Special Relativity for Beginners Part II

(The Meaning of *pc* in Einstein's Total Relativistic Energy Formula)

The purpose of this paper is to find out the physical meaning of the product pc in Einstein's Total Relativistic Energy Formula.

by Rodolfo A. Frino

Electronics Engineer
Degree from the National University of Mar del Plata - Argentina
rodolfo_frino@yahoo.com.ar
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1. The Meaning of *pc*

What is the physical meaning of the product *pc* in Einstein's total relativistic energy formula

$$E^2 = (pc)^2 + \left(m_0 c^2\right)^2 \tag{1.1}$$

We know, due to the dimensions of the above equation, that the product *pc* has to be an energy. But we want to be more specific. So let us consider the famous Einstein's equation

$$E = mc^2 \tag{1.2}$$

We shall multiply both sides by the velocity, *v*, of the body or particle and we shall swap sides. This yields

$$mvc^2 = vE \tag{1.3}$$

This equation can be written as

$$mvc = \left(\frac{v}{c}\right)E \tag{1.4}$$

But the momentum of the particle is

$$p = m\,v \tag{1.5}$$

Now from equations (1.4) and (1.5) we can write

$$pc = \left(\frac{v}{c}\right) E \qquad \text{The meaning of } pc \tag{1.6}$$

Thus we draw the following conclusion

The meaning of the product: *p* times *c*

The product of the relativistic momentum, *p,* of a body (or particle) times the speed of light, *c,* is *v/c* times the total relativistic energy of the body (or particle), *E*.

Equation (1.6) can be also written in terms of β , which is normally defined as

$$\beta \equiv \frac{v}{c} \tag{1.7}$$

This definition allow us to write equation (1.6) as follows

$$pc = \beta\, E \tag{1.8}$$

Now we can write Einstein's equation (1.1) as follows

$$E^2 = \beta^2 E^2 + \left(m_0 c^2\right)^2 \tag{1.9}$$

This formula leads to equation (1.2) as it should be.

2. Conclusions

In summary, we have proved that the product of the momentum of a particle times the speed of light is equal to *v/c* times the total relativistic energy of the particle.

Appendix 1
Nomenclature

The following are the symbols used in this paper

c = speed of light in vacuum
v = speed of a body or particle of mass m
m = relativistic mass of a body or particle
m_0 = rest mass of a body or particle
p = relativistic momentum of a body or particle

E = total relativistic energy (or simply relativistic energy) of a body or particle
β = ratio of the speed, v, of a massive body to the speed of light, c

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FURTHER READING

(1) A. Einstein, *On the Electrodynamics of Moving Bodies*, Annalen der Physik, (1905).
(2) R. A. Frino, *Special Relativity for Beginners – Part I*, vixra.org: vixra 1506.0161, (2015).