Modified Alcubierre Warp Drive II

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Abstract:

A solution of general relativity is presented that describes an Alcubierre [1] propulsion system in which it is possible to travel at superluminal speed while reducing the energy density and energy by an arbitrary value [16].Here we investigate the negative energy in the Pfenning zone [4] or warped region (appendix 3), and the quantum inequalities involved.

1. Introduction:

Alcubierre [1] in 1994 proposed a solution of the equations of general relativity which provides the only viable means to accelerate a spaceship up to superluminal velocities without using wormholes. A problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. Moreover he used quantum inequalities to show that this energy gets distributed at very short scale (about 100 times the Planck length) up to a multiplicative factor equal to the squared speed. In the previous part of this pubblication (part I), we presented a way to reduce the amount of energy involved and its spacial distribution within the warp bubble. In this second part we investigate the quantum inequalities.

Note: In the following we adopt the notation used by Landau and Lifshitz in the second volume ("The Classical Theory of Fields") of their well known Course of Theoretical Physics [15].

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1.1 Summary:

We start with the metric

$$ds^2=\left(1-v^2\frac{f(x,y,z-k(t))^2}{a(x,y,z-k(t))^2}\right)dt^2+2v\frac{f(x,y,z-k(t))}{a(x,y,z-k(t))}dt\,dz-dx^2-dy^2-dz^2 \qquad (1)$$

From the components of the Einstein tensor in contravariant form [11] for

- 1)-The Pfenning zone is the zone within the interval: $R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2}$ where $\Delta\ll 1$ R is the radius of the warp bubble and Δ is the wall thickness of the Warp bubble $R\gg\Delta$.

- 2)- $r=(x^2+y^2+(z-k(t))^2)^{\frac{1}{2}}$ and $\frac{dk(t)}{dt}=v=const$

- 3)-In the Pfenning zone we let $a=a(r)=a(x,y,z-k(t))\gg 1$ (there is the source of esotic matter),and $D_i a\leq a\,,D_{i\,,k}a\leq a$ or $da(r)/dr\leq a(r)$

can be reduced by an arbitrary value. G^{tt} is [11]:

where $D_i=\frac{\partial}{\partial x^i}\,,x^i=x\,,y\,,z\,(i=1,2,3)$

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}(v^2(f(x,y,z-k(t))^2\,D_2(a)(x,y,z-k(t))^2+f(x,y,z-k(t))^2\,D_1(a)(x,y,z-k(t))^2$$

$$+D_1(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2+D_2(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2$$

$$-2\,D_1(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,D_1(a)(x,y,z-k(t))$$

$$-2\,D_2(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,D_2(a)(x,y,z-k(t))))$$

Einstein Equations:

$$G^{ik}=\frac{8\pi G}{c^4}T^{ik}$$ [15] T^{ik} (energy-impulse tensor)

The functions $f(r)=f(x,y,z-k(t))$ and $a(r)=a(x,y,z-k(t))$ can assume the following values:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$
- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<1$, $f(r)$ is

$f(r)=-\frac{(r-R-\frac{\Delta}{2})}{\Delta}$ (Pfenning zone [4]) and $a=a(r)=a(x,y,z-k(t))\gg 1$

(possessing extremely large values) and $D_i a\leq a\,, D_{i\,,k} a\leq a$ or $da(r)/dr\leq a(r)$

- 1)-Internal metric of the Warp bubble $(0<r<R-\frac{\Delta}{2})$ is:

$$ds^2=dt^2-(dz-vdt)^2-dx^2-dy^2 \tag{2}$$

moving with velocity v (multiple of the speed of light c) along the z-axis.

- 2)-Metric outside of the bubble beyond the Pfenning zone $(r>R+\frac{\Delta}{2})$ is:

$$ds^2=dt^2-dx^2-dy^2-dz^2 \tag{3}$$

2 Computation of the negative energy in the Pfenning zone, its comparison with the Casimir effect (plane-parallel condenser) and quantum inequalities.

