Review Paper

From An Exact Solution of 2D Navier-Stokes Equations to a Navier-Stokes Cosmology on Cantor Sets

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ABSTRACT

In a recent paper I derived an exact analytical solution of Riccati form of 2D Navier-Stokes equations with Mathematica. Now I will present a possible route from an exact analytical solution of the Navier-Stokes equations to Navier-Stokes Cosmology on Cantor Sets. The route is by showing that Raychaudhury equation leads to Friedmann equation when the vorticity vector, shear tensor and tidal force tensor vanish. Then I show how one can generalize it further to Navier-Stokes systems on Cantor Sets. While this paper contains nothing new except for pedagogical purpose, it may serve as an outline towards Navier-Stokes Cosmology on Cantor Sets.

Key Words: Navier-Stokes equations, Raychaudhury equation, Navier-Stokes cosmology, Cantor sets, Mathematica.

Introduction

In a recent paper I derived an exact analytical solution of Riccati form of 2D Navier-Stokes equations with Mathematica[3], based on Argentini's paper [1]. Now I will present a possible route from an exact analytical solution of the Navier-Stokes equations to Navier-Stokes Cosmology on Cantor Sets. The route is by showing that Raychaudhury equation leads to Friedmann equation when the vorticity vector, shear tensor and tidal force tensor vanish. Then I show how one can generalize it further to Navier-Stokes systems on Cantor Sets.

Riccati form of 2D Navier-Stokes equations

The 2D Navier-Stokes equation for a steady viscous flow can be written as follows [6]:

$$\rho(\vec{\upsilon}\cdot\nabla)\vec{\upsilon} = -\nabla p + \rho\vec{f} + \mu\Delta\vec{\upsilon} \tag{1}$$

Argentini obtained a general exact solution of ODE version of 2D Navier-Stokes equation in Riccati form as follows [1][2]:

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$$\dot{u}_1 - \alpha . u_1^2 + \beta = 0 \text{,} \tag{2}$$

where:

$$\alpha = \frac{1}{2\upsilon},$$

and

$$\beta = -\frac{1}{\upsilon}(\frac{\dot{q}}{\rho} - f_1)s - \frac{c}{\upsilon}.$$

The solution of Riccati equation is notoriously difficult to find, so this author decides to use Mathematica software in order to get an exact analytical solution [4][5]. The result has been presented in a recent paper [3].

Vorticity as the driver of Accelerated Expansion

According to Ildus Nurgaliev [7], velocity vector V_α of the material point is projected onto coordinate space by the tensor of the second rank $H_{\alpha\beta}$:

$$V_\alpha = H_{\alpha\beta} R^\beta \tag{3}$$

Where the Hubble matrix can be defined as follows for a homogeneous and isotropic universe:

$$H_{\alpha\beta} = \begin{pmatrix} H & \pm\omega & \pm\omega \\ \mp\omega & H & \pm\omega \\ \mp\omega & \mp\omega & H \end{pmatrix} \tag{4}$$

Where the global average vorticity may be zero, though not necessarily [7].
Now we will use Newtonian equations to emphasize that cosmological singularity is consequence of the too simple model of the flow, and has nothing to do with special or general relativity as a cause [7]. Standard equations of Newtonian hydrodynamics in standard notations read:

$$\frac{d\vec{\upsilon}}{dt} = \frac{\partial\vec{\upsilon}}{\partial t} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\varphi + \frac{1}{\rho}\nabla\rho + \frac{\mu}{\rho}\Delta\vec{\upsilon} + ..., \tag{5}$$

$$\frac{\partial\rho}{\partial t} + \nabla\rho\vec{\upsilon} = 0, \tag{6}$$

$$\Delta\varphi = 4\pi G\rho \tag{7}$$

Procedure of separating of diagonal H, trace-free symmetrical σ, and anti-symmetrical ω elements of velocity gradient was used by Indian theoretician Amal Kumar Raychaudhury (1923-2005). The equation for expansion θ, sum of the diagonal elements of

$$\dot{\theta}+\frac{1}{3}\theta^2+\sigma^2-\omega^2=-4\pi G\rho+div(\frac{1}{\rho}\sum f) \tag{8}$$

is most instrumental in the analysis of singularity and bears the name of its author. [7]\`
System of (5)-(7) gets simplified up to two equations [7]:

$$\dot{\theta}+\frac{1}{3}\theta^2-\omega^2=0, \tag{9}$$

$$\dot{\omega}+\frac{2}{3}\theta\omega=0. \tag{10}$$

Recalling $\theta=3H$, the integral of (10) takes the form [7]:

