A Partial Proof of Carmichael Conjecture

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Abstract- In this note, we shall briefly survey The Carmichael Conjecture and give a partial proof.

1.Introduction

This note is divided into three sections, this section is a brief introduction of Euler phi function (or Euler totient function).

Definition -Euler 's function $\emptyset(n)$ is defined for natural 'n' as the number of positive integers less than or equal to 'n' and coprime to 'n'.It holds that

$\emptyset(n)= n(1-\frac{1}{P1}) \dots .(1-\frac{1}{Pk})$

where $n = {P_1}^{a_1} \dots . {P_k}^{a_k}$ is the factorization into primes.

In 1907 Robert Carmichael stated that, For every n there is at least one other integer $m \neq n$ such that $\emptyset(m) = \emptyset(n)$.

2.PROOF

Suppose n is an odd integer then $n = {P_1}^{a_1} \dots . {P_k}^{a_k}$,here all $P_1,,,P_k$ are odd primes.

So $\emptyset(n) = {P_1}^{a_1} \dots . {P_k}^{a_k}(1 - \frac{1}{P1}) \dots . (1 - \frac{1}{Pk})$

$$= 2.\frac{1}{2}.{P_1}^{a_1} \dots . {P_k}^{a_k}\left(1 - \frac{1}{P1}\right) \dots . \left(1 - \frac{1}{Pk}\right)$$

$$= 2.{P_1}^{a_1} \dots . {P_k}^{a_k}\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{P1}\right) \dots . \left(1 - \frac{1}{Pk}\right)$$

$$= \emptyset(2n)$$

Hence if n is odd then $\emptyset(n) = \emptyset(2n)$

Now we prove for even n.

The only known Fermat primes are ,S=3,5,17,257,65537

Now let $n = 2^p.{P_1}^{a_1} \dots . {P_k}^{a_k}$,where none of $P_1,,,,P_k$ belongs to S. So, $\emptyset(n) = 2^{p-1}.{P_1}^{a_1} \dots . {P_k}^{a_k}(1 - \frac{1}{P1}) \dots . (1 - \frac{1}{Pk})$ ----------------- (A)

If $p = 1$ then its same as previous,so $p \geq 2$.

Now from (A) we have ,

$$2^{p-2}.2.3.\frac{1}{3}P_1{}^{a_1}\ldots.P_k{}^{a_k}(1-\frac{1}{P1})\ldots.(1-\frac{1}{Pk})$$

$$= 3.2^{p-2}.\frac{2}{3}.P_1{}^{a_1}\ldots.P_k{}^{a_k}\left(1-\frac{1}{P1}\right)\ldots.\left(1-\frac{1}{Pk}\right)$$

$$= 3.2^{p-2}.P_1{}^{a_1}\ldots.P_k{}^{a_k}(1-\frac{1}{P1})\ldots.(1-\frac{1}{Pk})(1-\frac{1}{3})$$

$$= \emptyset(3.2^{p-1}.P_1{}^{a_1}\ldots.P_k{}^{a_k})$$

If $\emptyset(n) = 2^p 3^q P_1{}^{a_1}\ldots.P_k{}^{a_k}\left(1-\frac{1}{P1}\right)\ldots.\left(1-\frac{1}{Pk}\right)$ and none of $P_1,,,,P_k$ is equal to 7 then $n = 2^p 3^{q+1} P_1{}^{a_1}\ldots.P_k{}^{a_k}$ or $n =$ $7.2^{p-1}.3^q.P_1{}^{a_1}\ldots.P_k{}^{a_k}$.

But if one of $P_1,,,,P_k$ is equal to 7 (and all $a_1,\ldots.,a_k$ greater than 1) then one of $P_1-1,\ldots P_k-1$ is equal to 6. But $2.3.6 = 36; 36+1 = 37$ a prime.So we can set 37 as a factor of n.

But if we give similar argument that 37 already exists in $P_1,,,,P_k$ then we cant take 37 as a factor of n.

In this way we cant get a complete solution when $3^2 = 9$ divides n.

3.So our conjecture becomes, If $n = 9.4.k = 36k$ then there is at least one distinct integer m such that $\emptyset(m) = \emptyset(n)$.

References- Wikipedia_Carmichael conjecture