The energy is:

$$E=\int\int\int(-g)^{1/2}\,T^{tt}\,dx^3 \tag{4}$$

where the triple integral extends over all the volume

and the energy density is

$$T^{tt}=k\,G^{tt} \tag{5}$$

$(g\ \textit{determinant of the spacial metric})$ and $k=c^4/8\pi G$.

In our case we get, in xyz-coordinates:

$$G^{tt}=-\frac{1}{4}\frac{1}{a(x,y,z-k(t))^4}(v^2(f(x,y,z-k(t))^2\,\mathrm{D}_2(a)(x,y,z-k(t))^2+f(x,y,z-k(t))^2\,\mathrm{D}_1(a)(x,y,z-k(t))^2$$

$$+\mathrm{D}_1(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2+\mathrm{D}_2(f)(x,y,z-k(t))^2\,a(x,y,z-k(t))^2$$

$$-2\,\mathrm{D}_1(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_1(a)(x,y,z-k(t))$$

$$-2\,\mathrm{D}_2(f)(x,y,z-k(t))\,a(x,y,z-k(t))\,f(x,y,z-k(t))\,\mathrm{D}_2(a)(x,y,z-k(t))))$$

which, written in Alcubierre form becomes:

$$G^{tt}=-\frac{1}{4}v^2\frac{x^2+y^2}{r^2}g(r) \tag{6}$$

where $g(r)$ is given by:

$$g(r)=[\frac{1}{a(r)^2}(\frac{df(r)}{dr})^2+(\frac{f(r)^2}{a(r)^4})(\frac{da(r)}{dr})^2-2\frac{df(r)}{dr}\frac{f(r)}{a(r)^3}\frac{da(r)}{dr}] \tag{7}$$

In the simplified case:

- 1)-inside the warp bubble $(0<r<R-\frac{\Delta}{2})$ $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $(r>R+\frac{\Delta}{2})$ $f(r)=0$ and $a(r)=1$
- 3)-in the Alcubierre warped region $(R-\frac{\Delta}{2}<r<R+\frac{\Delta}{2})$ $0<f(r)<1$, $f(r)$ is

  $f(r)=-\frac{(r-R-\frac{\Delta}{2})}{\Delta}$ and $a(r)=a(x,y,z-k(t))=A=constant\gg 1$ possessing extremely large values

2.1 The energy E in the Pfenning zone in our case (see [4] for an example):

The energy given by (4), taking into account the conditions set there in, becomes:

$$E=-\frac{8\pi k v^2}{32\pi}\int\frac{x^2+y^2}{r^2}g(r)dx^3 \tag{8}$$

$$E=-\frac{8\pi k v^2}{12}\int r^2 g(r)dr \tag{9}$$

$$g(r)\approx\left(\frac{-1}{a(r)\Delta}\right)^2 \tag{10}$$

((10) is dominant term of (7) for $\Delta\ll 1$, $a=a(r)\gg 1$ and $D_i a\leq a, D_{i,k}a\leq a$ or $da(r)/dr\leq a(r)$, $a(r)>1/\Delta$

$$E=-\frac{8\pi k v^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} r^2 g(r)dr=-\frac{8\pi k v^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} r^2\left(\frac{-1}{a(r)\Delta}\right)^2 dr \tag{11}$$

$$E=-\frac{8\pi k v^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} r^2\left(\frac{-1}{A\Delta}\right)^2 dr \tag{12}$$

$$E=-\frac{8\pi k}{12}v^2\frac{(\frac{R^2}{\Delta}+\frac{\Delta}{12})}{A^2} \tag{13}$$

where $R \gg \Delta$ (very likely), R being the radius of the warp bubble. If Δ is about $1.6 \bullet 10^{-35} m$ (Planck lenght) as an absurd, setting $A=10^{50}$ and R = 100 m, the energy E gets reduced significantly. Using the chosen data the energy is $E=-3 \bullet 10^{-17} v^2 \, joule$ quite smaller than the U(d) due the the Casimir effect.