$$H^2=H_\infty^2-\frac{3\omega_0^2 R_0^4}{R^4}. \tag{11}$$

In this regards, it is interesting to remark here that Zalaletdinov has shown that Raychaudhury evolution equation can yield Friedman equation at certain limits. His expression of Raychaudhury evolution equation is as follows: [8, p. 26]

$$\dot{\theta}+\frac{1}{3}\theta^2+4\pi G\rho-\Lambda=0\,. \tag{11}$$

When the vorticity vector, shear tensor and tidal force tensor vanish, then (11) is equivalent to Friedman equation [8]:

$$\theta=\frac{3}{R}\frac{dR}{dt}. \tag{12}$$

One more thing is worth to remark here: if we compare equation (8) and (11), then one obtains a new dynamical expression of cosmological constant, as follows:

$$\Lambda=-\sigma^2+\omega^2+div(\frac{1}{\rho}\sum f)\,. \tag{13}$$

How to write down Navier-Stokes equations on Cantor Sets

Now we can extend further the Navier-Stokes equations to Cantor Sets, by keeping in mind their possible applications in cosmology.
By defining some operators as follows:

1. In Cantor coordinates [9]:

$$\nabla^{\alpha} \cdot u = div^{\alpha} u = \frac{\partial^{\alpha} u_1}{\partial x_1^{\alpha}} + \frac{\partial^{\alpha} u_2}{\partial x_2^{\alpha}} + \frac{\partial^{\alpha} u_3}{\partial x_3^{\alpha}}, \tag{14}$$

$$\nabla^{\alpha} \times u = curl^{\alpha} u = \left(\frac{\partial^{\alpha} u_3}{\partial x_2^{\alpha}} - \frac{\partial^{\alpha} u_2}{\partial x_3^{\alpha}} \right) e_1^{\alpha} + \left(\frac{\partial^{\alpha} u_1}{\partial x_3^{\alpha}} - \frac{\partial^{\alpha} u_3}{\partial x_1^{\alpha}} \right) e_2^{\alpha} + \left(\frac{\partial^{\alpha} u_2}{\partial x_1^{\alpha}} - \frac{\partial^{\alpha} u_1}{\partial x_2^{\alpha}} \right) e_3^{\alpha} \,. \tag{15}$$

2. In Cantor-type cylindrical coordinates [10, p.4]:

$$\nabla^{\alpha} \cdot r = \frac{\partial^{\alpha} r_R}{\partial R^{\alpha}} + \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} + \frac{r_R}{R^{\alpha}} + \frac{\partial^{\alpha} r_z}{\partial z^{\alpha}}, \tag{16}$$

$$\nabla^{\alpha} \times r = \left(\frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_{\theta}}{\partial \theta^{\alpha}} - \frac{\partial^{\alpha} r_{\theta}}{\partial z^{\alpha}} \right) e_R^{\alpha} + \left(\frac{\partial^{\alpha} r_R}{\partial z^{\alpha}} - \frac{\partial^{\alpha} r_z}{\partial R^{\alpha}} \right) e_{\theta}^{\alpha} + \left(\frac{\partial^{\alpha} r_{\theta}}{\partial R^{\alpha}} + \frac{r_R}{R^{\alpha}} - \frac{1}{R^{\alpha}} \frac{\partial^{\alpha} r_R}{\partial \theta^{\alpha}} \right) e_z^{\alpha} \,. \tag{17}$$

Then Yang, Baleanu and Machado are able to obtain a general form of the Navier-Stokes equations on Cantor Sets as follows [9, p.6]:

$$\rho \frac{D^{\alpha} \upsilon}{Dt^{\alpha}} = -\nabla^{\alpha} \cdot (pI) + \nabla^{\alpha} \left[2\mu \left(\nabla^{\alpha} \cdot \upsilon + \upsilon \cdot \nabla^{\alpha} \right) - \frac{2}{3} \mu \left(\nabla^{\alpha} \cdot \upsilon \right) I \right] + \rho b$$

The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.

Concluding remarks

This paper discusses a possible route from an exact analytical solution of the Navier-Stokes equations to Navier-Stokes Cosmology on Cantor Sets. The route is by showing that Raychaudhury equation leads to Friedmann equation when the vorticity vector, shear tensor and tidal force tensor vanish. Then I show how one can generalize it further to Navier-Stokes systems on Cantor Sets. While this paper contains nothing new except for pedagogical purpose, it may serve as an outline towards Navier-Stokes Cosmology on Cantor Sets.
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