Letters and Comments

Depicting of electric fields

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Abstract
Examples are presented that geometrical images of generated electromagnetic fields are emitted by the geometrical images of the electromagnetic fields, which are the sources

Keywords: electromagnetism, differential forms

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1. Problem

It is obviously that a charge density ρ is the *source* of the irrotational electric vector field $\mathbf{D}$, that ρ *generates* the field $\mathbf{D}$ according to the formula

$$\int \rho(x')\frac{\mathbf{r}(x,x')}{4\pi r^3(x,x')}dV' = \mathbf{D}(x) \qquad (1)$$

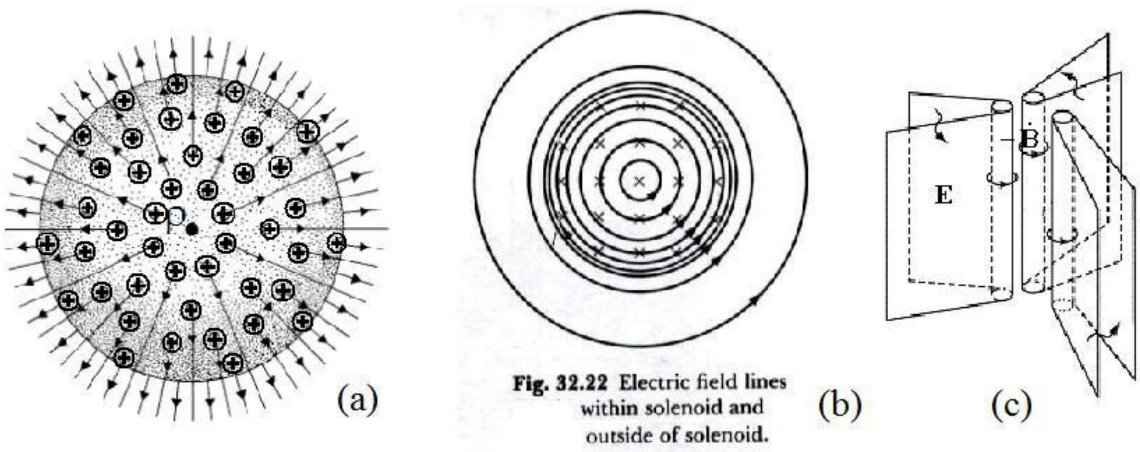
(here x denotes x, y, z). The field $\mathbf{D}$ is depicted by the field lines: the lines are always tangent to the field vectors $\mathbf{D}$. If the density of the field lines is proportional to the magnitude of the vector $\mathbf{D}$, the lines **emerge** from electric charges [1], i.e. from the charge density ρ (see Fig. 1a[1]). The charge density ρ emits the field lines of vectors $\mathbf{D}$.

At the same time, the derivative of magnetic field, $\dot{\mathbf{B}}$, is the source of the solenoidal electric field $\mathbf{E}$, and $\dot{\mathbf{B}}$ generates this field according to the formula

$$-\int \dot{\mathbf{B}}(x')\times\frac{\mathbf{r}(x,x')}{4\pi r^3(x,x')}dV' = \mathbf{E}(x)\,. \qquad (2)$$

But the field lines of the solenoidal field **do not emerge** from the derivative $\dot{\mathbf{B}}$. Instead, the lines are closed around the derivative, $\dot{\mathbf{B}}$, (see Fig. 1b[2]). The derivative, $\dot{\mathbf{B}}$, does not emit the field lines of the solenoidal field.

Why? What is the cause of the difference between generating of irrotational and solenoidal fields?



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[1] Figure 2.5 from [2] is used here, but its sense is modified
[2] This figure is from [3]

Fig.1 Generations of electromagnetic fields.
(a) Field lines of vectors $\mathbf{D}$ emerge from the charge density ρ, which is a source of vector field $\mathbf{D}$.
(b) The derivative, $\dot{\mathbf{B}}$, is a source of the solenoidal field $\mathbf{E}$.
(c) Field bisurfaces of covector field $\mathbf{E}$ emerge from the field tubes of the vector density $-\dot{\mathbf{B}}$.