2.2 The energy U(d) between the two plates in a plane-parallel condenser in empty space, due to the Casimir effect, is:

$$U(d)=-\pi^2\left[(\frac{h}{2\pi})\frac{c}{720\mathrm{d}^3}\right]L^2 \tag{14}$$

where d is the distance between the plates and L is the side of the square conducting plate. As can be seen, the energy is negative and this implies that the force (equal to the opposite of the derivative with respect to d) is attractive, as has been experimentally found ("Lamoreaux" [12]). In the case $d=1\mu m$ and L = 1 m (L is chosen to be quite large, but not as large as the Pfenning zone) it is found that $U(d)=-4\,10^{-10} \, joule$.

2.3 Quantum inequalities. Calculation for our solution:

The quantum inequalities are [13]:

$$\frac{t_0}{\pi}\int_{-\infty}^{+\infty}\frac{<T^{ik}u_i u_k>}{t^2+t_0^2}dt \geq -\frac{3}{32\pi^2 t_0^4} \tag{15}$$

for $(h, c, G=1)$ and u_i is the quadrivelocity in a Eulerian or moving reference

system. In our case using the International System of Units we get as Pfenning solution [4] :

$$\Delta \leq \frac{1}{\alpha^2} v^2 L_{Planck} \tag{16}$$

where Pfenning in his paper [4] chooses $\alpha = 1/10$ and therefore

.

$$\Delta \leq 10^2 v^2 L_{Planck} \tag{17}$$

I believe that the concentration of energy in a very small volume is due to the bad choice of the $f = f(r)$ function,or of the parameter α by Pfenning [4], since the quantum inequalities are valid for all values of t_0 [13] and this leads to $0 < \alpha \ll 1$ therefore α to an arbitrary value. We can conclude that α can assume an arbitrarily small value, and the results presented in [13] provide indirect evidence of this.

3 Appendix:The energy density,and energy E in the Alcubierre warped region in new case (no trivial):

- 1)-inside the warp bubble $f(r)=1$ and $a(r)=1$
- 2)-outside the warp bubble $f(r)=0$ and $a(r)=1$
- 3)-within the Alcubierre warped region $0 < f(r) < 1$ and $a(r) \gg 1$,
  $f(r)$ is for all r (example):

  $f(r) = (1/2)(1 - \tanh @(r-R))$, $(df/dr)^2 = (1/4)(@^2/[(\cosh @(r-R))^4])$ [17] .(18)

  $a(r)$ is for all r (example):

$$a(r)=(2^P)/[1+(\tanh[@(r-R)])^2]^P \qquad P\gg 1 \quad [17] \tag{19}$$

$$da(r)/dr=(-P)((1/2)[1+(\tanh[@(r-R)])^2])^{-(P+1)}(@\tanh[@(r-R)]/(\cosh[@(r-R)])^2)$$

$$(da(r)/dr)^2=(-P)^2((1/2)[1+(\tanh[@(r-R)])^2])^{(-2(P+1))}(@\tanh[@(r-R)]/(\cosh[@(r-R)])^2)^2$$

R radius of the warp bubble.

Equation (7) : $g(r)=[\frac{1}{a(r)^2}(\frac{df(r)}{dr})^2+(\frac{f(r)^2}{a(r)^4})(\frac{da(r)}{dr})^2-2\frac{df(r)}{dr}\frac{f(r)}{a(r)^3}\frac{da(r)}{dr}]$

$$g(r)=g_1(r)+g_2(r)+g_3(r) \tag{20}$$

$$g_1(r)=\frac{1}{a(r)^2}(\frac{df(r)}{dr})^2=(\frac{1}{4})\frac{@^2}{a(r)^2(\cosh[@(r-R)])^4} \tag{21}$$

$$g_2(r)=(\frac{f(r)^2}{a(r)^4})(\frac{da(r)}{dr})^2\approx(P@)^2\frac{(\tanh[@(r-R)])^2}{a(r)^2(\cosh[@(r-R)])^4} \tag{22}$$

$$g_3(r)=-2\frac{df(r)}{dr}\frac{f(r)}{a(r)^3}\frac{da(r)}{dr}\approx-(P@^2)\frac{\tanh[@(r-R)]}{a(r)^2(\cosh[@(r-R)])^4} \tag{23}$$

for P=280 ,@=5000 , $a(r)\gg 1$ in Alcubierre warped region (19), the energy density is,for equations (21),(22),(23),and paper [11],[16],[17]:

$$energy\, density=|-\frac{k}{4}v^2\frac{x^2+y^2}{r^2}g(r)|\ll 1 \tag{24}$$

in Alcubierre warped region.

