Solving Problems of Advance of Mercury's Perihelion and Deflection of Photon Around the Sun with New Newton's Formula of Gravity

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Abstract: According to the new Newton's formula of gravity (the original law of gravity plus a correction term), i.e., improved Newton's formula of gravity, applying the methods of classical mechanics to solve the problem of advance of Mercury's perihelion and the problem of deflection of photon around the Sun respectively, and the results are the same as given by general relativity. Pointing out that the further topic is based on deriving original law of gravity and original Newton's second law with law of conservation of energy to derive the new Newton's formula of gravity (the improved Newton's formula of gravity) with law of conservation of energy. To realize the purpose that partially replacing relativity and solving some problems that cannot be solved by relativity with the methods of classical mechanics.

Key words: Law of gravity, new Newton's formula of gravity (improved Newton's formula of gravity), advance of Mercury's perihelion, deflection of photon around the Sun, law of conservation of energy

Introduction

In references [1, 2], the new Newton's formula of gravity (the original law of gravity plus a correction term), i.e., the improved Newton's formula of gravity, was presented; but it did not present the detailed process to solve the problem of advance of Mercury's perihelion and the problem of deflection of photon around the Sun respectively with the methods of classical mechanics. While, in this paper the detailed process will be given (the results are the same as given by general relativity).

The improved Newton's formula of gravity is as follows

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \qquad (1)$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object with mass m moving around the object with mass M along a curve, and the value of p is given by: $p = a(1-e^2)$ (for ellipse), $p = a(e^2-1)$ (for hyperbola), $p = y^2/2x$ (for parabola).

1 Solving the problem of advance of Mercury's perihelion with new Newton's formula of gravity

In classical mechanics, acted by the central force, the orbit differential equation (Binet's formula) reads

$$h^2u^2(u''+u) = -\frac{F}{m} \qquad (2)$$

where: $u = \frac{1}{r}$.

As deriving Eq.(1), it already gives

$$h^2 = GMp \qquad (3)$$

Substituting Eq.(1) and Eq.(3) into Eq.(2), we have the following equation of planet's movement around the Sun

$$u''+u = \frac{1}{p} + \frac{3GMu^2}{c^2} \qquad (4)$$

For ellipse, $p = a$ (e^2-1), thus the approximate solution for Eq.(4) is as follows

$$u \approx \frac{GM}{a(1-e^2)c^2}[1+e\cos(1-\frac{3GM}{a(1-e^2)c^2})\varphi] \qquad (5)$$

Hence, the value of ε for advance of planetary perihelion for one circuit is as follows

$$\varepsilon = \frac{24\pi^3 a^2}{T^2c^2(1-e^2)} \qquad (6)$$

where: T, a, and e are orbital period, semi-major axis and eccentricity respectively.

Obviously, this result is the same as given by general relativity.

In addition, according to Eq.(1), for problem of planetary motion around the Sun, the improved Newton's formula of gravity reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2ma(1-e^2)}{c^2r^4}$$

2 Solving the problem of deflection of photon around the Sun with new Newton's formula of gravity

As solving this problem by using the improved formula of gravity, the method to be used is the same as presented in references [3], in which the original law of gravity was used.

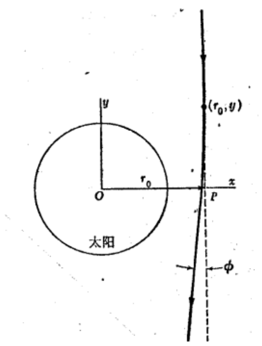

Fig. 1 Deflection of photon around the Sun

Supposing that m represents the mass of photon, for the reason that it will be eliminated, so it is not necessary to give its value. As shown in Fig.1, r_0 represents the nearest distance between the photon and the center of the Sun, for the reason that the deflection is very small, so actually the value of r_0 is the same as the photon is not deflected. When the photon is located at （r_0，y） (the value of y is measured from point P in Fig.1), the force acted on photon is as follows

$$F_x = \frac{Fr_0}{(r_0^2 + y^2)^{1/2}} \qquad (7)$$

where：$F = -\frac{GMm}{r_0^2 + y^2} - \frac{3G^2M^2mp}{c^2(r_0^2 + y^2)^2}$

Because

$$mv_x = \int F_x dt = \int F_x \frac{dy}{v_y} \approx \frac{1}{c}\int F_x dy$$

Therefore

$$v_x \approx -\frac{2GMr_0}{c}\int_0^\infty \frac{dy}{(r_0^2 + y^2)^{3/2}} - \frac{6G^2M^2pr_0}{c^3}\int_0^\infty \frac{dy}{(r_0^2 + y^2)^{5/2}}$$