2. Solution

The point is the field $\mathbf{E}$ in (2) is not a vector field. $\dot{\mathbf{B}}$ generates a *covector* field. $\mathbf{E}$ is a covector field. And covector fields are depicted not by field lines. Covector fields are depicted by bisurfaces. In the case of (2), field bisurfaces **emerge** from the field tubes, which represent the derivative of magnetic field, $\dot{\mathbf{B}}$, as is shown in Fig. 1c[3].

3. Geometrical quantities

It is important to recognize that the electromagnetism involves geometrical quantities of two different types [5]. These are: covariant (antisymmetric) tensors, e.g. $\mathbf{E} = E_\beta$, $\mathbf{B} = B_{\gamma\beta}$, which are named exterior differential forms or simply forms, and contravariant (antisymmetric) tensor *densities*, e.g. ρ, $\mathbf{D} = D^\alpha$, $\mathbf{H} = H^{\alpha\beta}$ (see Fig. 2[4])

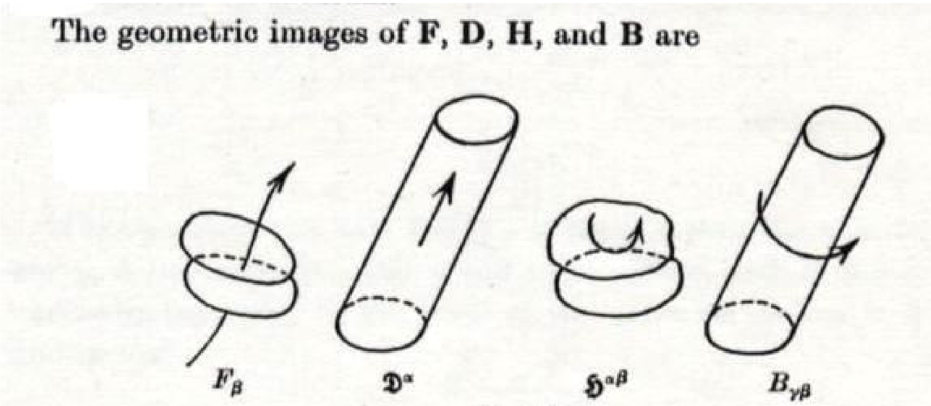

Fig. 2. Covector $\mathbf{E} = \mathbf{F}$ is represented by two parallel plane elements equipped with an outer orientation.
Vector density $\mathbf{D}$ is represented by a cylinder with an inner orientation.
Covariant bivector $\mathbf{B}$ is represented by a cylinder with an outer orientation.

Physicists and mathematics often use gotic fonts while writing densities. E.g. Schouten uses $\mathfrak{D}$ $\mathfrak{H}$ instead of $\mathbf{D}$ and $\mathbf{H}$. We do not use a gothic font.

So, according to Fig. 1c, the field tubes of the covariant bivector $-\dot{\mathbf{B}}$ emits field biplanes of covector $\mathbf{E}$. Their orientations are consistent.

References

[1] Feynman R P, Leighton R B, Sands M 1965 *The Feynman Lectures on Physics* (Addison–Wesley, London) Vol. 2, p. 4–11.
[2] Ohanian H C 1988 *Classical Electrodinamics* (Newton: Allyn and Bacon).
[3] Ohanian H C 1985 *Physics* (W. W. Norton, N. Y).
[4] Khrapko R I 2011 V*isible representation of exterior differential forms and pseudo forms. Electromagnetism in terms of sources and generation of fields. Наглядное представление дифференциальных форм и псевдоформ. Электромагнетизм в терминах источников и порождений полей.* (Saarbrucken: Lambert). http://khrapkori.wmsite.ru/ftpgetfile.php?id=105&module=files
[5] Schouten J A 1951 *Tensor Analysis for Physicists* (Oxford: Clarendon).

[3] This figure is from [4], p. 7.
[4] This is figure 23 from [5].

