adopting Planck units such that $\hbar = c = G = 1$, and, if relevant, may similarly eliminate Boltzmann's constant, k, and the Coulomb constant, ε_0.

Convenient as they are for theory, the Planck units have values that are strikingly inappropriate for phenomena at the atomic level: length and time are far too small; energy is ridiculously large. In a mathematical sense, this is trivial – merely the result of the weakness of gravity compared with the other forces of nature. Of course, if the strength of gravity were comparable to that of the electromagnetic interaction, we would not be here to debate the issue (an example of the much-abused Anthropic Principle).

Of fundamental importance, however, is the role of the Planck units in defining the scale at which classical General Relativity and traditional quantum mechanics must give way to a unified theory of Quantum Gravity. In this sense, the Planck scale defines the atomicity of space-time, the level of detail at which Nature paints her picture of reality. This detail seems incredibly fine-grained compared with the scale of atomic processes.

In recent decades, however, a paradigm has emerged which suggests limits on the atomicity of space-time and its information content that are far coarser than the Planck dimensions imply. The Holographic Principle constrains the entropy, degrees-of-freedom and information within a region [Bousso, 2002]. Counter-intuitively, the limit scales as the area of the region's boundary surface, rather than its contained volume.

A relationship of information to geometry was first proposed in black hole thermodynamics, where each bit requires an area of 4 l_P^2 on the event horizon [Bekenstein, 1973]. The universality of this relationship, implying dimensional reduction in Quantum Gravity, was first proposed by 't Hooft [1993], and through work of Thorn and Susskind, refined into a

Holographic Principle [Susskind, 1994], which has been found to be remarkably general and robust [Bousso, 2002; Bousso, Freivogel & Leichenauer, 2010].

Identifying the boundary which constrains the entropy and information contained within a region is straightforward for compact objects of simple geometry, such as black holes. Applying the Holographic Principle on a cosmological scale, however, for example to estimate the total entropy of the universe [Egan & Lineweaver, 2010], is anything but trivial. Cosmological horizons in an expanding universe cause endless misunderstandings, not merely in the popular press and textbooks, but also—regrettably—in the professional literature [Davis & Lineweaver, 2003, in a paper with the catchy title: *Expanding Confusion*].

Although the Hubble radius ($c/H_0 \approx 14$ Glyr $\approx 1.3\times10^{26}$ m), is of the same order of magnitude as the Cosmic Event Horizon and the Particle Horizon, these parameters are not equal. For precise computation, it is important to distinguish between them. Careful analysis of the mathematical foundations of the Holographic Principle identifies the boundary which constrains entropy and information content as the Event Horizon, the furthest present distance of points within whose *future* light cone we will lie [Bousso, 2002].

In the present epoch, the Cosmic Event Horizon lies at a distance, R_{CEH}, which is about 10% larger than the Hubble radius. In the future, these radii converge, being almost indistinguishable by twice the present age of the universe [see Davis & Lineweaver, 2003, Figure 1, top panel]. The Particle Horizon is at the significantly greater distance (*circa* 43 Glyr, roughly 3 times the Hubble radius) at which presently lie the most distant objects we can now observe by light they emitted in the remote past ($1/H_0 \approx 14$ Gyr ago). Such objects are now receding from us at superluminal velocity. This is possible in an expanding universe because these objects are no longer causally connected with us and are therefore exempt from the restrictions of Special Relativity, which applies only to inertial frames. Galaxies observed at redshifts exceeding $z \approx 1.46$ are presently receding superluminally [Davis & Lineweaver, 2003].

Uncertainty in H_0 contributes the greatest variance to the estimate of R_{CEH}. Determining the current value, H_0, of the Hubble parameter, H_t, and its time evolution, has been a dominant concern in astronomy for many decades. Telescopic observations provide measures of recession velocity (from Doppler shifts in spectra) versus distance (from the light of "standard candles," such as Cepheid variable stars and Type Ia supernovae). Detailed modelling of the fine structure of the Cosmic Microwave Background radiation provides independent estimates of cosmological parameters which have statistically impressive precision, but disagree significantly with the telescopic data [Planck Collaboration, 2013].

The latest CMB survey, by the ESA *Planck* space mission, indicates H_0 close to 68 Km/s/Mpc [Planck Collaboration, 2013]. The CMB is the oldest signal we can observe in our universe, being measured at millimeter wavelengths, with a redshift $z \approx 1100$. Telescopic studies continue to indicate higher values, in the low-mid 70s, based on a more "recent" view of objects in the mid-infrared [Freedman *et al.*, 2012]. Study of galaxies within 20 Mpc, (virtually the present on the cosmological timescale), give even higher values, 85 – 92, but, because the recession velocities are very small, are marred by the necessity of modelling out local gravitational effects [Willick & Batra, 2001].

Even the most extreme of these variations yields an estimate of R_{CEH} which puts l_H close to the r.m.s. electromagnetic diameter of the proton and varies less than one order of magnitude tracking back in time to the decoupling of the CMB and forward to the infinite future.

The importance of the proton is not merely that it is the majority constituent of the baryonic matter in the universe, but rather that the nucleons are the smallest entities of non-zero size found in nature (electrons and quarks remaining point-like at any experimentally-accessible resolution). Since the holographic length, l_H, is calculated from the cosmological parameter R_{CEH}, we are seeing a connection between the largest and smallest scales of our universe. Barring a spectacular coincidence, such a link must surely be of deep physical significance.

A connection between particle theory and the mathematical framework of cosmology does not emerge from the calculation of the holographic length, which proceeds purely from informational considerations. The derivation of quantum mechanics from information theory [Chiribella, D'Ariano & Perinotti, 2011], though of enormous significance for our understanding of the quantum domain, does not make the link we are seeking here. Instead, it turns out to be the existence of a non-zero cosmological constant, Λ, which makes the connection.

The cosmological constant, representing the dark energy which drives the accelerating expansion of our universe, renders R_{CEH} finite [equations 2 – 4], and therefore l_H finite [equation 1]. With a zero cosmological constant, we would have an infinite horizon. There would be no limit on the information content of the universe, and the notion of a holographic length would be meaningless.

The link between the cosmological constant and particle physics is the duality of quantum field theories describing elementary particles with theories of quantum gravity in anti-de Sitter spaces (AdS/CFT correspondence), first proposed by Maldacena [1998]. This was promptly recognized as a leap forward in quantum gravity, specifically super-symmetric gravity formulated in string (M) theory [Witten, 1998]. A holographic version of quantum chromodynamics (Holographic QCD) has become a productive approach to the nuclear strong interaction and its bound states, the hadrons [Erlich, 2014]. Realistic approximations for the radii of baryons can be calculated in this framework [Hashimoto, Sakai & Sugimoto, 2010].

As Smolin [2010] points out, in all of the theories based on AdS/CFT correspondence, we need a non-zero cosmological constant to set up the holographic description of gravity. This applies even in the application to Loop Quantum Gravity of Verlinde's [2010] concept of gravitation as an entropic force [Smolin, 2010].

It will be one of the most beautiful ironies in the history of physics if the cosmological constant, which Einstein introduced and later famously dubbed his "biggest blunder," turns out to be the link between our descriptions of the universe on the largest and the smallest scales.

Conclusion

The Holographic Principle, applied to the relevant cosmological horizon, dictates scales of length, time and energy which are concordant with those of nuclear particles and thus are some 20 orders of magnitude different from the Planck units. The connection between cosmology and particle physics is a non-zero value for the cosmological constant, essential in the AdS/CFT correspondence and the holographic theories which followed.

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