After calculating, it gives

$$v_x \approx -\frac{2GM}{cr_0} - \frac{4G^2M^2p}{c^3r_0^3} \qquad (8)$$

Hence, the deflection angle ϕ is as follows

$$\phi \approx tg\phi \approx \frac{|v_x|}{c} = \frac{2GM}{c^2r_0} + \frac{4G^2M^2p}{c^4r_0^3} \qquad (9)$$

While, the value of ϕ should be determined by iteration method.

Before determining the value of ϕ, firstly we will validate that the value of the second term in Eq.(9) is equal to the value of the first term, that means that the deflection given by the second term in Eq.(9) is equal to that given by the first term.

As solving problem of deflection of photon around the Sun with general relativity, the photon's orbit is a hyperbola, and its equation is as follows

$$u = u_0\cos\varphi + \frac{GMu_0^2(1+\sin^2\varphi)}{c^2} \qquad (10)$$

where：$u_0 = \frac{1}{r_0}$

Hence, the reciprocal of the half normal chord p is as follows

$$\frac{1}{p} = u\big|_{\varphi=\pi/2} = \frac{2GM}{c^2r_0^2} \qquad (11)$$

Substituting this value of the half normal chord p into Eq.（9）, it gives

$$\phi = \frac{4GM}{c^2 r_0} = \frac{4GM}{c^2 R_s} \qquad (12)$$

where: R_s is the radius of the Sun.

Thus, the value of the second term in Eq.(9) is really equal to the value of the first term.

Now we determine the value of ϕ in Eq.(9) by iteration method.

Suppose

$$\phi = \frac{KGM}{c^2 r_0} \qquad (13)$$

In order to apply iteration method, the relationship between ϕ and p should be given.

Considering two straight lines, the first one is $x = r_0$, the second one is passing through the origin O and the first quadrant, and it makes an angle of $\phi/2$ with the positive direction of Y axis; the value of the half normal chord p is equal to the distance between the origin O and the intersection of the two straight lines. Supposing that the intersection of the two straight lines is the point P_1 (it is not shown in Fig.1), then its coordinates are $(r_0, \sqrt{p^2 - r_0^2})$.

From the triangle formed by the three points of origin O, P, and P_1, it gives

$$\frac{r_0}{p} = \sin\frac{\phi}{2} \approx \frac{\phi}{2}$$

$$p = \frac{2r_0}{\phi} \qquad (14)$$

Now, considering the result of the value of ϕ given by the original law of gravity, it gives

$$K_0 = 2$$

Here, the deflection angle is as follows

$$\phi_0 = \frac{2GM}{c^2 r_0}$$

From Eq.（14）, its corresponding half normal chord p_0 is as follows

$$p_0 = \frac{c^2 r_0^2}{GM}$$

Substituting the value of p_0 into Eq.（9）, it gives

$$\phi_1 = \frac{6GM}{c^2 r_0}$$

Namely

$$K_1 = 6$$

Similarly, the values of K_2, K_3, and the like are as follows: 3.3333, 4.4000, 3.8182, 4.0952, 3.9535, 4.0234, 3.9883, 4.0059, 3.9971, 4.0015, 3.9993, 4.0004, 3.9998, 4.0001, 4.0000, 4.0000; finally it gives

$$K = 4$$

This result is also the same as given by general relativity.

According to Eq.(1), for problem of deflection of photon around the Sun, the improved Newton's formula of gravity reads

$$F = -\frac{GMm}{r^2} - \frac{1.5GMmr_0^2}{r^4}$$

where: r_0 is the shortest distance between the photon and the Sun, if the light and the Sun is tangent, it is equal to the radius of the Sun.

The interesting fact is that, for this problem, the maximum gravitational force given by the improved Newton's formula of gravity is 2.5 times of that given by the original Newton's law of gravity.

3 Further topic

In references [4-6], the original law of gravity and the original Newton's second law have been derived with law of conservation of energy, based on this, the further topic is to derive the new Newton's formula of gravity (the improved Newton's formula of gravity), i.e., Eq.(1), with law of conservation of energy. And, finally, to realize the purpose that partially replacing relativity and solving some problems that cannot be solved by relativity with the methods of classical mechanics.

References

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