And if $R=100\,m$ $P=280$, $@=5000$ [17],in Alcubierre warped region (there is the source of esotic matter),the energy is,for equation (9):

$$, \quad E=-\frac{8\pi k v^2}{12}\int_0^{+\infty} r^2 g(r)\,dr \qquad k=c^4/8\pi G \tag{25}$$

and for the same values of the parameters shown above the Energy is:

$$E\approx -4v^2 10^{-87}\,joule \tag{26}$$

energy is very small compared with Casimir effect.

And mass is : $$m\approx -4v^2 10^{-104}\,kg \tag{27}$$

If $R=100\,m$ $P=140$, $@=5000$ [17],in Alcubierre warped region (there is the source of esotic matter),the energy is:

$$E\approx -v^2 10^{-7}\,joule \tag{28}$$

energy is compared with Casimir effect,with d=100 nm and L=1 m,equation (14).

A modification of parameters P, @, in a(r) and for f (r), between those chosen, determines a greater or lesser reduction of the energy density and the energy,with the same radius value R of the warp bubble that contains the spaceship,and also changes the thickness of the warped region.

4 Conclusion: The calculations seem to suggest that the modified Alcubierre propulsion system allows to reach superluminal speeds and the negative energy can be arbitrarily reduced. This energy is compared with that of the Casimir effect in a parallel plane condenser, to investigate the experimental feasibility. Quantum inequalities are then calculated in orded to investigate the spacial distribution of the negative energy in the Pfenning zone. In the Alcubierre solution a event horizon is present at $v>1$ while in our solution it is present at $v\geq 1$. In our next paper this problem will be investigated .

References

[1] M. Alcubierre, Classical and Quantum Gravity **11**, L73 (1994).

[2] C. Barcelo, S. Finazzi, and S. Liberati, ArXiv e-prints (2010), arXiv:1001.4960 [gr-qc].

[3] C. Clark, W. A. Hiscock, and S. L. Larson, Classical and Quantum Gravity **16**, 3965 (1999).

[4] M. J. Pfenning and L. H. Ford, Classical and Quantum Gravity **14**, 1743 (1997).arXiv:9702026

[5] F. S. N. Lobo and M. Visser, Classical and Quantum Gravity **21**, 5871 (2004).

[6] F. S. N. Lobo, ArXiv e-prints (2007), arXiv:0710.4474 [gr-qc].

[7] Finazzi, Stefano; Liberati, Stefano; Barceló, Carlos (2009). "Semiclassical instability of dynamical warp drives".Physical Review D **79** (12): 124017. arXiv:0904.0141

[8] Van den Broeck, Chris (1999). "On the (im)possibility of warp bubbles". arXiv:gr-qc/9906050

[9] C. Van Den Broeck, Class. Quantum Grav. 16 (1999) 3973

[10] Hiscock, William A. (1997). "Quantum effects in the Alcubierre warp drive spacetime". Classical and Quantum Gravity **14** (11): L183–L188. arXiv gr-qc/9707024

[11] Perniciano G. (2015),viXra:1507.0165

[12] S. K. Lamoreaux,"Demonstration of the Casimir Force in the 0.6 to 6 μm Range",*Phys. Rev. Lett.***78**, 5–8 (1997)

[13] L.H. Ford and T.A. Roman, Phys. Rev. D 51, 4277 (1995)

[14] Ford L H and Roman T.A. 1996 Phys. Rev. D 53 p 5496 arXiv: gr-qc/9510071

[15] L D Landau and E M Lifshitz "Theory of Fields", Fourth Edition: Volume 2 (Course of Theoretical Physics Series)

[16] Perniciano G.(2015),viXra:1507.0193v7

[17] Loup F. (2015),hal-01183043,viXra:1508.0048

[18] Perniciano G.(2015),viXra:1507.0193v1