EJP quality

EJP Board does not know the difference between vector and covector and does not want to know. They rejected the paper **"Depicting of electric fields"** EJP-101291 and ignore author's objection. Please see

Sent: Monday, July 20, 2015
REFEREE REPORT(S):
This paper shows a difference between the electric field E and displacement vector D. Author is right that the two vectors have a different geometrical structure. However he is not right saying that the E vector is a covector. The argument for E being a vector is in fact very simple. Consider a point particle with charge q in electric field. The force F exerted on the particle is F = qE. The force must be a vector, since Newton's equations give a linear relation between the force and acceleration, hence also velocity. These mechanical quantities are vectors for sure. Thus E must be a vector.
This argument does not apply to the D field, which in fact is a covector.
Author's argument for E being a covector prove only that either B (magnetic induction) or E is a covector but do not prove that E must be a covector.
In fact there is a deep analogy between mechanical velocity v and vector E, and mechanical momentum p with vector D. In mechanics velocity is a vector and momentum – a covector. **In electromagnetism E is a vector and D – a covector. The main result of the paper is erroneous.**
The paper is not written in a transparent way, it is very hard to follow author's arguments.
I do not recommend this letter for publication.
EDITOR-IN-CHIEF COMMENTS:
The paper seems rather confusing (as detailed by **the Board Member**) and in my opinion does not help at all in the understanding or teaching of physics. Therefore I consider it inappropriate for EJP.

Author's objection.

Dear Editors, This Referee Report is unacceptable. See my notes (red)
REFEREE REPORT(S):
This paper shows a difference between the electric field E and displacement vector D.
No, the difference between the electric field **E** and displacement vector (density) **D** is well known and is not worth writing a paper. This paper shows that geometrical image o **D**, i.e. tubes with an inner orientation, emerge from the charge density ρ, which is the *source* of the field **D**. And the paper shows that geometrical image of E, i.e. bisurfaces with an outer orientation, emerge from the field tubes of the vector density $-\partial_t \mathbf{B}$, which are the *source* of the solenoidal electric field **E**
Author is right that the two vectors have a different geometrical structure. However he is not right saying that the E vector is a covector.
No, here the Referee is trivial mistaken. **E** is a covector, according to $-\partial_t \mathbf{B} = \mathrm{curl}\,\mathbf{E}$, because curl is applied only to covectors: $-\partial_t B_{ik} = \partial_i E_k - \partial_k E_i$.
Besides, **E** is a covector, according to $\mathbf{E} = -\mathrm{grad}\,\varphi$, because grad is a covector: $E_i = -\partial_i \varphi$.
The argument for E being a vector is in fact very simple. Consider a point particle with charge q in electric field. The force F exerted on the particle is F = qE. The force must be a vector, since Newton's equations give a linear relation between the force and acceleration, hence also velocity. These mechanical quantities are vectors for sure. Thus E must be a vector.
If Referee wants **E** as a vector, he must use the metric tensor: $E^i = g^{ik} E_k$, but a force considered as F = -grad U is a covector.
This argument does not apply to the D field, which in fact is a covector.
This is a trivial delusion. According to $\rho = \mathrm{div}\mathbf{D}$, **D** is a vector density because div does not apply to a covector, div applies to a vector density: $\rho = \partial_i D^i$.

Author's argument for E being a covector prove only that either B (magnetic induction) or E is a covector but do not prove that E must be a covector.

B is a covariant bivector B_{ik}, or, after dualization, is a pseudo vector: $B^{j} = B_{ik}\varepsilon^{ikj}$

In fact there is a deep analogy between mechanical velocity v and vector E, and mechanical momentum p with vector D. In mechanics velocity is a vector and momentum – a covector.

This is a monstrous muddle! The Referee forgot **p** = m**v.**

In electromagnetism E is a vector and D – a covector. The main result of the paper is erroneous.

The main result of the referee comments is the Referee's incompetence

The paper is not written in a transparent way, it is very hard to follow author's arguments.

This opinion confirms the Referee's incompetence

I do not recommend this letter for publication.

I recommend to change this Referee.